Chapter 13 - NUCLEI

Examples

Example 13.1 Given the mass of iron nucleus as 55.85u and $A = 56$, find the nuclear density?

Solution

$$m_{Fe} = 55.85, \quad u = 9.27 \times 10^{-26}\ \text{kg}$$

$$\text{Nuclear density} = \frac{\text{mass}}{\text{volume}} = \frac{9.27 \times 10^{-26}}{(4\pi/3)\left(1.2 \times 10^{-15}\right)^3} \times \frac{1}{56}$$

$$= 2.29 \times 10^{17}\ \text{kg m}^{-3}$$

This density of matter is comparable to that of neutron stars (an astrophysical phenomenon). This indicates that the matter in these objects has been compacted to the point that it resembles a large nucleus.

Example 13.2 Calculate the energy equivalent of 1g of substance.

Solution

$$\text{Energy, } E = 10^{-3} \times \left(3 \times 10^{8}\right)^2\ \text{J}$$

$$E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}\ \text{J}$$

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

Example 13.3 Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV. Using this, express the mass defect of ${}^{16}_{8}\text{O}$ in MeV/c^2.

Solution

$$1\text{u} = 1.6605 \times 10^{-27}\ \text{kg}$$

To convert it into energy units, we multiply it by c^2 and find that energy equivalent

$$= 1.6605 \times 10^{-27} \times \left(2.9979 \times 10^{8}\right)^2\ \text{kg m}^2/\text{s}^2$$

$$= 1.4924 \times 10^{-10}\ \text{J}$$

$$= \frac{1.4924 \times 10^{-10}}{1.602 \times 10^{-19}}\ \text{eV}$$

$= 0.9315 \times 10^9 \mathrm{eV}$

$= 931.5 \mathrm{MeV}$

or, $1\mathrm{u} = 931.5 \mathrm{MeV}/\mathrm{c}^2$

For ${}^{16}_{8}\mathrm{O}$, $\Delta M = 0.13691\mathrm{u} = 0.13691 \times 931.5 \mathrm{MeV}/\mathrm{c}^2$

$= 127.5 \mathrm{MeV}/\mathrm{c}^2$

The energy needed to separate ${}^{16}_{8}\mathrm{O}$ into its constituents is thus $127.5 \mathrm{MeV}/\mathrm{c}^2$.

Example 13.4 The half-life of ${}^{238}_{92}\mathrm{U}$ undergoing α-decay is 4.5×10^9 years. What is the activity of $1\mathrm{g}$ sample of ${}^{238}_{92}\mathrm{U}$?

Solution

$T_{1/2} = 4.5 \times 10^9 \mathrm{y}$

$= 4.5 \times 10^9 \mathrm{y} \times 3.16 \times 10^7 \mathrm{s}/\mathrm{y}$

$= 1.42 \times 10^{17} \mathrm{s}$

One kmol of any isotope contains Avogadro's number of atoms, and so $1\mathrm{g}$ of ${}^{238}_{92}\mathrm{U}$ contains

$\frac{10^{-3}}{238} \mathrm{kmol} \times 6.025 \times 10^{26}$ atoms /kmol $= 25.3 \times 10^{20}$ atoms. The decay rate R is $R = \lambda N$

$$= \frac{0.693}{T_{1/2}} N = \frac{0.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}} \mathrm{s}^{-1}$$

$= 1.23 \times 10^4 \mathrm{s}^{-1}$

$= 1.23 \times 10^4 \mathrm{Bq}$

Example 13.5 Tritium has a half-life of \$12.5\$ y undergoing beta decay. What fraction of a sample of pure tritium will remain undecayed after $25\mathrm{y}$.

Solution

By definition of half-life, half of the initial sample will remain undecayed after $12.5\mathrm{y}$. In the next $12.5\mathrm{y}$, one-half of these nuclei would have decayed. Hence, one fourth of the sample of the initial pure tritium will remain undecayed.

Example 13.6 We are given the following atomic masses:

$^{238}_{92}\mathrm{U} = 238.05079\mathrm{u}\ ^{4}_{2}\mathrm{He} = 4.00260\mathrm{u}$

$^{234}_{90}\mathrm{Th} = 234.04363\mathrm{u}\ ^{1}_{1}\mathrm{H} = 1.00783\mathrm{u}$

$^{237}_{91}\mathrm{Pa} = 237.05121\mathrm{u}$

Here the symbol Pa is for the element protactinium $(Z = 91)$.

(a) Calculate the energy released during the alpha decay of $^{238}_{92}\mathrm{U}$.

(b) Show that $^{238}_{92}\mathrm{U}$ can not spontaneously emit a proton.

Solution

(a) The alpha decay of $^{238}_{92}\mathrm{U}$ is given by Eq. (13.20). The energy released in this process is given by

$$Q = \left(m_{\mathrm{U}} - m_{\mathrm{Th}} - m_{\mathrm{He}}\right)c^2$$

Substituting the atomic masses as given in the data, we find

$$Q = (238.05079 - 234.04363 - 4.00260)\mathrm{u} \times c^2$$

$$= (0.00456\mathrm{u})c^2$$

$$= (0.00456\mathrm{u})(931.5\mathrm{MeV/u})$$

$$= 4.25\mathrm{MeV}.$$

(b) If $^{238}_{92}\mathrm{U}$ spontaneously emits a proton, the decay process would be

$$^{238}_{92}\mathrm{U} \rightarrow ^{237}_{91}\mathrm{Pa} + ^{1}_{1}\mathrm{H}$$

The Q for this process to happen is

$$= \left(m_{\mathrm{U}} - m_{\mathrm{Pa}} - m_{\mathrm{H}}\right)c^2$$

$$= (238.05079 - 237.05121 - 1.00783)\mathrm{u} \times c^2$$

$$= (-0.00825\mathrm{u})\mathrm{c}^2$$

$$= -(0.00825\mathrm{u})(931.5\mathrm{MeV/u})$$

$$= -7.68\mathrm{MeV}$$

Thus, the Q of the process is negative and therefore it cannot proceed spontaneously. We will have to supply an energy of $7.68 \mathrm{MeV}$ to a ${}^{238}_{92}\mathrm{U}$ nucleus to make it emit a proton.

Example 13.7 Answer the following questions:

(a) Are the equations of nuclear reactions (such as those given in Section 13.7) "balanced' in the sense a chemical equation (e.g., $2H_2 + O_2 \rightarrow 2H_2O$) is? If not, in what sense are they balanced on both sides?

(b) If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice-versa) in a nuclear reaction?

(c) A general impression exists that mass-energy interconversion takes place only in nuclear reaction and never in chemical reaction. This is strictly speaking incorrect. Explain.

Solution

(a) A chemical equation is balanced if the number of atoms of each element on both sides of the equation is the same. A chemical reaction just modifies the original atom combinations. Elements can be transmuted in a nuclear reaction. In a nuclear process, the number of atoms of each element is not always preserved. In a nuclear reaction, however, the number of protons and neutrons are both conserved individually. [Actually, even at very high energies, this isn't exactly true; the overall charge and total 'baryon number' are strictly conserved. This isn't something we need to discuss right now.] In nuclear reactions, the number of protons and neutrons on both sides of the equation are the same.

