Chapter 12: ATOMS

EXAMPLES

12.1 In the Rutherford's nuclear model of the atom, the nucleus (radius about $10^{-15}m$) is analogous to the sun about which the electron move in orbit (radius =$10^{-10}m$) like the earth orbits around the sun. If the dimension of the solar system had the same proportion as those of the atom, would the earth be closer to or farther away from the sun than actually it is? The radius of earth's orbit is about $1.5\times10^{11}m$. The radius of the sun is taken as $7\times10^{8}m$.

Solution:

The ratio of radius of the electron's orbit to the radius of nucleus is $\left(10^{-10}m\right)/\left(10^{-15}m\right)=10^{5}$, that is, the electron's orbit radius is 10^{5} times larger than the nucleus. If the radius of the earth's orbit would be $10^{5}\times7\times10^{8}m=7\times10^{13}m$. It will be more than 100 times greater than the actual orbital radius of the earth. Thus, we can say that the earth would be much farther away from the sun.

So it tells that an atom contains much greater fraction of empty space than that of our solar system.

12.2 In a Gieger - Marsden experiment, what is the distance of the closest approach to the nucleus of a $7.7MeV$ α particles before it comes momentarily to rest and reverse its direction?

Solution:

The idea over here is that while throughout the scattering process, conservation of the total mechanical energy of the system consisting of an α particle and gold nucleus took place. The initial mechanical energy of the system is E_i, before the particle and the nucleus interact, and it equals to its mechanical energy E_f when the α particles momentarily stop. The initial energy E_i is the kinetic energy K of the incoming α particle whereas the final energy E_f is the electric potential energy U of the system. We can calculate the potential energy U as

Let us consider d as the centre – to - centre distance between the α particle and the gold nucleus when the α particle is a t its stopping point. Then we can write the conversation of energy for $E_i=E_f$ as

$$K=\frac{1}{4\pi\in_0}\frac{(2e)(Ze)}{d}=\frac{2Ze^2}{4\pi\in_0 d}$$

Thus, the distance for closest approach d is given by

$$d=\frac{2Ze^2}{4\pi\in_0 K}$$

The maximum kinetic energy found in α particles of natural origin is $7.7MeV$ or $1.2\times10^{-12}J$. Since $1/4\pi\in_0=9.0\times10^{9}Nm^2/C^2$. Therefore with $e=1.6\times10^{-19}C$, we have,

$$d = \frac{(2)\left(9.0\times10^{9}\,Nm^{2}/C^{2}\right)\left(1.6\times10^{-19}C\right)^{2}Z}{1.2\times10^{-12}J}$$

$$= 3.84\times10^{-16}\,Zm$$

The atomic number of the foil material gold is $Z = 79$, so that

$$d(Au) = 3.0\times10^{-14}m = 30\,fm.\left(1\,fm\left(i.e.,\ fermi\right) = 10^{-15}m\right).$$

The radius of the gold nucleus her is less than $3.0\times10^{-14}m$. This is not a good arrangement while comparing with the observed result as the actual radius of gold nucleus is $6\,fm$. This cause for discrepancy is that for the distance of closest approach is considerably much larger than that of the sum of the radii of the gold nucleus and the α particle. Thus, the α particle reverses its motion without even exactly touching the gold nucleus.

12.3 It is found experimentally that $13.6eV$ energy is required to separate a hydrogen atom into a proton and an electron. Compute the orbital radius and the velocity of the electron in the hydrogen atom.

Solution:

Total energy of the electron in hydrogen atom is $-13.6eV = -13.6\times1.6\times10^{-19}J = -2.2\times10^{-18}J$. Thus from Eq. (12.4), we have

$$E = -\frac{e^{2}}{8\pi\epsilon_{0}r} = -2.2\times10^{-18}J$$

This gives the orbital radius

$$r = -\frac{e^{2}}{8\pi\epsilon_{0}E} = \frac{\left(9\times10^{9}\,Nm^{2}/C^{2}\right)\left(1.6\times10^{-19}C\right)^{2}}{(2)\left(-2.2\times10^{-18}J\right)}$$

$$= 5.3\times10^{-11}m.$$

The velocity for the revolving electron can be calculated with $m = 9.1\times10^{-31}kg$

$$v = \frac{e}{\sqrt{4\pi\epsilon_{0}mr}} = 2.2\times10^{6}m/s.$$

12.4 According to the classical electromagnetic theory, calculate the initial frequency of the light emitted by the electron revolving around a proton in hydrogen atom.

