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Chapter 11: Dual Nature of Radiation and Matter

Example Questions:-

Example 11.1 Monochromatic light of frequency 6.0×10^{14} Hz is produced by a laser. The power emitted is 2.0×10^{-3} W .
(a) What is the energy of a photon in the light beam?
(b) How many photons per second, on an average, are emitted by the source?

Solution:

(a) We know that a photon has energy which is found by the expression:

$$E = h\nu = \left(6.63\times10^{-34}\,Js\right)\left(6.0\times10^{14}\,Hz\right) = 3.98\times10^{-19}\,J$$

(b) Let N be the number of photons which are emitted by the source per second. Thus, we can say that the power transmitted through the beam equals N times the energy per photon E. therefore, we have P=NE. Then

$$N = \frac{P}{E} = \frac{2.0\times10^{-3}W}{3.98\times10^{-19}\,J} = 5.0\times10^{15}\ photons\ per\ second.$$

Example 11.2 The work function of cesium is 2.14 eV . Find
(a) The threshold frequency for cesium, and
(b) The wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60 V.

Solution:

(a) Considering the threshold frequency, the energy $h\nu_o$ of the incident radiation must be equal to the work function ϕ_o . Thus, we have

$$\nu_o = \frac{\phi_o}{h} = \frac{2.14eV}{6.63\times10^{-34}\,Js} = \frac{2.14\times1.6\times10^{-19}\,J}{6.63\times10^{-34}\,Js} = 5.16\times10^{14}\,Hz$$

Hence, we can infer that the frequencies which are less than the work function cannot eject electrons.

(b) The photocurrent equates to zero when the maximum kinetic energy of the ejected electrons is equal to eV_o , where V_p is the retarding potential.
Einstein's photoelectric equation is:

$$\lambda = \frac{hc}{\left(eV_o + \phi_o\right)} = \frac{\left(6.63\times10^{-34}\,Js\right)\left(3\times\frac{10^8\,m}{s}\right)}{\left(0.60eV + 2.14eV\right)} = \frac{19.89\times10^{-26}\,Jm}{2.74\times1.6\times10^{-19}\,J} = 454nm$$

Example 11.3 The wavelength of light in the visible region is about $390\ \mathrm{nm}$ for violet color, about $550\ \mathrm{nm}$ (average wavelength) for yellow green colour and about $760\ \mathrm{nm}$ for red color.
(a) What are the energies of photons in (eV) at the (i) violet end, (ii) average wavelength, yellow-greencolour, and (iii) red end of the visible spectrum? (Take $\mathrm{h}\ = 6.63\times10^{-34}\ \mathrm{J\ s}$ and $1\ \mathrm{eV}\ =\ 1.6\times10^{-19J}$.)
(b) From which of the photosensitive materials with work functions listed in Table 11.1 and using the results of (i), (ii) and (iii) of (a), can you build a photoelectric device that operates with visible light?

Solution:

(a) The energy associated with the incident photon is: $E = h\nu = \dfrac{hc}{\lambda}$

$$E = \frac{\left(6.63\times10^{-34}\,Js\right)\left(3\times10^{8}\,m/s\right)}{\lambda}$$

(i) Wavelength of violet light= $390nm$
Energy of photon:

$$E_1 = \frac{1.989\times10^{-25}}{390\times10^{-9}\,m} = 3.19eV$$

(ii) Wavelength of yellow-green light= $550nm$
Energy of photon:

$$E_2 = \frac{1.989\times10^{-25}}{550\times10^{-9}\,m} = 2.26eV$$

(iii) Wavelength of red light= $760nm$
Energy of photon:

$$E_1 = \frac{1.989\times10^{-25}}{760\times10^{-9}\,m} = 1.64eV$$

(b) If we want a photoelectric device to operate, we need that the frequency of the incident light must be equal or greater than the work function ϕ_o . Thus, the photoelectric device will operate with violet light (with $E =\ 3.19\ eV$) photosensitive material Na (with $\phi_0 =\ 2.75\ eV$), K (with $\phi_0 =\ 2.30\ eV$) and Cs (with $\phi_0 = 2.14\ eV$). It will also operate with yellow-green light (with $E =\ 2.26\ eV$) for Cs (with $\phi_0 =\ 2.14\ eV$) only. However, it will not operate with red light (with $E =\ 1.64\ eV$) for any of these photosensitive materials.

Example 11.4 What is the de Broglie wavelength associated with (a) an electron moving with a speed of $5.4\times10^{6}\,\mathrm{m/s}$, and (b) a ball of mass $150\ \mathrm{g}$ travelling at $30.0\ \mathrm{m/s}$?

Solution:

(a) For the electron, we know that:

Mass= $m = 9.11\times10^{(-31)} kg$

It is given that:

Speed= $v = 5.4\times10^{6} m/s$

Momentum:

$p = mv = 4.92\times10^{-24} kg\ m/s$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34} JS}{4.92\times10^{-24} Kg\frac{m}{s}} = 0.135nm$$

(b) For the ball, we are given that:

Mass= $m^{'} = 0.150Kg$

Speed= $v^{'} = 30.0\ m/s$

Then, the momentum associated with the ball is:

$$p' = m'v' = 4.50Kg\ m/s$$

De Broglie Wavelength:

$$\lambda' = \frac{h}{p'} = \frac{6.63\times10^{-34} JS}{4.50kg\ m/s} = 1.47\times10^{-34} m$$

Thus, we can say that the wavelength of the electron is comparable with X-rays wavelengths, but for the ball it is about 10^{-19} times the size of proton, which is beyond experimental measurements.

Example 11.5 An electron, an a-particle, and a proton have the same kinetic energy. Which of these particles has the shortest de Broglie wavelength?

Solution:

We know that, for a given particle, the associated De-Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

Kinetic energy is given by, $k = \frac{p^{m}}{2m}$

Thus, we can write that:

$$\lambda = \frac{h}{\sqrt{2mk}}$$

From this relation, we can infer that the for a kinetic energy, the associated De-Broglie wavelength of the particle is inversely proportional to the square root of their masses.

A proton $({}^{1}_{1}H)$is 1836 times massive than an electron and an a-particle$({}^{4}_{2}He)$.

Hence, a – particle has the shortest de Broglie wavelength.

Example 11.6 A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4} . Calculate the particle's mass and identify the particle.

Solution:

We know that the De-Broglie wavelength of particle with mass m and velocity v, is depicted by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Thus, the mass of an electron can be found out by the following expression:

$$m_e = \frac{h}{\lambda_e v_e}$$

Now, we are given that:

$$\frac{v}{v_e} = 3 and \frac{\lambda}{\lambda_e} = 1.813 \times 10^{-4}$$

Then, mass of the particle, $m = m_e \left(\frac{\lambda_e}{\lambda_e}\right)\left(\frac{v_e}{v}\right) = 1.675 \times 10^{-27} kg$.

Thus, the particle, with this mass could be a proton or a neutron.

Example 11.7 What is the de Broglie wavelength associated with an electron, accelerated through a potential difference of 100 volts ?

Solution:

We are given that the accelerating potential is $V = 100V$

Thus, De-Broglie wavelength λ is

$$\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{V}} nm = \frac{1.227}{\sqrt{100}} nm = 0.123 nm$$

Thus, the De-Broglie wavelength associated with the electron in this case is of the order of X-ray wavelengths.

Exercise Questions:-

11.1 Find the

(a) Maximum frequency, and

(b) Minimum wavelength of X-rays produced by $30\,\mathbf{kV}$ electrons.

Solution:

Electric potential of an electron has a value, $V = 30kV = 3\times10^4 V$

Hence, energy of the electrons is given by, $E = 3 \times 104\ eV$

e = Charge on an electron = 1.6 × 10−19 C

(a) Let the maximum frequency produced by the X-rays = υ

The energy of the electrons is given by the relation mentioned below:

$$E = h\upsilon$$

Where,

h = Planck's constant $= 6.626 \times 10-34\ Js$

Therefore, $\upsilon = \dfrac{E}{H}$

$$= \frac{1.6\times10^{-19}\times3\times10^4}{6.626\times10^{-34}} = 7.24\times10^{18}\,Hz$$

Hence, the maximum frequency of X-rays produced is given by = 7.24×10^{18}

(b) The minimum wavelength produced by the X-rays is given as:

$$\lambda = \frac{c}{\upsilon}$$

$$= \frac{3\times10^8}{7.24\times10^{18}} = 4.14\times10^{-11}m = 0.0414nm$$

Hence, the minimum wavelength of X-rays produced is $= 0.0414\ nm.$

11.2 The work function of cesium metal is 2.14 eV . When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) Maximum kinetic energy of the emitted electrons,

(b) Stopping potential, and

(c) Maximum speed of the emitted photoelectrons?