(b) We know that a nucleus' binding energy contributes negatively to the mass of the nucleus (mass defect). Because the amount of protons and neutrons in a nuclear reaction is preserved, the total rest mass of neutrons and protons is the same on both sides of the process. However, the total binding energy of nuclei on the left and right sides do not have to be the same. In a nuclear reaction, the difference between these binding energies manifests as energy released or absorbed. We argue that the difference in the total mass of nuclei on the two sides is transformed into energy or vice versa since binding energy contributes to mass. A nuclear reaction is an example of mass energy interconversion in this sense.

(c) In terms of mass-energy interconversion, a chemical reaction is conceptually analogous to a nuclear reaction. The difference in chemical (not nuclear) binding energies of atoms and molecules on the two sides of a reaction can be traced to the energy generated or absorbed in a chemical reaction. Because chemical binding energy has a negative contribution (mass defect) to an atom's or molecule's total mass, we can also state that the difference in total mass of atoms or molecules on the two sides of the chemical reaction is turned into energy or vice versa. A chemical reaction's mass flaws, on the other hand, are almost a million times smaller than those in a nuclear reaction. This is what gives the overall impression. In a chemical process, mass-energy interconversion does not occur (which is wrong).

Exercises

13.1 (a) Two stable isotopes of lithium ${}^{6}_{3}\mathrm{Li}$ and ${}^{7}_{3}\mathrm{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses $6.01512 \mathrm{u}$ and $7.01600 \mathrm{u}$, respectively. Find the atomic mass of lithium.

Ans: Mass of ${}_{3}^{6}\mathrm{Li}$ isotope, m_1=6.01512 u

Mass of ${}_{3}^{7}Li$ isotope, $m = 7.01600\mathrm{u}$

Abundance of ${}_{3}^{6}\mathrm{Li}$, $\eta_1 = 7.5\%$

Abundance of ${}_{3}^{7}Li$, $\eta_2 = 92.5\%$

The atomic mass of lithium

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2},$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934\mathrm{u}$$

(b) Boron has two stable isotopes, ${}_{5}^{10}\mathrm{B}$ and ${}_{5}^{11}\mathrm{B}$. Their respective masses are $10.01294\mathrm{u}$ and $11.00931\mathrm{u}$, and the atomic mass of boron is $10.811\mathrm{u}$. Find the abundances of ${}_{5}^{10}\mathrm{B}$ and ${}_{5}^{11}\mathrm{B}$.

Ans: Mass of boron isotope ${}_{5}^{10}\mathrm{B}$, $\mathrm{m} = 10.01294\mathrm{u}$

Mass of boron isotope ${}_{5}^{11}\mathrm{B}$, $\mathrm{m} = 11.00931\mathrm{u}$

Abundance of ${}_{5}^{10}\mathrm{B}, \eta_1 = x\%$

Abundance of ${}_{5}^{11}\mathrm{B}, \eta_2 = (100 - \mathrm{x})\%$

Atomic mass of boron, $m = 10.811u$

Atomic mass of boron atom:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637}$$

$$= 19.89\%$$

$$100 - x = 80.11\%$$

As a result,

Abundance of ${}^{10}_{5}\mathrm{B}, \eta_1 = 19.89\%$

Abundance of ${}^{11}_{5}\mathrm{B}, \eta_2 = (100 - 19.89)\%$

13.2 The three stable isotopes of neon: ${}^{20}_{10}\mathrm{Ne}$, ${}^{21}_{10}\mathrm{Ne}$ and ${}^{22}_{10}\mathrm{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are $19.99\mathrm{u}$, $20.99\mathrm{u}$ and $21.99\mathrm{u}$, respectively. Obtain the average atomic mass of neon.

Ans: Atomic mass of ${}^{20}_{10}\mathrm{Ne}$, $\mathrm{m}_1 = 19.99\mathrm{u}$

Abundance of ${}^{20}_{10}\mathrm{Ne}$, $\eta_1 = 90.51\%$

Atomic mass of ${}^{21}_{10}\mathrm{Ne}$, $\mathrm{m}_2 = 20.99\mathrm{u}$

Abundance of ${}^{21}_{10}\mathrm{Ne}$, $\eta_2 = 0.27\%$

Atomic mass of ${}^{22}_{10}\mathrm{Ne}$, $\mathrm{m}_3 = 21.99\mathrm{u}$

Abundance of ${}^{22}_{10}\mathrm{Ne}$, $\eta_3 = 9.22\%$

The average atomic mass of neon,

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771\mathrm{u}$$

13.3 Obtain the binding energy (in MeV) of a nitrogen nucleus $\begin{pmatrix}14\\7\end{pmatrix}$, given $m\left(\frac{14}{7}\mathrm{N}\right) = 14.00307\mathrm{u}$.

Ans: Atomic mass of $\left({}_{7}\mathrm{N}^{14}\right)$ nitrogen, $\mathrm{m} = 14.00307\mathrm{u}$

N^{14} nitrogen's nucleus has same number of protons & neutrons, i.e., 7

Hence, the mass defect of this nucleus, $\Delta m = 7m_1 + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$$\therefore \Delta \mathrm{m} = 7\times 1.007825 + 7\times 1.008665 - 14.00307$$
$$= 7.054775 + 7.06055 - 14.00307$$
$$= 0.11236\mathrm{u}$$

$$1\mathrm{u} = 931.5\mathrm{MeV}/\mathrm{c}^2$$
$$\therefore \Delta \mathrm{m} = 0.11236\times 931.5\mathrm{MeV}/\mathrm{c}^2$$

Binding energy of the nucleus:

$$E_b = \Delta mc^2$$

Where, $c =$ Speed of light,

$$\therefore \mathrm{E_b} = 0.11236\times 931.5\left(\frac{\mathrm{MeV}}{c^2}\right)\times c^2 = 104.66334\mathrm{MeV}$$

As a result, a nitrogen nucleus' binding energy is $104.66334\mathrm{MeV}$.

13.4 Obtain the binding energy of the nuclei $^{56}_{26}\mathrm{Fe}$ AND $^{209}_{83}\mathrm{Bi}$ in units of MeV from the following data:

$$m\left(^{56}_{26}\mathrm{Fe}\right) = 55.934939\mathrm{u}$$
$$m\left(^{209}_{83}\mathrm{Bi}\right) = 208.980388\mathrm{u}$$

Ans:

Atomic mass of $^{56}_{26}\mathrm{Fe}$, $\mathrm{m_1} = 55.934939\mathrm{u}$

$^{56}_{26}\mathrm{Fe}$ nucleus has 26 protons and $(56-26) = 30$ neutrons

So,the mass defect of the nucleus, $\Delta m = 26\times m_H + 30\times m_n - m_1$

Here,

Mass of a proton, $\mathrm{m_H} = 1.007825\mathrm{u}$

Mass of a neutron, $m_n = 1.008665u$

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$
$= 26.20345 + 30.25995 - 55.934939$
$= 0.528461u$

$1u = 931.5MeV / c^2$

$\therefore \Delta m = 0.528461 \times 931.5MeV / c^2$

The binding energy of this nucleus:

$E_{b1} = \Delta mc^2$

Where, $C =$ Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$

$= 492.26MeV$

Average binding energy/nucleon, $= 492.26MeV$.