Solution:

From example 12.3 it is already known that velocity of electron moving around a proton in hydrogen atom in a orbit of radius $5.3\times10^{-11}m$ is $2.2\times10^{-6}m/s$. Thus, we know the frequency of the electron moving around the proton is

$$v = \frac{v}{2\pi r} = \frac{2.2\times10^{6}ms^{-1}}{2\pi\left(5.3\times10^{-11}m\right)}$$

$\approx 6.6\times10^{15}\,Hz$.

According to the electromagnetic theory, the frequency of the electromagnetic waves emitted by the revolving electrons is same as that of the frequency of its revolution around the nucleus. Therefor the initial frequency of the light emitted is $6.6\times10^{15}\,Hz$.

12.5 A $10kg$ satellite circles earth once every $2h$ in a orbit having a radius of $8000km$. Assuming that Bohr's angular momentum postulate applies to satellite just as it does o an electron in the hydrogen atom, find the quantum number of the satellite.

Solution:

from Eq. (12.13), we have

$$mv_n r_n = nh/2\pi$$

Here $m = 10kg$ and $r_n = 8\times10^6 m$. We have the time period T of the circling satellite as $2h$. that is $T = 7200s$ $T = 7200s$.

Thus the velocity $v_n = 2\pi r_n / T$.

The quantum number of the orbit of satellite

$$n = \left(2\pi r_n\right)^2 \times m/(T\times h)$$

Substituting the values,

$$n = \left(2\pi\times8\times10^6 m\right)^2 \times 10/(7200s\times6.64\times10^{-34}\,Js)$$

$$= 5.3\times10^{45}$$

We should note that the quantum number for the satellite motion is much larger. For large quantum numbers the results of quantisation conditions will be similar to those of general physics.

12.6 Using the Rydberg formula, calculate the wavelengths of the first four spectral lines in the Lyman series of the hydrogen spectrum.

Solution:

The Rydberg's formula is

$$hc/\lambda_{if} = \frac{me^4}{8\in_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The wavelength of the first four lines in the Lyman series correspond to the transition from $n_i = 2,3,4,5$ to $n_f = 1$. We know that

$$\frac{me^4}{8\in_0^2 h^2} = 13.6eV = 21.76\times10^{-19}\,J$$

Therefore,

$$\lambda_{i1} = \frac{hc}{21.76\times10^{-19}\left(\frac{1}{1}-\frac{1}{n_i^2}\right)} m$$

$$= \frac{6.625\times10^{-34}\times3\times10^{8}\times n_i^2}{21.76\times10^{-19}\times(n_i^2-1)} m = \frac{0.9134n_i^2}{(n_i^2-1)}\times10^{-7} m$$

$$= 913.4n_i^2/(n_i^2-1)\dot{A}$$

Substituting $n_i = 2,3,4,5$ we get $\lambda_{21} = 1218\dot{A}, \lambda_{31} = 1028\dot{A}, \lambda_{41} = 974.3\dot{A}, \lambda_{51} = 951.4\dot{A}$

EXERCISES

12.1 Choose the correct alternative from the clues given at the end of each statement:

(a) The size of the atom in Thomson's model is ………… the atomic size in Rutherford's model. (much greater than/no different from/ much less than.)

(b) In the ground state of ……….. the electrons are in stable equilibrium, while in ………. electrons always experience a net force. (Thomson's model/ Rutherford's model.)

(c) An atom based on ………... is doomed to collapse. (Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a ………... but has a highly non-uniform mass distribution in ……….. (Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in ………… (Rutherford's model/ both the models.)

