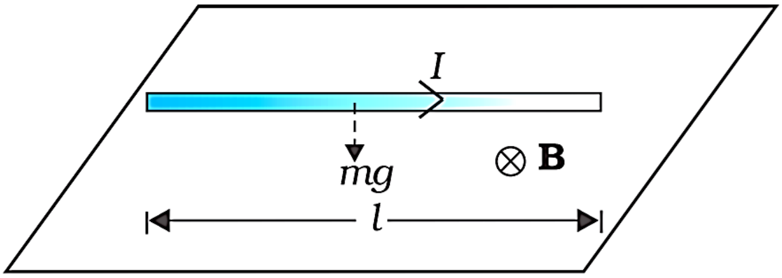
EXAMPLES

Example 4.1: A straight wire of mass $200g$ and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B as shown. What is the magnitude of the magnetic field?



Solution: From the equation: $F = Il \times B$, we see that there is an upward force F having magnitude $Il \times B$.

This must be balanced by the force of gravity, for mid-air suspension: $mg = IlB$

$$\Rightarrow \quad B = \frac{mg}{Il} = \frac{0.2 \times 9.8}{2 \times 1.5} = 0.65\text{T}$$

$$\therefore \boxed{B = 0.65\text{T}}$$

Note: It would have been sufficient to specify m/l, the mass per unit length of the wire

Example 4.2: If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis as shown, which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge).

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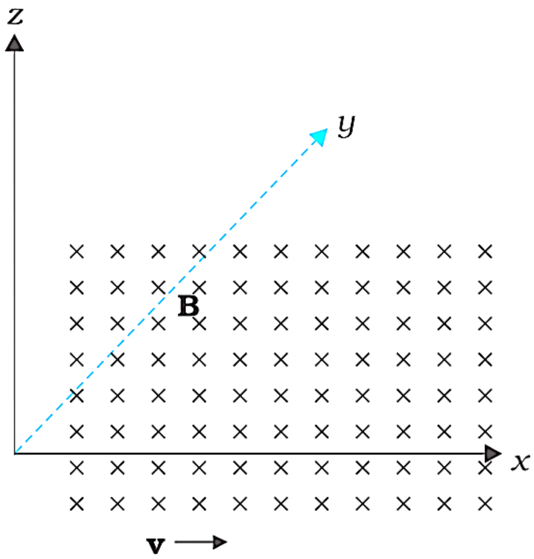

Solution: The particle's velocity v is along the x-axis, whereas B, the magnetic field, is along the y-axis (screw rule or right-hand thumb rule), therefore v B is along the z-axis. As a result,

(a) it will be along the –z axis for electrons.

(b) The force is along the +z axis for a positive charge (proton).

Example 4.3: What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^{7} m / s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. ($1\text{ eV}=1.6\times10^{-19}$ J).

Solution: Using formula: $r = mv/(qB)$

$$r=\frac{9\times10^{-31}\text{kg}\times3\times10^{7}\text{ms}^{-1}}{\left(1.6\times10^{-19}\text{C}\times6\times10^{-4}\text{T}\right)}$$

$$=28\times10^{-2}\text{m}=28\text{cm}$$

$$\nu=v/(2\pi r)=17\times10^{6}\text{s}^{-1}=17\times10^{6}\text{Hz}=17\text{MHz}$$

$$E=(\frac{1}{2})\text{mv}^{2}=(\frac{1}{2})9\times10^{-31}\text{kg}\times9\times10^{14}\text{m}^{2}/\text{s}^{2}=40.5\times10^{-17}\text{J}$$

$$\approx4\times10^{-16}\text{J}=2.5\text{keV}$$

Example 4.4: A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator.

$$\left(e=1.60\times10^{-19}\text{C}, m_{p}=1.67\times10^{-27}\text{kg},\quad 1\text{MeV}=1.6\times10^{-13}\text{J}\right)$$

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Solution: The frequency of the oscillator should be same as that of the frequency of the proton's cyclotron.