Atomic mass of ${}^{209}_{83}Bi$ $m_2 = 208.980388u$

So, the mass defect of this nucleus,

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Here,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 83.649475 + 127.091790 - 208.980388$
$= 1.760877u$

But $1u = 931.5MeV / c^2$

$\therefore \Delta m' = 1.760877 \times 931.5MeV / c^2$

Binding energy of this nucleus,

$E_{b2} = \Delta m' c^2$

$= 1.760877 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$

$= 1640.26\text{MeV}$

Average binding energy/nucleon $= \frac{1640.26}{209} = 7.848\text{MeV}$

13.5 A given coin has a mass of 3.0g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms (of mass 62.92960u).

Ans: Mass of a copper coin, $m' = 3\text{g}$

Atomic mass of $_{29}\text{Cu}^{63}$ atom, $m = 62.92960\text{u}$

Together $_{29}\text{Cu}^{63}$ atoms in coin $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

$N_A =$ Avogadro's number $= 6.023 \times 10^{23}$ atoms / g

Mass number $= 63\text{g}$

$\therefore N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$ atoms

$_{29}\text{Cu}^{63}$ nucleus has 29 protons and $(63-29)34$ neutrons

Mass Defect, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Mass of a proton, $m_H = 1.007825\text{u}$

Mass of a neutron, $m_n = 1.008665\text{u}$

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

$= 0.591935\text{u}$

All of the atoms in the coin have a mass defect. $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}\,\mathrm{u}$

$1\mathrm{u} = 931.5\,\mathrm{MeV}/c^2$

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5\,\mathrm{MeV}/c^2$

As a result, the binding energy of the coin's nucleus,

$E_b = \Delta mc^2$

$= 1.69766958 \times 10^{22} \times 931.5\left(\frac{\mathrm{MeV}}{c^2}\right) \times c^2$

$= 1.581 \times 10^{25}\,\mathrm{MeV}n$

But

$1\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}E_b$

$= 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}\,\mathrm{J}$

This is the amount of energy required to separate all of the neutrons and protons from the given coin.

13.6 Write nuclear reaction equations for

(i) α **-decay of** ${}^{226}_{88}\mathrm{Ra}$

Ans:

$${}_{88}\mathrm{Ra}^{226} \longrightarrow {}_{86}\mathrm{Rn}^{222} + {}_{2}\mathrm{He}^{4}$$

(ii) α **-decay of** ${}^{242}_{94}\mathrm{Pu}$

Ans:

$${}^{242}_{94}\mathrm{Pu} \longrightarrow {}^{238}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}$$

(iii) $\beta^- -$ **decay of** ${}^{32}_{15}\mathrm{P}$

Ans:

$${}^{32}_{15}\mathrm{P} \longrightarrow {}^{32}_{16}\mathrm{S} + e^- + \overline{\nu}$$

(iv) $\beta^- -$ **decay of** ${}^{210}_{83}\mathrm{Bi}$

Ans:

$$^{210}_{83}B \longrightarrow {}^{210}_{84}PO + e^- + \overline{\nu}$$

(v) β^+ **- decay of** $^{11}_{6}C$

Ans:

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^+ + \nu$$

(vi) β_+ **- decay of** $^{97}{}_{43}Tc$

Ans:

$$^{97}_{43}Tc \longrightarrow {}^{97}_{42}MO + e^+ + \nu$$

(vii) Electron capture of $^{120}_{54}Xe$

Ans:

$$^{120}_{54}Xe + e^+ \longrightarrow {}^{120}_{53}I + \nu$$

13.7 A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to

a) 3.125%

Ans: Half-life $= \mathrm{T}$ years

Original amount $= N_0$

The quantity of radioactive isotope remaining after decay $= N$

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$\lambda =$ Decay

$t =$ Time

$\therefore -\lambda t = \frac{1}{32}$

$-\lambda t = \ln 1 - \ln 32$

$-\lambda t = 0 - 3.4657$

$t = \frac{3.4657}{\lambda}$

$\lambda = \frac{0.693}{T}$

$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T$ years

b) 1% **of its original value?**

Ans: After decay, the amount of the radioactive isotope $= \text{N}$

$\frac{N}{N_0} = 1\% = \frac{1}{100}$

But $\frac{N}{N_0} = \text{e}^{-2t}$

$\therefore \text{e}^{-\lambda t} = \frac{1}{100}$

$-\lambda t = \ln 1 - \ln 100$

$-\lambda t = 0 - 4.6052 t = \frac{4.6052}{\lambda}$

Since, $\lambda = 0.693/\text{T}$

$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645 T years$

13.8 The normal activity of living carbon-containing matter is found to be about 15 **decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive** ${}^{14}{}_{6}\text{C}$ **present with the stable carbon isotope** ${}^{12}{}_{6}\text{C}$**. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life** $(5730$ **years) of** ${}^{14}{}_{6}\text{C}$**, and the measured activity, the age of the**

specimen can be approximately estimated. This is the principle of ${}_{6}^{14}\mathrm{C}$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Ans:

Decay rate of living carbon-containing matter, $R=15$ decay/min

Assume that N be the number of radioactive atoms present in a standard carbon-containing substance.

Half life of ${}^{14}{}_{6}\mathrm{C}, T_{1/2}=5730$ years

The degradation rate of the specimen collected from the Mohenjodaro site was calculated as follows:

$R'=9$ decays /min

Assume that N represent the number of radioactive atoms in the specimen during the Mohenjodaro era.

$$\frac{N}{N'}=\frac{R}{R'}=\mathrm{e}^{-\lambda t}$$

$$\mathrm{e}^{-\lambda t}=\frac{9}{15}=\frac{3}{5}$$

$$-\lambda t=\log_e\frac{3}{5}=-0.5108$$

$$\therefore t=\frac{0.5108}{\lambda}$$

$$\text{But } \lambda=\frac{0.693}{T_1/2}=\frac{0.693}{5730}$$

$$\therefore t=\frac{0.5108}{\frac{0.693}{5730}}=4223.5\text{ years}$$

As a result, the probable age of the Indus-Valley civilisation is 4223.5 years.

13.9 Obtain the amount of ${}_{27}^{60}\mathrm{Co}$ {necessary to provide a radioactive source of } $8.0\mathrm{mCi}$ strength. The half-life of ${}_{27}^{60}\mathrm{Co}$ is 5.3 years.

Ans: The strength,

$$\frac{dN}{dt}=8.0\mathrm{mCi}$$

$$=8\times10^{-3}\times3.7\times10^{10}$$

$= 29.6 \times 10^{7}$ decay $/ \mathrm{s}$

Where,

N= Required number of atoms

Half-life of ${}^{60}_{27}\mathrm{Co}$, $T_{1/2} = 5.3$ years $= 5.3 \times 365 \times 24 \times 60 \times 60 = 1.67 \times 10^{8}\,\mathrm{s}$

For decay constant λ, rate of decay,

$$\frac{dN}{dt} = \lambda N$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^{8}}\,\mathrm{s}^{-1}$$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^{7}}{\frac{0.693}{1.67 \times 10^{8}}}$$

$$= 7.133 \times 10^{16} \text{ atoms}$$

Mass of 6.023×10^{23} (Avogadro's number) atoms $= 60\,\mathrm{g}$

$$\therefore Mass of\, 7.133 \times 10^{16}\, atoms = \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}\,\mathrm{g}$$

As a result, amount of ${}^{60}_{27}\mathrm{Co}$ needed is 7.106×10^{-6} g.

13.10 The half-life of ${}^{90}_{38}\mathrm{Sr}$ is 28 years. What is the disintegration rate of $15\mathrm{mg}$ of this isotope?

