Chapter 1: Electric Charges and Fields

Question 1.1:

What is the force between two small charged spheres having charges of 2×10^{-7}C and 3×10^{-7}C placed 30 cm apart in air?

Answer:

Given:

Charge of the first sphere,

$$q_1 = 2 \times 10^{-7} C$$

Charge of the second sphere,

$$q_2 = 3 \times 10^{-7} C$$

Distance between the spheres, r = 30 cm = 0.3 m

According to Coulomb's law electrostatic force between the spheres is given by the relation,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Where, ε_0 = Permittivity of space and

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N.m}^2/\text{C}^2$$

Hence, Coulomb force,

$$F = \frac{9\times10^9\times2\times10^{-7}}{(0.3)^2}$$

$$= 6\times10^{-3} \text{ N}$$

Hence, force between the two small charged spheres is 6×10^{-3} N

Both charges are positive in nature. Therefore, force between spheres will be repulsive.

Question 1.2:

The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge − 0.8 μC in air is 0.2 N.

(a) What is the distance between the two spheres?
(b) What is the force on the second sphere due to the first?

Answer:

(a) Given,

Electrostatic force acting on the first sphere,

$F = 0.2\ \text{N}$

Charge on first sphere,

$q_1 = 0.4\ \mu C = 0.4 \times 10^{-6}\text{C}$

Charge on the second sphere,

$q_2 = -0.8\ \mu C = -0.8 \times 10^{-6}\text{C}$

According to Coulomb's law electrostatic force between the spheres is given by the relation

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Where, ε_0= Permittivity of free space and

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N.m}^2/\text{C}^2$$

Hence,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\Rightarrow r^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{F}$$

$$\Rightarrow r^2 = \frac{\left(9\times10^9\right)\times\left(0.4\times10^{-6}\right)\times\left(0.8\times10^{-6}\right)}{0.2}$$

$$\Rightarrow r^2 = 1.44\times10^{-2}$$

$$\Rightarrow r = \sqrt{1.44\times10^{-2}}$$

$$\Rightarrow r = 0.12\ \text{m} = 12\ \text{cm}$$

Therefore, the distance between the two charged spheres is 0.12 m.

(b) As the charge on the spheres are asymmetric in nature hence both the spheres attract each other with the force of same magnitude. Therefore, the force on the second sphere due to the first sphere is $0.2\ \text{N}$

Question 1.3:

Check that the ratio $ke^2/G\,m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Answer:

Given,

The ratio is $= \dfrac{ke^2}{G\ m_e m_p}$

Where,

G = Gravitational constant. Unit of G is $\mathrm{Nm^2kg^{-2}}$

m_e and m_p = **Masses of electron and proton respectively and their unit is kg.**

e = Electric charge. Its unit is C.

$$k = \frac{1}{4\pi\varepsilon_0}$$

and its unit is $\mathrm{N\ m^2C^{-2}}$.

Hence, the unit of the given ratio is,

$$\frac{ke^2}{G\ m_e m_p} = \frac{[Nm^2C^{-2}][C^2]}{[Nm^2kg^{-2}][kg][kg]} = \left[M^0L^0T^0\right]$$

Therefore, the given ratio has no unit so the ratio is dimensionless.

$e = 1.6 \times 10^{-19}\,\mathrm{C}$

$G = 6.67 \times 10^{-11}\,\mathrm{N\ m^2kg^{-2}}$

$m_e = 9.1 \times 10^{-31}\,\mathrm{kg}$

$m_p = 1.66 \times 10^{-27}\,\mathrm{kg}$

Hence, the numerical value of the ratio is,

$$\frac{ke^2}{G\ m_e m_p} = \frac{\left(9\times10^9\right)\times\left(1.6 \times 10^{-19}\right)^2}{\left(6.67 \times 10^{-11}\right)\times\left(9.1 \times 10^{-31}\right)\times\left(1.66 \times 10^{-27}\right)}$$
$$\approx 2.3\times10^{39}$$

This is the ratio represents the ratio of electric force to the gravitational force acting between a proton and an electron, when they are placed at same distance.

Question 1.4:

(a) Explain the meaning of the statement ‘electric charge of a body is quantised’.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Answer:

(a) The electric charge of a body is quantized. This means that the possessed electric charge of the body is integral multiple of electron's charge and only integral (1, 2, 3, 4,…..) number of electron could be transferred from one body to the other.

(b) Charges cannot be transferred in fraction charge of electron. Therefore, a body possesses total charge only in integral multiples of electric charge, so electric charge of a body is quantized.

(c) In macroscopic system or when large scale charges present in the system, the number of charges used are huge as compared to the magnitude of electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Hence, one can ignore and consider that electric charge is continuous.

