CHAPTER 15: WAVES

EXAMPLES

1: - Given below are some examples of wave motion. State in each case if the wave motion is transverse, longitudinal or a combination of both:

(a) Motion of a kink in a longitudinal spring produced by displacing one end of the spring sideways.

Solution: - (a) Motion of waves in a kink in a longitudinal spring produced by displacing one end of the spring sideways are the combination of both longitudinal and transverse wave motion.

(b) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

Solution: - (b) Longitudinal waves are created by rotating the piston back and forth in a cylinder carrying a liquid.

(c) Waves produced by a motorboat sailing in water.

Solution:- (c) Waves produced by a motorboat sailing in water are the combination of both longitudinal and transverse waves**.**

(d) Ultrasonic waves in air produced by a vibrating quartz crystal.

Solution: (d) Longitudinal ultrasonic waves are produced in the air by a vibrating quartz crystal.

2: - A wave travelling along a string is described by,

$Y(x, t) = 0.005 \sin(80.0\, x - 3.0\, t)$, **In which the numerical constants are in SI units $\left(\mathbf{0.005\ m,\ 80.0\ rad\ m^{-1},\ and\ 3.0\ rad\ s^{-1}}\right)$. Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also, calculate the displacement y of the wave at a distance $\mathbf{x = 30.0\ cm}$ and time $\mathbf{t = 20s}$?**

Solution: -On comparing this displacement equation with Equation

$y(x, t) = A \sin(kx - \omega t)$, we find

(a) The amplitude of the wave is $0.005\, m = 5\, mm$.

(b) The angular wave number k and angular frequency ω are $k = 80.0\, m^{-1}$

and $\omega = 3.0\, s^{-1}$

We, then, relate the wavelength λ to k through

$$\lambda = 2\pi / k$$
$$= \frac{2\pi}{1\ 80.0 m^{-1}}$$
$$= 7.85\ cm$$

(c) Now, we relate T to ω by the relation

$$T = 2\pi / \omega$$

$$\frac{2\pi}{3.0\ s^{-1}}$$

$$= 2.09\ s$$

and frequency, $v = \frac{1}{T} = 0.48\ Hz$

The displacement y at $x = 30.0$ cm and time $t = 20$ s is given by

$$y = (0.005\ m)\ sin\ (80.0 \times 0.3 - 3.0 \times 20)$$

$$= (0.005\ m)\ sin\ (-36 + 12\pi)$$

$$= (0.005\ m)\ sin\ (1.699)$$

$$= (0.005\ m)\ sin\ (97^0)$$

≈ 5 mm

3. A steel wire 0.72 m long has a mass of 5.0×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire?

Solution: - Mass per unit length of the wire

$$\mu = \frac{5.0 \times 10^{-3}\ \text{kg}}{0.72\ \text{m}}$$

$$= 6.9 \times 10^{-3}\ \text{kg m}^{-1}$$

Tension, $T = 60\ N$ The speed on the wave of the wire is given by

$$V = \sqrt{T / u}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60\ \text{N}}{6.9 \times 10^{-3}\ \text{kg m}^{-1}}}$$

$$= 93\ \text{m s}^{-1}$$

4. Estimate the speed of sound in air at standard temperature and pressure. The mass of 1 mole of air is $29.0 \times 10 - 3$ kg.

Solution: - We know that 1 mole of any gas occupies 22.4 litres at STP. As a result, the density of air at STP is: $\rho° = (mass\ of\ one\ mole\ of\ air) / (volume\ of\ one\ mole\ of\ air\ at\ STP)$

$= \dfrac{29\times10^{-3} kg}{22.4\times10^{-4} m^3}$

We get for the speed of sound in air at STP using Newton's formula for the speed of sound in a medium,

$$v = \left[\frac{1.01\times10^5 \text{ N m}^{-2}}{1.29 \text{ kg m}^{-3}}\right]^{\frac{1}{2}}$$

