CHAPTER-10: WAVE OPTICS

EXAMPLES

EXAMPLE 10.1 What speed should a galaxy move with respect to us so that the sodium line at $589.0\,\text{nm}$ is observed at $589.6\,\text{nm}$?

Solution: As $\nu\lambda = c, \frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda}$ (for small changes in ν and λ).

For

$$\Delta\lambda = 589.6 - 589.0 = +0.6\,\text{nm}$$

we get [using Eq. (10.9)]

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} = -\frac{v_{\text{radial}}}{c}$$

$$v_{\text{radial}} \equiv +c\left(\frac{0.6}{589.0}\right)$$

or, $= +3.06\times10^{5}\,\text{m}\,\text{s}^{-1}$

$= 306\,\text{km/s}$

As a result, the galaxy is accelerating away from us.

Example 10.2

(a) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency. Explain why?

Solution: The interaction of incoming light with the atomic components of matter causes reflection an d refraction. Atoms may be thought of as oscillators that absorb the frequency of an external source (li ght) and cause forced oscillations. The frequency of light emitted by a charged oscillator is the same a s its oscillation frequency. As a result, the frequency of dispersed light is identical to that of incoming light.

(b) When light travels from a rarer to a denser medium, the speed decreases. Does the reduction in speed imply a reduction in the energy carried by the light wave?

Solution:
No, the energy carried by a wave is determined by its amplitude, not its speed of propagation.

(c) In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity of light in the photon picture of light.

Solution:
The number of photons crossing a unit area per unit time determines the intensity of light in the photon image at a certain frequency.

Example 10.3 Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when bluegreen light of wavelength $500\,\mathrm{nm}$ is used?

Solution: Fringe separation,

$$= \frac{D\lambda}{d}$$
$$= \frac{1\times5\times10^{-7}}{1\times10^{-3}}\,\mathrm{m}$$
$$= 5\times10^{-4}\,\mathrm{m}$$
$$= 0.5\,\mathrm{mm}$$

Example 10.4 What is the effect on the interference fringes in a Young's double-slit experiment due to each of the following operations: (In each operation, take all parameters, other than the one specified, to remain unchanged.)

(a) the screen is moved away from the plane of the slits;

Solution: The fringes' angular separation stays constant $(=\lambda/d)$.

The real fringe separation grows in proportion to the screen's distance from the plane of the two slits.

(b) the (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength;

Solution: Fringe separation (as well as angular separation) diminishes.

However, take note of the requirement in (d) below.

(c) the separation between the two slits is increased,

Solution: The separation of the fringes (and also angular separation) decreases.
However, take note of the requirement in (d) below.

(d) the source slit is moved closer to the double-slit plane;

Solution: Let s be the size of the source and S be the distance between the two slits' planes.

The criterion $s/S < \lambda/d$ must be met in order to witness interference fringes;
otherwise, interference patterns created by various regions of the source overlap and no fringes are seen.
As S declines (i.e., the source slit is brought closer), the interference pattern becomes less and less acute, and the fringes disappear when the source is pushed too close for this condition to be true.
The fringe separation will remain constant till this happens.

(e) the width of the source slit is increased;

Solution: The same as before (d).

The fringe pattern becomes weaker and less crisp as the source slit width rises.

The interference pattern vanishes when the source slit is sufficiently large that the condition $s / S \leq \lambda / d$

(f) the monochromatic source is replaced by a source of white light?

Solution:
Interference patterns caused when distinct white light component colours overlap (incoherently). The centre brilliant fringes for the various colours are all in the same place. As a result, the white cent re fringe.

For a point P for which $S_2P - S_1P = \lambda_b / 2$, where $\lambda_b (= 4000 \mathring{A} A)$ represents the wavelength for the blue colour, the blue component will be missing, and the fringe will be reddish-brown in colour. Slightly farther away where $S_2Q - S_1Q = \lambda_b = \lambda_r / 2$ where $\lambda_r (= 8000 \mathring{A})$ is the wavelength for the red colour, the fringe will be predominantly blue.

As a result, the closest fringes on each side of the centre white fringe will be red, while the furthest wi ll be blue. After a few fringes, there is no discernible fringe pattern.

Example 10.5 In Example 10.3, what should the width of each slit be to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?