Question 1.5:

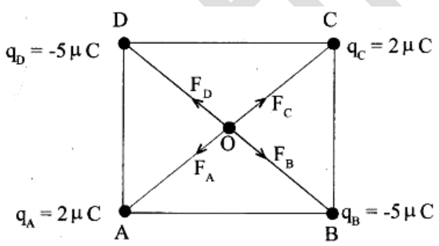
When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Answer: During rubbing the rod with silk charges of equal magnitude but of opposite nature produced on the two bodies acting in this process because charges are created in pairs. This process of charging is known as charging by friction. The net charge is zero on the system of two rubbed bodies. This is because equal amount of charge annihilate from one body and same amount gained by other body. When a glass rod is rubbed with a silk cloth, opposite natured charges appear on both. This phenomenon is in consistence with the law of conservation of energy because mechanical energy in this process transform into electrostatic energy and heat due to friction. A similar phenomenon is observed with many other pairs of bodies.

Question 1.6:

Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$ and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 µC placed at the centre of the square?

Answer: The given diagram shows a square of side 10 cm and four charges are placed at its corners. O is the centre of the square.



Where,

$$(\text{Sides are})\ AB = BC = CD = AD = 10\text{ cm}$$

$$(\text{Diagonals are})\ AC = BD = 10\sqrt{2}\text{cm}$$

$$AO = OC = DO = OB = 5\sqrt{2}\text{cm}$$

A charge of amount 1µC is placed at point O.

Force of repulsion between charges placed at corner A and centre O is equal in magnitude but opposite in direction relative to the repulsion force acting between the charges placed at corner C and centre O. Therefore, those two forces will cancel each other. Similarly, force of attraction between the charges placed at corner B and centre O is equal in magnitude but opposite in direction relative to the attraction force acting between the charges placed at corner D and centre O. Hence, those attractive forces will also neutralise each other. Therefore, net force acting on 1μC charge at centre O caused by the four charges placed at the corner of the square on is zero.

Question 1.7:

(a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two field lines never cross each other at any point?

Answer:

(a) An electrostatic field line is a continuous curve because a charge experiences a continuous force when they are in an electrostatic field. The field lines represent line cannot have sudden breaks or discontinuity because the motion of the charge is continuous and does not jump from one point to the other.

The tangent on electric field line at a point represents the direction of electric field at that point. If two field lines get cross each other at a point, then there will be two tangents at the point pointing two direction of electric field at that point. This is not possible. Therefore, two field lines never cross each other.

Question 1.8:

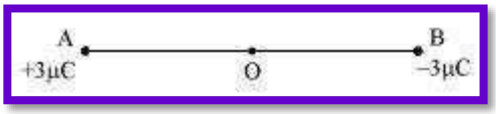
Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude $1.5 \times 10^{-9} C$ is placed at this point, what is the force experienced by the test charge?

Answer:

(a) The problem is represented by the given figure. Here O is the mid-point of line AB.



The distance between the two charges,

$AB = 20$ cm

$\therefore AO = OB = 10$ cm

Net electric field at point O = E

Electric field at point O caused by +3μC charge,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-6}}{(OA)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-6}}{(10\times10^{-2})^2} \text{ N/C} \qquad \text{along } OB$$

Where, ε_0 = Permittivity of free space and $\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ Nm}^2/\text{C}^2$

Hence,

The magnitude of electric field at point O caused by −3μC charge,

$$E_2 = \left|\frac{1}{4\pi\varepsilon_0}\frac{-3\times10^{-6}}{(OB)^2}\right|$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-6}}{(10\times10^{-2})^2} \text{ N/C} \qquad \text{along } OB$$

$$\therefore \quad E = E_1 + E_2$$

$$\Rightarrow E = 2\times E_1 \qquad (\because E_1 = E_2)$$

$$\Rightarrow E = 2\times\frac{1}{4\pi\varepsilon_0}\frac{3\times10^{-6}}{(10\times10^{-2})^2} \text{ N/C} \qquad \text{along } OB$$

Since both E_1 and E_2 in the direction of OB

$$E = 2\times9\times10^9\times\frac{3\times10^{-6}}{(10\times10^{-2})^2} \text{ N/C} \qquad \text{along OB}$$

$$= 5.4 \times 10^6 N/C$$

Therefore, the electric field at mid-point O is 5.4×10^6 N/C along OB.

(b) A negative test charge of amount 1.5×10^{-9} C **is placed at mid – point O.**

$q = -1.5 \times 10^{-9} C$

Force experienced by the test charge $= F$

$$\therefore F = -qE$$
$$= -1.5 \times 10^{-9} \times 5.4 \times 10^6$$
$$= -8.1 \times 10^{-3} \text{N}$$

Negative sign imply the direction of force is opposite to the direction of electric field. Hence, the force is directed along line OA. This is because the negative test charge is repelled by the negative charge at point B but attracted towards point A.

Therefore, the force experienced by the test charge is 8.1×10^{-3}N along OA.

Question 1.9:

A system has two charges $q_A = 2.5 \times 10^{-7}$C and $q_B = -2.5 \times 10^{-7}$C located at points A: (0, 0, − 15 cm) and B: (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?

Answer:

The given charges can be placed in a coordinate frame of reference.

At point A, amount of charge, $q_A = 2.5 \times 10^{-7} C$

At point B, amount of charge, $q_B = -2.5 \times 10^{-7} C$

Total charge ,

$$q = q_A + q_B$$

$$= 2.5 \times 10^{7} C - 2.5 \times 10^{-7} C$$

$$= 0$$

Distance between two charges at points A and B, $d = 15 + 15 = 30$ cm = 0.3 m

Electric dipole moment of the system is given by,

$$p = q_A \times d = q_B \times d = 2.5 \times 10^{-7} \times 0.3$$

$= 7.5 \times 10^{-8}$C.m along positive z-axis

Hence, the electric dipole moment of the system is 7.5×10^{-8}C.m along positive z−axis.