$$= 280 \text{ m s}^{-1}$$

5. A pipe, 30.0 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of sound in air as 330 m s^{-1}.

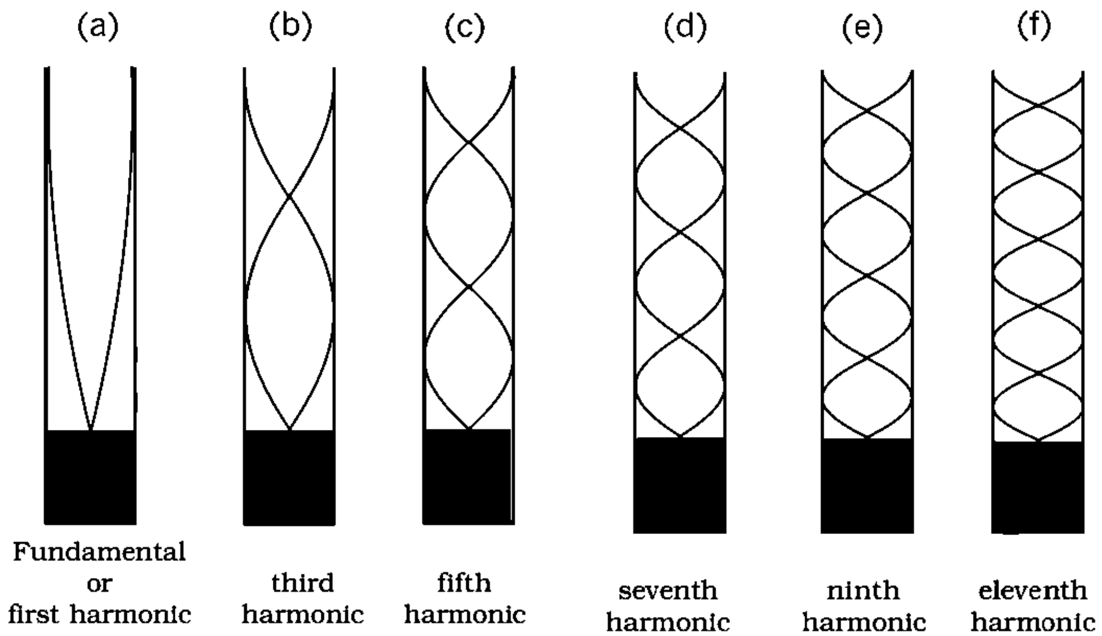
Solution: - The first harmonic frequency is calculated as follows: $\nu_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$ (open pipe)

where L is the pipe's length.

The nth harmonic's frequency is:

$$\nu_n = \frac{nv}{2L}, \text{ for } n = 1, 2, 3, \ldots \text{(open pipe)}$$

The following are the first few modes of an open pipe: -



For $L = 30.0\ cm,\ v = 330\ m\ s^{-1}$,

$\nu_n = 1330\ (m/\ s)\ /0.6\ (m)\ =\ 550\ m\ s^{-1}$

Clearly, a source of frequency $1.1\ kHz$ will resonate at ν_2, i.e., the second harmonic.

If one end of the pipe is now closed, the fundamental frequency will be

$\nu_1 =\ v/\lambda 1$

$= v/4L$ (pipe closed at one end)

Only the harmonics with odd numbers are present:

$\nu_3 =\ 3v/4L\ ,\ \nu_5 =\ 5v/4L$, and so on.

For $L\ =\ 30\ cm$ and $v\ =\ 330\ m\ s^{-1}$, the fundamental frequency of the pipe closed at one end is $275\ Hz$ and the source frequency corresponds to its fourth harmonic.

Because this harmonic isn't a conceivable mode, there will be no resonance with the source after one end is closed.

6. Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz?

Solution: -As we increase the tension of a string its frequency increases because tension is directly proportional to velocity and velocity is directly proportional to frequency. If the original frequency of B (ν_B) were greater than that of A (ν_A), further increase in(ν_B).The frequency of the beats should have increased as a result of this. However, the frequency of the beats appears to be decreasing. This shows that $\nu_B <\ \nu_A$. Since $\nu_A -\ \nu_B = 5\ Hz, =$ and $\nu_A = 427\ Hz$, we get $\nu_B = 422\ Hz.$

7. A rocket is moving at a speed of 200 m s^{-1} towards a stationary target. While moving, it emits a wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (1) the frequency of the sound as detected by the target and (2) the frequency of the echo as detected by the rocket.

Solution: -(1) The frequency of the beats should have increased as a result of this. However, the frequency of the beats appears to be decreasing. $330\ m\ s^{-1}$,. Since the source is approaching a stationary target, $v_o =\ 0$, and v_s must be replaced by v_s. Thus, we have

$$v = v_0\left(1-\frac{v_s}{v}\right)$$

$$v = 100Hz\times\left[1-\frac{200ms^{-1}}{330ms^{-1}}\right]$$

$$= 2540Hz$$

(2) Because it is the source of echo, the target is now the source, and the rocket's detector is now the detector or observer (because it detects echo). As a result, and has a positive value. The frequency of

sound released by the source (the target) is v, not v0, which is the frequency intercepted by the target. As a result, the frequency detected by the rocket is $v' = v(1+v_o/v)$

$$= 2540Hz \times (1+330/200)m/s$$

$$= 4080\ Hz$$

Exercise: -

1: - A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Solution: - Given, mass of the string, $M = 2.50\ kg$

Tension in the string, $T = 200\ N$

Length of the string, $l = 20.0\ m$

Mass per unit length, $\mu = \frac{M}{L}$

$$= \frac{2.50}{20}$$

$$= 0.125 kg\ m^{-1}$$

The velocity (v) of the transverse wave in the string is given by the relation:

$$V = \sqrt{\frac{t}{u}}$$

$$= \sqrt{\frac{200}{0.125}}$$

$$= 40\ m/s$$

Therefore, Time taken by the disturbance to reach the other end $t = \frac{l}{v}$

$$= \frac{20}{40}$$

$$= 0.5\ sec$$

2: - A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is $340\ m\ s^{-1}$? ($g = 9.8\ m\ s^{-2}$)