Solution:

Work function of cesium metal is given as, $\phi_{\circ} = 2.14\, eV$

Frequency of light is given by, $\upsilon = 6.0 \times 10^{14}\, Hz$

(a) The maximum kinetic energy is given by the photoelectric effect through the following expression:

$$K = h\upsilon - \phi_{\circ}$$

Where, we have

h = Planck's constant = $6.626 \times 10-34\ Js$

Therefore, $K = \dfrac{6.626 \times 10^{34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}}$

$= 2.485 - 2.140 = 0.345 eV$

Hence, the maximum kinetic energy of the emitted electrons ishas a value of $0.345\ eV$.

(b) For stopping potential $V_{\circ}$, we can write the equation for kinetic energy as the following:

$$K = eV_{\circ}$$

Therefore,

$$V_{\circ} = \frac{K}{e}$$

$$= \frac{0.345 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 0.345\, V$$

Hence, the stopping potential of the material is given by 0.345 V.

(c) Maximum speed of the emitted photoelectrons is denoted by = V

Hence, the relation for kinetic energy can be written as:

Where,

m = Mass of an electron = $9.1 \times 10-31\ kg$

$$v^2 = \frac{2K}{m}$$
$$= \frac{2\times 0.345\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}$$
$$= 0.1104\times 10^{12}$$
$$\therefore, \upsilon = 3.323\times 10^5\ m/s = 332.3 km/s$$

Hence, the maximum speed of the emitted photoelectrons is given by 332.3 km/s.

11.3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Solution:

Photoelectric cut-off voltage has a value of, $V_\circ = 1.5\ V$

The maximum kinetic energy of the emitted photoelectrons is given by the following expression:

$$K = eV_\circ$$

Where, we have

e = Charge on an electron = $1.6 \times 10-19\ C$

$$\therefore, K_e = 1.6\times 10^{-19}\times 1.5$$
$$= 2.4\times 10^{-19}\ J$$

Therefore, the maximum kinetic energy of the photoelectrons emitted in the given experiment is $2.4 \times 10-19\ J$.

11.4 Monochromatic light of wavelength $632.8\ nm$ **is produced by a helium-neon laser. The power emitted is** $9.42\ mW$.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Solution:

Wavelength of the monochromatic light is given by, $\lambda = 632.8\ nm = 632.8 \times 10-9\ m$

Power emitted by the laser is given by, $P = 9.42\ mW = 9.42 \times 10-3\ W$

Planck's constant has a value, $h = 6.626 \times 10-34\ Js$

Speed of light is known to be, $c = 3 \times 108\ m/s$

Mass of a hydrogen atom is known to be, $m = 1.66 \times 10-27\ kg$

(a) The energy of each photon is given as:

$$E = \frac{hc}{\lambda}$$
$$= \frac{6.626\times10^{-34}\times3\times10^{4}}{632.8\times10^{-9}}$$
$$= 3.141\times10^{-19}\,J$$

The momentum of each photon is given as:

$$P = \frac{h}{\lambda}$$
$$= \frac{6.626\times10^{-34}}{632.8\times10^{-9}}$$
$$= 1.047\times10^{-27}\,kg\ ms^{-1}$$

(b) Assuming that the number of photons arriving per second, at a target irradiated by the beam = n. Assume that the beam has a uniform cross-section that is less than the target area. Hence, the equation for power can be written as:

$$P = nE$$
$$\therefore n = \frac{p}{E}$$
$$= \frac{9.42\times10^{-3}}{3.141\times10^{-19}} = 3\times10^{16}\ photons/s$$

(c) Momentum of the hydrogen atom is the same as the momentum of the photon, Momentum is given as:

$$p = 1.047 \times 10^{-27}\ kg\ ms^{-1}$$

Momentum is given by:

$$p = mv$$

Where,

v = speed of the hydrogen atom

$$\therefore v = \frac{p}{m}$$

$$= \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621\, m/s$$

11.5 The energy flux of sunlight reaching the surface of the earth is $1.388 \times 103\ \mathrm{W/m^2}$. How many photons (nearly) per square meter are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of $550\ \mathrm{nm}$.

Solution:

Energy flux of sunlight reaching the surface of earth is given by, $\Phi = 1.388 \times 103\ W/m^2$

Hence, power of sunlight per square meter is given by, $P = 1.388 \times 103\ W$

Speed of light is known to be, $c = 3 \times 108\ m/s$

Planck's constant is known to be, $h = 6.626 \times 10-34\ Js$

Average wavelength of photons present in sunlight is given by,

$$\lambda = 550nm = 550 \times 10^{-9} m$$

Number of photons per square meter incident on earth per second = n. Hence, the equation for power can be written as the following:

$$P = nE$$

$$\therefore n = \frac{P}{E} = \frac{P\lambda}{hc}$$

$$= \frac{1.388 \times 10^{3} \times 550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = 3.84 \times 10^{21}\ photons/m^2/s$$

Therefore, every second, 3.84×10^{21} photons are incident per square meter on earth.

11.6 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Solution:

The slope of the cut-off voltage (V) versus frequency (ν) of an incident light is given as:

$$\frac{V}{v} = 4.12 \times 10^{-15}\ Vs$$

V is related to frequency by the equation below:

$$h\upsilon = eV$$

Where, we have

e = Charge on an electron $= 1.6 \times 10^{-19}\ C$

h = Planck's constant

$$\therefore h = e \times \frac{V}{v}$$
$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34}\ Js$$

Therefore, the value of Planck's constant is $= 6.592 \times 10^{-34}$.

11.7: A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm.

(a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere?

Solution:

Power of the sodium lamp is given by, $P = 100\ W$

Wavelength of the emitted sodium light is given by, $\lambda = 589\ nm = 589 \times 10^{-9} m$

Planck's constant is known to be, $h = 6.626 \times 10^{-34}\ Js$

Speed of light is known to be, $c = 3 \times 108\ m/s$

(a) The energy per photon associated with the sodium light is given as the following:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626\times10^{-34}\times3\times10^{8}}{589\times10^{-9}} = 3.37\times10^{-19}\,J$$

$$= \frac{3.37\times10^{-19}}{1.6\times10^{-19}} = 2.11eV$$

(b) Number of photons delivered to the sphere = n.

The equation for power can be written as the following:

$$P = nE$$

$$\therefore n = \frac{P}{E}$$

$$= \frac{100}{3.37\times10^{-19}} = 2.96\times10^{20}\ \ photons/s$$

Therefore, 2.96×10^{20} photons are delivered to the sphere every second.

11.8: The threshold frequency for a certain metal is 3.3×1014 Hz. If light of frequency 8.2×1014 Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Solution:

Threshold frequency of the metal is given by, $\upsilon_{\circ} = 3.3\times10^{14}\,Hz$

Frequency of light incident on the metal is given by, $\upsilon = 8.2\times10^{14}\,Hz$

Charge on an electron is known to be, $e = 1.6 \times 10^{-19}\ C$

Planck's constant is known to be, $h = 6.626 \times 10^{-34}\ Js$

Cut-off voltage for the photoelectric emission from the metal = $\upsilon_{\circ}$

The equation for the cut-off energy is given as the following:

$$e\upsilon_{\circ} = h(\upsilon - \upsilon_{\circ})$$
$$\upsilon_{\circ} = \frac{h(\upsilon - \upsilon_{\circ})}{e}$$
$$= \frac{6.626\times10^{-34}\times(8.2\times10^{14} - 3.3\times10^{14})}{1.6\times10^{-19}} = 2.0292\,V$$

Therefore, the cut-off voltage for the photoelectric emission is $2.0292\,V$.