Solution:

(a) The size of the atom taken in Thomson's model and Rutherford's model have the same order of magnitude.

(b) In the ground state of Thomson's model, the electrons are in stable equilibrium, while in Rutherford's model the electrons always experience a net force.

(c) An atom based on Rutherford's model is doomed to collapse.

(d) An atom has a continuous mass distribution in Thomson's model, but has a highly non uniform mass distribution in Rutherford's model.

(e) The positively charged part of the atom possesses most of the mass in both the models.

12.2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below $14K$.) What results do you expect?

Solution:

In the experiment of alpha-particle scattering, if a thin sheet of solid hydrogen is replaced for a gold foil, then the angle of scattering will not be large. It is because the mass of hydrogen ($1.67\times10^{-27} kg$) is less than that of mass of incident α particles ($6.64\times10^{-27} kg$). Thus, the mass of the scattering particle will be more than the target nucleus (hydrogen). As a result, the α particles cannot bounce back if solid hydrogen is used in the experiment of α particle scattering.

12.3 What is the shortest wavelength present in the Paschen series of spectral lines?

Solution:

Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[\frac{1}{n_1^2} - \frac{1}{n^2_2} \right]$$

Were,

h = Planck's constant $= 6.6 \times 10^{-34} Js$

c = Speed of light $= 3 \times 10^8 m/s$

(n_1 and n_2 are integers)

The shortest wavelength as in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2} \right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{-7} m$$

$$= 818.9 nm$$

12.4 A difference of $E = 2.3 eV$ separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Solution:

In an atom the separation of two energy level is given by,

$$E = 2.3 eV$$

$$= 2.3 \times 1.6 \times 10^{-19}$$

$$= 3.68 \times 10^{-19} J$$

Let v be the frequency of radiation emitted by the atom when get transited from the upper level to the lower level.

We have the relation for the energy as:

$$E = hv$$

Were,

h $=$ Planck's constant $= 6.62\times10^{-32}\,Js$

$$\therefore v=\frac{E}{h}$$

$$=\frac{3.68\times10^{-19}}{6.62\times10^{-32}}=5.55\times10^{14}\,Hz$$

Hence, the frequency of the radiation is $5.6\times10^{14}\,Hz$.

12.5 The ground state energy of hydrogen atom is $-13.6eV$. What are the kinetic and potential energies of the electron in this state?

Solution:

Ground state energy of hydrogen atom, $E=-13.6eV$

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy $=-E=-(-13.6)=13.6eV$

Potential energy equals to the twice of negative of kinetic energy.

Potential energy $=-2\times(13.6)=-27.2eV$

12.6 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

Solution:

For ground level, $n_1=1$

Let E_1 be the energy of this level. So E_1 is related with n_1 as:

$$E_1=\frac{-13.6}{n_1^{\,2}}eV$$

$$=\frac{-13.6}{1^2}=-13.6eV$$

The atom gets excited and jumps to higher level, $n_2=4$

Let E_2 be the energy of this level.

$$\therefore E_2=\frac{-13.6}{n_2^{\,2}}eV$$

$$=\frac{-13.6}{4^2}=-\frac{13.6}{16}eV$$

The energy absorbed by the photon is:

$E = E_2 - E_1$

$$= \frac{-13.6}{16} - \left(\frac{-13.6}{1}\right)$$

$$= \frac{13.6 \times 15}{16} eV$$

$$= \frac{13.6 \times 15}{16} \times 1.6 \times 10^{-19} = 2.04 \times 10^{-19} J$$

For a photon of wavelength λ, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Were,

h = Planck's constant $= 6.6 \times 10^{-34} Js$

c = Speed of light $= 3 \times 10^{8} m/s$

$$\therefore \lambda = \frac{hc}{E}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2.04 \times 10^{-18}}$$

$$= 9.7 \times 10^{-8} m = 97 nm$$

And, frequency of a photon is given by the relation,

$$v = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^{8}}{9.7 \times 10^{-8}} \approx 3.1 \times 10^{15} Hz$$

Hence, the wavelength of the photon is $97 nm$ while the frequency is $3.1 \times 10^{15} Hz$.