Solution: -Height of the tower, $s = 300\ m.$

Initial velocity of the stone, $u = 0$

Acceleration, $a = g = 9.8\ m/s^2$

Speed of sound in air $= 340\ m/s$

The second equation of motion can be used to calculate the time (t) it takes for the stone to strike the water in the pond:

$$s = ut_1 + \frac{1}{2}gt^2$$

$$300 = 0 + \frac{1}{2} \times 9.8t^2$$

Therefore, $t_1 = 7.82\ sec$

Time taken by the sound to reach the tower, $t_2 = \dfrac{300}{340}$

$= 0.88\ sec$

Therefore, total time taken $= 7.82 + 0.88\ sec$

$= 8.7\ sec$

3: - A steel wire has a length of $12.0\ \text{m}$ and a mass of $2.10\ \text{kg}$. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20\ °\text{C} = 343\ \text{m s}^{-1}$

Solution: -Given, length of the steel wire, $l = 12\ m$

Mass of the steel wire, $m = 2.10\ kg$

Velocity of the transverse wave, $v = 343\ m/s$

Mass per unit length, $\mu = \dfrac{M}{L}$

$= 2.10/12$

$= 0.175\ kg\ m^{-1}$

For tension T, velocity of the transverse wave can be obtained using the relation:

$$V = \sqrt{\frac{t}{u}}$$

Therefore, $T = v^2 u$

$= (343)^2 \times 0.175 = 20588.575$

$$\approx 2.06 \times 104\ N$$

4: -Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air is independent of pressure, increases with temperature, and increases with humidity.

Solution: - As we know that density is mass per unit volume

Putting this in the formula we get,

$$v = \sqrt{\frac{\gamma PV}{M}}$$

Now from the ideal gas equation for $n = 1$: $PV = RT$

For constant T,

PV= Constant

Since both M and γ are constants,

v = Constant

As a result, under constant temperature, the speed of sound in a gaseous medium is unaffected by changes in the gas's pressure. For one mole of an ideal gas,

the gas equation can be written as: $PV = RT$

$$P = RT/V \ --- \ (1)$$

Substituting the eqn 1 in the given formula we get

$$v = \sqrt{\frac{\gamma RT}{V\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Where, Mass, $M = \rho V$ is a constant

γ and R are also constants We conclude from equation that

$$v \propto \sqrt{T}$$

As a result, the speed of sound in a gas is proportional to the square root of the gaseous medium's temperature, i.e., the speed of sound increases as the temperature of the gaseous medium rises and vice versa.

Let V_m and V_d represent the sound speeds in moist and dry air, respectively.

Let p_m and p_d be the relative densities of moist and dry air.

Hence the speed of sound in moist air will be

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \text{ - (1)}$$

In dry air, the sound speed would be

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \quad \text{-(2)}$$

Dividing eqn 1and 2 we get

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma P}{\rho_m} \times \frac{\rho_d}{\gamma P}} = \sqrt{\frac{\rho_d}{\rho_m}}$$

The presence of water vapour, on the other hand, reduces the density of air, i.e.

$\rho d < \rho m$

Therefore, $.V_m \angle V_d$

As a result, sound travels faster in moist air than in dry air. As a result, the speed of sound increases with humidity in a gaseous medium.

5: -You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - v\,t$ or $x + v\,t$, i.e. $y = f(x \pm v\,t)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave:

Solution: -

(a) $(\mathbf{x} - \mathbf{vt})^2$

For x=0 and t=0 the equation $(x - vt)^2$ becomes 0

Hence for $x = 0$ *and* $t = 0$ this equation represents a point not a wave.

(b) $\log\left[(\mathbf{x} + \mathbf{vt}) / \mathbf{x_0}\right]$

For $x = 0$ *and* $t = 0$ the equation

$log\left[(x + vt) / x_0\right] = log 0 = \infty$

since the function does not have finite values of x and t it does not represents a travelling wave.

(c) $\mathbf{1} / (\mathbf{x} + \mathbf{vt})$

For $x = 0$ *and* $t = 0$, the function

$1/(x + vt) = 0$

The function does not describe a travelling wave since it does not converge to a finite value for.

No, these equations do not represent a wave, and the reverse of the supplied equation is incorrect, because the most important criteria for representing a travelling wave is that the x and t numbers remain finite**.**

6: - A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s^{-1} and in water 1486 m s^{-1}.