Ans: Half life of ${}^{90}_{38}\mathrm{Sr}$, $t^{1/2} = 28$ years

$$= 28 \times 365 \times 24 \times 60 \times 60$$
$$= 8.83 \times 10^{8}\,\mathrm{s}$$

Mass of the isotope, $m = 15\mathrm{mg}$

$90\,\mathrm{g}$ of ${}^{90}_{38}\mathrm{Sr}$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, $15\mathrm{mg} of {}^{90}_{38}\mathrm{Sr}$ contains:

$$\frac{6.023\times10^{23}\times15\times10^{-3}}{90}, i.e., 1.0038\times10^{20}$$ number of atoms

Rate of disintegration, $\frac{dN}{dt}=\lambda N$

Where,

$$\lambda = \text{ Decay constant } = \frac{0.693}{8.83\times10^{8}} s^{-1}$$

$$\therefore \frac{dN}{dt}=\frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}}$$

$$=7.878\times10^{10} \text{ atoms / s}$$

So, the disintegration rate $7.878\times10^{10} atoms/s$.

13.11 Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Ans: Nuclear radius of the gold isotope $_{79}Au^{197}=R_{Au}$

Nuclear radius of the silver isotope $_{47}Ag^{107}=R_{Ag}$

Mass number of gold, $A_{Au}=197$

Mass number of silver, $A_{Ag}=107$

The ratio of the radii,

$$\frac{R_{Au}}{R_{Ag}}=\left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}=\left(\frac{197}{107}\right)^{\frac{1}{3}}=1.2256$$

13.12 Find the Q-value and the kinetic energy of the emitted a-particle in the a-decay of

Given,

$m\left(\begin{matrix}226\\88\end{matrix}\text{Rn}\right)=226.02540\text{u},$

$m\left(\begin{matrix}222\\86\end{matrix}\text{Rn}\right)=222.01750\text{u},$

$m\left(\begin{matrix}220\\86\end{matrix}\text{Rn}\right)=220.01137\text{u},$

$m\left(\begin{matrix}216\\84\end{matrix}\text{Po}\right)=216.00189\text{u}$

(a) ${}^{226}_{88}\text{Ra}$ **and**

Ans: α particle decay of ${}^{226}_{88}\text{Ra}$ emits a He nucleus.

${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Ra} + {}^{4}_{2}\text{He}$

Q-value of emitted $\alpha-$particle = Sum (initial mass - final mass)

c^2 Where, $c=$ Speed of light

$m\left(\begin{matrix}26\\88\end{matrix}Ra\right)=226.02540u$

$m\left({}^{4}_{2}\text{He}\right)=4.002603\text{u}$

Q-value $=[226.02540-(222.01750+4.002603)]\text{u}c^2$

$=0.005297\text{uc}^2$

But $1\text{u}=931.5\text{MeV}/\text{c}^2n$

$\therefore \text{Q}=0.005297\times 931.5\approx 4.94\text{MeV}$

$K.E.$ of the $\alpha-$particle

$=\frac{222}{226}\times 4.94=4.85\text{MeV}$

(b) ${}^{220}_{86}\text{Rn}$**.**

Ans: Alpha particle decay of $\left({}^{220}_{86}\text{Rn}\right)^{220}_{86}\text{Rn} \rightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\text{He}$

Mass $\begin{pmatrix} 220 \\ 86 \end{pmatrix}$ Rn of $= 220.01137\text{u}$

Mass $\begin{pmatrix} 216 \\ 84 \end{pmatrix}$ Po of $= 216.00189\text{u}$

$\therefore$ Q-value $= [220.01137 - (216.00189 + 4.00260)] \times 931.5\$ \approx 641\text{MeV}$

Kinetic energy of the α -particle $\left(\frac{220-4}{220}\right) \times 6.41 = 6.29\text{MeV}$

13.13 The radionuclide ^{11}C decays according to $^{11}_{6}\text{C} \rightarrow ^{11}_{5}\text{B} + e^{+} + v$: $\text{T}_{1/2} = 20.3\text{min}$

The maximum energy of the emitted positron is 0.960MeV.

Given the mass values:

$m\left(^{11}_{6}\text{C}\right) = 11.011434\text{u}$ **and** $m\left(^{11}_{6}\text{B}\right) = 11.009305\text{u}$,

calculate Q and compare it with the maximum energy of the positron emitted.

Ans:

$$m\left(^{11}_{6}\text{C}\right) = 11.011434\text{u}$$
$$m\left(^{11}_{6}\text{B}\right) = 11.009305\text{u}$$
$$^{11}_{6}\text{C} \rightarrow ^{11}_{5}\text{B} + e^{+} + v$$

Half life of $^{11}_{6}\text{C}$ nuclei, $T_{1/2} = 20.3\text{min}$

Maximum energy $= 0.960\text{MeV}$

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}\text{C}$

$$\Delta Q = \left[m'\left({}_{6}\text{C}^{11}\right) - \left[m'\begin{pmatrix} 11 \\ 5 \end{pmatrix}B + m_e\right]\right]c^2 \ldots(1)$$

Where,

$\text{m}_\text{e} =$ Mass of an electron or positron $= 0.000548\text{u}$

$\text{c} =$ Speed of light

$\text{m}' =$ Respective nuclear masses

If atomic masses are used, then we have to add $6\mathrm{m_e}$ in the case of $^{11}C and 5\mathrm{m_e}$ in the case of ^{11}B

So, equation (1) be:

$$\Delta Q=\left[m\left({}_6\mathrm{C}^{11}\right)-m\left(\begin{matrix}11\\5\end{matrix}B\right)-2m_c\right]c^2$$

Here, $m\left({}_0\mathrm{C}^{11}\right)$ and $m\left({}^{11}\mathrm{B}\right)$ are the atomic masses.

$$\therefore \Delta\mathrm{Q}=[11.011434-11.009305-2\times 0.000548]\mathrm{c}^2$$
$$=\left(0.001033\mathrm{c}^2\right)\mathrm{u}$$

But $1\mathrm{u}=931.5\mathrm{Mev}/\mathrm{c}^2$

$$\therefore \Delta\mathrm{Q}=0.001033\times 931.5\approx 0.962\mathrm{MeV}$$

Q is about equal to the maximal energy of the released positron.

13.14 The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$m\left({}^{23}_{10}\mathrm{Ne}\right)=22.994466\mathrm{u}$$
$$m\left({}^{23}_{11}\mathrm{Na}\right)=22.989770\mathrm{u}$$

Ans: In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission $^{23}_{10}\mathrm{Ne}$ $^{23}_{10}\mathrm{Ne}\rightarrow^{23}_{11}\mathrm{Na}+e^-+\overline{v}+Q$

Atomic mass $m\left({}^{23}_{10}\mathrm{Ne}\right)of=22.994466\mathrm{u}$

Atomic mass $m\left(\begin{matrix}23\\11\end{matrix}\mathrm{Na}\right)\ of=22.989770\mathrm{u}$

Mass of an electron, $\mathrm{m_e}=0.000548\mathrm{u}$

Q-value,

$$Q=\left[m\left({}^{23}_{10}\mathrm{Ne}\right)-\left[m\left({}^{23}_{11}\mathrm{Na}\right)+m_c\right]\right]c^2$$

$\therefore Q=[22.994466-22.989770]c^2$

$=\left(0.004696c^2\right)\mathrm{u}$

But $1\mathrm{u}=931.5\mathrm{MeV}/c^2$

$\therefore Q=0.004696\times 931.5=4.374\mathrm{MeV}$

So, The maximal kinetic energy of the released electrons is almost equal to the Q-value.i.e., $4.374\mathrm{MeV}$.