Solution: $a\theta = \lambda, \theta = \dfrac{\lambda}{a}$

$$10\frac{\lambda}{d} = 2\frac{\lambda}{a} a = \frac{d}{5} = 02\,\text{mm}$$

The wavelength of light and the screen's distance are not taken into account in the a computation.

Example 10.6 Assume that light of wavelength $6000 \mathring{A}$ is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of

100 inch ?

Solution: A 100 inch telescope implies that $2a = 100 inch = 254\,\text{cm}$.

$\lambda = 6000 \mathring{A} A = 6 \times 10^{-5}$ cm then

$$\Delta\theta \approx \frac{0.61 \times 6 \times 10^{-5}}{127} \approx 2.9 \times 10^{-7} radians$$

Example 10.7 For what distance is ray optics a good approximation when the aperture is $3\,\mathrm{mm}$ wide and the wavelength is $500\,\mathrm{nm}$?

Solution: $z_F = \frac{a^2}{\lambda} = \frac{\left(3\times10^{-3}\right)^2}{5\times10^{-7}} = 18\,\mathrm{m}$

$18m$ distance is ray optics a good approximation.

Example 10.8 Discuss the intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids?

Solution: Let I_0 be the intensity of polarised light after passing through the first polariser P_1. Then the intensity of light after passing through second polariser P_2 will be

$$I = I_0 \cos^2\theta$$

where θ is the angle between pass axes of $P_1 and$ P_2. Since P_1 and P_3 are crossed the angle between the pass axes of P_2 and P_3 will be $(\pi/2-\theta)$. Hence the intensity of light emerging from P_3 will be

$$I = I_0\cos^2\theta\cos^2\left(\frac{\pi}{2}-\theta\right)$$
$$= I_0\cos^2\theta\sin^2\theta$$
$$= \left(I_0/4\right)\sin^2 2\theta$$

As a result, the transmitted intensity will be at its peak when $\theta = \pi/4$.

Example 10.9 Unpolarised light is incident on a plane glass surface. What should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other?

Solution: For $i + r = \pi/2$,

$\tan i_B = \mu = 1.5$.

$i_B = 57^\circ$.

For the air-glass interface, this is the Brewster's angle.

CHAPTER-10 (WAVE OPTICS)

CLASS XII

EXERCISES

10.1 Monochromatic light of wavelength $589\,\mathrm{nm}$ is incident from air on a water surface. What are the wavelength, frequency and speed of

(a) reflected,

Answer: Wavelength of incident monochromatic light is , $\lambda = 589\,\text{nm} = 589 \times 10^{9}\ \text{m}$

Speed of light in air, $\text{c} = 3 \times 10^{8}\ \text{m}$

Refractive index of water, $\mu = 1.33$

The ray will reflect in the same medium as the incident ray in this situation.
As a result, the reflected ray will have the same wavelength, speed, and frequency as the incident light .

Frequency of light,

$$v = \frac{c}{\lambda}$$

$v =$ Frequency of light

$\text{c} =$ Speed of light

$\lambda =$ Wavelength of light

$$\Rightarrow v = \frac{3 \times 10^{8}}{589 \times 10^{-9}}$$

$$\Rightarrow v = 5.09 \times 10^{14}\ \text{Hz}$$

As a result, the speed $= 3 \times 10^{8}\ \text{m/s}$

Frequency $= 5.09 \times 10^{14}\ \text{Hz}$

wavelength of the reflected light $= 589\,\text{nm}$

(b) refracted light? Refractive index of water is 1.33.

Answer: Light's frequency is unaffected by the properties of the medium through which it travels.
As a result, the refracted beam in water will have the same frequency as the incident or reflected light in air.