Solution: -(a) Frequency of the ultrasonic sound, $\nu = 1000\ kHz = 10^6\,Hz$

Speed of sound in air, $v_a = 340\ m/s$

The wavelength (λ) of the reflected sound is given by the relation $\lambda = \frac{v}{\text{v}}$

$= 340/10^6$

$= 3.4\times10^{-4}$

(b)Frequency of the ultrasonic sound, $\nu = 1000\ kHz = 10^6\,Hz$

Speed of sound in water, $v_w = 1486\ m/s$

The wavelength of the transmitted sound is given as: $\lambda = \frac{v}{\text{v}}$

$= \frac{1486}{10^6}$

$= 1.49\times10^{-3}\,m$

7: -A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is

1.7 km s^{-1}? The operating frequency of the scanner is 4.2 MHz

Solution:-Speed of sound in the tissue, $v = 1.7\ km/s = 1.7 \times 103\ m/s$

Operating frequency of the scanner, $\nu = 4.2\ MHz = 4.2 \times 106\ Hz$

The wavelength of sound in the tissue is given as $\lambda = \frac{v}{\text{v}}$

$= \frac{1.7\times10^3}{4.2\times10^6}$

$= 4.1\times10^{-4}\,m$

8: - A transverse harmonic wave on a string is described by

$$y(x,\ t)\ =\ 3.0\ sin\ (36\ t\ +\ 0.018\ x\ +\ \pi/4)$$

Where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

Solution: -(a) Yes, it is a travelling wave.

Since speed $=\dfrac{\omega}{k}$

$=\dfrac{36}{0.018}$

$=\ 20\ m/s$

And its direction is from right to left.

(b) What are its amplitude and frequency?

Solution:-(b) Amplitude of the given wave, $a\ =\ 3\ cm$

Frequency of the given wave:

$V=\dfrac{\omega}{k}$

$=\dfrac{36}{2\times 3.14}$

$=\ 5.73\ Hz$

(c) What is the initial phase at the origin?

Solution:-(c)Initial phase at the origin is $\dfrac{\pi}{4}$

(d) What is the least distance between two successive crests in the wave?

Solution:-(d) The wavelength of the wave is equal to the distance between two successive crests or troughs. $\lambda=\dfrac{2\pi}{k}$

$$=\ 2\times 3.14/0.018$$
$$=\ 3.49\ m$$

9:- For the wave described in $y(x,\ t)\ =\ 3.0\ sin\ (36\ t\ +\ 0.018\ x\ +\ \pi/4)$ **, plot the displacement (y) versus (t) graphs for** $\mathbf{x\ =\ 0,\ 2\ and\ 4\ cm}$**. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?**

Solution: - All the waves have different phases.

The given transverse harmonic wave is:

$$y(x,\ t)\ =\ 3.0\ sin\ (36\ t\ +\ 0.018\ x\ +\ \pi/4)$$

For $x\ =\ 0,$ the equation reduces to:

$$y(x,\ t)\ =\ 3.0\ sin\ (36\ t+\ \pi/4)$$

Also, $\omega=\dfrac{2\pi}{t}$

$= 36\ rad\ /\ s^{-1}$

Therefore, $T\ =\dfrac{\pi}{8} sec$

Now, plotting y vs. t graphs using the different values of t, as listed in the given table

t (sec)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
Y(cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$-\frac{3\sqrt{2}}{2}$	-3	$-\frac{3\sqrt{2}}{2}$	0

For $x\ =\ 0,\ x\ =\ 2,\ and\ x\ =\ 4$, the phases of the three waves will get changed. This is due to the fact that amplitude and frequency are unaffected by changes in x.

10 :-For the travelling harmonic wave

.$y(x,\ t)\ =\ 2.0\ cos\ 2\pi\ (10t\ -\ 0.0080\ x\ +\ 0.35)$ where x and y are in cm and t in s.
Calculate the phase difference between oscillatory motion of two points separated by a distance of (a) 4 m

(b) 0.5 m

(c) $\lambda/2$

(d) $3\lambda/4$

Solution:-Equation for a travelling harmonic wave is given as:

$$y\ (x,\ t)\ =\ 2.0\ cos\ 2\pi\ (10t\ -\ 0.0080x\ +\ 0.35)$$

$$=\ 2.0\ cos\ (20\pi t\ -\ 0.016\pi x\ +\ 0.70\ \pi)$$

Where, Propagation constant, $k\ =\ 0.0160\ \pi$

Amplitude, $a = 2\ cm$

Angular frequency, $\omega = 20\ \pi\ rad / s$

Phase difference is given by the relation $\varnothing = kx = \frac{2\pi}{\lambda}$

(a) For $x = 4m = 400cm$

$\varnothing = 0.016\pi \times 400 = 6.4\pi\ rad$

For $x = 0.5m = 50\ cm$

$\varnothing = 0.016\pi \times 50 = 0.8\pi\ rad$

(b) For $X = \frac{\lambda}{2}$

(c) $\varnothing = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$

$= \pi\ rad$

For

$X = \frac{3\lambda}{4} \varnothing = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4}$

$= 1.5\pi\ rad$

11: -The transverse displacement of a string (Clamped at its both ends) is given by $\mathbf{Y\ (x,\ t) = 0.06\ sin(2\pi / 3x)\ cos\ (120\ \pi t)}$ where x and y are in m and t in s. The length of the string is $\mathbf{1.5\ m}$ and its mass is $\mathbf{3.0\ \times 10^{-2} kg}$. Answer the following:

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

Solution: - The general equation representing a stationary wave is given by the displacement function : $y\ (x,\ t) = 2a\ sin\ kx\ cos\ \omega t$.