Question 11.9: The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution:

No

Work function of the metal is given by, $\phi_{\circ} = 4.2eV$

Charge on an electron is known to be, $e = 1.6 \times 10^{-19}\ C$

Planck's constant is known to be, . $h = 6.626 \times 10^{-34}\ Js$.

Wavelength of the incident radiation is given by, $\lambda = 330\ nm = 330 \times 10^{-9}\ m$

Speed of light is known to be, $c = 3 \times 108\ m/s$

The energy of the incident photon is given as the following:

$$E = \frac{hc}{\lambda}$$
$$= \frac{6.626\times10^{-34}\times3\times10^{8}}{330\times10^{-9}} = 6.0\times10^{-19}\,J$$
$$= \frac{6.0\times10^{-19}}{1.6\times10^{-19}} = 3.76eV$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no photoelectric emission will take place.

11.10: Light of frequency 7.21×1014 Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×105 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Solution:

Frequency of the incident photon is given by, $\upsilon = 488nm = 488\times10^{-9}$

Maximum speed of the electrons is given by, $v = 6.0 \times 105\ m/s$

Planck's constant is known to be, $h = 6.626 \times 10-34\ Js$

Mass of an electron is known to be, $m = 9.1 \times 10-31\ kg$

For threshold frequency $\upsilon_\circ$, the relation for kinetic energy is written as the following:

$$\frac{1}{2}mv^2 = h(\upsilon - \upsilon_\circ)$$

$$\upsilon_\circ = \upsilon - \frac{mv^2}{2h}$$

$$= 7.21\times10^{14} - \frac{(9.1\times10^{-31})\times(6\times10^5)^2}{2\times(6.626\times10^{-34})}$$

$$= 7.21\times10^{14} - 2.472\times10^{14}$$

$$= 4.738\times10^{14}\ Hz$$

Therefore, the threshold frequency for the photoemission of electrons is $4.738 \times 1014\ Hz$.

Question 11.11: Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Solution:

Wavelength of light produced by the argon laser is given by, $\lambda = 488\ nm = 488 \times 10^{-9}\ m$

Stopping potential of the photoelectrons is given by, $V_\circ = 0.38\ V$

$$1eV = 1.6 \times 10^{-19}\ J$$

$$V_\circ = \frac{0.38}{1.6\times10^{-19}}\ eV$$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

Charge on an electron is known to be, $e = 1.6 \times 10^{-19}\ C$

Speed of light is known to be, $c = 3 \times 10\ m/s$

From Einstein's photoelectric effect, we have the relation involving the work function $\phi_\circ$ of the material of the emitter as:

$$eV_\circ = \frac{hc}{\lambda} - \phi_\circ$$
$$\phi_\circ = \frac{hc}{\lambda} - eV_\circ$$
$$= \frac{6.626\times10^{-34}\times3\times10^{4}}{1.6\times10^{-19}\times488\times10^{-4}}$$
$$= 2.54 - 0.38 = 2.16\ eV$$

Therefore, the material with which the emitter is made has the work function of $2.16\ eV$.

11.12: Calculate the

(a) Momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V .

Solution:

Potential difference is given by, $V = 56\ V$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

Mass of an electron is known to be, $m = 9.1 \times 10^{-31}\ kg$

Charge on an electron is known to be, $e = 1.6 \times 10^{-19}\ C$

(a) At equilibrium, the kinetic energy of each electron is equal to the accelerating potential, i.e., we can write the relation for velocity (v) of each electron as:

$$\frac{1}{2}mv^2 = eV$$
$$v^2 = \frac{2eV}{m}$$
$$\therefore v^2 = \sqrt{\frac{2\times1.6\times10^{-19}\times56}{9.1\times10^{-31}}}$$
$$= \sqrt{19.69\times10^{12}} = 4.44\times10^{6}\ m/s$$

The momentum of each accelerated electron is given as:

$p = mv$

$= 9.1 \times 10^{-31} \times 4.44 \times 10^{6}$

$= 4.04 \times 10^{-24}\ kg\ m\ s^{-1}$

Therefore, the momentum of each electron is $4.04 \times 10^{-24}\ kg\ m\ s^{-1}$.

(c) De Broglie wavelength of an electron accelerating through a potential V, is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}} \overset{\bullet}{\text{A}}$$

$$= \frac{12.27}{\sqrt{56}} \times 10^{-10} m$$

$$= 0.1639\ nm$$

Therefore, the de Broglie wavelength of each electron is $0.1639\ nm$.

11.13: What is the

(a) Momentum,

(b) Speed, and

(c) de Broglie wavelength of an electron with kinetic energy of $120\ \text{eV}$.

Solution:

Kinetic energy of the electron is given by, $E_k = 120\ eV$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

Mass of an electron is known to be, $m = 9.1 \times 10^{-31}\ kg$

Charge on an electron is known to be, $e = 1.6 \times 10^{-19}\ C$

(a) For the electron, we can write the relation for kinetic energy as:

$$E_k = \frac{1}{2} mv^2$$

Where, v = Speed of the electron

$$\therefore v^2 = \sqrt{\frac{2eE_k}{m}}$$
$$= \sqrt{\frac{2\times1.6\times10^{-19}\times120}{9.1\times10^{-31}}}$$
$$= \sqrt{42.198\times10^{12}}$$
$$= 6.496\times10^{6}\ m/s$$

Momentum of the electron is given by,

$$p = mv = 9.1 \times 10^{-31} \times 6.496 \times 10^{6}$$
$$= 5.91 \times 10^{-24}\ kg\ ms^{-1}$$

Therefore, the momentum of the electron is $5.91 \times 10^{-24}\ kg\ m\ s^{-1}$.

(b) Speed of the electron is given by, $v = 6.496 \times 10^{6}\ m/s$

(c) De Broglie wavelength of an electron having a momentum p , is given as:

$$\lambda = \frac{h}{p}$$
$$= \frac{6.6\times10^{-34}}{5.91\times10^{-24}} = 1.116\times10^{-10} m$$
$$= 0.112\ nm$$

Therefore, the de Broglie wavelength of the electron is $0.112\ nm$.

11.14: The wavelength of light from the spectral emission line of sodium is $589\ \text{nm}$. Find the kinetic energy at which

(a) An electron, and

(b) A neutron would have the same de Broglie wavelength.

Solution:

Wavelength of light of a sodium line is given by, $\lambda = 589\ nm = 589 \times 10^{-9}\ m$

Mass of an electron is known to be, $m_e = 9.1 \times 10^{-31}\ kg$

Mass of a neutron is known to be, $m_n = 1.66 \times 10^{-27}\ kg$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

(a) For the kinetic energy K , of an electron accelerating with a velocity v , we have the relation given below:

$$K=\frac{1}{2}m_e v^2 \qquad \ldots(1)$$

We have the relation for de Broglie wavelength as the following:

$$\lambda=\frac{h}{m_e v}$$

$$\therefore v^2=\frac{h^2}{\lambda^2 m_e^{\ 2}} \qquad \ldots(2)$$

Substituting equation (2) in equation (1), we get the relation:

$$K=\frac{1}{2}\frac{m_e h^2}{\lambda^2 m_e^{\ 2}}=\frac{h^2}{2\lambda^2 m_e} \qquad \ldots(3)$$

$$=\frac{(6.6\times10^{-34})}{2\times(589\times10^{-9})^2\times9.1\times10^{-31}}$$

$$\approx 6.9\times10^{-25}J$$

$$=\frac{6.9\times10^{-25}}{1.6\times10^{-19}}=4.31\times10^{-6}eV=4.31\mu eV$$

Hence, the kinetic energy of the electron is $6.9\ \times\ 10^{-25}\ J\ or\ 4.31\ \mu eV$.

(b) Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$$\frac{h^2}{2\lambda^2 m_n}=\frac{(6.6\times10^{-34})^2}{2\times(589\times10^{-9})^2\times1.66\times10^{-27}}$$

$$=3.78\times10^{-28}\ J$$

$$=\frac{3.78\times10^{-28}}{1.6\times10^{-19}}=2.36\times10^{-9}neV$$

Hence, the kinetic energy of the neutron ishas a value of $3.78\ \times\ 10^{-28}\ J\ or\ 2.36\ neV$.