13.15 The Q value of a nuclear reaction $A+b\rightarrow C+d$ is defined by

$\mathrm{Q}=[\mathrm{m_A}+\mathrm{m_b}-\mathrm{m_C}-\mathrm{m_d}]\mathrm{c}^2$ **where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.**

Atomic masses are given to be

$m\left({}_1^2\mathrm{H}\right)=2.014102\mathrm{u}$

$m\left({}_1^3\mathrm{H}\right)=3.016049\mathrm{u}$

$m\left({}_6^{12}\mathrm{C}\right)=12.000000\mathrm{u}$

$m\left({}_{10}^{20}\mathrm{Ne}\right)=19.992439\mathrm{u}$

(i) ${}_1^1\mathrm{H}+{}_1^3\mathrm{H}\rightarrow{}_1^2\mathrm{H}+{}_1^2\mathrm{H}$

Ans: ${}_1\mathrm{H}^1+{}_1^3\mathrm{H}\rightarrow{}_1^2\mathrm{H}+{}_1^2\mathrm{H}$

Atomic mass $m\left({}_1^1\mathrm{H}\right)=1.007825\mathrm{u}$

Atomic mass $m\left({}_1^3\mathrm{H}\right)=3.016049\mathrm{u}$

Atomic mass $m\left({}_1^2\mathrm{H}\right)=2.014102\mathrm{u}$

Q-value is,

$Q=\left[m\left({}_1^1\mathrm{H}\right)+m\left({}_1^3\mathrm{H}\right)-2m\left({}_1^2\mathrm{H}\right)\right]c^2$

$=[1.007825+3.016049-2\times 2.014102]c^2$

$Q=\left(-0.00433c^2\right)\mathrm{u}$

But $1\mathrm{u}=931.5\mathrm{MeV}/\mathrm{c}^2$

$\therefore Q=-0.00433\times 931.5=-4.0334\mathrm{MeV}$

The reaction's negative Q-value indicates that it is endothermic.

(ii) ${}^{12}_{6}\mathrm{C}+{}^{12}_{6}\mathrm{C}\rightarrow{}^{20}_{10}\mathrm{Ne}+{}^{4}_{2}\mathrm{He}$

Ans: ${}^{12}_{6}\mathrm{C}+{}^{12}_{6}\mathrm{C}\rightarrow{}^{20}_{10}\mathrm{Ne}+{}^{4}_{2}\mathrm{He}$

Atomic mass of $m\left({}^{12}_{6}\mathrm{C}\right)=12.0\mathrm{u}$

Atomic mass of $m\left({}^{20}_{10}\mathrm{Ne}\right)=19.992439\mathrm{u}$

Atomic mass of $m\left({}^{4}_{2}\mathrm{He}\right)=4.002603\mathrm{u}$

The Q-value,

$$Q=\left[2m\left({}^{12}_{6}\mathrm{C}\right)-m\left({}^{20}_{10}\mathrm{Ne}\right)-m\left({}^{4}_{2}\mathrm{He}\right)\right]c^2$$
$$=[2\times 12.0-19.992439-4.002603]c^2$$
$$=\left(0.004958c^2\right)\mathrm{u}$$
$$=0.004958\times 931.5$$
$$=4.618377\mathrm{MeV}$$

The reaction's positive Q-value indicates that it is exothermic.

13.16 Suppose, we think of fission of ${}^{56}_{26}\mathrm{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\mathrm{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $m\left({}^{56}_{26}\mathrm{Fe}\right)=55.93494\mathrm{u}$ and $m\left({}^{28}_{13}\mathrm{Al}\right)=27.98191\mathrm{u}$.

Ans: The fission of ${}^{56}_{26}\mathrm{Fe}$:

${}^{56}_{13}\mathrm{Fe}\rightarrow 2\,{}^{28}_{13}\mathrm{Al}$

Atomic mass of $m\left(\frac{56}{26}\mathrm{Fe}\right)=55.93494\mathrm{u}$

Atomic mass of $m\left({}^{28}_{13}\mathrm{Al}\right)=27.98191\mathrm{u}$

The Q-value,

$Q=\left[m\left({}^{56}_{26}\mathrm{Fe}\right)-2m\left({}^{28}_{13}\mathrm{Al}\right)\right]c^2$

$=[55.93494-2\times 27.98191]c^2$

$=\left(-0.02888c^2\right)\mathrm{u}$

But $1\mathrm{u}=931.5\mathrm{MeV}/\mathrm{c}^2$

$\therefore Q=-0.02888\times 931.5=-26.902\mathrm{MeV}$

The fission has a negative Q-value. As a result, energetically, fission is not conceivable.

The Q-value must be positive in order for a fission reaction to be energetically feasible.

13.17 The fission properties of ${}^{239}{}_{94}\mathrm{Pu}$ are very similar to those of ${}^{235}_{92}\mathrm{U}$.

The average energy released per fission is $180\mathrm{MeV}$. How much energy, in MeV, is released if all the atoms in $1\mathrm{kg}$ of pure ${}^{239}{}_{94}\mathrm{Pu}$ undergo fission?

Ans: Average energy released per fission of ${}^{239}{}_{94}\mathrm{Pu}, E_{av}=180\mathrm{MeV}$

Amount of pure ${}_{94}\mathrm{Pu}^{239}, \mathrm{m}=1\mathrm{kg}=1000\mathrm{g}$

$\mathrm{N_A}=$ Avogadro number $=6.023\times 10^{23}$

Mass number of ${}^{239}_{94}\mathrm{Pu}=239\mathrm{g}$

1 mole of ${}_{94}\mathrm{Pu}^{239}$ contains $\mathrm{N_A}$ atoms.

${}_{94}\mathrm{Pu}^{239} \mathit{contains}\left(\frac{\mathrm{N_A}}{\text{Mass number}}\times m\right) \mathit{atoms}$

$=\frac{6.023\times 10^{23}}{239}\times 1000$

$=2.52\times 10^{24}\,\mathit{atoms}$

Total energy released during the fission of $1\mathrm{kg}\,\mathit{of}\,{}^{239}{}_{94}\mathrm{Pu}$

$E=E_{av}\times 2.52\times 10^{24}$

$=180\times 2.52\times 10^{24}$

$=4.536\times 10^{26}\,\mathrm{MeV}$

So, $4.536\times 10^{26}\,\mathrm{MeV}$ is released if all the atoms in $1\mathrm{kg}$ of pure ${}_{94}\mathrm{Pu}^{239}$ undergo fission.

13.18 A $1000 MW$ fission reactor consumes half of its fuel in $5.00 y$. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Ans: Fuel's Half life, $t_{\frac{1}{2}} = 5 = 5 \times 365 \times 24 \times 60 \times 60 s$

We know that in the fission of $1g$ of $^{235}_{92}U$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., $235 g$ of contains 6.023×10^{23} atoms.

$\therefore 1g$ of $^{235}_{92}U$ $\frac{6.023 \times 10^{23}}{235}$ atoms .