This equation is similar to the given equation:

$$\mathbf{Y\ (x,\ t) = 0.06\ sin(2\pi / 3x)\ cos\ (120\ \pi t)}$$

As a result, the supplied function is a stationary wave.

A wave that travels in the positive x-direction is denoted by

$y_1 = a\sin(\omega t - kx)$

The wave that travels in the negative x-direction is denoted by:

$$y_2 = a\sin(\omega t - kx)$$

The result of superposing these two waves is: -

$$y = y_1 + y_2$$

$$= a\sin(\omega t - kx) - a\sin(\omega t + kx)$$

$$= a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t) - a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t)$$

$$= -2a\sin(kx)\cos(\omega t)$$

$$= -2a\sin(\frac{2\pi}{\lambda}x)\cos(2\pi vt)$$

The transverse displacement of the string is given as

$$\mathbf{Y\ (x,\ t)\ =\ 0.06\ sin(2\pi/3x)\ cos\ (120\ \pi t)} \ \ldots\ldots (2)$$

Comparing equations (i) and (ii), we have, $\frac{2\pi}{\lambda} = \frac{2\pi}{3}$

Therefore wavelength $= 3m$

It is given that: $120\pi = 2\pi v$

Frequency, $v = 60\ Hz$

Wave speed, $v = v\lambda$

$= 60 \times 3 = 180\ m/s$

In a string, the velocity of a transverse wave is given by

V = $\sqrt{t/u}$

Where, Velocity of the transverse wave, $v = 180\ m/s$

Mass of the string, $m = 3.0 \times 10^{-2} kg$

Length of the string, $L = 1.5\ m$

Mass per unit length of the string,

$$\mu = \frac{m}{l}$$

$$= \frac{3}{1.5 \times 10^{-2}}$$

$= 2 \times 10^{-2} kgm^{-1}$

Tension in the string = T

From equation, tension can be obtained as

$$T = v^2\mu$$

$$= (180)^2 \times 2 \times 10^{-2}$$

$$= 648\ N$$

12:- For the wave on a string described in Ques 11, do all the points on the string oscillate with the same.

(a) **Frequency**

Solution: - Yes, except at the nodes because all the points on the string oscillate with the same frequency, except at the nodes which have zero frequency.

(b) phase

Solution:- All the points in any vibrating loop have the same phase, except at the nodes.

(c) amplitude

Solution:- All the points in any vibrating loop have different amplitudes of vibration.

(ii) What is the amplitude of a point $0.375\ \mathrm{m}$ away from one end?

Solution :- For $x = 0.375\ m\ and\ t = 0$

Amplitude = displacement = $\mathbf{0.06\sin(2\pi/3x)\cos}0$

$$= \mathbf{0.06\ \sin}(\mathbf{2}\pi/\mathbf{3}\times 0.375)$$

$$= 0.042m$$

13:- Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent a travelling wave,a stationary wave ornone at all:

(a) $\mathbf{y = 2\cos(3x)\sin(10t)}$

Solution:-(a) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation.

(b) $y = 2\sqrt{x - vt}$

Solution :-(b)The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave.

(c) $\mathbf{y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)}$

Solution :-(c) The given equation represents a travelling wave as the harmonic terms kx and ωt are in the combination of $kx - \omega t$.

(d) $\mathbf{y = \cos x \sin t + \cos 2x \sin 2t}$

Solution :-(d) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves.

14: -A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

Solution: -Mass of the wire, $m = 3.5 \times 10-2\ kg$

Linear mass density,

$$\mu = \frac{m}{l}$$

$$= 4 \times 10^{-2}\ kgm^{-1}$$

Frequency of vibration, $\nu = 45\ Hz$

Length of the wire, $l = \dfrac{m}{\mu}$

$$= 3.5 \times 10^{-2} / 4 \times 10^{-2}$$

$$= 0.875\ m$$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:

$$\lambda = \frac{2l}{n}$$

Where n is the no. of nodes in the wire

For fundamental node, $n = 1$:

$$\lambda = 2L$$

$$\lambda = 2 \times 0.875 = 1.75\ m$$

The speed of the transverse wave in the string is given as: $v = \nu\ \lambda = 45 \times 1.75 = 78.75\ m/s$

The tension produced in the string is given by the relation:

$$T = v^2\mu = (78.75)^2 \times 4.0 \times 10^{-2}$$

$$= 248.06\ N$$

15:- A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency $340\ \text{Hz}$) when the tube length is $25.5\ \text{cm}$ or $79.3\ \text{cm}$. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Solution:-Frequency of the turning fork, $\nu = 340\ Hz$

Such a system produces odd harmonics.