Question 1.10:

An electric dipole with dipole moment 4×10^{-9}C.m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^{4} N C^{-1}. Calculate the magnitude of the torque acting on the dipole.

Answer:

Given,

Electric dipole moment, $p = 4 \times 10^{-9}$C.m

Angle between dipole moment p with a uniform electric field, $\theta = 30°$

∴ Electric field,

$$E = 5 \times 10^{4} \text{N.C}^{-1}$$

Torque acting on the dipole is given by the relation ,

$$\tau = pE \sin\theta$$

$$\Rightarrow \tau = 4 \times 10^{-9} \times 5 \times 10^{4} \times \sin 30°$$

$$\Rightarrow \tau = 20 \times 10^{-5} \times \frac{1}{2} = 10^{-4} \text{ N.m}$$

Hence, the magnitude of the torque acting on the dipole is 10^{-4} N.m

Question 1.11:
A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)
(b) Is there a transfer of mass from wool to polythene?

Answer:

(a) When polythene is rubbed with wool, a number of electrons get transferred from wool to polythene. Therefore, wool gets positively charged and polythene becomes negatively charged.

Amount of charge on the polythene piece, $q = -3 \times 10^{-7}$ C

Amount of charge on an electron, $e = -1.6 \times 10^{-19}$ C

Let, number of electrons transferred from wool to polythene $= n$

The value of n can be calculated using the relation,

$$q = ne$$

$$\Rightarrow n = \frac{q}{e}$$

$$\Rightarrow n = \frac{-3\times10^{-7}}{-1.6\times10^{-19}} = 1.87\times10^{12}$$

Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}.

(b) Yes.
A negligible transfer of mass taking place in this process. This is because an electron has mass,

$m_e = 9.1 \times 10^{-3}$kg

$\therefore$ Total mass transferred to polythene from wool,

$$m = m_e \times n$$
$$= 9.1 \times 10^{-31} \times 1.85 \times 10^{12}$$
$$= 1.706 \times 10^{-18}\, kg$$

Therefore, a negligible amount of mass is transferred from wool to polythene during rubbing.

Question 1.12:

(a) Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.
(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Answer:

(a)

Charge on sphere A, $q_A = 6.5 \times 10^{-7} C$
Charge on sphere B, $q_B = 6.5 \times 10^{-7} C$
Distance between the spheres A and B is,
$r = 50$ cm = 0.5 m
Coulomb force between the two spheres,

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r^2}$ Where, ε_0 = Permittivity of free space

$$\Rightarrow F = 9\times10^9 \times \frac{(6.5 \times 10^{-7})\times(6.5 \times 10^{-7})}{(0.5)^2}$$

$$\Rightarrow F = 1.52 \times 10^{-2} \text{ N}$$

Therefore, the force between the two spheres 1.52×10^{-2} N

(b) After doubling the charge,

Charge on sphere A, $q_A = 2\times6.5 \times 10^{-7} = 1.3 \times 10^{-6} C$
Charge on sphere B, $q_B = 2\times6.5 \times 10^{-7} = 1.3 \times 10^{-6} C$
Now distance becomes half, therefore distance between the spheres A and B is,

$r = \frac{50}{2}$ cm = 0.25 m

Repulsion force between the two spheres,

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r^2}$ Where, ε_0 = Permittivity of free space

$$\Rightarrow F = 9\times10^9 \times \frac{(1.3 \times 10^{-6})\times(1.3 \times 10^{-6})}{(0.25)^2}$$

$$\Rightarrow F = 16\times1.52 \times 10^{-2} \text{ N}$$

$$\Rightarrow F = 0.243 \text{ N}$$

Hence, the force between the two spheres is 0.243 N

Question 1.13:

Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Answer:

Distance between the charged spheres, A and B, $r = 0.5$ m

Initially, the charge on each sphere,

$q = 6.5 \times 10^{-7}$ C

When sphere A is touched with an uncharged sphere C,

Charge q equally distributed between A and C , therefore $\frac{q}{2}$ amount of charge from A will transfer to sphere C.

Hence, now charge on each of the spheres, A and C, is $=\frac{q}{2}$

When sphere C with charge $\frac{q}{2}$ is brought in contact with sphere B with charge q,

total charges on the system will divide into two equal halves given as,

$$\frac{\frac{q}{2}+q}{2}=\frac{3q}{4}$$

Hence, now charge on each of the spheres, C and B, is $\frac{3q}{4}$.

