CHAPTER-9: MECHANICAL PROPERTIES OF SOLIDS)

Example

9.1 **A structural steel rod has a radius of $10\ \text{mm}$ and a length of $1.0\ \text{m}$. A $100\ \text{kN}$ force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus, of structural steel is $2.0 \times 10^{11}\ \text{N m}^{-2}$**

Answer

We assume that the rod is held by a clamp at one end, and therefore the force F is applied at the opposite end, parallel to the length of the rod. Then the strain on the rod is given by

$$\text{Stress } = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$= \frac{100 \times 10^3\ \text{N}}{3.14 \times \left(10^{-2}\ \text{m}\right)^2}$$

$$= 3.18 \times 10^8\ \text{N m}^{-2}$$

$$\Delta L = \frac{(F/A)L}{Y}$$

$$= \frac{\left(3.18 \times 10^8\ \text{N m}^{-2}\right)(1\text{m})}{2 \times 10^{11}\ \text{N m}^{-2}}$$

$$= 1.59 \times 10^{-3}\ \text{m}$$

$$= 1.59\ \text{mm}$$

The strain is given by

$$\text{Strain } = \frac{\Delta L}{L}$$

$$= \frac{\left(1.59 \times 10^{-3}\ \text{m}\right)}{1\text{m}}$$

$$= 1.59 \times 10^{-3}$$

$$= 0.16\%$$

9.2

A copper wire of length $2.2\ \text{m}$ and a steel wire of length $1.6\ \text{m}$, both of diameter $3.0\ \text{mm}$, are connected end to end. When stretched by a load, the net elongation is found to be $0.70\ \text{mm}$. Obtain the load applied.

Answer

The copper and steel wires are under a tensile stress because they need an equivalent tension (equal to the load W) and therefore the same area of cross-section A. From Eq. (9.7) we've stress = strain × Young's modulus. Therefore

$$\frac{W}{A} = Yc \times (\Delta Lc \ / Lc)$$
$$= Ys \times (\Delta Ls \ / Ls)$$

where the subscripts c and s refer to copper and stainless steel respectively. Or,

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{Y_s}{Y_c}\right) \times \left(\frac{L_c}{L_s}\right) Given$$
$$L_c = 2.2\,\text{m}, L_s = 1.6\,\text{m}$$
$$FromTable 9.1 Y_c = 1.1\times10^{11}\ \text{N}\cdot\text{m}^{-2},$$
$$and Y_s^c = 2.0\times10^{11}\ \text{N}\cdot\text{m}^{-2}$$

$$\frac{\Delta L}{\Delta L_{s\ c}} = \left(2.0\times10^{11} / 1.1\times10^{11}\right)\times(2.2/1.6) = 2.5$$

The total elongation is given to be

$$\Delta L_c + \Delta L_s = 7.0\times10^{-4}\ \text{m}$$

Solving the above equations,

$$\Delta L_c = 5.0\times10^{-4}\ \text{m}, and \Delta L_s = 2.0\times10^{-4}\ \text{m}$$

$$W = \frac{\left(A\times Y_c \times \Delta L_e\right)}{L_c}$$
$$= \pi\left(1.5\times10^{-3}\right)^2 \times \left[\frac{\left(5.0\times10^{-4}\times1.1\times10^{11}\right)}{2.2}\right]$$
$$= 1.8\times10^2\ \text{N}.$$

9.3

In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer who is lying on his back (as shown in Fig. 9.5). The combined mass of all the persons performing the act, and the tables, plaques etc. involved is $280\ \text{kg}$ The mass of the performer lying on his back at the bottom of the pyramid is $60\ \text{kg}$. Each thighbone (femur) of this performer has a length of $50\ \text{cm}$ and an effective radius of $2.0\ \text{cm}$. Determine the amount by which each thighbone gets compressed under the extra load.

Answer

Total mass of all the performers, tables, plaques etc is $= 280\ kg$

Mass of the performer $= 60\ kg$

Mass supported by the legs of the performer at the lowest of the pyramid

$= 280 - 60 = 220\ kg$

Weight of this supported mass

$= 220\ kg\ wt$

$= 220 \times 9.8\ N = 2156\ N..$

Weight supported by each thighbone of the performer

$= ½\ (2156)$

$N = 1078\ N$

From Table 9.1, the Young's modulus for the bone is given by

$Y = 9.4 \times 109\ N\ m^{-2}$.

Length of each thighbone $L = 0.5\ m$

the radius of thighbone $= 2.0\ cm$

Thus the cross-sectional area of the thighbone

$A = \pi \times \left(2 \times 10^{-2}\right)\ 2\ m^2$

$= 1.26 \times 10^{-3}\ m^2$.