The fundamental note in a closed pipe is given by the relation:

$L_1 = \dfrac{\lambda}{4}$ where length of the pipe $L_1 = 0.255\ m$

Therefore, $\lambda = 4l1$

$= 4 \times 0.255$

$= 1.02\ m$

Speed of the sound $v = f = 340 \times 1.02 = 346.8\ m/s$

16:-A steel rod $100\ \text{cm}$ long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be $2.53\ \text{kHz}$. What is the speed of sound in steel?

Solution:-Length of the steel rod, l = $100\ cm = 1\ m$

Fundamental frequency of vibration, $\nu = 2.53\ kHz = 2.53 \times 10^3\ Hz$

When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (n) are formed at its two ends.

The distance between two successive node $l = \dfrac{\lambda}{2}$

Wavelength = $2l = 2m$

The speed of sound in steel is given by the relation:

$v = f\lambda$

$= 2.53 \times 10^3 \times 2$

$= 5.06 \times 10^3\ m/s$

$= 5.06\ km/s$

17:-A pipe $20\ \text{cm}$ long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a $430\ \text{Hz}$ source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air is $340\ \text{m s}^{-1}$).

Solution:-Length of the pipe, $l = 20\ cm = 0.2\ m$

Source frequency = nth normal mode of frequency,

$v_n = 430\ Hz$

Speed of sound,

$v = 340\ m/s$

In a closed pipe, the nth normal mode of frequency is given by the relation:

$$v_n = (2n-1)v/4l$$

$$430 = (2n-1)340/4\times 0.2$$

$$(2n-1) = 430\times 4\times 0.2/340 = 1.01$$

$$2n = 2.01$$

$$n \sim 1$$

Hence, the first mode of vibration frequency is resonantly excited by the given source. In a pipe open at both ends, the nth mode of vibration frequency is given by the relation:

$$v_n = nv/2l$$

$$v_n = nv/2l$$

$$n = 0.5$$

The supplied source does not produce a resonant vibration in an open pipe because the number of the mode of vibration (n) must be an integer.

18:- Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency $6\ \text{Hz}$. The tension in the string A is slightly reduced and the beat frequency is found to reduce to $3\ \text{Hz}$. If the original frequency of A is $324\ \text{Hz}$, what is the frequency of B?

Solution:-Frequency of string A, $f_A = 324\ Hz$

Frequency of string B = f_B

Beat's frequency, $n = 6\ Hz$

As we know that Beat frequency is given as:-

$$n = \ Frequency\ of\ A \pm Frequency\ of\ B$$

$$6 = 324 \pm f_b$$

$$.fb = 330Hz \text{ or } 318Hz$$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension,

Hence Beat frequency $318\ Hz$

19: - Explain why (or how):

(a) In a sound wave, a displacement node is a pressure antinode and vice versa,

Solution:- (a)A node is a place where the amplitude of vibration is the smallest and the pressure is the highest.

An antinode, on the other hand, is a place where the amplitude of vibration is greatest and the pressure is lowest.

As a result, a displacement node is nothing more than a pressure antinode, and vice versa.

(b) Bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",

Solution:- (b) Bats produce ultrasonic sound waves at an extremely high frequency. They are reflected back at them by these waves.by stumbling blocks When a bat gets a reflected wave (frequency), it calculates the distance, direction, and strength of the wave.With the help of its cerebral sensors, it can determine the nature and size of a barrier.

(c) A violin note and sitar note may have the same frequency, yet we can distinguish between the two notes

Solution:- (c) The overtones produced by a sitar and a violin are distinct, as are the strengths of these overtones. As a result, even if they generate the same sounds, a sitar and a violin can be distinguished even if they have same frequency of vibration.

(d) Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

Solution:- (d) Shear modulus is a property of solids. They are able to withstand shearing force. Fluids yield to shearing stress because they have no defined shape. A transverse wave propagates in a medium in such a way that it creates shearing stress. This type of wave can only go through solids, not gases. Solids and fluids have varying bulk moduli. They are able to withstand compressive stress. As a result, longitudinal waves can travel through both solids and liquids.

(e) The shape of a pulse gets distorted during propagation in a dispersive medium

Solution:- (e)A pulse is basically a collection of waves of various wavelengths. Depending on the nature of the medium, these waves travel at varying speeds in a dispersive medium. The shape of a wave pulse is distorted as a result of this.