11.15: What is the de Broglie wavelength of

(a) A bullet of mass 0.040 kg travelling at the speed of 1.0 km / s ,

(b) A ball of mass 0.060 kg moving at a speed of 1.0 m / s , and

(c) A dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Solution:

(a) Mass of the bullet, $m = 0.040\ kg$

Speed of the bullet has a value of, $v = 1.0\ km/s = 1000\ m/s$

Planck's constant has a value of, $h = 6.6 \times 10^{-34}\ Js$

De Broglie wavelength of the bullet is given by the relation below:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{0.040\times1000} = 1.65\times10^{-35}\,m$$

(b) Mass of the ball has a value of, $m = 0.060\ kg$

Speed of the ball has a value of, $v = 1.0\ m/s$

De Broglie wavelength of the ball is given by the relation below:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{0.060\times1} = 1.1\times10^{-32}\,m$$

(c) Mass of the dust particle has a value of, $m = 1 \times 10^{-9} kg$

Speed of the dust particle has a value of, $v = 2.2\ m/s$

De Broglie wavelength of the dust particle is given by the relation below:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{2.2\times1\times10^{-9}} = 3.0\times10^{-25}\,m$$

11.16: An electron and a photon each have a wavelength of 1.00 nm. Find

(a) Their momenta,

(b) The energy of the photon, and

(c) **The kinetic energy of electron.**

Solution:

Wavelength of an electron (λ_e) and a photon (λ_p) have the values:

$$(\lambda_e) = (\lambda_p) = 1nm = 1\times10^{-9}m$$

Planck's constant is known to be, $h = 6.63 \times 10^{-34}\ Js$

(a) The momentum of an elementary particle is given by de Broglie relation below:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

It is evident by this expression above that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, we can say thattheir momenta are also equal.

$$\therefore p = \frac{6.63\times10^{-34}}{1\times10^{-9}} = 6.63\times10^{-25}\ kgm/s$$

(b)The energy of a photon is given by the relation below:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light is known to be, $c = 3\times10^{8} m/s$
$$\therefore E = \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{-9}\times1.6\times10^{-19}}$$
$$= 1243.1\,eV = 1.243\,keV$$

Therefore, the energy of the photon has a value of $1.243\ keV$.

(c)The kinetic energy (K) of an electron having momentum p , is given by the relation below:

$$K = \frac{1}{2}\frac{p^2}{m}$$

Where,

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Mass of the electron is known to be, $m = 9.1 \times 10^{-31}\ kg$

$p = 6.63 \times 10^{-25}\ kg\ m\ s^{-1}$

$$\therefore K = \frac{1}{2} \times \frac{(6.63 \times 10^{-25})^2}{9.1 \times 10^{-31}} = 2.415 \times 10^{-19} J$$

$$= \frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 eV$$

Hence, the kinetic energy of the electron has a value of $1.51\ eV$.

11.17:

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m ? (b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

Solution:

(a) De Broglie wavelength of the neutron is given by, $\lambda = 1.40 \times 10^{-10}\ m$

Mass of a neutron is known to be, $m_n = 1.66 \times 10^{-27}\ kg$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

Kinetic energy (K) and velocity (v) are related as the following:

$$K = \frac{1}{2} m_n v^2 \qquad \ldots(1)$$

De Broglie wavelength (λ) and velocity (v) are related as:

$$\lambda = \frac{h}{m_n v} \qquad \ldots(2)$$

Using equation (2) in equation (1), we get the following:

$$K = \frac{1}{2}\frac{m_n h^2}{\lambda^2 m_n{}^2} = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{(6.63\times10^{-34})^2}{2\times(1.40\times10^{-10})^2\times1.66\times10^{-27}} = 6.75\times10^{-21} J$$

$$\frac{6.75\times10^{-21}}{1.6\times10^{-19}} = 4.219\times10^{-2} eV$$

Hence, the kinetic energy of the neutron is given by $6.75 \times 10^{-21}\ J\ or\ 4.219 \times 10^{-2}\ eV$.

(b) Temperature of the neutron, $T = 300\ K$

Boltzmann constant is known to be, $k = 1.38 \times 10^{-23}\ kg\ m^2\ s^{-2}\ K^{-1}$

Average kinetic energy of the neutron is known to be:

$$K' = \frac{3}{2}\times kT$$

$$= \frac{3}{2}\times1.38\times10^{-23}\times300 = 6.21\times10^{-21} J$$

The relation for the de Broglie wavelength is given as:

$$\lambda' = \frac{h}{\sqrt{2K'm_n}}$$

Where,

$$m_e = 1.66\times10^{-27} kg$$

$$h = 6.6\times10^{-34}\ Js$$

$$K' = 6.75\times10^{-21}\ J$$

$$\therefore \lambda' = \frac{6.63\times10^{-34}}{\sqrt{2\times6.21\times10^{-21}\times1.66\times10^{-27}}} = 1.46\times10^{-10} m$$

$$= 0.146\ nm$$

Therefore, the de Broglie wavelength of the neutron is $0.146\ nm$.

11.18: Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution:

The momentum of a photon having energy $(h\nu)$ is given as:

$$p = \frac{h\upsilon}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} \qquad \ldots (i)$$

Where,

λ = Wavelength of the electromagnetic radiation

c = Speed of light
h = Planck's constant

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

But, $p = mv$

$$\therefore \lambda = \frac{h}{p} \qquad \ldots (ii)$$

Where,

m = Mass of the photon

v = Velocity of the photon

Hence, it can be inferred from equations (i) and (ii) that the wavelength of the electromagnetic radiation is equal to the de Broglie wavelength of the photon.

11.19 What is the de Broglie wavelength of a nitrogen molecule in air at 300 K ? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Solution:

Temperature of the nitrogen molecule is given as, $T = 300\ K$

Atomic mass of nitrogen is known to be= 14.0076 u

Hence, mass of the nitrogen molecule will be given by,

$$m = 2 \times 14.0076 = 28.0152\ u$$

But, it is known that $1\ u = 1.66 \times 10^{-27}\ kg\ m = 28.0152 \times 1.66 \times 10^{-27}\ kg$

Planck's constant is known to be, $h = 6.63 \times 10^{-34}\ Js$

Boltzmann constant is known to be, $k = 1.38 \times 10^{-23}\ J\ K^{-1}$

We have the expression which connects mean kinetic energy $(\frac{3}{2}kT)$ of the nitrogen molecule with the root mean square speed (υ_{rms}) as:

$$\frac{1}{2}mv_{rms}^{2} = \frac{3}{2}kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Hence, the de Broglie wavelength of the nitrogen molecule is given as:

$$\lambda = \frac{h}{m\upsilon_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times28.0152\times1.66\times10^{-27}\times1.38\times10^{-23}\times300}}$$

$$= 0.028\times10^{-9}\,m$$

$$= 0.028\ nm$$

Therefore, the de Broglie wavelength of the nitrogen molecule is $0.028\ nm$.

Additional Exercise:

11.20:

(a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The specific charge of the electron, i.e., its e/m is given to be $1.76 \times 1011\ C\ kg^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for a collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Solution:

(a) The potential difference developed across the vacuumed tube has a value of, $V = 500\,V$

The specific charge on an electron is known to be, $e/m = 1.76 \times 10^{11} Ckg^{-1}$

The speed with which the electrons are emitted is given by the formula given below:

$$K.E. = \frac{1}{2}mv^2 = eV$$

$$\therefore v = \left(\frac{2eV}{m}\right)^{\frac{1}{2}} = \left(2V \times \frac{e}{m}\right)^{\frac{1}{2}}$$

$$= \left(2 \times 500 \times 1.76 \times 10^{11}\right)^{\frac{1}{2}}$$

$$= 1.327 \times 10^7\ m/s$$

(b) Electric potential developed in anode has a value of, $V = 10MV = 10 \times 10^6 V$

The speed with which the electrons are emitted is given by the following expression:

$$v = \left(2V\frac{e}{m}\right)^{\frac{1}{2}}$$

$$= \left(2 \times 10^7 \times 1.76 \times 10^{11}\right)^{\frac{1}{2}}$$

$$= 1.88 \times 10^9\, m/s$$

Since, nothing can move faster than light, therefore, our above result is wrong. The formula used is the above problem can only be used in non-relativistic approach, where v<<<c,

Where, we have

v = speed of the particle or matter

c = speed of light in vacuum

For the cases where speed of particle or matter is comparable with the speed of light, the following formula is used:

$$E = mc^2$$

Where,

m = relativistic mass

$$i.e.\ m = m_{\circ}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$m_{\circ}$ = mass of particle or matter at rest.