12.7 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$ and 3 levels. (b) Calculate the orbital period in each of these levels.

Solution:

(a) Let v_1 be the orbital speed of the electron in the hydrogen atom which is in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 is given by,

$$v_1 = \frac{e^2}{n_1 4\pi \in_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2 \in_0 h}$$

Where, $e = 1.6 \times 10^{-19} C$

$\in_0$ = Permittivity of free space = $8.85\times10^{-12}N^{-1}C^2m^{-2}$

h = Planck's constant = $6.62\times10^{-34}Js$

$=0.0218\times10^8=2.18\times10^6 m/s$

For level $n_2=2$, the relatable orbital speed is:

$$v_2=\frac{e^2}{n_2 2\in_0 h}$$

$$=\frac{(1.6\times10^{-19})^2}{2\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$=1.09\times10^{-6}m/s$$

For $n_3=3$, we can write the relation for the corresponding orbital speed as:

$$v_3=\frac{e^2}{n_3 2\in_0 h}$$

$$=\frac{(1.6\times10^{-19})^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$=7.27\times10^5 m/s$$

Hence, the speed of the electron in a hydrogen atom in n=1, n =2 and n =3 is $2.18\times10^6 m/s, 1.09\times10^6 m/s, 7.27\times10^5 m/s$ respectively.

(b) Let T_1 be the orbital period of the electron when it is in level $n_1=1$.

Orbital period is related to orbital speed as:

$$T_1=\frac{2\pi r_1}{v_1}$$

Were, r_1 = Radius of the orbit

$$\in_0=\frac{n_1^2h^2\in_0}{\pi me^2}$$

h = Planck's constant = $6.62\times10^{-34}Js$

e = Charge of an electron = $1.6\times10^{-19}C$

$\in_0$ = Permittivity of free space = $8.85\times10^{-12}N^{-1}C^2m^{-2}$

m = Mass of an electron = $9.1\times10^{-31}kg$

$$\therefore T_1 = \frac{2\pi r_1}{v_1}$$

$$= \frac{2\pi \times (1)^2 \times \left(6.62\times10^{-34}\right)^2 8.85\times10^{-12}}{2.18\times10^6 \times 9.1\times10^{-31} \times \left(1.6\times10^{-19}\right)^2}$$

$$= 15.27\times10^{-17} = 1.527\times10^{-16} s$$

For level $n_2 = 2$, the time period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Were, $r_2 =$ Radius of the electron for level $n_2 = 2$

$$= \frac{(n_2)^2 h^2 \in_0}{\pi m e^2}$$

$$\therefore T_2 = \frac{2\pi r_2}{v_2}$$

$$= \frac{2\pi \times (2)^2 \times (6.62\times10^{-34})^2 \times 8.85\times10^{-12}}{1.09\times10^6 \times \pi \times 9.1\times10^{-31} \times \left(1.6\times10^{-19}\right)^2}$$

$$= 1.22\times10^{-15} s$$

For level $n_3 = 3$, the time period will be:

$$T_3 = \frac{2\pi r_3}{v_3}$$

Were,

$r_3 =$ Electron's radius for level $n_3 = 3$,

$$= \frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

$$= \frac{2\pi \times (3)^2 \times \left(6.62\times10^{-34}\right)^2 \times 8.85\times10^{-12}}{7.27\times10^5 \times \pi \times 9.1\times10^{-31} \times \left(1.6\times10^{-19}\right)^2}$$

$$= 4.12\times10^{-15} s$$

Hence, the orbital period in each of these levels is $1.52\times10^{-16} s, 1.22\times10^{-15}$ *and* $4.12\times10^{-15} s$ respectively.

12.8 The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11} m$. What are the radii of the n $= 2$ and n $= 3$ orbits?

Solution:

The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11} m$.