20: - A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10\ \text{m s}^{-1}$, (b) recedes from the platform with a speed of $10\ \text{m s}^{-1}$? (ii) What is the speed of sound in each case? The speed of sound in still air can be taken as $340\ \text{m s}^{-1}$. Solution: -(a)Frequency of the whistle, $\nu = 400\ Hz$

Speed of the train, $v = 10\ m/s$

Speed of sound, $v = 340\ m/s$

As the train approaches the platform, the apparent frequency (v) of the whistle is given by the relation:

$$v' = \left(\frac{v}{v - v_t}\right) v$$

$$= \left(\frac{340}{340 - 10}\right) 400$$

$= 412.12 Hz$

The apparent frequency of the whistle as the train recedes from the platform is given by the relation:

$$v'' = \left(\frac{v}{v + v_t}\right) v'$$

$$= \left(\frac{340}{340 + 10}\right) 400$$

$= 388.57 Hz$

The relative motions of the source and observer produce the apparent shift in frequency of sound. These relative motions produce no effect on the speed of sound. Therefore, the speed of sound in air in both the cases remains the same, i.e., $340\ m/s$.

21: - A train, standing in a station-yard, blows a whistle of frequency $400\ \mathrm{Hz}$ in still air. The wind starts blowing in the direction from the yard to the station with at a speed of $10\ \mathrm{m\ s^{-1}}$. What is the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of $10\ \mathrm{m\ s^{-1}}$? The speed of sound in still air can be taken as $340\ \mathrm{m\ s^{-1}}$?

Solution: - Frequency of the sound produced by the whistle, $v = 400\ Hz$

Speed of sound = $340\ m/s$

Velocity of the wind, $v = 10\ m/s$

The frequency of the sound heard by the observer will be the same as that produced by the source because there is no relative motion between the source and the observer

$= 400\ Hz.$

The wind is blowing toward the observer. As a result, the sound's effective speed increases by $10\ units$, i.e., Effective speed of the sound,

$v_e = 340 + 10 = 350\ m/s$

The wavelength (λ) of the sound heard by the observer is given by the relation:

$$\lambda = \frac{v_e}{v}$$

$$= 350/400$$

$$= 0.875\ m$$

For the running observer:

Velocity of the observer, $v_o = 10\ m/s$

The observer is moving toward the source. There is a shift in frequency as a result of the relative motions of the source and the observer. (v'). This is given by the relation:

$$v' = \left(1 + \frac{v_o}{v}\right)v$$

$$= \left[\frac{(340+10)}{340}\right] \times 400$$

$$= 411.76\ Hz$$

Because the air is motionless, the sound's effective speed equals

$340 + 0 = 340\ m/s$. The source is at a standstill. As a result, the wavelength of the sound will not change, i.e. $0.875\ m$. As a result, the two scenarios are not exactly the same.

Additional Exercises: -

1: - A travelling harmonic wave on a string is described by

$$\mathbf{y(x,\ t) = 7.5\ sin\ (0.0050x\ + 12t\ +\ \pi/4)}$$

(a)what are the displacement and velocity of oscillation of a point at $\mathbf{x = 1\ cm}$, and $\mathbf{t = 1\ s}$? Is this velocity equal to the velocity of wave propagation? (b)Locate the points of the string which have the same transverse displacements and velocity as the $\mathbf{x = 1\ cm}$ point at $\mathbf{t = 2\ s,\ 5\ s}$ and $\mathbf{11\ s}$.

Solution: - For $x = 1\ cm\ and\ t = 1s,$

$$y(1,1) = 7.5\sin(0.0050 + 12 + \pi/4)$$

$$= 7.5\sin(12.0050 + \pi/4)$$

$$= 7.5\sin\theta$$

Where $\theta = 12.0050 + \pi/4$

$= 12.79 rad$

$= 732.81^0$

$\therefore y(1,1) = 7.5\sin\left(732.81^0\right)$

$= 7.5\sin\left(90\times 8 + 12.81^0\right)$

$= 7.5\sin\left(12.81^0\right)$

$= 7.5\times 0.2217$

$\approx 1.663 cm$

The velocity of the oscillation at a given point and time is given as

$$v = \frac{d}{dx}y(x,t) = \frac{d}{dx}\left[7.5\sin\left(0.0050x + 12t + \pi/4\right)\right]$$

$$= 7.5\times 12\cos\left(0.0050x + 12t + \pi/4\right)$$

At $x = 1m \ and \ t = 1 \ sec$

$$v = y(1,1) = 90\cos\left(12.005 + \pi/4\right)$$

$$= 90\cos(732.81^0)$$

$$= 90\cos(90\times 8 + 12.81^0)$$

$$= 90\cos(12.81^0)$$

$$= 87.75 m/s$$

As speed $= \dfrac{\omega}{k}$

$$= \frac{12}{0.0050}$$

$$= 2400 \ cm/sec$$

As a result, the wave oscillation velocity at $x = 1$ cm and $t = 1$ s is not equal to the wave propagation velocity.