Frequency of the refracted light ray, $v = 5.09 \times 10^{14}\ \text{Hz}$

The refractive index of water is related to the speed of light in water, as shown in the formula below:

$$\text{v} = \frac{\text{c}}{\mu}$$

$$\Rightarrow \text{v} = \frac{3 \times 10^{8}}{1.33}$$

$$\Rightarrow \text{v} = 2.26 \times 10^{8}\ \text{m/s}$$

The relationship between the wavelength of light in water and the speed and frequency of light is sho wn in the formula below.

$$\lambda = \frac{v}{v}$$

$$\Rightarrow \lambda = \frac{2.26 \times 10^{8}}{5.09 \times 10^{14}}$$

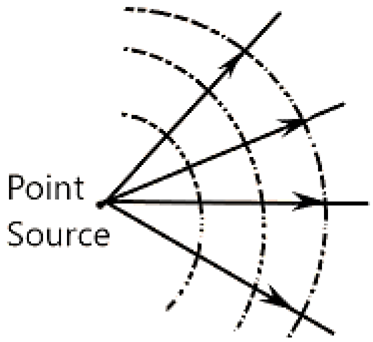
$$\Rightarrow \lambda = 444.007 \times 10^{-9}\ \text{m}$$

$$\Rightarrow \lambda = 444.01\text{nm}$$

10.2 What is the shape of the wavefront in each of the following cases:

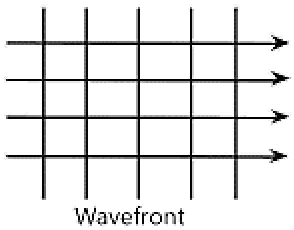
(a) Light diverging from a point source.

Answer:
When light diverges from a point source, the wavefront takes on a spherical form. The wavefront of a point source is seen in the illustration.



(b) Light emerging out of a convex lens when a point source is placed at its focus.

Answer:
When a point source is placed at the focus of a convex lens, the wavefront takes on the form of a paral lel grid. As illustrated in the diagram, this may be expressed in a variety of ways.



(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Answer:
In this situation, the plane is the fraction of the wavefront of light intercepted by the Earth from a distant star.

10.3

(a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is $3.0\times10^{8}\ \mathrm{m\,s^{-1}}$)

Answer: Refractive index of glass, $\mu = 1.5$

Speed of light, $c = 3.0\times10^{8}\ \mathrm{m/s}$

Speed of light in glass is given by the formula,

$$v = \frac{c}{\mu}$$

$$\Rightarrow v = \frac{3\times10^{8}}{1.5} = 2\times10^{8}\ \mathrm{m/s}$$

As a result, the speed of light in glass $= 2\times10^{8}\ \mathrm{m/s}$.

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Answer: The colour of light affects the speed of light in glass.

A violet component of white light has a refractive index that is higher than that of a red component.
As a result, the speed of violet light in glass is slower than the speed of red light, because speed and refractive index are inversely linked.
As a result, violet light passes slower through a glass prism than red light.

10.4 In a Young's double-slit experiment, the slits are separated by $0.28\ \mathrm{mm}$ and the screen is placed $1.4\ \mathrm{m}$ away. The distance between the central bright fringe and the fourth bright fringe is measured to be $1.2\ \mathrm{cm}$. Determine the wavelength of light used in the experiment.

Answer: Distance between the slits, $d = 0.28\ \mathrm{mm} = 0.28\times10^{-3}\ \mathrm{m}$

Distance between the slits and the screen, $D = 1.4\ \mathrm{m}$

Distance between the central fringe and the fourth $(n = 4)$ fringe,

$u = 1.2\ \mathrm{cm} = 1.2\times10^{-2}\ \mathrm{m}$

In the case of constructive interference, the distance between the two fringes may be calculated as follows:

$$u = n\lambda\frac{D}{d}$$

where,

$n =$ Order of fringes $= 4$

$\lambda =$ Wavelength of light used

$$\lambda = \frac{ud}{nD}$$

$$\Rightarrow \lambda = \frac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4}$$

$$\Rightarrow \lambda = 6\times10^{-7}$$
$$\Rightarrow \lambda = 600\,\text{nm}$$

As a result, the wavelength of the light $= 600\,\text{nm}$.