Using formula: $B = 2\pi mv/q$

$$B = \frac{6.3\times1.67\times10^{-27}\times10^{7}}{\left(1.6\times10^{-19}\right)}$$

$= 0.66\text{T}$

Therefore, final velocity of proton is: $v = r\times2\pi v$

$v = 0.6\text{m}\times6.3\times10^{7} = 3.78\times10^{7}\,\text{m/s}$

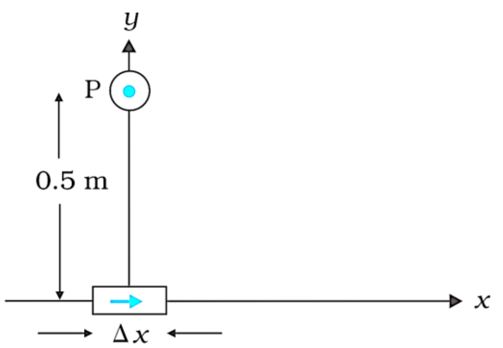
Now,

$$E = \frac{1}{2}\text{mv}^2 = \frac{1.67\times10^{-27}\times14.3\times10^{14}}{\left(2\times1.6\times10^{-13}\right)}$$

$= 7\text{MeV}$

Therefore, $E = 7MeV$.

Example 4.5: An element $\Delta \boldsymbol{l} = \Delta x\hat{\mathbf{i}}$ is placed at the origin and carries a large current $I = 10A$ as shown. What is the magnetic field on the y-axis at a distance of $0.5m$. $\Delta \mathbf{x} = \mathbf{1}$ cm.



Solution: Using equation: $|\text{d}\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

$\text{d}l = \Delta x = 10^{-2}\text{m}, I = 10\text{A}, \quad r = 0.5\text{m} = y, \mu_0/4\pi = 10^{-7}\frac{\text{Tm}}{\text{A}}$

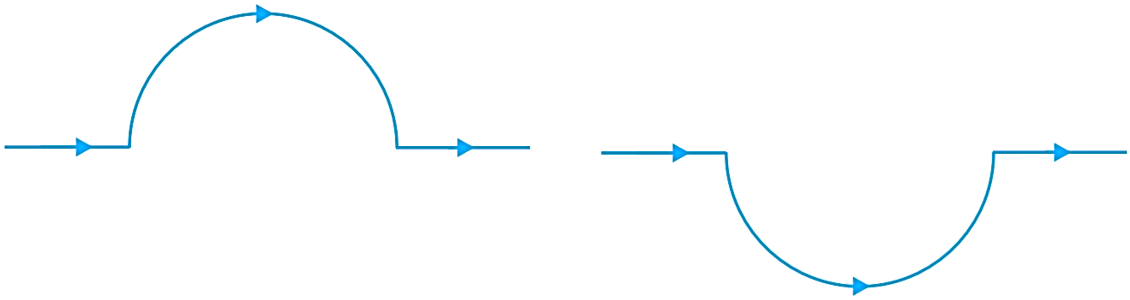
$\theta = 90^{\circ}; \sin\theta = 1$

$$\Rightarrow \quad |\text{d}\mathbf{B}| = \frac{10^{-7}\times10\times10^{-2}}{25\times10^{-2}} = 4\times10^{-8}\text{T}$$

The direction of the field is in the +z-direction. This is so since

$\text{d}\boldsymbol{l}\times\mathbf{r} = \Delta x\hat{\mathbf{i}}\times y\hat{\mathbf{j}} = y\Delta x(\hat{\mathbf{i}}\times\hat{\mathbf{j}})' = y\Delta x\hat{\mathbf{k}}$

Example 4.6: A straight wire carrying a current of $12A$ A is bent into a semi-circular arc of radius $2.0cm$ as shown in figure (a). Consider the magnetic field B at the centre of the arc. (a) What is the magnetic field due to the straight segments? (b) In what way the contribution to B from the semicircle differs from that of a circular loop and in what way does it resemble? (c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in figure (b)?



(a) (b)

Solution:

(a) dl and r for each element of the straight segments are parallel. Therefore, $dl \times r = 0$. $|B|$.

(b) $dl, dl \times r$ are all parallel to each other for all segments of the semi-circular arc (into the plane of the paper). All of these contributions add up to a significant amount. As a result, the right-hand rule determines the direction of B for a semi-circular arc, and its magnitude is half that of a circular loop. As a result, B is 12 normal to the plane of the paper.