Using Eq. (9.8), the compression in each thighbone (ΔL) are often computed as

$$\Delta L = \left[\frac{(F \times L)}{(Y \times A)}\right]$$

$$= \left[\frac{(1078 \times 0.5)}{\left(9.4 \times 109 \times 1.26 \times 10^{-3}\right)} /\right]$$

$= 4.55 \times 10^{-5}\ m\ or\ 4.55 \times 10^{-3}\ cm$.

This is a very small change! The fractional decrease in the thighbone is

$$\frac{\Delta L}{L} = 0.000091\ or\ 0.0091\%$$

9.4

A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of 9.0×104 N. The lower edge is riveted to the floor. How much will the upper edge be displaced?

Answer

The lead of the arbor is fixed and thus the force is applied resemblant to the narrow face as shown in Fig.9.7.

The area of the face parallel to which this force is applied is

$A = 50\ cm \times 10\ cm$

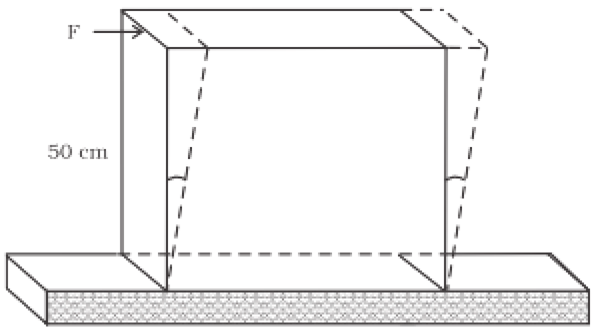
$= 0.5\ m \times 0.1\ m$

$= 0.05\ m2$

Therefore, the stress applied is

$= \left(9.4 \times \dfrac{104\ N}{0.05}\ m^2\right).$

$= 1.80 \times 106\ Nm^{-2}$



We know that shearing strain $= \left(\dfrac{\Delta x}{L}\right) = \dfrac{Stress}{G}$

Therefore the displacement

$\Delta x = \left(\dfrac{Stress \times L}{G}\right)$

$= \dfrac{(1.8 \times 106\ N\ m^{-2} \times 0.5m)}{(5.6 \times 109\ N\ m-2)} = 0.16\ mm$

$= 1.6 \times 10^{-4}\ m$

9.5

The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, $\dfrac{\Delta V}{V}$, of water at the bottom of the ocean, given that the bulk modulus of water is $2.2 \times 109\ N\ m^{-2}$.. (Take $g = 10\ m\ s^{-2}$).

Answer

The pressure exerted by a $3000\ m$ column of water on the bottom layer

$$p = h\rho g = 3000\ m \times 1000\ kg\ m^{-3} \times 10\ m\ s^{-2}$$

$$= 3 \times 107\ kg\ m^{-1}\ s^{-2}$$

$= 3 \times 107\ N\ m^{-2}$ Fractional compression $\frac{\Delta V}{V}$, $is\ \frac{\Delta V}{V} = \frac{stress}{B}$

$$= \frac{(3 \times 107\ N\ m-2)}{(2.2 \times 109\ N\ m-2\ )}$$

$$= 1.36 \times 10^{-2}\ or\ 1.36\ \%$$

EXERCISES

9.1

A steel wire of length $4.7\ \mathrm{m}$ and cross-sectional area $3.0 \times 10-5\ \mathrm{m2}$ stretches by the same amount as a copper wire of length $3.5\ \mathrm{m}$ and cross-sectional area of $4.0 \times 10-5\ \mathrm{m2}$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Answer:

the Length of the steel wire $L_1 = 4.7\ m$

Area of the cross-section of the steel wire, $A_1 = 3.0 \times 10^{-5} m^2$

the Length of the copper wire, $L_2 = 3.5\ m$

Area of the cross-section of the copper wire, $A_2 = 4.0 \times 10^{-5} m^2$

the Change in length $= \Delta L_1 = \Delta L_2 = \Delta L$

the Force applied in both the cases $= F$

the Young's modulus of the steel wire is **:** $y_1 = f1$

$$y_1 = \frac{f_1}{A_2} \times \frac{L_1}{\Delta L}$$

$$= \frac{F \times 4.7}{3.O \times 10^{-5} \times \Delta L}$$ **... (i)**

Young's modulus of the copper wire is :

$$Y_2=\frac{F_2}{A_2}\times\frac{L_2}{\Delta L}$$

$$=\frac{F\times3.5}{4.0\times10^{-5}\times\Delta L}.......(2)$$

Dividing (i) by (ii), we will get:

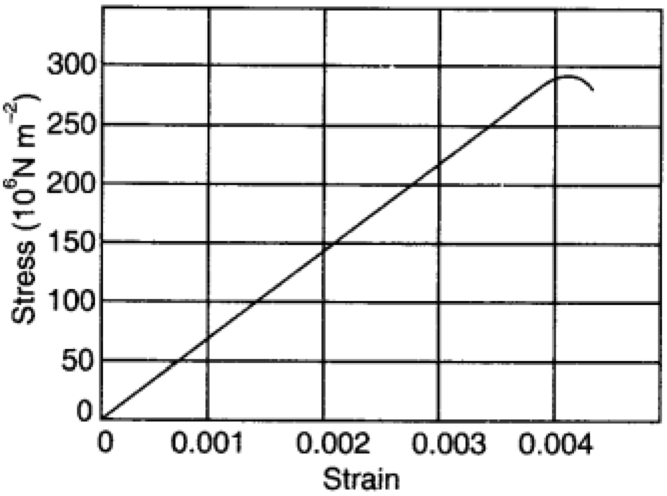
$$\frac{Y_1}{Y_2}=\frac{4.7\times4.0\times10^{-5}}{3.0\times10^{-5}\times3.5}=1.79:1$$

The ratio of Young's modulus of steel thereto of copper is $1.79:1$.

9.2

Figure 9.11 shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

ANSWER;



It is the clear from that the given graph that for stress $\frac{150}{106}\frac{N}{M^2}$

the strain is 0.002

Young's modulus,

$$Y=\frac{Stress}{Strain}$$

$$\frac{105\times10^6}{0.002}=7.5\times10^{10}\frac{N}{M^2}$$

Hence, the Young's modulus for the given material is $7.5\times\frac{1010\,N}{m2}$

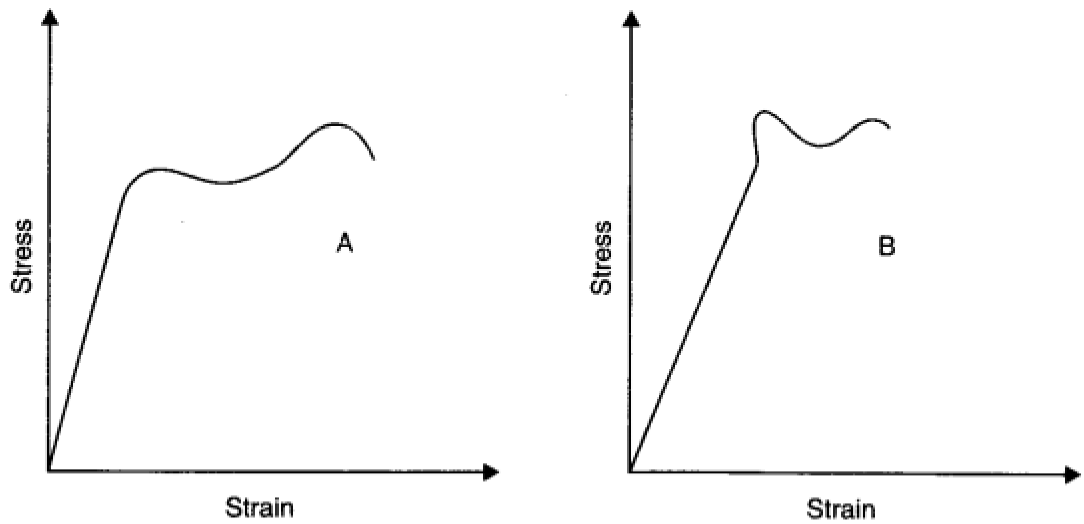
The yield strength of the fabric is that the maximum stress that the fabric are often sustain without crossing the elastic limit.

It is clear from the given graph that the approximate yield strength of the material is

$$300\times 106\ Nm/2\ or\ 3\ \times \frac{108\ N}{m2}$$

9.3

The stress-strain graphs for materials A and B are shown in Fig. 9.12.



The graphs are drawn to the same scale.

a) Which of the materials has the greater Young's modulus?

Answer:

A

b) Which of the two is the stronger material?

Answer:

A

For a given strain, the stress for material A is more than it is for material B, as shown in the two graphs.

$$YoungModulus = \frac{stress}{strain}$$

For a given strain, if the strain for a cloth is more, then Young's modulus is additionally greater for that material. Therefore, Young's modulus for material A is bigger than it is for material B.

The amount of stress required for fracturing a cloth , like its fracture point, gives the strength of that material. Fracture point is that the extreme during a stress-strain curve. It are often observed that material A can withstand more strain than material B.

9.4

Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel:

Answer- False

(b) The stretching of a coil is determined by its shear modulus

Answer

True

For a given stress, the strain in rubber is quite like it's in steel.