Force of repulsion between sphere A having charge $\frac{q}{2}$ and sphere B having charge $\frac{3q}{4}$ is

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r^2}$$ Where, ε_0 = Permittivity of free space

$$\Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{\frac{q}{2}\times\frac{3q}{4}}{(0.5)^2}$$

$$\Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{8\times(0.5)^2}$$

$$\Rightarrow F = 9\times10^9\times\frac{3\times(6.5\times10^{-7})^2}{8\times(0.5)^2}$$

$$\Rightarrow F = 5.703\times10^{-3}\,\text{N}$$

Therefore, the force of repulsion acting between the two spheres is 5.703×10^{-3} N

Question 1.14:

Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

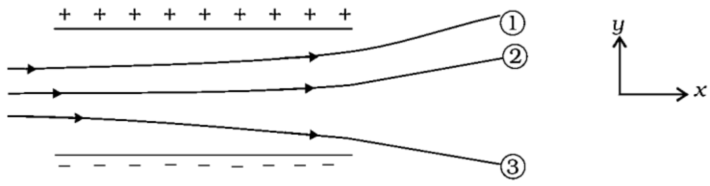


FIGURE 1.33

Answer:

Opposite charges attract each other and same charges repel each other. In given figure it showed that particles 1 and 2 both bend towards the positively charged plate and repelled away from the negatively charged plate. Therefore, these two particles are negatively charged.

It is also observed that particle 3 largely bends towards the negatively charged plate and repelled away from the positive charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the amount of deflection for a given velocity. As it is observed that the deflection of particle 3 is relatively maximum, hence it has the highest charge to mass ratio.

Question 1.15:

Consider a uniform electric field $\mathbf{E} = 3 \times 10^3 \,\hat{\mathbf{i}}$ N/C

(a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz - plane?

(b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

Answer:

(a) Electric field intensity, $\mathbf{E} = 3 \times 10^3 \,\hat{\mathbf{i}}$ N/C

Side of the square, $s = 10 \text{ cm} = 0.1 \text{ m}$

Area of the square, $A = s^2 = 0.01 \text{ m}^2$

As the plane of the square is parallel to the y-z plane
Therefore, angle between the
unit vector normal to the plane and electric field, θ = 0°

Flux through the surface,

$$\Phi = EA\cos\theta$$
$$= 3 \times 10^3 \times 0.01 \times \cos 0°$$
$$= 30 \text{ N.m}^2\text{/C}$$

(b) When plane makes an angle of 60° with the x – axis.

Therefore,

$\theta = 60°$

Now flux through the surface,

$\Phi = EA\cos\theta$

$= 3 \times 10^3 \times 0.01 \times \cos 60°$

$= 15 \text{ N.m}^2/\text{C}$

Question 1.16:

What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Answer:

All the surfaces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube will be equal to the number of field lines piercing out of the opposite surface of cube. Therefore, net flux through the cube is zero.

Question 1.17:

Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{ N.m}^2/\text{C}$.

(a) What is the net charge inside the box?
(b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Answer:

(a) Net outward flux through the surface of the box,

$\Phi = 8.0 \times 10^3 \text{ N m}^2/\text{C}$

For a body containing net charge q, flux is given by the relation,

$$\Phi = \frac{q}{\varepsilon_0}$$

$\varepsilon_0 =$ Permittivity of free space

$= 8.854 \times 10^{-12} \text{ N}^{-1}\text{C}^2\text{m}^{-2}$

$\therefore q = \varepsilon_0 \Phi$

$= 8.854 \times 10^{-12} \times 8.0 \times 10^3 \text{C}$

$= 7.08 \times 10^{-8} \text{C}$

$= 0.07 \ \mu\text{C}$

Therefore, the net charge inside the box is 0.07 μC.

(b) No

Net flux coming out through a body depends on the net charge contained in the surface of the body. If net flux is zero, then it can imply that net charge inside the body is zero. The body might have equal amount of positive and negative charges.

Question 1.18:

A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.)

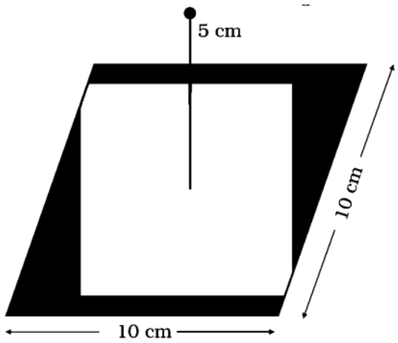


FIGURE 1.34

Answer:

The square can be considered as one face of a cube of edge 10 cm where charge q is placed at the centre of the cube.

According to Gauss's theorem, total electric flux through all six faces of the cube is,

$$\phi_{Total} = \frac{q}{\varepsilon_0}$$

Hence, electric flux through one face of the cube is

$$\phi = \frac{\phi_{Total}}{6} = \frac{q}{6\varepsilon_0}$$

Where,

ε_0 = Permittivity of free space = $8.854 \times 10^{-12}\ N^{-1}C^{2}m^{-2}$

$q = 10\ \mu C = 10 \times 10^{-6} C$

$$\therefore \phi = \frac{10\times10^{-6}}{6\times8.854 \times 10^{-12}}$$
$$= 1.88 \times 10^{5}\ N\,m^{2}C^{-1}$$

Hence, electric flux through the given square is 1.88×10^{5} N m^{2}C^{-1}

Question 1.19:

A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Answer:

The net electric flux (ϕ_{Net}) through the all cubic surface is given by

$$\phi_{Net} = \frac{q}{\varepsilon_0}$$

Where,

ε_0 = Permittivity of free space = 8.854×10^{-12} N^{-1}C^{2}m^{-2}

q = Net charge enclosed inside the cube = 2.0 μC = 2×10^{-6}C

$$\phi_{Net} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \phi_{Net} = \frac{2 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$\Rightarrow \phi_{Net} = 2.26 \times 10^{5} \text{ N m}^2\text{C}^{-1}$$

The net electric flux through all the surfaces is 2.26×10^{5} N m^{2}C^{-1}

Question 1.20:

A point charge causes an electric flux of -1.0×10^{3} Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(b) What is the value of the point charge?