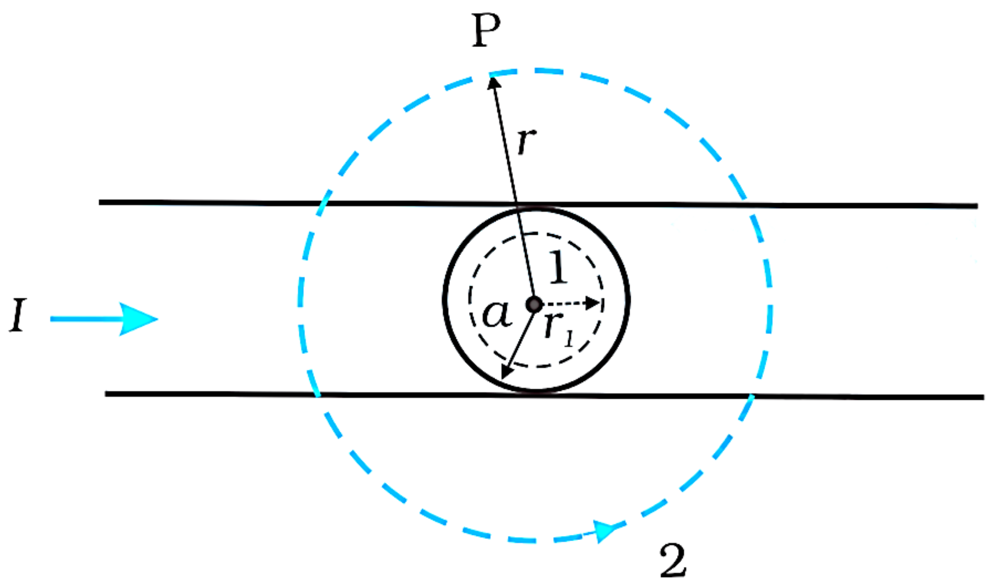
(c) The magnitude of B is same but direction is opposite.

Example 4.7: Consider a tightly wound 100 turn coil of radius $10cm$, carrying a current of $1A$. What is the magnitude of the magnetic field at the centre of the coil?

Solution: We can assume that each circular element has the same radius $R = 10\ cm = 0.1\ m$ because the coil is tightly wrapped. $N = 100$ is the number of turns. The magnetic field's magnitude is,

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 10^2 \times 1}{2 \times 10^{-1}} = 2\pi \times 10^{-4} = 6.28 \times 10^{-4}\text{T}$$

Example 4.8: Figure shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region r < a and r > a.



Solution: (a) Consider the case r > a. The Amperian loop, denoted by the number 2, is a circle that is concentric to the cross-section.

For this loop: $L = 2\pi r$

As a result, we get the well-known term for a long straight wire: $B(2\pi r) = \mu_0 I$

$$= B = \frac{\mu_0 I}{2\pi r}$$

$$= B \propto \frac{1}{r} \quad (r > a)$$

(a) Consider the situation r a. The Amperian loop is a circular with the number 1 on it. Using r as the radius of the circle for this loop, $L = 2\pi r$

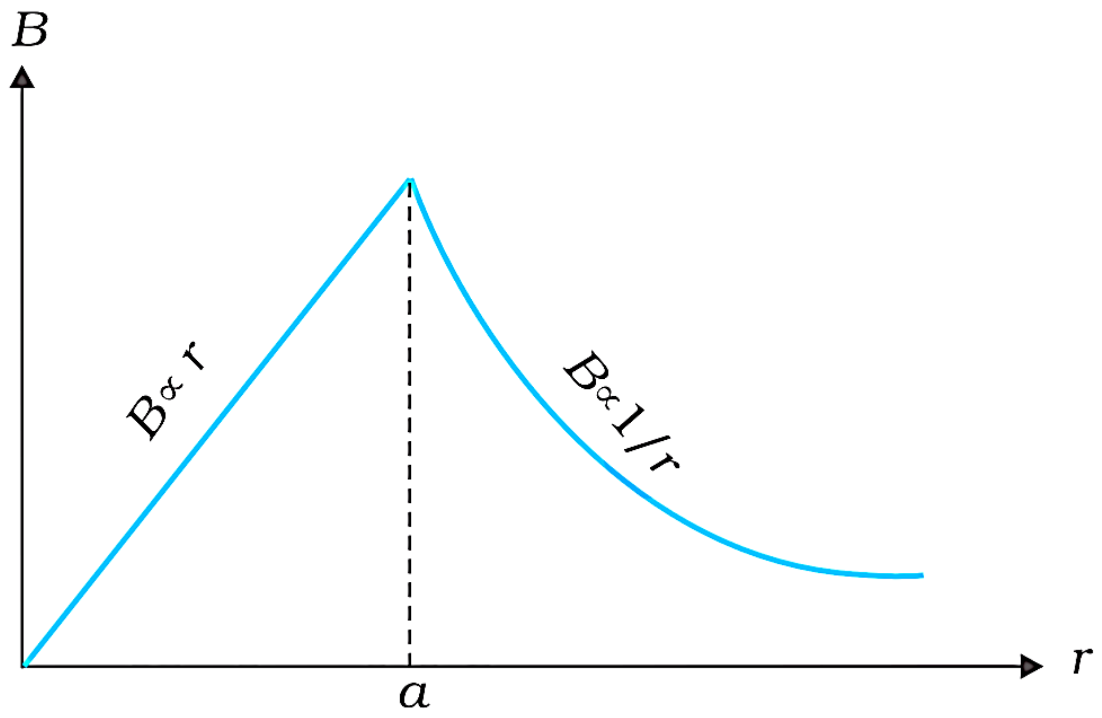
The current enclosed I e is no longer equal to I, but it is now smaller than this number because the present distribution is uniform.The current enclosed is,