Therefore, form the above result we have,

Kinetic energy is given by:

$$K.E. = mc^2 - m_{\circ}c^2$$

11.21:

(a) A mono-energetic electron beam with electron speed of $5.20 \times 106\ \mathrm{m\ s^{-1}}$ is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals $1.76 \times 1011\ \mathrm{C\ kg^{-1}}$.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified? [Note: Exercises 11.20(b) and 11.21(b) take you to relativistic mechanics which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

Solution:

(a) Given speed of the electron has a value of, $v = 5.20 \times 10^6\ m/s$

Magnetic field experienced by the electron has a value of, $B = 1.30 \times 10^{-4}\ T$

Specific charge of an electron is known to be, $e/m = 1.76 \times 1011\ C\ kg^{-1}$

Where, Charge on the electron is known to be, $e = 1.6 \times 10^{-19}\ C$

Mass of the electron is known to be, $m = 9.1 \times 10^{-31}\ kg^{-1}$

The experienced by electron due to the magnetic field is given as below:

$$F = e\left|\vec{v} \times \vec{B}\right|$$
$$= evB\sin\theta$$

θ = Angle between the magnetic field and the beam velocity. The magnetic field is normal to the direction of beam.

$$\therefore \theta = 90^\circ$$
$$F = evB \qquad \ldots(1)$$

The beam rotates in a circular path of radius, r. Due to its bending nature, centripetal force $\left(F = \frac{mv^2}{r}\right)$ is provided to the beam.

Hence, equation (1) reduces to:

$$evB = \frac{mv^2}{r}$$
$$\therefore r = \frac{mv}{eB} = \frac{v}{\left(\frac{e}{m}\right)B}$$
$$= \frac{5.20 \times 10^6}{\left(1.76 \times 10^{11}\right) \times 1.30 \times 10^{-4}}$$
$$= 0.227\ m$$
$$= 22.7\ cm$$

Hence, a circular path with radius $22.7\ cm$ is traced.

(b) Energy of the electron beam has a value of, $E = 20\ MeV = 20 \times 10^6 \times 1.6 \times 10^{-19}\ J$

The formula for energy of an electron is given below:

$$E=\frac{1}{2}mv^2$$

$$\therefore v=\left(\frac{2E}{m}\right)^{\frac{1}{2}}$$

$$=\sqrt{\frac{2\times 20\times 10^6\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}}$$

$$=2.652\times 10^9\ m/s$$

Since, nothing can move faster than light, therefore, our above result is wrong. The formula used is the above problem can only be used in non-relativistic approach, where v<<<c,

Where, we have

v = speed of the particle or matter

c = speed of light in vacuum

For the cases where speed of particle or matter is comparable with the speed of light, the following formula is used is given below:

$$E=mc^2$$

Where,

m = relativistic mass

$$i.e.\ m=m_{\circ}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$m_{\circ}$ = mass of particle or matter at rest.

Therefore, the circular path's radius is given by given below:

$$r=\frac{mv}{eB}$$

$$=\frac{m_{\circ}v}{eB\sqrt{\frac{c^2-v^2}{c^2}}}$$

11.22: An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (10^{-2} mm of Hg). A magnetic field of

2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method. Determine e/m from the data.

Solution:

Potential of an anode has a value, $V = 100\ V$

Magnetic field experienced by the electrons has a value, $B = 2.83 \times 10^{-4}\ T$

Radius of the circular orbit has a value of : $r = 12.0\ cm = 12.0 \times 10^{-2}\ m$

Let, Mass of each electron = m

Let, Charge on each electron = e

Let, Velocity of each electron = v

The energy of each electron is equal to its kinetic energy, i.e.

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m} \qquad \ldots(1)$$

It is the magnetic field, due to its bending nature, that provides the centripetal $\left(F = \frac{mv^2}{r}\right)$ force for the beam. Hence, we can write:

Centripetal force = Magnetic force

$$evB = \frac{mv^2}{r}$$

$$\therefore eB = \frac{mv}{r}$$

$$v = \frac{eBr}{m} \qquad \ldots(2)$$

Putting the value of v in equation (1), we get the following:

$$\frac{2eV}{m}=\frac{e^2B^2r^2}{m^2}$$
$$\frac{e}{m}=\frac{2V}{B^2r^2}$$
$$=\frac{2\times100}{\left(2.83\times10^{-4}\right)^2\times\left(12\times10^{-2}\right)^2}$$
$$=1.73\times10^{11}\,Ckg^{-1}$$

Therefore, the specific charge ratio (e/m) is has value of $1.73\times10^{11}\,Ckg^{-1}$.

11.23:

(a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at $0.45\,\AA$. What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Solution:

(a) Wavelength produced by an X-ray tube has a value of, $\lambda=0.45\,\mathring{A}=0.45\times10^{-10}m$

Planck's constant is known to be, $h = 6.626 \times 10^{-34}\ Js$

Speed of light is known to be, $c = 3 \times 10^{8}\ m/s$

The maximum energy of a photon is given as by the following expression:

$$E=\frac{hc}{\lambda}$$
$$=\frac{6.626\times10^{-34}\times3\times10^{8}}{0.45\times10^{-10}\times1.6\times10^{19}}$$
$$=27.6\times10^{3}eV=27.6\,keV$$

Therefore, the maximum energy of an X-ray photon has a value $27.6\,keV$.

(b) Accelerating voltage provides energy to the electrons for producing X-rays. To get an X-ray of $27.6\,keV$, the incident electrons must possess at least $27.6\,keV$ of kinetic electric energy. Hence, an accelerating voltage of the order of $30\,keV$ is required for producing X-rays.

11.24: In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 109 eV)

Solution:

Total energy of two γ-rays is given by:

$$E = 10.2\ BeV$$
$$= 10.2 \times 109\ eV$$
$$= 10.2 \times 109 \times 1.6 \times 10-10\ J$$

Hence, the energy of each γ-ray is given by:

$$E' = \frac{E}{2}$$
$$= \frac{10.2 \times 1.6 \times 10^{-10}}{2}$$
$$= 8.16 \times 10^{-10}\ J$$

Planck's constant is known to be, $h = 6.626 \times 10^{-34}\ Js$

Speed of light is known to be, $c = 3 \times 10^{8}\ m/s$

Energy is related to wavelength by the expression given below:

$$E' = \frac{hc}{\lambda}$$
$$\therefore \lambda = \frac{hc}{E'}$$
$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{8.16 \times 10^{-10}}$$
$$= 2.436 \times 10^{-16} m$$

Therefore, the wavelength associated with each γ-ray has a value $2.436 \times 10^{-16} m$.

11.25: Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radio waves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive $(\sim 10^{-10}\ \mathrm{W\ m^{-2}})$. Take the area of the pupil to be about 0.4 cm2, and the average frequency of white light to be about 6×1014 Hz.

Solution:

(a) Power of the medium wave transmitter is given by,

$P = 10\ kW = 10^4\ W = 10^4\ J/s$

Hence, energy emitted by the transmitter per second is given by, $E = 10^4$

Wavelength of the radio wave is given b, $\lambda = 500\ m$

The energy of the wave is given as:

Planck's constant is known to be, $h = 6.626 \times 10^{-34}\ Js$

Speed of light is known to be, $c = 3 \times 10^8\ m/s$

$$\therefore E_1 = \frac{6.626\times10^{-34}\times3\times10^8}{500}$$
$$= 3.96\times10^{-28} J$$

Let n be the number of photons emitted by the transmitter.

$$\therefore nE_1 = E$$
$$n = \frac{E}{E_1}$$
$$= \frac{10^4}{3.96\times10^{-28}}$$
$$= 2.525\times10^{31}$$
$$\approx 3\times10^{31}$$

The energy (E1) of a radio photon is very less, but the number of photons (n) emitted per second in a radio wave is very large.