Answer:

(a) Given,
Electric flux,

$\Phi = -1.0 \times 10^{3}$ N m^2/C

Radius of the Gaussian surface, r = 10.0 cm

Electric flux emerging out from a surface depends on the net charge enclosed inside a body. It is independent of the size of the body. If the radius of the Gaussian surface is doubled and the charge enclosed by the enlarged surface remain same, then the net flux passing through the surface will also remain the same i.e., -10^{3} N m^2/C .

(b) According to Gauss's theorem electric flux through a closed surface is given by

$\phi = \frac{q}{\varepsilon_0}$

Where,

ε_0 = Permittivity of free space = $8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

q = Net charge enclosed by the spherical surface= $\phi \varepsilon_0$

$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$

$= -8.854 \times 10^{-9} C$

$= -8.854$ nC

Hence, the magnitude of the point charge is -8.854 nC

Question 1.21:

A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Answer:

Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge q is given by,

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{d^2} \quad ....(1)$$

Where

E = Electric field = 1.5×10^3 N/C

d = Distance from the centre = 20 cm = 0.2 m

ε_0 = Permittivity of free space = $8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

From eq (1) we get,

$\therefore$ q = Net charge enclosed by the surface

$= E(4\pi\varepsilon_0)d^2$

$= \left(1.5 \times 10^3\right) \times \frac{1}{9 \times 10^9} \times (0.2)^2$

$= 6.67 \times 10^{-9}$ C

$= 6.67$ nC

ε_0 = Permittivity of free space and

Hence, the net charge on the sphere is 6.67 nC

Question 1.22:

A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0\ \mu C/m^2$.

(a) Find the charge on the sphere.

(b) What is the total electric flux leaving the surface of the sphere?

Answer:

(a) Given,

Diameter of the sphere, $d = 2.4$ m

Radius of the sphere, $r = \frac{d}{2} = 1.2$ m

Surface charge density, $\sigma = 80.0\ \mu C/m^2 = 80 \times 10^{-6} C/m^2$

Total charge on the surface of the sphere,

Q = Charge density × Surface area

$= \sigma \times 4\pi r^2$

$= 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$

$= 1.447 \times 10^{-3} C$

Hence, the charge on the sphere is $1.447 \times 10^{-3} C$

(b) Total electric flux (ϕ_{Total}) piercing out the surface of a sphere enclosed net charge Q is given by the relation,

$$\phi_{Total} = \frac{Q}{\varepsilon_0}$$

Where,

$\varepsilon_0 =$ Permittivity of free space

$= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

$Q = 1.447 \times 10^{-3} C$

$$\therefore \phi_{Total} = \frac{1.447 \times 10^{-3}}{8.854 \times 10^{-12}} = 1.63 \times 10^{8} N\ C^{-1} m^2$$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^{8} N\ C^{-1} m^2$

Question 1.23: An infinite line charge produces a field of $9 \times 10^4 N/C$ at a distance of 2 cm. Calculate the linear charge density.

Answer: Electric field at a distance d produced by the infinite line charges having linear charge density λ is given by,

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$\Rightarrow \lambda \quad = \quad \left(2\pi\varepsilon_0 d\right) E$$

Where,

Distance, d = 2 cm = 0.02 m

Electric field ,E = 9×10^4 N/C

ε_0 = Permittivity of free space

$$\text{Now, } 2\pi\varepsilon_0 = \frac{1}{2\times9\times10^9}\ \text{C}^2/\text{N.m}^2$$

$$\therefore \lambda = \frac{9\times10^4}{2\times9\times10^9} = 10\ \mu\text{C/m}$$

Therefore, the linear charge density is $10\ \mu$C/m

Question 1.24:

Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2

. What is E:

(a) In the outer region of the first plate,
(b) In the outer region of the second plate, and
(c) Between the plates?

Answer: Since, as per the question the two parallel plates A and B close to each other. Outer region of plate A is marked as I, outer region of plate B is marked as III, and the region between A and B plates, is labelled as II.

Charge density of plate A, $\sigma = 17.0 \times 10^{-22}$ C/m^2

Charge density of plate B, $\sigma = -17.0 \times 10^{-22}$ C/m^2

Electric field E is zero in region I and III, because charge is not enclosed by the respective plates.