$$I_e = I\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{Ir^2}{a^2}$$

Using the Amperian law: $B(2\pi r) = \mu_0 \frac{Ir^2}{a^2}$

$$B = \left(\frac{\mu_0 I}{2a^2}\right) r$$

$$B \propto r \quad (r < a)$$



The magnitude of B is plotted against the distance r from the wire's centre in Figure. The field's direction is determined by the right-hand rule, which is tangential to the relevant circular loop (1 or 2) in this section.

Example 4.9: A solenoid of length 0.5 m has a radius of 1 cm and is made up of $500\,turns$. It carries a current of $5A$. What is the magnitude of the magnetic field inside the solenoid?

Solution: Per unit length, the number of turns is, $n = \frac{500}{0.5} = 1000$ turns / m

The length $l = 0.5\,m$ *and radius* $r = 0.01\,m$.

Thus $l/a = 50$ i.e. $l >> a$.

So, we can use formula: $B = \mu_o nI$

$$B = 4\pi \times 10^{-7} \times 10^{3} \times 5$$
$$B = 6.28 \times 10^{-3}\,\text{T}$$

Example 4.10 The horizontal component of the earth's magnetic field at a certain place is 3.0×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west; (b) south to north?

Solution: $\mathbf{F} = I\mathbf{l} \times \mathbf{B}$

$F = IlB\sin\theta$

Now, force per unit length: $f = F/l = IB\sin\theta$

(a) When the direction of current is from east to west:

$\theta = 90°$

$f = IB = 1\times3\times10^{-5} = 3\times10^{-5}\,\text{Nm}^{-1}$

Thus it is greater than $2\times10^{-7}\ Nm^{-1}$. As a result, in standardising the ampere, it is critical to eliminate the effect of the earth's magnetic field and other stray fields.

The force is in a downward direction. The directional attribute of the cross product of vectors can be used to determine this direction.

(b) When the direction of the current is from south to north:

$\theta = 0^{\circ}$

$\therefore f = 0$

Therefore, there is no force acting on the conductor.

Example 4.11 A 100 turn closely wound circular coil of radius $10\,\text{cm}$ carries a current of $3.2\,\text{A}$.

(a) What is the field at the centre of the coil?

(b) What is the magnetic moment of this coil?

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of $2\,\text{T}$ in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field.

(c) What are the magnitudes of the torques on the coil in the initial and final position?

(d) What is the angular speed acquired by the coil when it has rotated by 90°? The moment of inertia of the coil is $0.1\,\text{kg m}^2$.

Solution: (a) Since,

$$B = \frac{\mu_0 NI}{2R}$$

Here, $N = 100; I = 3.2\,\text{A}$, and $R = 0.1\text{m}$.

Hence,

$$B = \frac{4\pi\times10^{-7}\times10^{2}\times3.2}{2\times10^{-1}}$$

$$= \frac{4\times10^{-5}\times10}{2\times10^{-1}} \quad (\text{using } \pi\times3.2 = 10)$$

$$= 2\times10^{-3}\,\text{T}$$

The right-hand thumb rule determines the direction.

(b) Magnetic Moment is given by: $m = NIA = NI\pi r^2$

$$m = NIA = NI\pi r^2 = 100\times 3.2\times 3.14\times 10^{-2} = 10\,\text{Am}^2$$

The right-hand thumb rule provides the direction once more.

(c) $\tau = |\mathbf{m}\times\mathbf{B}|$

$$= mB\sin\theta$$

Initially, $\theta = 0$. Thus, initial torque $\tau_i = 0$. Finally, $\theta = \frac{\pi}{2}(or 90^{\circ})$.