For a continuing stress

$$YoungModulus = \frac{stress}{strain}$$

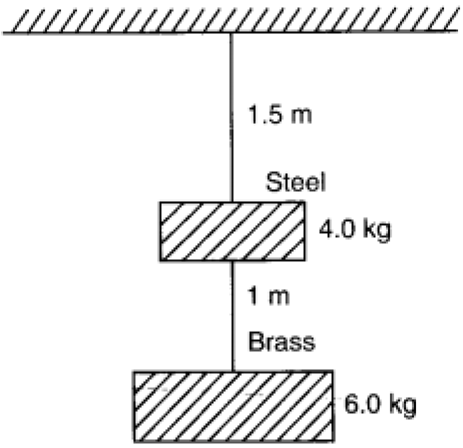
$$y \propto \frac{1}{strain}$$

Hence, Young's modulus for rubber is a smaller amount than it's for steel.

Shear modulus is that the ratio of the applied stress to the change within the shape of a body. The stretching of a coil changes its shape. Hence, shear modulus of elasticity is involved in this process.

9.5

Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. 9.13. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.



ANSWER:

The Elongation of the steel wire is $= 1.49 \times 10^{-4}\, m$

The Elongation of the brass wire is $= 1.3 \times 10^{-4}\, m$

The Diameter of the wires is, d $= 0.25\, m$

Hence, the radius of the wires is, $r = \frac{d}{2} = 0.125\, cm$

The Length of the steel wire is, $L_1 = 1.5\, m$

The Length of the brass wire is, $L_2 = 1.0\, m$

The Total force exerted on the steel wire is:

$$F1 = (4 + 6)\, g = 10 \times 9.8 = 98\, N$$

The Young's modulus for steel is :

Wherein,

$$Y_1 = \frac{\left(\frac{f_1}{A_1}\right)}{\left(\frac{\Delta L}{L_1}\right)}$$

ΔL Change in the length of the steel wire A_1

A_1 =Area of cross section of the steel wire = π_1^2

Younge modulus of steel $\mathrm{Y}_1 = 2.0\times 10^{11}\ pa$

$$\Delta L_1 = \frac{F_1 \times L_1}{A_1 \times Y_1} = \frac{F_1 \times L_1}{\pi r_1^2 \times Y_1}$$

$$= \frac{98\times 1.5}{(0.125\times 10^{-2})}\times \pi r_1^2 \times 10^{11}$$

$$= 1.49\times 10^{-4}\,m$$

Total force on the wire

$F_2 = 6\ \times 9.8 = 58.8N$

Young modulus for brass

$$Y_2 = \frac{\left(\frac{F_2}{A_2}\right)}{\left(\frac{\Delta L_2}{L_2}\right)}$$

Where as,

L_2 =Change in length

A_2 area of cross section of the bress wire

$$\Delta L = \frac{F2\times L2}{A2\times Y2} = \frac{F2\times L2}{\pi r_2^2 \times y2}$$

$$\frac{58.8\times 1.0}{\pi(0.125\times 10^{-2})\times\left(0.91\times 10^{11}\right)}$$

$$= 1.3\times 10^{-4}\,m$$

Totalforce on the bress wire

$F_2 = 6\times 9.8 = 58.8N$

Younge modulus for brass

The Elongation of the steel wire is $=1.49 \times 10^{-4}\ m$

The Elongation of the brass wire is $= 1.3 \times 10^{-4}\ m$

9.6

The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

ANSWER:

Edge of the cube $L = 10\ CM = 0.1M$ to the cube m= 100kg

The mass attached to the cube m $= 100kg$

Shear modulus of aluminium $= 25GPa = 25\times10^{9}\ Pa$

$$\frac{shearstress}{shearstrain} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{L}{\Delta L}\right)}$$

Where,

F = the Applied force

$= mg = 100 \times 9.8 = 980\ N$

A = Area of the one of the faces of the cube

$= 0.1 \times 0.1 = 0.01\ m^2$

ΔL = Vertical deflection of the cube

$mg = 100 \times 9.8 = 980\ N$

$$\Delta L = \frac{F}{A\eta}$$

$$= \frac{980\times0.1}{10^{-2}\times(25\times109)}$$

The vertical deflection of the face of the cube is $3.92 \times10^{-7}\ m$

9.7

Four identical hollow cylindrical columns of mild steel support a big structure of mass $50,000$ kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column

ANSWER:

The Mass of the large structure, $M = 50,000\ kg$

The Inner radius of the column is, $r = 30\ cm = 0.3\ m$

The Outer radius of the column is, $R = 60\ cm = 0.6\ m$

The Young's modulus of steel is, $Y = 2 \times 10^{11}\ Pa$

The Total force exerted is, $F = Mg = 50000 \times 9.8\ N$

$$= \frac{5000 \times 9.8}{4}$$

$$= 122500 N$$

Stress = the Force exerted on a single column

Young's modulus is ,

Y = stress\strain

$$\text{Strain} = \frac{F \setminus A}{Y}$$

whereas,

Area of $A = \left(R^2 - r^2\right) = (0.6) - (0.3)^2$

$$\text{Strain} = \frac{122500}{\pi(0.6)^2 - (0.3)^2 \times 2 \times 10^{11}}$$

Hence the compressional of each column is 7.2210^{-7}

9.8

A piece of copper having a rectangular cross-section of 15.2 mm $\times$ 19.1 mm is pulled in tension with $44,500$ N force, producing only elastic deformation. Calculate the resulting strain?