Electric field E in region II is given by the relation,

$$E = \frac{\sigma}{\varepsilon_0}$$

Where,

ε_0 = Permittivity of free space = 8.854×10^{-12} N^{-1}C^{2}m^{-2}

$$\therefore E \quad = \quad \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-12}} = 1.92 \times 10^{-10}\ \text{N/C}$$

Hence, electric field between the plates is 1.92×10^{-10} N/C

Question: 1.25

An oil drop of 12 excess electrons is held stationary under a constant electric field 2.55×10^4 N C^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g.cm^{-3}. Estimate the radius of the drop. $\left(g = 9.81\ ms^{-2};\ e = 1.60 \times 10^{-19} C\right)$.

Answer:

Given

Excess electrons on an oil drop, $n = 12$

Electric field intensity, $E = 2.55 \times 10^4$ N C^{-1}

Density of oil, $\rho = 1.26\ gm/cm^3 = 1.26 \times 10^3 kg/m^3$

Acceleration due to gravity, $g = 9.81$ m s^{-2}

Charge on an electron, $e = 1.6 \times 10^{-19}$C

Let, radius of the oil drop $= r$

Force (F) due to electric field E is equal to the weight of the oil drop (W),

$$F = W$$

$$\Rightarrow Eq = mg$$

$$\Rightarrow neE = \frac{4}{3}\pi r^3 \times \rho \times g$$

Where,

q = Net charge on the oil drop $= ne$

Mass of the oil drop

= Volume of the drop × Density of oil

$$= \frac{4}{3}\pi r^3 \times \rho \times g$$

Radius of the drop,

$$r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$$

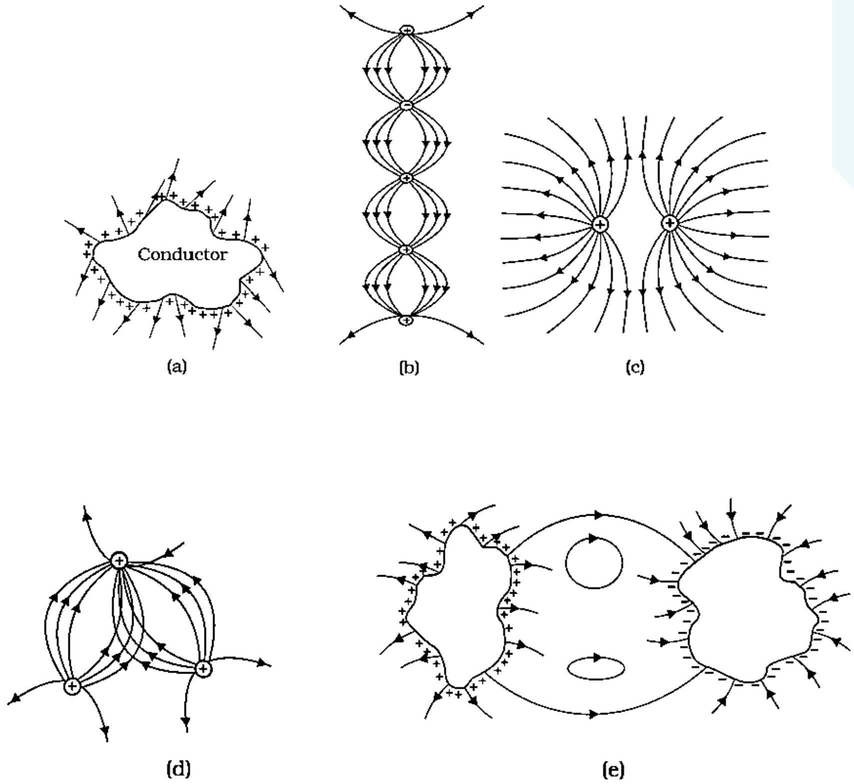
$$= \left[\frac{3 \times 2.55 \times 10^4 \times 12 \times 1.6 \times 10^{-19}}{4 \times \pi \times 1.26 \times 10^3 \times 9.81}\right]^{1/3}$$

$$= 9.82 \times 10^{-7}\ m$$

Hence, the radius of the oil drop is 9.82×10^{-7} m .

Question 1.26

Which among the curves shown in Figure cannot possibly represent electrostatic field lines?



Answer 1.26:

(a) The field lines showed in (a) do not represent electrostatic field lines because field lines always be normal to the surface of the conductor they normally emerge and enter the conductor.

(b) The field lines showed in figure (b) do not represent electrostatic field lines because the field lines could not be emerged from a negative charge and cannot end at a positive charge.

(c) The field lines showed in figure (c) represent an electrostatic field lines. This is because the field lines emerge from the positive charges and field lines repel each other.

(d) The field lines showed in figure (d) do not represent electrostatic field lines be as the field lines can not intersect each other.

(e) The field lines showed in (e) do not represent electrostatic field lines because closed loops are not developed in the area between the field lines.

Question 1.27:

In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of $10^5\,\mathrm{NC^{-1}}$ per metre. What are the force and torque experienced by a system having a total dipole moment equal to $10^{-7}\,\mathrm{Cm}$ in the negative z-direction?