Thus, final torque $\tau_f = mB = 10\times 2 = 20\,\text{N m}$.

d) From Newton's second law,

$$g\frac{\mathrm{d}\omega}{\mathrm{d}t} = mB\sin\theta$$

where g is the moment of inertia of the coil. From chain rule, $\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}\theta}\omega$

Using this, $g\omega\mathrm{d}\omega = mB\sin\theta\mathrm{d}\theta$

Integrating from

$$\theta = 0 to \theta = \pi/2,$$

$$g\int_0^{\omega_f}\omega\mathrm{d}\omega = mB\int_0^{\pi/2}\sin\theta\mathrm{d}\theta$$

$$g\frac{\omega_f^2}{2} = -mB\cos\theta\Big|_0^{\pi/2} = mB$$

$$\omega_f = \left(\frac{2mB}{g}\right)^{1/2}$$

$$= \left(\frac{2\times 20}{10^{-1}}\right)^{1/2}$$

$$= 20\,\text{s}^{-1}$$

Example 4.12 (a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis).

(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Solution:

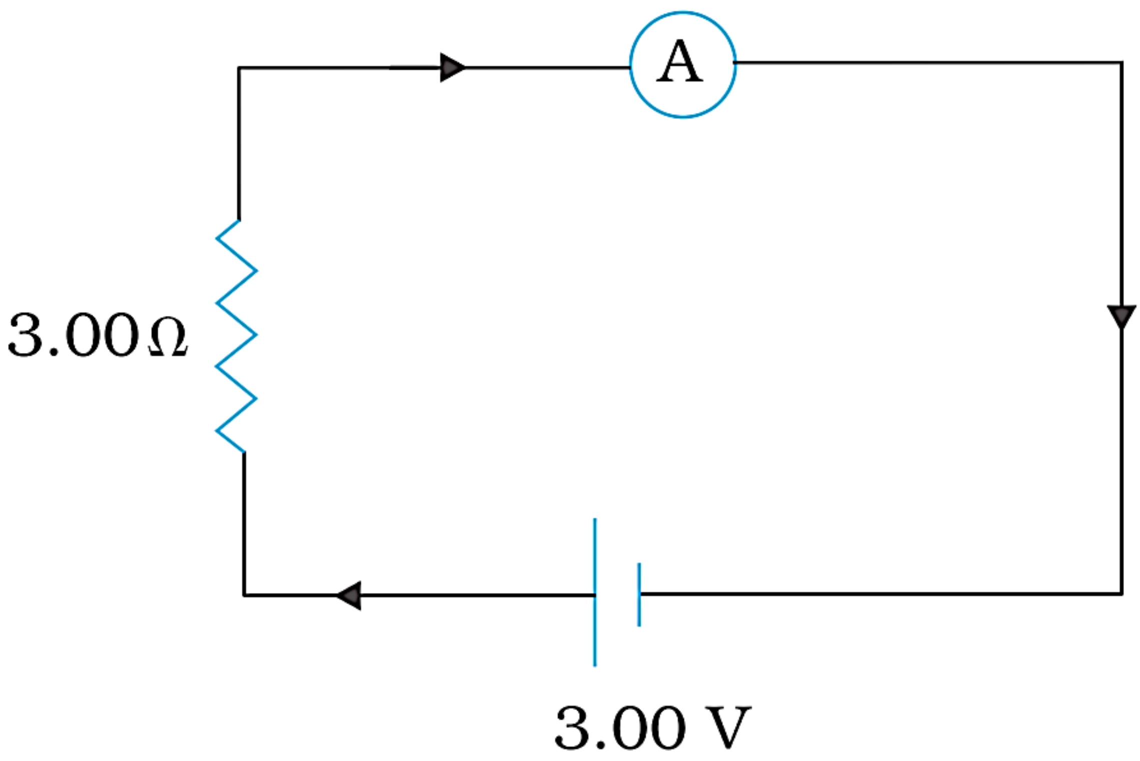
(a) No, because that would necessitate a vertical orientation. But $\tau = I\mathbf{A}\times\mathbf{B}$, and since $\mathbf{A}$ of the horizontal loop is in the vertical direction, τ would be in the plane of the loop for any $\mathbf{B}$.

(b) The area vector
$\mathbf{A}$ of the loop must point in the direction of the external magnetic field for stable equilibrium to exist.

The magnetic field created by the loop is in the same direction as the external field, both normal to the loop's plane, giving rise to the greatest flux of the total field in this orientation.

(c)To maximise flux, it takes a circular form with its plane normal to the field, because a circle enclos es more area for a given perimeter than any other shape.

Example 4.13 In the circuit the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance $R_G = 60.00\Omega$; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance $r_s = 0.02\Omega$; (c) is an ideal ammeter with zero resistance?