Answer:

The Length of the piece of copper is, $l = 19.1\ mm = 19.1 \times 10^{-3} m$

The Breadth of the piece of copper is, $b = 15.2\ mm = 15.2 \times 10^{-3} m$

The Area of the copper piece is:

$A = l \times b = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4} m^2$

The Tension force applied on the piece of copper is, $F = 44500\ N$

The Modulus of elasticity of copper is, $\eta = 42 \times 109\ N/m^2$

Modulus of elasticity = stress\ strain

$$= \frac{44500}{2.9 \times 10^{-4} \times 42 \times 10^{9}}$$

$$strain = \frac{F}{A\eta}$$

$$= 3.65 \times 10^{-3}$$

9.9

A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, what is the maximum load the cable can support ?

Answer:

Radius of the steel cabler = $1.5 = 0.015 mm$

Maximum allowable stress = $10^8\ NM^{-2}$

maximum stress= maximum fource\ area of cross section

maximum force =maximum stress area if cross section

$$= 10^{8} (0.015)^2$$

$$= 7.06510^4\ N$$

Hence the cable can support the maximum load of $7.06510^4\ N$

9.10

A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Answer:

The tension force is working on each wire is that the same. Thus, the extension in each case is the same. Since the wires are of an equivalent length, the strain also will be an equivalent .

The relation for Young's modulus is given as:

$$Y = \frac{stress}{strain} = \frac{\frac{F}{A}}{strain} = \frac{\frac{4f}{rd}}{strain}$$

whereas ,

F= The tension force

A= The area of cross section

D= The Diameter of the wire

It can be inferred from equation $(i) that Y \propto \frac{1}{d^2}$

The Young's modulus for iron is $Y_1 = 190 \times 10^9$ Pa

The Diameter of the iron wire is $= d_1$

The Young's modulus for copper is, $Y_2 = 110 \times 10^9$ Pa

The Diameter of the copper wire is $Y_2 = 110 \times 10^9$ Pa

Therefore, the ratio of their diameters is given as:

$$\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.31:1$$

9.11

A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m is whirled in a vertical circle with an angular velocity of 2 rev / s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

ANSWER-

The Mass is, $m = 14.5$ kg

The Length of the steel wire is, $l = 1.0$ m

The Angular velocity is, $\omega = 2$rev / s

The Cross-sectional area of the wire is, $a = 0.065$cm^2

Let δl be the elongation of the wire when the mass is at rock bottom point of its path.

When the mass is placed at the position of the great circle , the entire force on the mass is:

$$F = mg + ml\omega^2$$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times (2)^2 = 200.1\text{N}$$

The Young's modulus is $= \dfrac{\text{Stress}}{\text{Strain}}$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{FI}{A\Delta l}$$

$$\therefore \Delta l = \frac{Fl}{AY}$$

The Young's modulus for steel is $= 2 \times 10^{11}$ Pa

$$\therefore \Delta l = \frac{200.1 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} = 1539.23 \times 10^{-7}$$

$$= 1.539 \times 10^{-4}\ \text{m}$$

Hence, the elongation of the wire is= 1.539×10^{-4} m

9.12

Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increases = 100.0 atm $(1\ \text{atm} = 1.013 \times 10^5\ \text{Pa})$, Final volume 100.5 litre.

Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Answer:

The Initial volume is $= V_1 = 100.0\text{l} = 100.0 \times 10^{-3}\ \text{m}^3$

The Final volume is $V_2 = 100.5\text{l} = 100.5 \times 10^{-3}\ \text{m}^3$

The Increase in volume is $\Delta V = V_2 - V_1 = 0.5 \times 10^{-3}\ \text{m}^3$

The Increase in pressure is $\Delta p = 100.0\,\text{atm} = 100 \times 1.013 \times 10^5$ Pa

Bulk modulus $=\dfrac{\Delta p}{\dfrac{\Delta V}{V_1}}$

$=\dfrac{\Delta p\times V_1}{\Delta V}n$

$=\dfrac{100\times 1.013\times 10^5\times 100\times 10^{-3}}{0.5\times 10^{-3}}n=$

2.026×10^9 Pa

The Bulk modulus of air $=1.0\times 10^5$ Pa

$\therefore \dfrac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}}$

$=\dfrac{2.026\times 10^9}{1.0\times 10^5}$

$=2.026\times 10^4$

This ratio is very high because The air is more compressible than The water.

9.13

What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×103 kg m^{-3}?