10.5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Answer: The intensity of the two light waves be I_1 and I_2. Their resultant intensities can be evaluated as: $I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Where,

$\phi =$ The phase difference between two waves for monochromatic light waves,

$$I_1 = I_2$$
$$I' = 2I_1 + 2I_1\cos\phi$$

The phase difference formula is as follows:

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

Since, path difference λ,

Phase difference is $\phi = 2\pi$

$$I' = 2I_1 + 2I_1 = 4I_1$$

Given,

$$I_1 = \frac{K'}{4}\ldots\ldots(1)$$

When path difference $= \frac{\lambda}{3}$

phase difference, $\phi = \frac{2\pi}{3}$

As a result, resultant intensity,

$$I_R = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\frac{2\pi}{3}$$

$$\Rightarrow I_R = 2I_1 + 2I_2\left(-\frac{1}{2}\right) = I_1$$

Using equation (1), we can state that

$I_R = I_1 = \frac{K}{4}$ As a result, the intensity of light at a place where the path difference is zero is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

10.6 A beam of light consisting of two wavelengths, $650\,\text{nm}$ and $520\,\text{nm}$, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength $650\,\text{nm}$.

Answer:

Wavelength of the first light beam, $\lambda_1 = 650\,\text{nm}$

Wavelength of second light beam, $\lambda_2 = 520\,\text{nm}$

Distance of the slits from the screen $= D$

Distance between the two slits $= d$

Distance of the n^{th} bright fringe on the screen from the central maximum is,

$$x = n\lambda_1\left(\frac{D}{d}\right)$$

For the third bright fringe, $n = 3$

$$x = 3 \times 650 \times \frac{D}{d} = 1950\left(\frac{D}{d}\right)\text{nm},$$

This is the distance between the central maxima and the third brilliant fringe on the screen.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Answer: Suppose the n^{th} bright fringe due to wavelength λ_2 and $(n-1)^{th}$ bright fringe due to wavelength λ_1 coincide on the screen. Assume the following criteria for brilliant fringes:

$n\lambda_2 = (n-1)\lambda_1$

$\Rightarrow 520n = 650n - 650$

$\Rightarrow 650 = 130n$

$\Rightarrow n = 5$

As a result, the relation may be used to find the shortest distance from the centre maximum:

$$x = n\lambda_2 \frac{D}{d}$$

$$\Rightarrow x = 5 \times 520 \frac{D}{d} = 2600 \frac{D}{d} nm$$

Because the actual solution of d and D isn't stated in the question,

it's impossible to find it.

10.7 In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed $1m$ away. The wavelength of light used is $600\,nm$. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be $4/3$.

Answer:

Distance of the screen from the slits, $D = 1m$

Wavelength of light, $\lambda_1 = 600\,nm$

Angular width of the fringe in air, $\theta_1 = 0.2^\circ$

Angular width of the fringe in water $= \theta_2$

Refractive index of water is $\frac{4}{3}$.

The refractive index is related to angular width is:

$$\mu = \frac{\theta_1}{\theta_2}$$

$$\Rightarrow \theta_2 = \frac{3}{4}\theta_1$$

$$\Rightarrow \theta_2 = \frac{3}{4} \times 0.2 = 0.15^\circ$$

As a result, the angular width of the fringe in this case θ in water will reduce to 0.15°.

10.8 What is the Brewster angle for air to glass transition? (Refractive index of glass $= 1.5$.)

Answer: Refractive index of glass, $=1.5$

Brewster angle $=\theta$

As seen in the formula below, Brewster angle is related to refractive index:

$$\tan\theta=\mu$$
$$\Rightarrow\theta=\tan^{-1}(1.5)=56.31^{\circ}$$

As a result, the Brewster angle for air-to-glass transition is 56.31°.

10.9 Light of wavelength $5000 \AA$ falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Answer: Wavelength of incident light,

$$\lambda=5000\,\mathrm{A}=5000\times10^{-10}\,\mathrm{m}$$
$$\text{Speed of light, } \mathrm{c}=3\times10^{8}\,\mathrm{m/s}$$

Frequency of incident light,

$$v=\frac{c}{\lambda}$$

$$\Rightarrow v=\frac{3\times10^{8}}{5000\times10^{10}}=6\times10^{14}\,\mathrm{Hz}$$

The incident light has the same wavelength and frequency as the reflected beam.

As a result, the wavelength of reflected light is $5000\,\mathrm{A}^{\circ}$ and its frequency is 6×10^{14} Hz.

When the reflected ray is normal to the incident light, the angle of incidence is added as,

$\angle\mathrm{i}$ and angle of reflection, $\angle\mathrm{r}$ is 90°.