The existence of a minimum quantum of energy can be ignored and the total energy of a radio wave can be treated as being continuous.

(b) Intensity of light perceived by the human eye is given by, $I = 10^{-10}\ W\ m^{-2}$

Area of a pupil has a value of, $A = 0.4\ cm^2 = 0.4 \times 10^{-4}\ m^2$

Frequency of white light has a value of, $\nu = 6 \times 10^{14}\ Hz$

The energy emitted by a photon is given as:

$$E = h\nu$$

Where,

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ Js$

$$E = 6.6 \times 10^{-34} \times 6 \times 10^{14} = 3.96 \times 10^{-19}\ J$$

Let n be the total number of photons falling per second, per unit area of the pupil.

The total energy per unit for n falling photons is given as the following:

$$E = n \times 3.96 \times 10^{-19}\ J\ s^{-1}\ m^{-2}$$

The energy per unit area per second is the intensity of light.

$$\therefore E = I$$
$$n \times 3.96 \times 10^{-19} = 10^{-10}$$
$$n = \frac{10^{-10}}{3.96 \times 10^{-19}}$$
$$= 2.52 \times 10^{8}\ m^{2}\ s^{-1}$$

The total number of photons entering the pupil per second is given by the following expression:

$$n_A = n \times A$$
$$= 2.52 \times 10^{8} \times 0.4 \times 10^{-4}$$
$$= 1.008 \times 10^{4}\ s^{-1}$$

This number is not as large as the one found in problem (a), but it is large enough for the human eye to never see the individual photons.

11.26: Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photocell made of molybdenum metal. If the stopping potential is -1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity $\left(10^{5}\ \text{W m}^{-2}\right)$ red light of wavelength 6328 Å produced by a He-Ne laser?

Solution:

Wavelength of ultraviolet light is given by, $\lambda = 2271\,Å = 2271 \times 10^{-10}\ m$

Stopping potential of the metal is given by, $V_{\circ} = 1.3\ V$

Planck's constant is given by, $h = 6.6 \times 10^{-34}\ J$

Charge on an electron is given by, $e = 1.6 \times 10^{-19}\ C$

Let work function of the metal = ϕ

Let frequency of light = ν

We have the photo-energy relation from the photoelectric effect as the following:

$$\phi_{\circ} = h\upsilon - eV_{\circ}$$
$$= \frac{hc}{\lambda} - eV_{\circ}$$
$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2271\times10^{-10}} - 1.6\times10^{-19}\times1.3$$
$$= 8.72\times10^{-19} - 2.08\times10^{-19}$$
$$= 6.64\times10^{-19}\,J$$
$$= \frac{6.64\times10^{-19}}{1.6\times10^{-19}} = 4.15eV$$

Let $\upsilon_{\circ}$ be the threshold frequency of the metal.

$$\therefore \phi_{\circ} = h\upsilon_{\circ}$$
$$\upsilon_{\circ} = \frac{\phi_{\circ}}{h}$$
$$= \frac{6.64\times10^{-19}}{6.6\times10^{-34}} = 1.006\times10^{15}\,Hz$$

Wavelength of red light is known to be, $\lambda_r = 6326\ \mathring{A}$

Hence, Frequency of red light,

$$\upsilon_r = \frac{c}{\lambda_r} = 6328\times10^{-10}\,m$$
$$= \frac{3\times10^{8}}{6328\times10^{-34}} = 4.74\times10^{14}\ Hz$$

Since v0>vr, the photocell will not respond to the red light produced by the laser.

11.27: Monochromatic radiation of wavelength $640.2\ nm\ (1nm = 10^{-9}\ m)$ from a neon lamp irradiates photosensitive material made of cesium on tungsten. The stopping voltage is measured to be $0.54\ V$. The source is replaced by an iron source and its $427.2\ nm$ line irradiates the same photo-cell. Predict the new stopping voltage.

Solution:

Wavelength of the monochromatic radiation has a value of,

$\lambda = 640.2\ nm = 640.2 \times 10^{-9}\ m$

Stopping potential of the neon lamp has a value of, $V_0 = 0.54\ V$

Charge on an electron has a value of, $e = 1.6 \times 10^{-19}\ C$

Planck's constant has a value of, $h = 6.6 \times 10^{-34}\ J$

Let ϕ_0 be the work function and ν be the frequency of emitted light.

We have the photo-energy relation from the photoelectric effect given by the following expression:

$$\phi_\circ = h\upsilon - eV_\circ$$
$$= \frac{hc}{\lambda} - eV_\circ$$
$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{640.2\times10^{-9}} - 1.6\times10^{-19}\times10.54$$
$$= 3.093\times10^{-19} - 0.864\times10^{-19}$$
$$= 2.229\times10^{-19}\ J$$
$$= \frac{2.229\times10^{-19}}{1.6\times10^{-19}} = 1.39eV$$

Wavelength of the radiation emitted from an iron source, λ' = 427.2 nm = 427.2 × 10−9 m

let, is the new stopping potential. Hence, photo-energy is given as:

$$eV_0 = \frac{hc}{\lambda'} - \phi_0$$

$$\frac{6.6\times10^{-34}\times3\times10^{8}}{427.2\times10^{-9}} - 2.229\times10^{-19}$$

$$= 4.63\times10^{-19} - 2.229\times10^{-19}$$

$$= 2.401\times10^{-19} J$$

$$= \frac{2.401\times10^{-19}}{1.6\times10^{-19}} = 1.5eV$$

Hence, the new stopping potential is $1.50\ eV$.

11.28: A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used: $\lambda_1 = 3650$ Å, $\lambda_2 = 4047$ Å, $\lambda_3 = 4358$ Å, $\lambda_4 = 5461$ Å, $\lambda_5 = 6907$ Å, The stopping voltages, respectively, were measured to be: $V_{01} = 1.28$ V, $V_{02} = 0.95$ V, $V_{03} = 0.74$ V, $V_{04} = 0.16$ V, $V_{05} = 0$ V Determine the value of Planck's constant h, the threshold frequency and work function for the material.

[Note: You will notice that to get h from the data, you will need to know e (which you can take to be 1.6 × 10−19 C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of e (from the oil-drop experiment) confirmed Einstein's photoelectric equation and at the same time gave an independent estimate of the value of h.]

Solution:

Einstein's photoelectric equation is given as by the following expression:

$$eV_0 = h\upsilon - \phi_0$$

$$V_0 = \frac{h}{e}v - \frac{\phi_0}{e} \qquad \ldots(1)$$

Where, V0 = Stopping potential

h = Planck's constant

e = Charge on an electron

ν = Frequency of radiation

Φ_0 = Work function of a material

It can be concluded from equation (1) that potential V0 is directly proportional to frequency ν. Frequency is also given by the relation:

$$\upsilon = \frac{speed\ of\ light(c)}{wavelength(\lambda)}$$

This relation can be used to obtain the frequencies of the various lines of the given wavelengths.

$$\upsilon_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3650\times10^{-10}} = 8.219\times10^{14}\,Hz$$

$$\upsilon_2 = \frac{c}{\lambda_2} = \frac{3\times10^8}{4047\times10^{-10}} = 7.412\times10^{14}\,Hz$$

$$\upsilon_3 = \frac{c}{\lambda_3} = \frac{3\times10^8}{4358\times10^{-10}} = 6.884\times10^{14}\,Hz$$