As wavelength $= \dfrac{2\pi}{k}$

$$= (2\times 3.14)\div 0.0050$$

$= 12.56 m$

Therefore, all the points at distances $n\lambda$, *i.e.*, $\pm\ 12.56\ m$, $\pm\ 25.12\ m$, … and so on for $x = 1\ cm$, will have the same displacement as the $x = 1\ cm$ points at $t = 2\ s$, $5\ s$, *and* $11\ s$.

2: - A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium

. (a) Does the pulse have a definite

(i) frequency,

Solution: - (i)No

(ii) wavelength

Solution: - (ii)No

, (iii) speed of propagation?

Solution: - (iii)Yes

(b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to $1/20$ or 0.05 Hz ?

Solution: - (b) No

Explanation: The wavelength and frequency of the narrow sound pulse are not fixed. However, the sound pulse's speed remains constant, i.e., it is equal to the speed of sound in that medium. The short pip produced after every 20 s does not mean that the frequency of the whistle is $0.05\ Hz$.

It denotes the $0.05\ Hz$ frequency with which the pip of the whistle is repeated.

3: - One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg . The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement $(y = 0)$ and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

Solution: -The equation of a travelling wave propagating along the positive y-direction is given by the displacement equation:

$$y\ (x,\ t) = a \sin\ (wt - kx)\ \ldots\ (i)$$

Linear mass density, = $8.0 \times 10-3\ kg\ m^{-1}$

Frequency of the tuning fork, $\nu = 256\ Hz$

Amplitude of the wave, $a = 5.0\ cm = 0.05\ m$ … (ii)

Mass of the pan, $m = 90\ kg$

Tension in the string,

$$T = mg = 90 \times 9.8 = 882\ N$$

The velocity of the transverse wave v, is given by the relation:

$V = \sqrt{\frac{T}{u}}$

$= \sqrt{\frac{882}{0.008}}$

$= 332\ m/sec$

Angular frequency $\omega = 2\pi v$

$= 2 \times 3.14 \times 256$

=\`$1608.5\ rad\ /\ sec$

$Wavelength\ =\ V\ /\ v$ $= 332/256$

$= 1.29m$

Propagation constant $k = 2\pi/\lambda$

$=\ 4.84\ m^{-1}$

Substituting the values, we get the displacement equation:

$y\ (x,\ t)\ =\ 0.05\ sin\ (1.6\ \times\ 103\ t\ -\ 4.84\ x)\ m$

4: - A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m s^{-1}.

Solution: -Operating frequency of the SONAR system, $\nu\ =\ 40\ kHz$

Speed of the enemy submarine, $v_e\ =\ 360\ km/h\ =\ 100\ m/s$

Speed of sound in water, $v\ =\ 1450\ m/s$

The observer (enemy submarine) is approaching the source, which is at rest. As a result, the relationship between the apparent frequency (v') received and reflected by the submarine is:

$v' = \left(1 + \frac{v_o}{v}\right) v$

$= \left(1 + \frac{100}{1450}\right) \times 40$

$= 42.76 KHz$

The frequency (v'') received by the enemy submarine is given by the relation:

$$v'' = \left(\frac{v}{v - v_s}\right)v'$$

$$v'' = \left(\frac{1450}{1450 - 100}\right) \times 42.76$$

$= 45.93 kHz$

5: - Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically, the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Solution: -Let v_S and v_P be the velocities of S and P waves respectively.

Let L be the distance between the epicentre and the seismograph.

We have: $L = v_S t_S$ …. (i)

$L = v_P t_P$ ……(ii)

Where, t_S and t_P are the respective times taken by the S and P waves to reach the seismograph from the epicentre

It is given that: $v_P = 8 \; km/s$

$v_S = 4 \; km/s$

From equations (i) and (ii),

we have: $v_S t_S = v_P t_P$

$4tS = 8 \; t_P t_S$

$= 2 \; t_P$ …… (iii)

It is also given that: $t_S - t_P = 4 \; min$

$= 240 \; s$

$2t_s - t_P = 240$

$t_P = 240$ and

$t_S = 2 \times 240 = 480 \; s$

From equation (ii)

, we get: $L = 8 \times 240 = 1920 \; km$

Hence, the earthquake occurs at a distance of $1920\ km$ from the seismograph.

6: - A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is $40\ \text{kHz}$. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Solution: -Ultrasonic beep frequency emitted by the bat,

Velocity of the bat, $v_b = 0.03v$

Where, v = velocity of sound in air

The apparent frequency of the sound striking the wall is given as:

$$v' = \left(\frac{v}{v - v_b}\right) v$$

$$= \left(\frac{v}{v - 0.003v}\right) \times 40$$

$$= \frac{40}{0.97}\ KHz$$

This frequency is reflected by the stationary wall, toward the bat. The frequency (v'') of the received sound is given by the relation:

$$v'' = \left(1 + \frac{v_b}{v}\right) v'$$

$$= \left(1 + \frac{0.003v}{v}\right) \frac{40}{0.97}$$

$$= 42.47\ KHz$$