The angle of incidence is always the same as the angle of reflection, according to the rule of reflection .

As a result, the total may be expressed as:

$$\angle\mathrm{i}+\angle\mathrm{r}=90^{\circ}$$
$$\Rightarrow\angle\mathrm{i}+\angle\mathrm{i}=90^{\circ}$$
$$\Rightarrow 2\angle\mathrm{i}=90^{\circ}$$

$$\Rightarrow\angle\mathrm{i}=\frac{90^{\circ}}{2}=45^{\circ}$$

As a result, for the provided condition in the question, the angle of incidence is 45°.

10.10 Estimate the distance for which ray optics is good approximation for an aperture of $4\,\text{mm}$ and wavelength $400\,\text{nm}$.

Answer: Fresnel's distance $\left(Z_F\right)$ can be defined as the distance for which the ray optics is a good approximation.

$$Z_F = \frac{a^2}{\lambda}$$

Here,

Aperture width $= a$,

Wavelength of light $= \lambda$.

Now, $$\begin{aligned} a &= 4\,\text{mm} = 4 \times 10^{-3}\,\text{m} \\ \lambda &= 400\,\text{nm} = 400 \times 10^{-9}\,\text{m} \end{aligned}$$

$$Z_F = \frac{\left(4 \times 10^{-3}\right)^2}{400 \times 10^{-9}} = 40\,\text{m}$$

As a result, the distance for which ray optics provides a reasonable approximation is $40\,\text{m}$.

CHAPTER-10 (WAVE OPTICS)

CLASS XII

ADDITIONAL EXERCISES

10.11 The $6563 \mathring{A} AH\alpha$ line emitted by hydrogen in a star is found to be redshifted by $15\ \mathrm{A}$. Estimate the speed with which the star is receding from the Earth.

Answer: Wavelength of $\mathrm{H_a}$ line emitted by hydrogen,

$$\lambda = 6563 \mathring{A}$$

$$\Rightarrow \lambda = 6563 \times -10^{-10}\ \mathrm{m}$$

Red shift of the star,

$$\left(\lambda' - \lambda\right) = 15 \mathring{A}^{\circ} = 15 \times 10^{-10}\ \mathrm{m}$$

Speed of light, $\mathrm{c} = 3 \times 10^{8}\ \mathrm{m/s}$

Let us suppose that the velocity of the star receding away from the Earth be v

The red shift is associated with velocity as given:

$$\lambda' - \lambda = \frac{\mathrm{v}}{\mathrm{c}} \lambda$$

$$\Rightarrow \mathrm{v} = \frac{\mathrm{c}}{\lambda} \times \left(\lambda' - \lambda\right)$$

$$\Rightarrow \mathrm{v} = \frac{3 \times 10^{8} \times 15 \times 10^{-10}}{6563 \times 10^{-10}} = 6.87 \times 10^{5}\ \mathrm{m/s}$$

As a result, the rate at which the star moves away from the Earth is $6.87 \times 10^{5}\ \mathrm{m/s}$.

10.12 Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

Answer:
No, the experimental finding of the speed of light in water does not support the prediction, and the the ory, which is an alternative image of light, is consistent with wave theory.
When light corpuscles touch the interface of two media, from a rarer (air) to a denser (water), the parti cles feel forces of attraction normal to the surface, according to Newton's corpuscular theory of light.

As a result, the velocity's normal component increases while the component along the surface remains same.

So,

$c \sin i = v \sin r \ldots\ldots(1)$ Where,

$\mathrm{i} =$ Angle of incidence

$\mathrm{r} =$ Angle of reflection

c = Velocity of light in air

v = Velocity of light in water

The formula for the relative refractive index of water in relation to air is:

$$\mu = \frac{v}{c}$$

As a result, equation (1) reduces to $\Rightarrow \frac{v}{c} = \frac{\sin i}{\sin r} = \mu \ldots\ldots(2)$

But $\mu > 1$.

As a result, it can be observed from equation (2) that $v > c$. This is not possible because this prediction is opposite to the experimental results of $c > v$. Clearly, the wave picture of light is consistent with the experimental results.

But $\mu > 1$

Hence, it can be observed from equation (2) that $v > c$.

This is impossible since this prediction contradicts the outcomes of the experiments. $c > V$.