Whole energy produced/gram of $^{235}_{92}U$,

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 MeV / g$$

$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^6}{235}$$

$$= 8.20 \times 10^{10} J / g$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}U$ consumed in 5 years by the $1000 MW$ fission reactor is:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^6}{100 \times 8.20 \times 10^{10}} g$$

$$\approx 1538 kg$$

Initial amount of $^{235}_{92}U = 2 \times 1538 = 3076 kg$

13.19 How long can an electric lamp of $100 W$ be kept glowing by fusion of $2.0 kg$ of deuterium?

Take the fusion reaction as

$$^2_1H + ^2_1H \rightarrow ^3_1He + n + 3.27 MeV$$

Ans: $_1^2H + _1^2H \rightarrow _2^3He + n + 3.27 MeV$

Amount of deuterium, $m = 2\,kg$

1 mole, i.e., $2\,g$ of deuterium contains 6.023×10^{23} atoms.

$\therefore 2.0\,kg$ of deuterium contains $= \dfrac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

When two atoms of deuterium fuse, $3.27\ MeV$ energy generated.

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26}\,MeV$$
$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

Total energy emitted per nucleus during the fusion reaction: $= 1.576 \times 10^{14}\ J$

Power of the electric lamp, $P = 100\,W = 100\,J/s$

As a result, the lamp's energy consumption per second $= 100\,J$

$$\frac{1.576 \times 10^{14}}{100}\,s$$
$$\frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365}$$

The entire time the electric bulb will be lit is determined as follows: $\approx 4.9 \times 10^{4}$ years

13.20 Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Ans: $d =$ Radius of 1^{st} deuteron + Radius of 2^{nd} deuteron (d is dis. b/w centres)

Radius of a deuteron nucleus $= 2$ fm $= 2 \times 10^{-15}$ m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}\,m$

Charge on a deuteron nucleus $=$ Charge on an electron $= e = 1.6 \times 10^{-19}\,C$

Potential energy of the two-deuteron system:

$$V=\frac{e^2}{4\pi\epsilon_0 d}$$

Where,

$\epsilon_0 = Permittivityoffreespace$

$$\frac{1}{4\pi\epsilon_0}=9\times10^9\ \mathrm{N\,m^2C^{-2}}$$

$$\therefore V=\frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}}\mathrm{J}$$

$$=\frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}\times\left(1.6\times10^{-19}\right)}\mathrm{eV}$$

$$=360\mathrm{keV}$$

As a result, the possible barrier height of the two-deuteron system is $360\mathrm{keV}$.

13.21 From the relation $R=R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Ans: $\mathrm{R}=\mathrm{R}_0\mathrm{A}^{1/3}$

Where,

$\mathrm{R}_0=$ Constant. $\mathrm{A}=$ Mass number of the nucleus

Nuclear matter density, $\rho=$ Mass of the nucleus Volume of the nucleus

Assume that m $=$ avg. mass of nucleus.

Hence, mass of the nucleus $=\mathrm{mA}$

$$therefore\,\rho=\frac{mA}{\frac{4}{3}\pi R^3}=\frac{3mA}{4\pi\left(R_0A^{\frac{1}{3}}\right)^3}=\frac{3mA}{4\pi R_0^3A}=\frac{3m}{4\pi R_0^3}$$

So, the nuclear matter density is independent of A. It is nearly constant.

13.22 For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the $\mathrm{K}-$ shell, is captured by the nucleus and a neutrino is emitted).

$$e^{+}+\frac{1}{Z}X \rightarrow {}^{A}_{Z-1}Y+v$$

Show that if β^{+} emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Ans: Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$$e^{+}+{}^{1}_{z}X \rightarrow {}^{1}_{z-1}Y+v+Q_1$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction is:

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y+e^{+}+v+Q_2$$

$m_N\left({}^{A}_{z}X\right)=$ Nuclear mass of ${}^{A}X$

$m_N\left({}^{A}_{z-1}Y\right)=$ Nuclear mass of ${}^{A}_{z-1}Y$

$m\left({}^{A}_{z}X\right)=$ Atomic mass of ${}^{A}_{Z}X$

$m\left({}^{A}_{z-1}Y\right)$ Atomic mass of ${}^{A}_{z-1}Y$

$\mathrm{m_e}=$ Mass of an electron

$\mathrm{c}=$ Speed of light

Q-value of the electron capture reaction is:

$$Q_1=\left[m_N\left({}^{A}_{z}X\right)+m_e-m_N\left({}^{A}_{z-1}Y\right)\right]c^2$$
$$=\left[m\left({}^{A}_{z}X\right)-Zm_v+m_e-m\left({}^{A}_{z-1}Y\right)+(Z-1)m_e\right]c^2$$
$$=\left[m\left({}^{A}_{z}X\right)-m\left({}^{A}_{z-1}Y\right)\right]c^2$$

Q-value of the positron capture reaction is:

$$Q_2=\left[m_N\left({}^{A}_{z}X\right)-m_N\left({}^{A}_{z-1}Y\right)-m_e\right]c^2$$
$$=\left[m\left({}^{A}_{z}X\right)-Zm_e-m\left({}^{A}_{z-1}Y\right)+(Z-1)m_e-m_e\right]c^2$$
$$=\left[m\left({}^{A}_{z}X\right)-m\left({}^{A}_{z-1}Y\right)-2m_e\right]c^2$$

$Q_2 > 0,$
$Q_1 > 0$
$Q_1 > 0$

it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is due to the fact that for an energetically permissible nuclear reaction, the Q-value must be positive.

Additional Question

23. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}\mathrm{Mg}\quad(23.98504\mathrm{u}), {}^{25}_{12}\mathrm{Mg}\quad(24.98584\mathrm{u})$ and ${}^{26}_{12}\mathrm{Mg}\quad(25.98259\mathrm{u})$. The natural abundance of ${}^{24}_{12}\mathrm{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Answer

Average atomic mass of magnesium, $m = 24.312\mathrm{u}$

Mass of magnesium ${}^{24}_{12}\mathrm{Mg}$ isotope, $m_1 = 23.98504\mathrm{u}$
Mass of magnesium ${}^{25}_{12}\mathrm{Mg}$ isotope, $m_2 = 24.98584\mathrm{u}$
Mass of magnesium ${}^{26}_{12}\mathrm{Mg}$ isotope, $m_3 = 25.98259\mathrm{u}$

Abundance of, ${}^{24}_{12}\mathrm{Mg}$ η_1 $\eta = 78.99\%$

Abundance of ${}^{25}_{12}\mathrm{Mg}, \eta_2 = x\%$

Hence, abundance of ${}^{26}_{12}\mathrm{Mg}, \eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_1}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

And $21.01 - x = 11.71\%$

Hence, the abundance of ${}^{25}_{12}\text{Mg}$ is 9.3% and that of ${}^{26}_{12}\text{Mg}$ is 11.71%.