Solution:

(a) Total resistance in the circuit is, $R_G + 3 = 63\Omega$. Hence, $I = \dfrac{3}{63} = 0.048\,\text{A}$

(b) The galvanometer's resistance when converted to an ammeter is,

$$\frac{R_G r_s}{R_G + r_s} = \frac{60 \quad \Omega \times 0.02\Omega}{(60+0.02)\Omega} = 0.02\Omega$$

Total Resistance: $0.02\Omega + 3\Omega = 3.02\Omega$. Hence, $I = \frac{3}{3.02} = 0.99\,\text{A}$

(c) The ideal ammeter is one that has no resistance, $I = \frac{3}{3} = 1.00\,\text{A}$

Exercise

4.1: A circular coil of wire consisting of 100 turns, each of radius $8.0\,cm$ carries a current of $0.40A$. What is the magnitude of the magnetic field B at the centre of the coil?

Solution:

Number of turns, $n = 100$

Radius of each turn, $r = 8.0\ cm = 0.08\ m$

Current flowing, $I = 0.4\ A$

Magnitude of the magnetic field at the center of the coil is given by the relation, $|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2\pi nl}{r}$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\text{TmA}^{-1}$

So, $|\vec{B}| = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2\pi \times 100 \times 0.4}{r}$

$|\vec{B}| = 3.14 \times 10^{-4}\,T$

Therefore, the magnitude of the magnetic field is $3.14 \times 10^{-4}\,T$

4.2: A long straight wire carries a current of $35A$. What is the magnitude of the field B at a point $20cm$ from the wire?

Solution: Current in the wire, $I = 35\ A$

Distance of a point from the wire, $r = 20\ cm = 0.2\ m$

Magnetic field at this point is given as: $|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2l}{r}$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\text{TmA}^{-1}$

So,

$$|\vec{B}| = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times35}{0.2}$$
$$= 3.5\times10^{-5}T$$

Therefore, the magnitude of the magnetic field at a point $20cm$ from the wire is $\theta = 60°$ $3.5\times10^{-5}T$

4.3: A long straight wire in the horizontal plane carries a current of $50A$ in north to south direction. Given the magnitude and direction of B at a point $2.5cm$ east of the wire.

Solution: Current flowing, I = 50 A

A point is $2.5cm$ away from the East of the wire.

∴Distance of the point from the wire, r = $2.5cm$.

Magnetic field at that point is given by:

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2l}{r}$$

where, $\mu_0 =$ Permeability of free space $= 4\pi\times10^{-7}\,\mathrm{TmA}^{-1}$

So, $|\vec{B}| = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times50}{2.5} = 4\times10^{-6}T$

Therefore, the point is located normal to the wire length at a distance of 2.5 m. The current in the wire flows in a vertically downward direction.

As a result, the magnetic field at the given position is vertically upward, according to Maxwell's right hand thumb rule.

4.4 A horizontal overhead power line carries a current of $90A$ in east to west direction. What is the magnitude and direction of the magnetic field due to the current $1.5m$ below the line?

Solution: Current in the line, I = $90A$

Point is at distance, r = $1.5m$

Magnetic field at that point is given by: $|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2l}{r}$

where, $\mu_0 =$ Permeability of free space $= 4\pi\times10^{-7}\,\mathrm{TmA}^{-1}$

so,

$$|\vec{B}| = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times90}{1.5}$$
$$= 1.2\times10^{-5}T$$

The flow of the current is from East to West. The point is situated beneath the power line. As a result, according to Maxwell's right hand thumb rule, the magnetic field is directed southward.

4.5 What is the magnitude of magnetic force per unit length on a wire carrying current of $8A$ and making an angle of $30°$ with the direction of a uniform magnetic field of $0.15T$?

Solution: Current, I = $8A$

Magnetic field, B = $0.15T$

Angle between the wire and magnetic field, θ = $30°$.

Magnetic force per unit length on the wire: $F = BI\sin\theta$

⇨ $F = 0.15\times 8\times 1\times \sin 30^{\circ}$

⇨ $F = 0.6\text{Nm}^{-1}$

Therefore, the magnetic force per unit length on wire is 0.6Nm^{-1}

4.6A $3.0\,cm$ wire carrying a current of $10A$ is placed inside a solenoidperpendicular to its axis. The magnetic field inside the solenoid is given tobe $0.27T$. What is the magnetic force on the wire?

Solution: Length of the wire, $l = 3\ cm = 0.03\ m$

⇨ Current, $I = 10A$

⇨ Magnetic field, B = $0.27T$

⇨ Angle between the current and magnetic field, $\theta = 90°$

Magnetic force exerted on the wire: $F = BIl\sin\theta$

$F = 0.27\times 10\times 0.03\sin 90^{\circ}$

$F = 8.1\times 10^{-2}\,\text{N}$

Magnetic force on the wire is $8.1\times 10^{-2}\,\text{N}$. The direction of the

force can be obtained from Fleming’s left hand rule.