The experimental results are clearly congruent with the wave image of light.

10.13 You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

Answer: Suppose an object at O be placed in front of a plane mirror MO' at a distance r (as in figure 1).

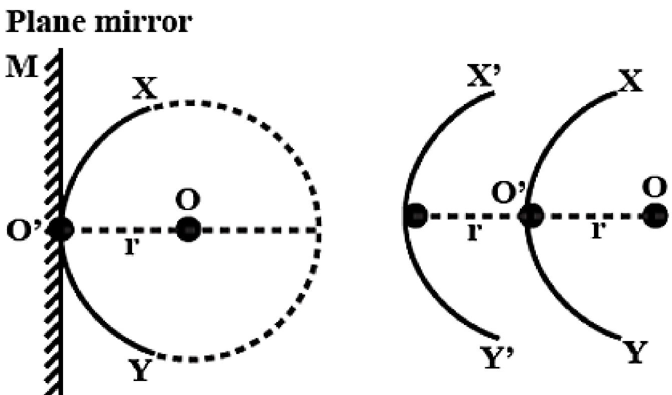


Fig.1 Fig.2

A circle is drawn from the centre (O) such that it just touches the plane mirror at point O' on the mirror.

According to Huygens' Principle, XY is the wavefront of incident light.

If the mirror is not present, then a similar wavefront $X'Y'$ (as XY) would form behind the point O' at distance r (as shown in fig. 2).

For the plane mirror, $X'Y'$ may be thought of as a virtual reflected ray.

As a result, a point object put in front of a plane mirror produces a virtual picture with the same distan ce from the mirror as the item r.

10.14 Let us list some of the factors, which could possibly influence the speed of wave propagation:

(i) nature of the source.

(ii) direction of propagation.

(iii) motion of the source and/or observer.

(iv) wavelength.

(v) intensity of the wave.

On which of these factors, if any, does

(a) the speed of light in vacuum,

Answer: The speed of light in a vacuum i.e., $c = 3\times10^8$ m/s (approximately) is a universal constant. The speed of light is unaffected by the velocity of the source, observer, or both. As a result, the menti oned parameters have no bearing on the speed of light in a vacuum.

(b) the speed of light in a medium (say, glass or water), depend?

Answer:
The wavelength of light in a medium influences the speed of light in that medium, among other things .

10.15 For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

Answer: No, the formulas for the two circumstances aren't exactly the same.

The propagation of sound waves necessitates the use of a medium.
Scientifically, the two scenarios are not equivalent since the motion of an observer relative to a mediu m is not the same in both. As a result, the doppler formulae for the two scenarios differ. Light waves a re capable of travelling in vacuum. Because the speed of light is independent of the velocity of the obs erver and the motion of the source when light travels in a medium, the above two scenarios are equiva lent in vacuum. Because the speed of light is dependent on the wavelength of the medium, the two sce narios are not identical.

10.16 In double-slit experiment using light of wavelength $600\,\text{nm}$, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Answer: Wavelength of light, $\lambda = 600\,\text{nm} = 600\times10^{-9}\ \text{m}$

Angular width of fringe, $\theta = 0.1^{\circ} = 0.1\times\ frac\pi180 = \frac{3.14}{1800}\,\text{rad}$

Angular width of a fringe is related to slit spacing (d) as:

$$\theta = \frac{\lambda}{\text{d}}$$

$$\Rightarrow \text{d} = \frac{\lambda}{\theta}$$

$$\Rightarrow \text{d} = \frac{600\times10^{-9}}{\frac{3.14}{1800}} = 3.44\times10^{-4}\ \text{m}$$

As a result, the spacing between the slits is 3.44×10^{-4} m.

10.17 Answer the following questions:

(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?

Answer:
When the width of a single slit diffraction experiment is increased by two times its original width, the size of the centre diffraction band shrinks to half its original size and the intensity of the central diffra ction band increases by four times.

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?

Answer: Diffraction from each slit modulates the interference pattern observed in a double-slit experiment.

The interference of the diffracted waves from each slit creates the pattern.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

Answer:
When a small circular obstruction is placed in the path of light from afar, a brilliant spot appears in th e centre of the obstacle's shadow. This happens because light waves are diffracted from the circular ob stacle's edge, which interferes constructively at the shadow's centre. This is a brilliant spot because of the constructive interference.