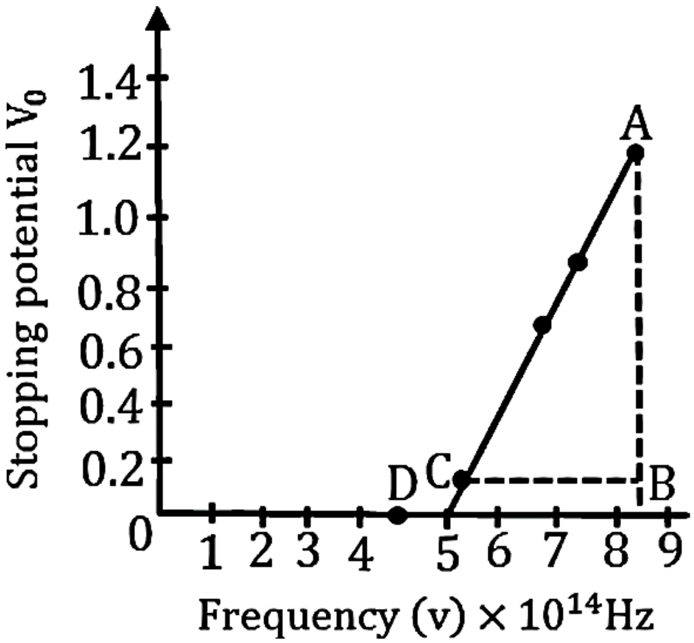
$$\upsilon_4 = \frac{c}{\lambda_4} = \frac{3\times10^8}{5461\times10^{-10}} = 5.439\times10^{14}\,Hz$$

$$\upsilon_5 = \frac{c}{\lambda_5} = \frac{3\times10^8}{6907\times10^{-10}} = 4.343\times10^{14}\,Hz$$

The given quantities can be listed in tabular form as:

Frequency $\times 10^{14}$ Hz	8.219	7.412	6.884	5.493	4.343
Stopping potential V_0	1.28	0.95	0.74	0.16	0

The following figure shows a graph between ν and V_0.



It can be observed that the obtained curve is a straight line. It intersects the ν-axis at 5×10^{14} Hz, which is the threshold frequency (ν_0) of the material. Point D corresponds to a frequency less than the threshold frequency. Hence, there is no photoelectric emission for the λ5 line, and therefore, no stopping voltage is required to stop the current.

Slope of the straight line = $\frac{AB}{CB}=\frac{1.28-0.16}{(8.241-5.493)\times10^{14}}$

From equation (1), the slope can be written as:

$$\frac{h}{e}=\frac{1.28-0.16}{(8.241-5.493)\times10^{14}}$$

$$\therefore h=\frac{1.12\times1.6\times10^{-19}}{2.726\times10^{14}}$$

$$=6.573\times10^{-34}\ Js$$

The work function of the metal is given as:

$$\phi_0 = hv_0$$
$$= 6.573\times10^{-34}\times5\times10^{14}$$
$$= 3.286\times10^{-19}\,J$$
$$= \frac{3.286\times10^{-19}}{1.6\times10^{-18}}$$
$$= 2.054\,eV$$

11.29: The work function for the following metals is given: Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Solution:

Mo and Ni will not show photoelectric emission in both cases

Wavelength for a radiation, $\lambda = 3300\,\mathring{A} = 3300 \times 10^{-10}\ m$

Charge on an electron, $e = 1.6 \times 10^{-19}\ C$

Planck's constant, $h = 6.6 \times 10^{-34}\ J$

The energy of incident radiation is given as:

$$E = \frac{hc}{\lambda}$$
$$= \frac{6.626\times10^{-34}\times3\times10^{8}}{3300\times10^{-10}}$$
$$= 6\times10^{-19}\,J$$
$$= \frac{6\times10^{-19}}{1.6\times10^{-19}} = 3.158\,eV$$

It can be observed that the energy of the incident radiation is greater than the work function of Na and K only. It is less for Mo and Ni. Hence, Mo and Ni will not show photoelectric emission.

If the source of light is brought near the photocells and placed $50\ cm$ away from them, then the intensity of radiation will increase. This does not affect the energy of the radiation. Hence, the result will be the same as before. However, the photoelectrons emitted from Na and K will increase in proportion to intensity.

11.30: Light of intensity $10^{-5}\ \text{W m}^{-2}$ falls on a sodium photo-cell of surface area 2cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric

emission in the wave-picture of radiation. The work function for the metal is given to be about $2\ eV$. What is the implication of your answer?

Solution:

Intensity of incident light, $I = 10^{-5}\ W\ m^{-2}$

Surface area of a sodium photocell, $A = 2\ cm^2 = 2 \times 10^{-4}\ m^2$

Incident power of the light, P = I × A

$= 10^{-5} \times 2 \times 10^{-4}$
$= 2 \times 10^{-9}\ W$

Work function of the metal,

$\phi_0 = 2\ eV$
$= 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}\ J$

Number of layers of sodium that absorbs the incident energy, n = 5

We know that the effective atomic area of a sodium atom, Ae is $10^{-20}\ m^2$.

Hence, the number of conduction electrons in n layers is given as:

$$n' = n \times \frac{A}{A_e}$$
$$= 5 \times \frac{2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

The incident power is uniformly absorbed by all the electrons continuously. Hence, the amount of energy absorbed per second per electron is:

$$E = \frac{P}{n'}$$
$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26}\ J/s$$

Time required for photoelectric emission:

$$t = \frac{\phi_0}{E}$$

$$= \frac{3.2 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^{7}\, s \approx 0.507\ years$$

The time required for the photoelectric emission is nearly half a year, which is not practical. Hence, the wave picture is in disagreement with the given experiment.

11.31: Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to $1\ \text{Å}$, which is of the order of inter-atomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31}$ kg).

Solution:

An X-ray probe has a greater energy than an electron probe for the same wavelength.

Wavelength of light emitted from the probe, $\lambda = 1\,\text{Å} = 10^{-10}\ m$

Mass of an electron, $m_e = 9.11 \times 10^{-31}\ kg$

Charge on an electron, $e = 1.6 \times 10^{-19}\ C$

Planck’s constant, $h = 6.6 \times 10^{-34}\ J$

The kinetic energy of the electron is given as:

$$E = \frac{1}{2} m v^2$$

$$m_e v = \sqrt{2 E m_e}$$

Where, v = Velocity of the electron

$m_e v$= Momentum (p) of the electron

According to the de Broglie principle, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2Em_e}}$$

$$\therefore E = \frac{h^2}{2\lambda^2 m_e}$$

$$= \frac{(6.6\times10^{-34})^2}{2\times\left(10^{-10}\right)^2\times9.11\times10^{-31}}$$

$$= 2.39\times10^{-17}\,J$$

$$= 149.375 eV$$

Energy of photon is given by:

$$E' = \frac{hc}{\lambda_e} ev$$

$$= \frac{6.626\times10^{-34}\times3\times10^8}{10^{-10}\times1.6\times10^{-10}}$$

$$= 12.375\times10^3\ eV$$

$$= 12.375\ keV$$

Hence, a photon has a greater energy than an electron for the same wavelength.

11.32: (a) Obtain the de Broglie wavelength of a neutron of kinetic energy $150\ eV$ **. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain.** $\left(m_n = 1.675 \times 10^{-27}\ kg\right)$

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 ºC). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Solution:

(a) De Broglie wavelength, $\lambda = 2.327\times10^{-12}\,m$

Neutron is not suitable for the diffraction experiment Kinetic energy of the neutron,

$$K = 150\ eV$$

$$= 150 \times 1.6 \times 10^{-19}$$

$$= 2.4 \times 10^{-17}\ J$$

Mass of a neutron, $m_n = 1.675 \times 10^{-27}\ kg$

The kinetic energy of the neutron is given by the relation:

$$E = \frac{1}{2}mv^2$$
$$m_n v = \sqrt{2Em_n}$$

Where, v = Velocity of the electron

m_n= Momentum (p) of the electron

According to the de Broglie principle, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2Em_e}}$$

From the above equation we get that wavelength is inversely proportional to the square root of the mass.

Therefore, wavelength decreases when mass increases and vice versa.

$$\therefore \lambda = \frac{6.6 \times 10^{34}}{\sqrt{2 \times 2.4 \times 10^{-17} \times 1.675 \times 10^{-27}}}$$
$$= 2.327 \times 10^{-12} m$$

It is given in the previous problem that the inter-atomic spacing of a crystal is about $1\,\mathring{A}$, i.e.,$10^{-10}\ m$. Hence, the inter-atomic spacing is about a hundred times greater. Hence, a neutron beam of energy $150\ eV$ is not suitable for diffraction experiments.