Answer:

Given,

Dipole moment of the system,

$$p = q \times dl = -10^{-7}\,\mathrm{C.m}$$

The rate of increase in electric field per unit length is,

$$\frac{dE}{dl} = 10^5 \text{ NC}^{-1}$$

Force (F) experienced by the system is given by the relation,

$$F = qE = q \times dl \times \frac{dE}{dl}$$

$$\Rightarrow F \quad = \quad p \times \frac{dE}{dl}$$

$$= -10^{-7} \times 10^5 = -10^{-2} \text{ N}$$

The force is -10^{-2} N -direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.

Torque (τ) is given by the relation, $\tau \;=\; pE\sin 180^\circ \;=\; 0$

Therefore, the torque experienced by the system is zero.

Question 1.28:

(a) A conductor A with a cavity as shown in Figure (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor.

(b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Figure (b)].

(c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

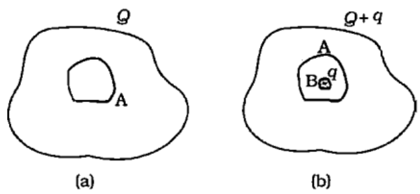


FIGURE 1.36

Answer:

(a) Let consider a Gaussian surface which is lying wholly within a conductor enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and ϵ_0 = the permittivity of free space.

$$\Phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

Here $\vec{E} = 0$

Hence, according to Gauss's law, Flux,

$$\Phi = 0.d\vec{s} = 0$$

$$\Rightarrow \frac{q}{\varepsilon_0} = 0$$

$$\Rightarrow q = 0 \qquad [\because \varepsilon_0 = \text{ constant}]$$

Hence, the charge inside the conductor is zero.

Therefore, the entire charge Q appears on the outer surface of the conductor.

(b) The outer surface of conductor A has a charge of amount Q. Another conductor B having charge +q is kept inside conductor A and it is insulated from A. Hence, a charge of amount −q will be induced in the inner surface of conductor A and +q is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is Q + q.

(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

Question 1.29:

A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\frac{\sigma}{2\varepsilon_0}\hat{n}$,where $\hat{n}$ is the unit vector in the outward normal direction and σ is the surface charge density near the hole.

Answer:

Let consider a conductor with a cavity or a hole. Inside the cavity is electric field zero.

Let E is the electric field just outside the conductor, q is the electric charge, σ is the charge density and ε_0 is the permittivity of free space

Charge $q = \sigma \times$ ds

According to Gauss's law, flux through the surface

$$\Phi = \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

Putting the value of q we get,

$$\Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

Hence, the electric field just outside the conductor is $\frac{\sigma}{2\varepsilon_0}\hat{n}$. This field is a superposition or resultant of field due to the cavity **E`** and the field due to the charged conductor **E`**. These fields are equal and opposite inside the conductor and equal in magnitude and direction pointing outside the conductor

$\therefore$ **E`+ E` = E**

$$\Rightarrow E' = \frac{E}{2} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

Therefore, the field due to the rest of the conductor is $\frac{\sigma}{\varepsilon_0}\hat{n}$.

Question 1.30

Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Hint: Use Coulomb's law directly and evaluate the necessary integral.]

Answer:

Let consider a long thin wire XY (as shown in the following figure) of uniform linear charge density λ.

Consider a point A at a perpendicular distance *l* from the mid-point O of the wire.
Let **E** be the electric field at point A due to the wire, XY.

Consider a small length element *dx* on the wire section with $OZ = x$

Let q be the charge on this piece.

$\therefore q = \lambda dx$

Electric field due to the piece,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(AZ)^2}$$

where AZ $= \sqrt{l^2 + x^2}$

$$\therefore dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(l^2 + x^2)}$$

The electric field will be resolved into two rectangular components. The perpendicular component is $dE\cos\theta$ and $dE\sin\theta$ is the parallel component. When the whole wire XY is considered, the component $dE\sin\theta$ is cancelled. Only the perpendicular component $dE\cos\theta$ affects point A. Hence, effective electric field at point A due to the element dx is dE_1.

$$\therefore dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx\cos\theta}{(l^2 + x^2)}\text{.....................(1)}$$

In ΔAZO,

$$\tan\theta = \frac{x}{l}$$

$$\Rightarrow x = l\tan\theta \text{ ....(2)}$$

On differentiating eq (2), we get

$$\frac{dx}{d\theta} = l\sec^2\theta d\theta \text{ ....(3)}$$

From equation (2) we have,

$$x^2 + l^2 = l^2\left(\tan^2\theta + 1\right) = l^2\sec^2\theta \text{ ....(4)}$$

Putting eq(3) and (4) in equation (1), we get

$$dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda l\sec^2\theta d\theta.\cos\theta}{l^2\sec^2\theta}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta d\theta}{l}$$

As the wire is so long θ tends to $\frac{-\pi}{2}$ to $\frac{\pi}{2}$

By integrating equation (5), we obtain the value of field E_1 as

$$\int_{-\pi/2}^{\pi/2} dE_1 = \frac{1}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}\frac{\lambda\cos\theta d\theta}{l}$$

$$\Rightarrow E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{l}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$\Rightarrow E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{l}\times 2$$

$$\Rightarrow E_1 = \frac{\lambda}{2\pi\varepsilon_0 l}$$

Hence, the electric field due to long wire is $\frac{\lambda}{2\pi\varepsilon_0 l}$.