Question 13.24: The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}\text{Ca}$ and ${}^{27}_{13}\text{Al}$ from the following data:

$$m\left({}^{40}_{20}\text{Ca}\right) = 39.962591\text{u}$$
$$m\left({}^{41}_{20}\text{Ca}\right) = 40.962278\text{u}$$
$$m\left({}^{26}_{13}\text{Al}\right) = 25.986895\text{u}$$
$$m\left({}^{27}_{13}\text{Al}\right) = 26.981541\text{u}$$

Answer

For ${}^{41}_{20}$ Ca: Separation energy $= 8.363007\text{MeV}$

${}^{27}_{13}\text{Al}$: Separation energy $= 13.059\text{MeV}$

$\left({}_{0}n'\right)$ is removed from a ${}^{41}_{20}\text{Ca}$

For A neutron nucleus. The corresponding nuclear reaction can be written as:

$${}^{41}_{20}\text{Ca} \longrightarrow {}^{40}_{20}\text{Ca} + {}^{1}_{0}\text{n}$$

It is given that:

$$m\left({}^{40}_{20}\text{ Ca }\right) \quad \text{Mass } = 39.962591\text{u}$$
$$m\left({}^{41}_{20}\text{ Ca }\right) \quad \text{Mass }\Big) = 40.962278\text{u}$$
$$m\left({}_{0}n^{1}\right) \quad \text{Mass } = 1.008665\text{u}$$

The mass defect of this reaction is given as:

$$\Delta\text{m} = m\left({}^{40}_{20}\text{Ca}\right) + \left({}^{1}_{0}\text{n}\right) - m\left({}^{41}_{20}\text{Ca}\right)$$
$$= 39.962591 + 1.008665 - 40.962278$$
$$= 0.008978\text{u}$$

But

$$1\text{u} = 931.5\text{MeV}/\text{c}^2$$
$$\therefore \Delta\text{m} = 0.008978 \times 931.5\text{MeV}/\text{c}^2$$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.008978 \times 931.5$

$= 8.363007\text{MeV}$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as: ${}^{27}_{13}\text{Al}$

It is given that:

$m\left({}^{27}_{13}\text{Al}\right)$ Mass $= 26.981541\text{u}$

$m\begin{pmatrix} 26 \\ 13 \end{pmatrix}\text{Al}$ Mass $= 25.986895\text{u}$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

It is given that:

The mass defect of this reaction is given as:

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.014019 \times 931.5$

$= 13.059\text{MeV}$

13.25 A source contains two phosphorous radio nuclides ${}^{32}_{15}\text{P}\left(T_{1/2} = 14.3\text{d}\right)$ and ${}^{33}_{15}\text{P}\left(T_{1/2} = 25.3\text{d}\right)$. Initially, 10% of the decays come from ${}_{15}\text{P}$. How long one must wait until 90% do so?

Answer:

Let the number of atoms of ${}^{32}_{15}\text{P}$ be N_1 and atoms of ${}^{33}_{15}\text{P}$ be N_2.

The decay constants of two atoms be λ_1 and λ_2 respectively.

The initial activity of ${}^{33}_{15}\text{P}$ is $\frac{1}{9}$ times that of ${}^{32}_{15}\text{P}$.

Therefore, we can write the decay constants as:

$$\text{N}_1\lambda_1 = \frac{\text{N}_2\lambda_2}{9}$$

Let after time t the activity of ${}^{33}_{15}\text{P}$ be 9 times that of ${}^{32}_{15}\text{P}$

$$N_1\lambda_1 e^{-\lambda_1 t} = 9N_2\lambda_2 e^{-\lambda_2 t}$$

Now. Divide equation (ii) by (i) and taking the natural log of both sides we get

$$-\lambda_1 t = \ln 81 - \lambda_2 t$$

$$t = \frac{\ln 81}{\lambda_2 - \lambda_1}$$

where $\lambda_2 = 0.048/$ day and $\lambda_1 = 0.027/$ day

$$t = \frac{4.394}{0.048 - 0.027} = 209.2 \text{ days}$$

t comes out to be 209.2 days.

13.26 Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}\text{Ra} \rightarrow ^{200}_{82}\text{Pb} + ^{14}_{6}\text{C}$$

$$^{223}_{88}\text{Ra} \rightarrow ^{219}_{88}\text{Rn} + ^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Answer:

(i) In case of first process,

$$^{223}_{88}\text{Ra} \rightarrow ^{200}82\,\text{Pb} + ^{14}_{6}\text{C} + \text{Q}$$

The mass defect occurred in reaction is,

$$\Delta m = \text{ mass of Ra}^{223} - \left(\text{ mass of Pb}^{209} + \text{ mass of C}^{14}\right)$$

$$= 223.01850 - (208.98107 + 14.00324)$$

$$= 0.03419\text{u}$$

So, the amount of energy released is given by:

$$Q = \Delta m \times 931\text{MeV}$$

$$Q = 0.03419 \times 931\text{MeV} = 31.83\text{MeV}$$

(ii) Now, in case of second reaction,

$$^{223}_{88}Ra \rightarrow ^{219}_{86}Rn + ^{4}_{2}He + Q$$

Mass defect is given as:

$$\Delta m = \text{ mass of } Ra^{223} - \left(\text{ mass of } Rn^{219} + \text{ mass of } He^{4}\right)$$

$$= 223.01850 - (219.00948 + 4.00260)$$

$$= 0.00642u$$

$\therefore$ the Energy released in the reaction is:

$$Q = 0.00642 \times 931 MeV = 5.98 MeV$$

Since both reactions have positive Q-value so both reactions are possible.

13.27 Consider the fission of $^{23/2}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $^{140}_{5s}Ce$ and $^{90}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$$m\left(^{238}_{22}U\right) = 238.05079u$$

$$m\left(^{140}_{58}Ce\right) = 139.90543u$$

$$m\left(^{90}_{44}Ru\right) = 98.90594u$$

Answer:

The reaction for the fission of $^{238}_{93}U$ by fast neutrons is as shown $^{238}_{93}U + ^{1}_{0}n \rightarrow ^{140}_{58}Ce + ^{99}_{44}U$

$$Q = \left[m\left(^{238}_{92}U\right) + m_N - m\left(^{140}_{58}Ce\right) - m\left(^{99}_{44}Ru\right)\right] \times 931.5 MeV$$

$$Q = [238.05079 + 1.00893 - 139.90543 - 98.90594] \times 931.5 MeV$$

$$Q = 0.24835 \times 931.5 MeV$$

$$Q = 231.1 MeV$$

13.28 Consider the D-T reaction (deuterium-tritium fusion)

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m\left({}_{1}^{2}\mathrm{H}\right)=2.014102\mathrm{u}$

$m\left({}_{1}^{3}\mathrm{H}\right)=3.016049\mathrm{u}$

(b) Consider the radius of both deuterium and tritium to be approximately $2.0\mathrm{fm}$. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

(Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles $=2(3kT/2); k=$ Boltzmann's constant, $T=$ absolute temperature.)

Answer:

(a) The reaction process is,

$${}_{1}^{2}\mathrm{H}+{}_{1}^{3}\mathrm{H}\rightarrow{}_{2}^{4}\mathrm{He}+\mathrm{n}+\mathrm{Q}$$

$$\mathrm{Q}\text{-value }=\left[\text{ mass of }{}_{1}^{2}\mathrm{H}+\text{ mass of }{}_{1}^{3}\mathrm{H}-\text{ mass of }{}_{2}^{4}\mathrm{He}-\text{ mass of n}\right]\times 931\mathrm{MeV}$$

$$=(2.014102+3.016049-4.002603-1.00867)\times 931\mathrm{MeV}$$

$$=0.018878\times 931=17.58\mathrm{MeV}$$

(b) Repulsive potential energy of two nuclei when they almost touch each othe is given by.

$$\frac{q^2}{4\pi\varepsilon_0(2r)}=\frac{9\times 10^9\left(1.6\times 10^{-19}\right)^2}{2\times 2\times 10^{-15}}\text{ joule}$$

$$=5.76\times 10^{-14}\text{ J}$$

Classically, this amount of K.E. is at least required to overcome coulomb repulsion.