Infinity Learn by Sri Chaitanya Educational Institutions

(d) Two students are separated by a $7\,\text{m}$ partition wall in a room $10\,\text{m}$ high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.

Answer:
Waves can be bent at a considerable angle by impediments, which is conceivable when the obstacle's size is similar to the wavelength of the waves. The wavelength of the light waves, on the other hand, i s too little in relation to the size of the obstruction. As a result, the diffraction angle will be less. As a r esult, the pupils are unable to see one another.

The size of the wall, on the other hand, is equivalent to the wavelength of the sound waves. The wave s bend at a tremendous angle as a result of this. As a result, the pupils can hear each other.

(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

Answer:
The foregoing remark is justified by the fact that in common optical equipment, the aperture size is su bstantially larger than the wavelength of the light employed.

10.18 Two towers on top of two hills are $40\,\text{km}$ apart. The line joining them passes $50\,\text{m}$ above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

Answer:

Distance between the towers, $d = 40\,\text{km}$

Height of the line joining the hills, $50\,\text{m}$

As a result, the radio waves' radial spread should not exceed $50\,\text{km}$.

Fresnel's distance may be represented as: since the slope is midway between the towers:
$Z_{\text{p}} = 20\,\text{km} = 20 \times 10^{4}\,\text{m}$

Aperture: $\text{a} = \text{d} = 50\,\text{m}$

Fresnel's distance, $Z_p = \dfrac{a^2}{\lambda}$

Here,

$\lambda =$ Wavelength of radio waves

$$\lambda = \frac{a^2}{Z_p}$$

$$\Rightarrow \lambda = \frac{(50)^2}{2\times10^4} = 1250\times10^{-4} = 0.1250\,\text{m}$$

$$\Rightarrow \lambda = 12.5\,\text{cm}$$

As a result, the wavelength of the radio waves is $12.5\,\text{cm}$.

10.19 A parallel beam of light of wavelength $500\,\text{nm}$ falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1m away. It is observed that the first minimum is at a distance of 2.5mm from the centre of the screen. Find the width of the slit.

Answer:

Wavelength of light beam is $\lambda = 500\,\text{nm} = 500\times10^{-9}\,\text{m}$

Distance of the screen from the slit, $D = 1\text{m}$

For first minima, $\text{n} = 1$

Distance between the slits $= d$

The initial minimal distance from the screen's centre may be calculated as follows:
$x = 2.5\text{mm} = 2.5\times10^{-3}\,\text{m}$

Now,

$$\text{n}\lambda = \text{x}\frac{\text{d}}{\text{D}}$$

$$\Rightarrow \text{d} = \frac{\text{n}\lambda\text{D}}{\text{x}}$$

$$\Rightarrow \text{d} = \frac{1\times500\times10^{-9}\times1}{2.5\times10^{-3}}$$

$$\Rightarrow \text{d} = 2\times10^{-4}\,\text{m} = 0.2\,\text{mm}$$

So, the width of the slits is $0.2\,\text{mm}$.

10.20 Answer the following questions:

(a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.

Answer:

A lowflying aircraft's weak radar emissions might interfere with the TV signals received by the TV antenna. The TV signals may get distorted as a result of this. As a result,when a lowflying aircraft passes by, the picture on our TV screen may shake somewhat.

(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Answer:

Our comprehension of intensity distributions and interference patterns requires the idea of linear super position of wave displacement.

This is because the linear aspect of a differential equation that drives wave motion leads to superpositi on.
If the solutions of the second order wave equation are y_1 and y_2,

then a linear combination of y_1 and y_2 is also a solution of the wave equation.

10.21 In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $n\lambda / a$. Justify this by suitably dividing the slit to bring out the cancellation.

Answer: Suppose a single slit of width d is divided into n smaller slits.

Width of each slit is $d' = \frac{d}{n}$

Angle of diffraction is given by the formula,

$$\theta = \frac{\frac{d}{d}\lambda}{d} = \frac{\lambda}{d}$$

In the direction of θ,
each of these infinitesimally small holes now transmits zero intensity.
As a result, combining these slits would result in zero intensity.