Let r_2 be the radius of the orbit at $n = 2$. The radius of the innermost orbit is:

$$r_2 = (n)^2 r_1$$

$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} m$$

For $n = 3$, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1$$

$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} m$$

Hence, the radii of an electron for $n = 2$ and $n = 3$ orbits are $2.12 \times 10^{-10} m and 4.77 \times 10^{-10} m$ respectively.

12.9 A $12.5 eV$ electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelength will be emitted?

Solution:

It is given that the energy of the electron beam used to bombard gaseous hydrogen in room temperature is $12.5 eV$. Also, the energy of the gaseous hydrogen in ground state at room temperature is $-13.6 eV$.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes $-13.6 + 12.5 eV$ i.e., $-1.1 eV$.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2} eV$$

For, $n = 3$, $E = \frac{-13.6}{9} = -1.5 eV$

This energy is approximately equals the energy of the gaseous hydrogen. So we can conclude that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, the electrons can jump from $n = 3$ to $n = 1$ directly, by which it forms a line of the Lyman series of the hydrogen spectrum.

The relation for wave number in Lyman series as:

$$\frac{1}{\lambda} = R_Y \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

Were,

R_Y = Rydberg constant $= 1.097 \times 10^7 m^{-1} \lambda$ = Wavelength of radiation emitted by the transition of the electron for n $= 3, \lambda$ can be obtained as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 1.097 \times 10^7 \left(1 - \frac{1}{9}\right) = 1.097 \times 10^7 \times \frac{8}{9}$$

$$\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55nm$$

If the electron jumps from n $= 2$ to n $= 1$, then the wavelength for the radiation given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 1.097 \times 10^7 \left(1 - \frac{1}{4}\right) = 1.097 \times 10^7 \times \frac{3}{4}$$

$$\lambda = \frac{4}{1.097 \times 10^7 \times 3} = 121.54nm = 2$$

If the transition takes place from n $= 3$ to n $= 2$, then the wavelength for the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{5 \times 1.097 \times 10^7} = 656.33nm$$

Thus, the radiation is similar to the Balmer series for the hydrogen spectrum.

Hence, in Lyman series, two wavelengths i.e., 102.5 and $121.5nm$ are emitted.

12.10 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5 \times 10^{11} m$ with the orbital speed $3 \times 10^4 m/s$. (Mass of earth $= 6.0 \times 10^{24} kg$.)

Solution:

Radius of the Earth's orbit around the Sun, $r = 1.5 \times 10^{11} m$

Orbital speed of the earth, v $= 3 \times 10^4 m/s$

Mass of the Earth, m $= 6.0 \times 10^{24} kg$

According to Bohr's model, quantised angular momentum is given by:

$$mvr = \frac{nh}{2\pi}$$

Were,

h = Planck's constant $= 6.62 \times 10^{-34}\ Js$

n = Quantum number

$$\therefore n = \frac{mvr2\pi}{h}$$

$$= \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^{4} \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$

$$= 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Hence, the quantum number which characterizes for the Earth's revolution is 2.6×10^{74}

ADDITIONAL EXERCISE

12.11 Answer the following questions, which will help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α particles at angles greater than 90^{o}) predicted by Thomson's model much less, about the same, or much greater than that predicted b Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α particles scattered at moderate angle is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle od scattering of α particles by a thin foil?

Solution:

(a) About the same.

The average angle of deflection of α particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. It is the reason for which average angle is considered for both the models.

(b) Much less

The probability of scattering α particles at angles greater than 90^{o} predicted by Thomson's model is much less than that predicted by Rutherford's model.

(c) Scattering is mainly due to single collision. The chances for a single collision to take place increases with the number of target atoms. Since the number of target atoms increasers with an increase in thickness of the target.

(d) Thomson's model

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α particles by a thin foil. It is because in models a very little deflection take place for a single collision. Hence, the multiple scattering is used to explain the observed average scattering angle.