Now, using the relation,

$$\mathrm{KE}=\frac{3}{2}\mathrm{kT}$$

$$\Rightarrow \mathrm{T}=\frac{2\mathrm{K}\cdot\mathrm{E}\cdot}{3\mathrm{k}}$$

$$=\frac{2\times 5.76\times 10^{-14}}{3\times 1.38\times 10^{-23}}$$

$$=2.78\times 10^9\text{ K}$$

13.29 Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

$$m\left({}^{198}\mathrm{Au}\right)=197.968233\mathrm{u}$$

$$m\left({}^{198}\mathrm{Hg}\right)=197.966760\mathrm{u}$$

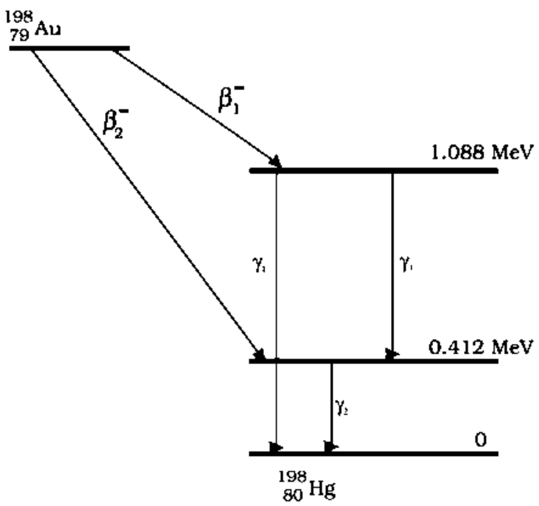

FIGURE13.6

Answer:

It can be observed from the given γ decay diagram that γ_1 decays from the $1.088\mathrm{MeV}$ energy level to the o MeV energy level.

Hence, the energy corresponding to γ_1 decays is given as

$$\mathrm{E}_1=1.088-0=1.088\mathrm{MeV}$$

$$\mathrm{hv}_1=1.088\times1.6\times10^{-19}\times10^6\ \mathrm{J}$$

Where,

$\mathrm{h}=$ Plank's constant and $\mathrm{v}_1=$ frequency of radiation radiated by γ_1 decay.

$$\therefore \mathrm{v}_1=\frac{\mathrm{E}_1}{\mathrm{h}}=\frac{1.088\times1.6\times10^{19}\times10^6}{6.6\times10^{-34}}=2.637\times10^{20}\ \mathrm{Hz}$$

It can be observed the given gamma decay diagram that γ_2 decays from the $0.412\mathrm{MeV}$ energy level to the OMeV energy level.

Hence, the energy corresponding to γ_2 decay is given as

$E_2 = 0.412 - 0 = 0.412\text{MeV}$

$hv_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6\ \text{J}$

Where v_2 is the frequency of the radiation radiated by γ_2 decay.

$\therefore v_2 = \dfrac{E_2}{h} = 9.988 \times 10^{19}\ \text{Hz}$

It can be observed from the given gamma decay diagram that γ_3 decays from the 1.088MeV energy level to the 0.412MeV energy level. Hence, the energy corresponding to γ_3 is given as

$E_3 = 1.088 - 0.412 = m0.676\text{MeVJ}$

$hv_3 = 0.676 \times 10^{-19} \times 10^6\ \text{J}$

Where $v_3 =$ frequency of the radiation radiated by γ_3 decay

$\therefore v_3 = \dfrac{E_3}{h} = 1.639 \times 10^{20}\ \text{Hz}$

Now mass of Au is 197.968u and mass of $\text{Hg} = 197.9667\text{u}$

$1\text{u} = 931.5\text{MeV}/c^2$

Energy of the highest level is given as :

$$E = \left[m\left(\frac{198}{78}\text{Au}\right) - m\left(\frac{190}{80}\text{Hg}\right)\right] = 197.968233 - 197.96676 = 0.001473\text{u} =$$

$0.001473 \times 931.5 = 1.3720995\text{MeV}$

β_1 decays from the 1.3720995MeV level to the 1.088MeV level

$\therefore$ maximum kinetic energy of the β_1 particle $= 1.3720995 - 1.088 =$

0.2840995MeV

β_2 decays from the 1.3720995MeV level to the 0.412MeV level

$\therefore$ Maximum kinetic energy of the β_2 particle =1.3720995-0.412=0.9600995

13.30 Calculate and compare the energy released by a) fusion of $1.0\,\text{kg}$ of hydrogen deep within Sun and b) the fission of $1.0\,\text{kg}$ of ^{235}U in a fission reactor.

Answer:

A.

Amount of hydrogen, $m = 1\text{kg} = 1000\text{g}$

1 mole, i.e.e, $1 g$ of hydrogen $\left({}_{1}^{1}\text{H}\right)$ contains 6.023×10^{23} atoms. Therefore 1000 g of hydrogen contains $6.023\times10^{23}\times1000$ atoms.

Within the Sun, four ${}_{1}^{1}\text{H}$ nuclei combine and form one ${}_{2}^{4}\text{He}$ nucleus. In this process 26MeV of energy is released. Hence the energy released from fusion of $1\text{kg}{}_{1}^{1}\text{H}$ is

$$E_1 = \frac{6.023\times10^{23}26\times10^{3}}{4} = 39.149\times10^{26}\text{MeV}$$

B:

Amount of ${}_{92}^{235}\text{U} = 1\text{kg} = 1000\text{g}$

One mole i.e. 235g of ${}_{92}^{225}\text{U}$ contains 6.023×10^{23} atoms. Therefore 1000g of ${}_{92}^{235}\text{U}$ contains $\frac{6.023\times10^{23}\times1000}{235}$ atoms.

It is known that the amount of energy released in the fission of one atom of ${}_{92}^{235}$ is 200MeV

Hence, energy released from the fission of 1kg of ${}_{92}^{235}$ is

$$E_2 = \frac{6.023\times10^{23}\times1000\times200}{235} = 5.106\times10^{26}\text{MeV}$$

$$\frac{E_1}{E_2} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1kg of hydrogen is nearly 8 times the energy released in the fission of one kg of uranium.

13.31 Suppose India had a target of producing by $2020\text{AD}, 200,000\text{MW}$ of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficlency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25\%. How much amount of fissionable uranium would our country need per year by 2020 ? Take the heat energy per fission of ${}^{235}\text{U}$ to be about 200MeV.

Answer:

Total electric power which is aimed to be produced by $2020 = 2\times10^5\,\text{MW}$

Power obtained from the nuclear power plant $=10\%$ of $2\times10^5\,\text{MW} = 2\times\ 10^4\,\text{MW} = 2\times10^{10}\ \text{W}$

Energy required from the nuclear plant in one year= Power $\times$ time $= 2\times$ $10^{10}\times365\times24\times60\times60 = 6.3\times10^{17}$ J

Available electric energy per fission $=25\%$ of $200\text{MeV} = (0.25\times200\times\ 1.602\times10^{-13}\ )$ $\text{J} = 8\times10^{-12}\ \text{J}$

Required no. of fission per year $=\dfrac{6.3\times10^{17}}{8\times10^{-12}} = 0.7875\times10^{29}$

Now, 6.023×10^{23} nuclei of ${}^{235}_{92}\text{U}$ have mass $=235\,\text{g}$

$\therefore$ Mass required to produce 7.9×10^{28} nuclei $=\dfrac{235}{6.023\times10^{23}}\times3.95\times10^{28}\ \text{g} \approx\ 3.084\times10^4\ \text{kg}$