4.7 Two long and parallel straight wires A and B carrying currents of $8.0A$ and $5.0A$ in the same direction are separated by a distance of $4.0\,cm$. Estimate the force on a $10cm$ section of wire A.

Solution: Current in wire A, $I_A = 8.0\ A$

⇨ Current in wire B, $I_B = 5.0\ A$

⇨ Distance between the two wires, $r = 4.0\ cm = 0.04\ m$

⇨ Length of a section of wire A: $L = 10\ cm = 0.1\ m$

Force exerted on length L due to the magnetic field: $F = \dfrac{\mu_0 I_A I_B L}{2\pi r}$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\text{TmA}^{-1}$

$$F = \frac{4\pi \times 10^{-7} \times 8 \times 5 \times 0.1}{2\pi \times 0.04} = 2 \times 10^{-5}\,N$$

Hence, the magnitude of force is $2 \times 10^{-5}\,\text{N}$. This is an attractive force normal to A towards B because the direction of current in wire is same.

4.8: A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is $1.8\ cm$. If the current carried is $8.0A$, estimate the magnitude of B inside the solenoid near its centre.

Solution: Length of the solenoid, $l = 80\ cm = 0.8\ m$

There are five layers of windings of 400 turns each on the solenoid.

- ⇨ Total number of turns: $N = 5 \times 400 = 2000$
- ⇨ Diameter of the solenoid: $D = 1.8\ cm = 0.018\ m$
- ⇨ Current carried: I = 8.0 A

Magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\text{TmA}^{-1}$

⇨ so, $$B = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{0.8}$$

$$B = 2.5 \times 10^{-2}\,T$$

Therefore, the magnitude of the magnetic field inside the solenoid near its centre is $2.5 \times 10^{-2}\,T$.

4.9: A square coil of side $10cm$ consists of 20 turns and carries a current of $12A$. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude $0.8T$. What is the magnitude of torque experienced by the coil?

Solution: Length of a side of the coil, $l = 10\ cm = 0.1\ m$

Current flowing in the coil, $I = 12\ A$

Number of turns on the coil, n = 20

Angle made by the plane of the coil with magnetic field, $\theta = 30°$

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Magnetic field, B = $0.8T$

Magnetic torque experienced by the coil in the magneticfield is given by the relation,

$\tau = nBIA\sin\theta$

where, A = Area of the square coil $= 1\times 1 = 0.1\times 0.1 = 0.01\text{m}^2$

$$\tau = 20\times 0.8\times 12\times 0.01\times \sin 30^\circ$$
$$= 0.96\text{Nm}$$

Hence, the magnitude of the torque experienced by the coil is 0.96Nm.

4.10: Two moving coil meters, M1 and M2 have the following particulars:

$R_1 = 10\Omega, N_1 = 30,$
$A_1 = 3.6\times 10^{-3}\text{m}^2, B_1 = 0.25\text{T}$
$R_2 = 14\Omega, N_2 = 42,$
$A_2 = 1.8\times 10^{-3}\text{m}^2, B_2 = 0.50\text{T}$

(The spring constants are identical for the two meters).

Determine the ratio of

(a) current sensitivity and

(b) voltage sensitivity of M2 and M1.

Solution: For moving coil meter M1:

Resistance, $R_1 = 10\ \Omega$

Number of turns, $N_1 = 30$

Area of cross-section, $A_1 = 3.6\ \times\ 10-3\ m^2$

Magnetic field strength, $B_1 = 0.25\ T$

Spring constant $K_1 = K$

For moving coil meter M2:

Resistance, $R_2 = 14\ \Omega$

Number of turns, $N_2 = 42$

Area of cross-section, $A_2 = 1.8\ \times\ 10^{-3}\ m^2$

Magnetic field strength, $B_2 = 0.50\ T$

Spring constant, $K_2 = K$

(a) Current sensitivity of M1: $I_{s1} = \dfrac{N_1 B_1 A_1}{K_1}$

And, current sensitivity of M2: $I_{s2} = \dfrac{N_2 B_2 A_2}{K_2}$

Therefore, $\dfrac{I_{s2}}{I_{s1}} = \dfrac{N_2 B_2 A_2}{N_1 B_1 A_1} = \dfrac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$

Hence the ratio of current sensitivity of M_2 to M_1 is1.4

(b) Voltage sensitivity for M2: $V_{s2} = \dfrac{N_2 B_2 A_2}{K_2 R_2}$

Voltage sensitivity for M1: $V_{s1} = \dfrac{N_1 B_1 A_1}{K_1 R_1}$

Therefore, ratio: $\dfrac{I_{s2}}{V_{s1}} = \dfrac{N_2 B_2 A_2 K_1 R_1}{N_1 B_1 A_1 K_2 R_2} = \dfrac{42 \times 0.5 \times 1.8 \times 10^{-3} \times 10 \times K}{K \times 14 \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1$

Hence, the ratio of voltage sensitivity of M_2 to M_1 is 1.