(b) De Broglie wavelength , $\lambda = 2.327 \times 10^{-12} m$

Room temperature, $T = 27°C = 27 + 273 = 300\ K$

The average kinetic energy of the neutron is given as:

$$E = \frac{3}{2}kT$$

Where, Boltzmann constant, $k = 1.38 \times 10^{-23}\ J\ mol^{-1}\ K^{-1}$

The wavelength of the neutron is given as:

$$\lambda = \frac{h}{\sqrt{2m_e E}} = \frac{h}{\sqrt{3m_n kT}}$$
$$= \frac{6.6\times10^{-34}}{\sqrt{3\times1.675\times10^{-27}\times1.38\times10^{-23}\times300}}$$
$$=1.447\times10^{-10}\,m$$

This wavelength is comparable to the inter-atomic spacing of a crystal. Hence, the high energy neutron beam should first be thermalised, before using it for diffraction.

11.33: An electron microscope uses electrons accelerated by a voltage of $50\,kV$. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution:

Electrons are accelerated by a voltage has a value of, $V = 50\ kV = 50 \times 10^3\ V$

Charge on an electron has a value of, $e = 1.6 \times 10^{-19}\ C$

Mass of an electron has a value of, $m_e = 9.11 \times 10^{-31}\ kg$

Wavelength of yellow light has a value of, $\lambda = 5.9 \times 10^{-7}\ m$

The kinetic energy of the electron is given as:

$$E = eV$$
$$= 1.6 \times 10^{-19} \times 50 \times 10^3$$
$$= 8 \times 10^{-15}\ J$$

De Broglie wavelength is given by the relation below:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2Em_e}}$$
$$\frac{6.6\times10^{-34}}{\sqrt{2\times10^{-31}\times9.11\times10^{-15}\times8}}$$
$$=5.467\times10^{-12}\,m$$

This wavelength is nearly 105 times less than the wavelength of yellow light. The resolving power of a microscope is inversely proportional to the wavelength of light used. Thus, the resolving power of an electron microscope is nearly 105 times that of an optical microscope.

11.34: The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Solution:

Wavelength of a proton or a neutron is known to be, $\lambda \approx 10^{-15}\ m$

Rest mass energy of an electron:

$$m_0c^2 = 0.511\,MeV$$

$$= 0.511 \times 10^6 \times 1.6 \times 10^{-19}$$
$$= 0.8176 \times 10^{-13}\,J$$

Planck's constant is known to be, $h = 6.6 \times 10^{-34}\ J$

Speed of lightis known to be, $c = 3 \times 10^8\ m/s$

The momentum of a proton or a neutron is given asis known to be:

$$p = \frac{h}{\lambda}$$
$$= \frac{6.6\times10^{-34}}{10^{-15}} = 6.6\times10^{-19}\,kgm/s$$

The relativistic relation for energy (E) is known to be:

$$E^2 = p^2c^2 + m_0{}^2c^4$$
$$= \left(6.6\times10^{-19}\times3\times10^4\right)^2 + \left(0.8176\times10^{-13}\right)^2$$
$$= 392.04\times10^{-22}$$
$$\therefore\ E = 1.98\times10^{-10}\,J$$
$$= \frac{1.98\times10^{-10}}{1.6\times10^{-19}}$$
$$= 1.24\times10^9\,eV = 1.24\,BeV$$

Thus, the electron energy emitted from the accelerator at Stanford, USA might be of the order of $1.24\ BeV$.

11.35: Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Solution:

De Broglie wavelength associated with He atom known to be, $\lambda = 0.7268 \times 10^{-19}$

Room temperature, $T = 27°C = 27 + 273 = 300\ K$

Atmospheric pressure has a value of, $= 1.01 \times 10^{5}\ Pa$

Atomic weight of a He atom has a value of = 4

Avogadro's number has a value of, $N_A = 6.023 \times 10^{23}$

Boltzmann constant has a value of, $k = 1.38 \times 10^{-23}\ J\ mol^{-1}\ K^{-1}$

Average energy of a gas at temperature T, is given as:

$$E = \frac{3}{2}KT$$

De-Broglie wavelength is given by the relation below:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

And here,

$$m = mass\ of\ He\ atom = \frac{Atomic\ weight}{N_A} = \frac{4}{6.023 \times 10^{23}} = 6.64 \times 10^{-24} g = 6.64 \times 10^{-27} Kg$$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 6.64 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} = 0.7268 \times 10^{-10} m$$

The ideal gas formula is given by the following expression:

$$PV = RT = kNT$$

$$\frac{V}{N} = \frac{kT}{P}$$

Where, we have

V=Volume of gas

N=Number of moles of the gas.

Mean separation between two atoms of the gas is given by the relation which is given in the following:

$$r=\left(\frac{V}{N}\right)^{\frac{1}{3}}=\left(\frac{kT}{P}\right)^{\frac{1}{3}}=\left(\frac{1.38\times10^{-23}\times300}{1.01\times10^{5}}\right)^{\frac{1}{3}}=3.35\times10^{-9}m$$

Therefore, the mean separation between the atoms is much greater than the given de Broglie Wavelength.

11.36: Compute the typical de Broglie wavelength of an electron in a metal at 27 ºC and compare it with the mean separation between two electrons in a metal which is given to be about2 × 10^{-10} m. [Note: Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishibility has many fundamental implications which you will explore in more advanced Physics courses.]

Solution:

Temperature is given by, $T=27°C=27+273=300k$

Mean separation between two electrons is given by, $r=2\times10^{-10}m$

De Broglie wavelength associated with an electron is given by this expression: $\lambda=\frac{h}{\sqrt{3mkT}}$

Here we have,

Planck's constant is known to be, $h = 6.6 \times 10^{-34} J$

Mass of an electron is given by, $m_e = 9.11 \times 10^{-31} kg$

Boltzmann constant is known to be, $k = 1.38 \times 10^{-23} J\ mol^{-1} K^{-1}$

$$\therefore\lambda=\frac{h}{\sqrt{3mkT}}=\frac{6.6\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}\approx6.2\times10^{-9}m$$

Thus, we can now infer that the given de Broglie wavelength is much greater than the given inter-electron separation.

11.37: Answer the following questions:
(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)e ; (−1/3)e]. Why do they not show up in Millikan's oil-drop experiment?
(b) What is so special about the combination e/m? Why do we not simply talk of eand m separately?
(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?
(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?
(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations' = hν, p = $\frac{h}{\lambda}$
But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed νλ) has no physical significance. Why?

Solution:

(a) Quarks which are located inside protons and neutrons carry fractional charges. This happens because nuclear force increases extremely high if they are pulled apart. Therefore, fractional charges may exist in nature; observable charges are still the integral multiple of an electrical charge.

(b) The basic relations between electric field and magnetic field is given by the expression:

$$eV = 1/2mv^2 and$$

$$eBV = \frac{mv^2}{r} respectively$$

These relations include these terms: e (electric charge), v (velocity), m (mass), V (potential), r(radius),and B (magnetic field). These relations will give the value of velocity of an electron.

$$v = \sqrt{2V\left(\frac{e}{m}\right)} = Br\left(\frac{e}{m}\right) respectively$$

It can be clearly seen from these relations that the dynamics of an electron is not determined by e and m separately, but by the ratio e/m.

(c) At atmospheric pressure, the ions of gases have no chance of reaching the irrespective electrons. This happens because of collision and recombination with other gas molecules. Consequently, gases are insulators at atmospheric pressure. At low pressures, ions have a chance of reaching their respective electrodes and constitute a current. Hence, they conduct electricity at these pressures.

(d) The work function of a metal is defined as the minimum energy required for a conduction electron to get out of the metal surface. All the electrons in an atom do not have the same energy level, thus when a ray with some energy is incident on a metal surface, the electrons are ejected from different levels with different energies. Hence, these emitted electrons show different energy distributions.

(e) The absolute value of energy of a particle is arbitrary within the additive constant. Hence, wavelength (λ) is significant, but the frequency (ν) associated with an electron has no direct physical significance. Therefore, the product νλ (phase speed) has no physical significance.

Group speed is given as:

$$v_G = \frac{d\nu}{dk} = \frac{d\nu}{d\left(\frac{1}{\lambda}\right)} = \frac{dE}{dp} = \frac{d\left(\frac{p^2}{2m}\right)}{dp} = \frac{p}{m}$$

This quantity has a physical meaning.