Answer:

The given depth be h

The Pressure at the given depth is, $p=80.0\,\text{atm}=80\times 1.01\times 10^5$ Pa

Density of water at the surface, $\rho_1=1.03\times 10^3\,\text{kg m}^{-3}$

Let ρ_2 be the density of water at the depth h.

Let V_1 be the quantity of water of mass m at the surface.

Let V_2 be the quantity of water of mass m at the depth **h**.

Let ΔV be the change in volume.

$$\Delta V = V_1 - V_2 = m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$$\therefore Volumetricstrain = \frac{\Delta V}{V_1}$$

$$= m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \times \frac{\rho_1}{m}$$

$$\therefore \frac{\Delta V}{V_1} = 1 - \frac{\rho_1}{\rho_2}$$

The Bulk modulus, $B = \frac{pV_1}{\Delta V}$

$$\frac{\Delta V}{V_1} = \frac{p}{B}$$

The Compressibility of water $= \frac{1}{B} = 45.8 \times 10^{-11}\ \text{Pa}^{-1}$

$$\therefore \frac{\Delta V}{V_1} = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.71 \times 10^{-3}$$

For equations (i) and (ii), we will get:

$$1 - \frac{\rho_1}{\rho_2} = 3.71 \times 10^{-3}$$

$$\rho_2 = \frac{1.03 \times 10^3}{1 - \left(3.71 \times 10^{-3}\right)}$$

$$= 1.034 \times 10^3\ \text{kg m}^{-3}$$

as a result, the density of water at the given depth (h) is $= 1.034 \times 10^3\ \text{kg m}^{-3}$.

9.14

Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

ANSWER:

The Hydraulic pressure exerted on the glass slab is, $\mathrm{p} = 10\,\mathrm{atm} = 10 \times 1.013 \times 10^5\ \mathrm{Pa}$

The Bulk modulus of glass is $B\ = 37 \times 10^9\ \mathrm{Nm}^{-2}$

The Bulk modulus is, $B = \dfrac{p}{\frac{\Delta V}{V}}$

Whereas,

$$\frac{\Delta V}{V} = \text{Fractional change in volume}$$

$$\therefore \frac{\Delta V}{V} = \frac{p}{B}$$

$$= \frac{10 \times 1.013 \times 10^5}{37 \times 10^9}$$

$$= 2.73 \times 10^{-5}$$

consequently, the fractional change in the volume of the glass slab is 2.73×10^{-5}

9.15

Determine the volume contraction of a solid copper cube, $10\ \mathrm{cm}$ on an edge, when subjected to a hydraulic pressure of $7.0\ \times\ 106$ Pa

Answer:

The Length of an edge of the solid copper cube is, $l = 10\,\mathrm{cm} = 0.1\mathrm{m}$

The Hydraulic pressure is $p = 7.0 \times 10^6\ \mathrm{Pa}$

The Bulk modulus of copper is, $B = 140 \times 10^9\ \mathrm{Pa}$

Whereas,

Bulk modulus, $B = \dfrac{p}{\frac{\Delta V}{V}}$

$$\frac{\Delta V}{V} = \text{The Volumetric strain}$$

$\Delta V =$ The Change in volume

V= The Original volume $\Delta V = \dfrac{pV}{B}$

The Original volume of the cube is, $\mathrm{V} = \mathrm{l}^3$

$$\therefore \Delta V = \frac{pl^3}{B}$$

$$= \frac{7\times10^6\times(0.1)^3}{140\times10^9}$$

$$= 5\times10^{-8}\ \mathrm{m}^3$$

$$= 5\times10^{-2}\ \mathrm{cm}^{-3}$$

accordingly, the volume contraction of the solid copper cube is $5\times10^{-2}\ \mathrm{cm}^{-3}$

9. 16

How much should the pressure on a litre of water be changed to compress it by 0.10%?

ANSWER:

The Volume of water, $V = 1\mathrm{L}$

it's given that water is to be compressed by **0.10** $\backslash\%$

$$\therefore Fractionalchange, \frac{\Delta V}{V}$$

$$= \frac{0.1}{100\times1}$$

$$= 10^{-3}$$

The Bulk modulus is, $B = \dfrac{\rho}{\frac{\Delta V}{V}}$

$$p = B\times\frac{\Delta V}{V}$$

The Bulk modulus of water is, $B = 2.2\times10^9\ \mathrm{Nm}^{-2}$

$$= 2.2\times10^{y}\times10^{-3}$$

$$= 2.2\times10^6\ \mathrm{Nm}^{-2}$$

on account of, the pressure on water should be $2.2\times10^{6}\,\text{Nm}^{-2}$

Additional Exercises:

9.17

Anvils made of single crystals of diamond, with the shape as shown in $\text{Fig. }9.14$, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of $50,000\text{ N}$. What is the pressure at the tip of the anvil?