12.12 The gravitational attraction between electron and proton in an hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Solution:

Radius of the first Bohr orbit is,

$$r_1 = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \qquad \ldots(1)$$

Were,

$\in_0$ = Permittivity of free space

H = Planck's constant = $6.63\times10^{-34}\,Js$

m_e = Mass of an electron = $9.1\times10^{-31}\,kg$

e = Charge of an electron = $1.9\times10^{-19}\,C$

m_p = Mass of proton = $1.67\times10^{-27}\,kg$

r = Distance between an electron and a proton

Coulomb's attraction between an electron and a proton is:

$$F_C = \frac{e^2}{4\pi \in_0 r^2} \qquad \ldots(2)$$

Attractional force of gravitation between an electron and a proton is as:

$$F_G = \frac{Gm_p m_e}{r^2} \qquad \ldots(3)$$

Were,

G = Gravitational constant $= 6.67\times10^{-11}\,Nm^2/kg^2$

If the Coulomb's electrostatic force and the force of gravity between an electron and a proton are equal, then:

$\therefore F_G = F_C$

$$\frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi \in_0 r^2}$$

$$\therefore \frac{e^2}{4\pi \in_0} = Gm_p m_e \qquad \ldots(4)$$

Putting the values of equation (4)in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^{21}}{Gm_p m_e^{\,2}}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times(9.1\times10^{-31})^2} \approx 1.21\times10^{29}\,m$$

We know that the universe is 156 billion light years wide or $1.5\times10^{27}\,m$ wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than that of the estimated size of the whole universe.

12.13 Obtain an expression for the frequency of radiation emitted when a hydrogen atom deexcites from level n to level (n −1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Solution:

It is given that a hydrogen atom de excites from upper level (n) to a lower level (n−1). The equation of energy (E_1) for radiation at level n is:

$$E_1 = hv_1 = \frac{hme^4}{(4\pi)^3 \in_0^{\,2} \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \qquad \ldots(i)$$

Were,

v_1 = Frequency of radiation at level

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

$\in_0$ = Permittivity of free space

The equation of energy (E_2) for radiation at level (n−1) is as:

$$E_2 = hv_2 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \ldots(ii)$$

Were,

v_2 = Frequency of radiation at level (n − 1)

Energy (E) released as a result of de-excitation:

$$E = E_2 - E_1, hv = E_2 - E_1 \quad \ldots(iii)$$

Were,

v = Frequency of radiation emitted

Putting values from equation (i) and (ii) in equation (iii), we get:

$$v = \frac{me^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4(2n-1)}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$

$$\therefore v = \frac{me^4}{32\pi^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \ldots(iv)$$

Equation for frequency of revolution of an electron is given as:

$$v_c = \frac{v}{2\pi r} \quad \ldots(v)$$

Were,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi \in_0 \left(\frac{h}{2\pi}\right) n} \quad \ldots(vi)$$

Radius of the n^{th} orbit is:

$$r=\frac{4\pi\epsilon_0\left(\frac{h}{2\pi}\right)^2}{me^2}n^2 \quad \ldots(vii)$$

Substituting the above two equations in equation (v), then:

$$v_c=\frac{me^4}{32\pi^3\epsilon_0^2\left(\frac{h}{2\pi}\right)^3 n^3} \quad \ldots(viii)$$

Hence, the frequency of the radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

12.14 Classically an electron can be in any orbit around the nucleus of the atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom at you might have learnt in the text. To stimulate what he might will have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimension of length that is roughly equal to the know size of an atom $\left(\sim 10^{-10}m\right)$.