Question 1.31:

It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge (+2/3) e, and the 'down' quark (denoted by d) of charge (−1/3) e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Answer:

A proton has three quarks. Let there is n up quarks in a proton, each having a charge of $(+2/3)\ e$

Charge due to n up quarks $= \left(\frac{+2e}{3}\right) n$

Number of down quarks in a proton $= 3 - n$

Each down quark has a charge of $= \left(\frac{-1}{3} e\right)$

Charge due to $(3 - n)$ down quarks $= \left(\frac{-1}{3} e\right) \times (3-n)$

Total charge on a proton $= +e$

$$\therefore e = \left(\frac{2e}{3}\right) n + \left(-\frac{1}{3} e\right)(3-n)$$

$$\Rightarrow e = \frac{2ne}{3} - e + \frac{ne}{3}$$

$$\Rightarrow 2e = ne$$

$$\Rightarrow n = 2$$

Number of quarks in a proton, n = 2

Number of down quarks in a proton = 3 - n = 3 - 2 = 1

Hence, a proton can be represented as 'uud'.

A neutron also has three quarks. Let there be n up quarks in a neutron.

Charge on a neutron due to n up quarks $= \left(+\frac{2e}{3}\right) n$

Number of down quarks is 3 – n, and each having a charge of $\left(-\frac{1}{3} e\right)$

Charge of a neutron due to (3 – n) down quarks $\left(-\frac{1}{3} e\right)(3-n)$

Total charge on a neutron = 0

$$\therefore 0 = \left(\frac{2e}{3}\right) n + \left(-\frac{1}{3} e\right)(3-n)$$

$$\Rightarrow e = ne$$

$$\Rightarrow n = 1$$

Number of up quarks in a neutron, n = 1

Number of down quarks in a neutron = $3 - n = 2$

Therefore, quark representation is 'udd'.

Question 1.32:

(a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where E = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Answer:

(a) Let consider the equilibrium of the test charge be stable. If a test charge is in equilibrium and displaced from its mean position in any direction, then it felt a restoring force towards a null point, where the electric field is zero. All the field lines near this null point are directed inwards to the null point. There is a net inward flux of electric field through a closed surface around the null point. Hence, according to Gauss's law, the flux of electric field through a surface, which is not enclosing any charge, is zero. Therefore, the equilibrium of the test charge can be stable.

(b) If two charges of same magnitude and same sign are placed at a certain distance, the mid-point of the joining line of the charges is the null point as electric field lines repel each other at mid-point there will be no resultant field. When a test charged is displaced from mean position along the line, it experiences a restoring force. If it is displaced normal to the joining line, then the net force takes it away from the null point. Hence, the charge is unstable because stability of equilibrium requires restoring force in all directions.

Question 1.33: A particle of mass m and charge (−q) enters the region between the two charged plates initially moving along x-axis with speed v_x (like particle 1 in Figure). The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2mv_x^2)$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Answer:

Charge on a particle of mass $m = -q$

Velocity of the particle $= v_x$

Length of the plates $= L$

Magnitude of the uniform electric field between the plates $= E$

Mechanical force,

$$F = Mass\ (m) \times Acceleration\ (a)$$

$$\Rightarrow a = \frac{F}{m}$$

$$\Rightarrow a = \frac{qE}{m} \text{ ....(1)} \qquad [\because \text{ As electric field } F = qE]$$

Time taken to cross the field of length L is,

$$t = \frac{\text{Length of the plate}}{\text{Velocity of the particle}} = \frac{L}{v_x} \text{..........................(2)}$$

In the vertical direction, initial velocity, $u = 0$

According to the third equation of motion,

vertical deflection s of the particlecan be obtained as,

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$$\Rightarrow s = \frac{qEL^2}{2mv_x^{\ 2}}$$

Therefore, vertical deflection of the particle at the far edge of the plate is $\frac{qEL^2}{2mv_x^{\ 2}}$.

This is similar to motion of horizontal projectiles under gravity.

Question 1.34:

Suppose that the particle in Exercise in 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6 m\ s^{-1}$.If E between the plates separated by 0.5 cm 9.1×10^2N/C, where will the electron strike the upper plate?

$$\left(|\ e\ | = 1.6 \times 10^{-19}C,\ m_e = 9.1 \times 10^{-31}kg.\right)$$

Answer:

Velocity of the particle, $v_x = 2.0 \times 10^6$ m/s

Separation of the two plates, $d = 0.5$ cm = 0.005 m

Electric field between the two plates, $E = 9.1 \times 10^2\ N/C$

Charge on an electron, $q = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Let the electron will strike the upper plate at the end of plate L, when deflection be s.
Therefore,

$$s=\frac{qEL^2}{2mv_x^{\;2}}$$

$$\Rightarrow L=\sqrt{\frac{2smv_x}{qE}}$$

$$\Rightarrow L=\sqrt{\frac{2\times 0.005\times 9.1\times 10^{-31}}{1.6\times 10^{-19}\times 9.1\times 10^{2}}}$$

$$=\sqrt{0.00025}$$

$$=0.016\text{ m}$$

Hence, the electron will strike the upper plate after travelling 0.016 m or 1.6 cm.