ANSWER:

The Diameter of the cones at the narrow ends is, $\text{d}=0.50\,\text{mm}=0.5\times10^{-3}\,\text{m}$

The Radius is, $r=\dfrac{d}{2}=0.25\times10^{-3}\,\text{m}$

The Compressional force is, $\text{F}=50000\,\text{N}$

The Pressure at the tip of the anvil is:

$$=\frac{50000}{\pi\left(0.25\times10^{-3}\right)^{2}}$$

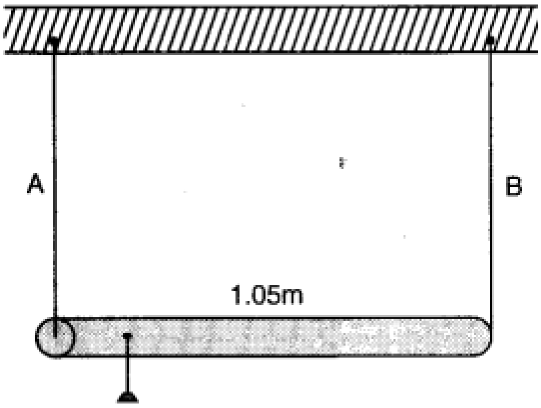
$$=2.55\times10^{11}\,\text{Pa}$$

on account of, the pressure at the tip of the anvil is $2.55\times10^{11}\,\text{Pa}$

9.18

A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.15. The cross-sectional areas of wires A and B are $1.0\text{ mm2 and }2.0\text{ mm2}$, respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires

ANSWER:



(a) $0.7\,\text{m}$ from the steel wire end is $0.432\,\text{m}$ from the steel-wire end

The Cross-sectional area of wire is $\mathbf{A}, a_1 = 1.0\,\text{mm}^2 = 1.0\times10^{-6}\,\text{m}^2$.

The Cross-sectional area of wire is $\mathbf{B}, a_2 = 2.0\,\text{mm}^2 = 2.0\times10^{-6}\,\text{m}^2$

The Young's modulus for steel is $Y_1 = 2\times10^{11}\,\text{Nm}^{-2}$

The Young's modulus for aluminium, $Y_2 = 7.0\times10^{10}\,\text{Nm}^{-2}$

allow a small mass m be suspended to the rod at a distance y from the end where wire $\mathbf{A}$ is attached

The Stress in the wire is $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{a}$

If the two wires have equal stresses, then:

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

Where, $F_1 =$ Force exerted on the steel wire

$F_2 =$ Force exerted on the aluminum wire

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} = \frac{1}{2}$$

The situation is shown in the following figure.

Taking torque about the point of suspension, we have:

$$F_1 y = F_2(1.05 - y)\frac{F_1}{F_2}$$

$$= \frac{(1.05 - y)}{y}$$

Using equations (i) and (ii), we can write:

$$\frac{(1.05 - y)}{y}$$

$$= \frac{1}{2}2(1.05 - y)$$

$$= y2,1 - 2y = y$$

$$3y = 2.1$$

$$\therefore y = 0.7\,\text{m}$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of $0.7\,\text{m}$ from the end where wire $\mathbf{A}$ is attached. Young's modulus $= \frac{\text{Stress}}{\text{Strain}}$

$$\text{Strain } = \frac{\text{Stress}}{\text{Young}'\text{smodulus}} = \frac{\frac{F}{a}}{Y}$$

whether the strain in the two wires is equal, then:

$$\frac{\frac{F_1}{a_1}}{Y_1}$$

$$= \frac{\frac{F_2}{a_2}}{Y_2}\frac{F_1}{F_2}$$

$$= \frac{a_1}{a_2}\frac{Y_1}{Y_2}$$

$$= \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}}$$

$$= \frac{10}{7}$$

Taking torque about the point where mass m, is suspended at a distance y_1 from the side where wire $\mathbf{A}$ attached, we get:

F_1y_1

$= F_2(1.05 - y_1)\frac{F_1}{F_2}$

$= \frac{(1.05 - y_1)}{y_1} \ldots$

Using equations (iii) and (iv), we get:

$\frac{(1.05 - y_1)}{y_1}$

$= \frac{10}{7}7(1.05 - y_1)$

$= 10y_1 17y_1$

$= 7.35$

$\therefore y_1 = 0.432\,\text{m}$

To produce an equal strain within the two wires, the mass should be suspended at a distance of $0.432\,\text{m}$ from the end where wire $\mathbf{A}$ is attached.

9.19

A mild steel wire of length $1.0\ \text{m}$ and cross-sectional area $0.50 \times 10-2\ \text{cm}^2$ is stretched, well within its elastic limit, horizontally between two pillars. A mass of $100\ \text{g}$ is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

ANSWER:

The Length of the steel wire is $= 1.0\,\text{m}$

The Area of cross-section is

$A = 0.50 \times 10^{-2}\,\text{cm}^2$

$= 0.50 \times 10^{-6}\,\text{m}^2$

The mass $100\,\text{g}$ is suspended from its midpoint.