(a) Construct a quantity with the dimension of length from the fundamental constants e, m_e **and c. Determine its numerical value**

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for `something else' to get the right atomic size. Now, the Planck's constant h had already made it appearance elsewhere. Bohr's insight lay in recognising that h, m_e **and e and confirm that its numerical value has indeed the correct order of magnitude.**

Solution:

(a) Charge on an electron, $e=1.6\times10^{-19}C$

Mass of an electron, $m_e=9.1\times10^{-31}kg$

Speed of light, $c=3\times10^8 m/s$

Considering a quantity involving in the given quantities is $\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)$

Were,

ϵ_0 = Permittivity of free space

And, $\frac{1}{4\pi\epsilon_0}=9\times10^9 Nm^2C^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0} \times \frac{e^2}{m_e c^2}$$

$$= 9\times10^9 \times \frac{\left(1.6\times10^{-19}\right)^2}{9.1\times10^{-31}\times\left(3\times10^8\right)^2}$$

$$= 2.81\times10^{-15} m$$

Hence, when compared to the normal size of an atom this numerical value of the quantity considered is smaller.

(b) Charge on an electron, $e = 1.6\times10^{-19} C$

Mass of an electron, $m_e = 9.1\times10^{-31} kg$

Planck's constant, $h = 6.63\times10^{-34} Js$

Let us take a quantity involving the given quantities as $\dfrac{4\pi \in_0 \left(\dfrac{h}{2\pi}\right)^2}{m_e e^2}$

Were,

$\in_0$ = Permittivity of free space

And, $\dfrac{1}{4\pi \in_0} = 9\times10^9 Nm^2C^{-2}$

The numerical value of the taken quantity will be:

$$\frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2}$$

$$= \frac{1}{9\times10^9} \times \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)}{9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$

$$= 0.53\times10^{-10} m$$

Hence, the value of the quantity taken is of the order of the atomic size.

12.15 The total energy of an electron in the first excited state of the hydrogen atom is about $-3.4 eV$.

(a) What is the kinetic energy of the electron in this state?

(b) what is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of the potential energy is changed?

Solution:

(a) Total energy of the electron, $E = -3.4eV$

The electron's kinetic energy equals to negative of the total energy.

$$\Rightarrow K = -E$$
$$= -(-3.4) = +3.4eV$$

Hence, the kinetic energy of the electron in the given state is $+3.4eV$.

(b) The electron's potential energy (U) equals to the negative of two times its kinetic energy.

$$\Rightarrow U = -2K$$
$$= -2 \times 3.4 = -6.8eV$$

Hence, the potential energy of the electron in the given state is $-6.8eV$.

(c) A system's potential energy depends on the reference point considered. Here, the potential energy considered is zero. If the reference point is changed, then the value of the potential energy of the system also gets changed. Since, total energy is the sum of kinetic and potential energies, total energy of the system will also get changed.

12.16 If Bohr's quantisation postulate (angular momentum = $\frac{nh}{2n}$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Solution:

Quantization of orbits of planets around the Sun is a topic which is not much discussed because the angular momentum associated with the planetary motion is very much related to the value of Planck's constant (h). The angular momentum of the Earth in its orbit is of order 10^{70}. For large values of n, the value of successive energies and angular momentum will be relatively small. Hence, the quantum levels for planetary motion are considered as continuous.

12.17 Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [I.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton].

Solution:

Mass of a negatively charged muon, $m_\mu = 207m_e$

According to Bohr's model,

Bohr's radius, $r_e \propto \left(\frac{1}{m_e}\right)$

Energy of an electronic hydrogen atom in its ground state, $E_e \propto m_e$

Energy of a muonic hydrogen atom in the ground state, $E_\mu \propto m_\mu$

The value of the first Bohr orbit, $r_e = 0.53A = 0.53\times10^{-10} m$

Let r_μ be the radius of muonic hydrogen atom.

At equilibrium, we can write the relation as:

$$m_\mu r_\mu = m_e r_e$$
$$207 m_e r_\mu = m_e r_e$$
$$\therefore r_\mu = \frac{0.53\times10^{-10}}{207} = 2.56\times10^{-13} m$$

Thus, in a muonic hydrogen atom the value of the first Bohr radius is

$2.56\times10^{-13} m$

We have,

$E_e = -13.6 eV$

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$
$$E_\mu = 207 E_e$$
$$= 207\times(-13.6) = -2.81 keV$$

Hence, the ground state energy of a muonic hydrogen atom is $-2.81 keV$.