$m = 100\,\text{g}$

$= 0.1\text{kg}$

therefore, the wire dips, as shown in the given figure.

Original length $= \mathrm{XZ}$

Depression = 1

The length after mass m is attached to the wire $= \mathrm{XO}+\mathrm{OZ}$

Increase in the length of the wire: $\Delta l=(XO+OZ)-XZ$

Where,

$$XO=OZ=\left[(0.5)^2+l^2\right]^{\frac{1}{2}}$$

$$\therefore \Delta l=2\left[(0.5)^2+(l)^2\right]^{\frac{1}{2}}-1.0n$$

$$=2\times 0.5\left[1+\left(\frac{l}{0.5}\right)^2\right]^{\frac{1}{2}}-1.0$$

Expanding and neglecting higher terms, we get:

$$\Delta l=\frac{l^2}{0.5}$$

$$\text{Strain}=\frac{\text{Increase in length}}{\text{Original length}}$$

Let T be the tension in the wire.

$$\therefore \mathrm{mg}=2\mathrm{T}\cos\theta$$

Using the figure, it can be written as:

$$\cos\theta=\frac{l}{\left((0.5)^2+l^2\right)^{\frac{1}{2}}}$$

$$=\frac{l}{(0.5)\left(1+\left(\frac{l}{0.5}\right)^2\right)^{\frac{1}{2}}}$$

Expanding the expression and eliminating the higher terms: $\cos\theta = \dfrac{l}{(0.5)\left(1+\dfrac{l^2}{2(0.5)^2}\right)}$

$$\left(1+\frac{I^2}{0.5}\right)=1\,forsmall$$

$$\therefore \cos\theta = \frac{l}{0.5}$$

$$\therefore T = \frac{mg}{2\left(\dfrac{l}{0.5}\right)}$$

$$= \frac{mg\times 0.5}{2l}$$

$$= \frac{mg}{4l}$$

$$Stress = \frac{\text{Tension}}{\text{Area}}$$

$$= \frac{mg}{Y}oung'smodulus$$

$$= \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{mg\times 0.5}{4l\times A\times l^2}$$

$$I = \sqrt[3]{\frac{mg\times 0.5}{4YA}}$$

Young's modulus of steel. $Y = 2\times 10^{11}$ Pa

$$\therefore l = \sqrt{\frac{0.1\times 9.8\times 0.5}{4\times 2\times 10^{11}\times 0.50\times 10^{-6}}}$$

$$= 0.0106\,\text{m}$$

therefore, the depression at the midpoint is $0.0106\,\text{m}$

9.20

Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm.. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on

the rivet is not to exceed 6.9×107 Pa? Assume that each rivet is to carry one quarter of the load.

ANSWER:

The Diameter of the metal strip is, $d = 6.0\,mm = 6.0 \times 10^{-3}\,m$

The Radius is, $r = \frac{d}{2} = 3.0 \times 10^{-5}\,m$

The Maximum shearing stress is $= 6.9 \times 10^{7}\,Pa$

The Maximum stress is $= \frac{\text{Maximum load or force}}{\text{Area}}$

The Maximum force $= Maximumstress \times \text{Area}$

$= 6.9 \times 10^{7} \times \pi \times (r)^{2}$

$= 6.9 \times 10^{7} \times \pi \times \left(3 \times 10^{-3}\right)^{2}$

$= 1949.94\,N$

Each rivet carries one-fourth of the load.

$\therefore$ Maximum tension on each rivet $= 4 \times 1949.94 = 7799.76\,N$

9.21:

The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about $1.1 \times$ 108 Pa A steel ball of initial volume 0.32 m3 is dropped into the ocean and falls to thebottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

ANSWER:

Water pressure at the bottom, $p = 1.1 \times 10^{8}\,Pa$

Initial volume of the steel ball, $V = 0.32\,m^{3}$

Bulk modulus of steel $B = 1.6 \times 10^{11}\,Nm^{-2}$

The ball falls at the bottom of the Pacific Ocean, which is $11\,km$ beneath the surface

Let the change within the volume of the ball on reaching to bottom of the ditch be ΔV

Bulk modulus, $B = \frac{p}{\frac{\Delta V}{V}}$

$$= \frac{1.1 \times 10^{8} \times 0.32}{1.6 \times 10^{11}} = 2.2 \times 10^{-4}\ \text{m}^{3}$$

in consequence, the change within volume of the ball on reaching the bottom of the ditch is $2.2 \times 10^{-4}\ \text{m}^{3}$.

Hence, material A is stronger than material B.