4.11: In a chamber, a uniform magnetic field of $6.5\ G\ (1\ G = 10^{-4}\ T)$ **is maintained.**

An electron is shot into the field with a speed of $4.8 \times 10^6\ m\ s^{-1}$ **normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit.** $\left(\mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}, \mathrm{m_e} = 9.1 \times 10^{-31}\mathrm{kg}\right)$

Solution: Magnetic field: $B = 6.5\ G = 6.5 \times 10^{-4}\ T$

- ⇨ Speed of the electron, v = $4.8 \times 10^6\ m\ s^{-1}$
- ⇨ Charge on the electron, $e = 1.6 \times 10^{-19}\ C$
- ⇨ Mass of the electron, $m_e = 9.1 \times 10^{-31}\ kg$
- ⇨ Angle between the shot electron and magnetic field, $\theta = 90°$

Magnetic force exerted on the electron in the magnetic field is given as: $F = evB\sin\theta$

The travelling electron receives centripetal force from this force. As a result, the electron begins to move in a circular route with radius r.

Therefore, centripetal force exerted on the electron, $F_e = \dfrac{mv^2}{r}$

The centripetal force imparted on the electron is equal to the magnetic force in equilibrium, i.e.

$$F_e = F$$

$$\Rightarrow \frac{mv^2}{r} = evB\sin\theta$$

$$\Rightarrow r = \frac{mv}{eB\sin\theta}$$

So, $r = \frac{9.1\times10^{-31}\times4.8\times10^{6}}{6.5\times10^{-4}\times1.6\times10^{-19}\times\sin 90^{\circ}} = 4.2\times10^{-2}\,\text{m} = 4.2\text{cm}$

Therefore, the radius of the circular orbit of the electron is 4.2 cm.

4.12: In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Solution: Magnetic field strength, $B = 6.5\ G = 6.5\ \times\ 10^{-4}\ T$

Charge of the electron, $e\ =\ 1.6\ \times\ 10^{-19}\ C$

Mass of the electron, $m_e\ =\ 9.1\ \times\ 10^{-31}\ kg$

Velocity of the electron, $4.8\ \times\ 10^{6}\ m\ s^{-1}$

Radius of the orbit, $r\ =\ 4.2\ cm\ =\ 0.042\ m$

Frequency of revolution of the electron $= v$

Angular frequency of the electron $= \omega = 2\pi v$

$\therefore$ Velocity of the electron; $v = r\omega$

The centripetal force balances the magnetic field on the electron in a circular orbit.

So, $\frac{mv^2}{r} = evB$

$$\Rightarrow \quad eB = \frac{mv}{r} = \frac{m(r\omega)}{r} = \frac{m(r.2\pi v)}{r}$$

$$\Rightarrow \quad v = \frac{Be}{2\pi m}$$

This expression for frequency is independent of the speed of the electron.

On putting the values in this expression, $v = \frac{6.5\times10^{-4}\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}} = 1.82\times10^{6}\,\text{Hz} \approx 18\text{MHz}$

Therefore, the frequency of the electron is around 18 MHz and is independent

of the speed of the electron.

4.13

(a) A circular coil of 30 turns and radius $8.0cm$ carrying a current of $6.0\,A$ is suspended vertically in a uniform horizontal magnetic field of magnitude $1.0T$. The field lines

make an angle of 60^o with the normal of the coil. Calculate the magnitude of the counter torque that must be applied toprevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Solution:

(a) Number of turns: $n = 30$

Radius of the coil, $r = 8.0\ cm = 0.08\ m$

Area of the coil $\pi r^2 = \pi(0.08)^2 = 0.0201\ m^2$

Current flowing: $I = 6.0\ A$

Magnetic field: $B = 1\ T$

Angle between the field lines and normal with the coil surface, $\theta = 60°$

In the magnetic field, the coil is subjected to a torque. As a result, it turns. The counter torque applied to keep the coil from spinning can be calculated using the following formula:
$\tau = nIBA\sin\theta$

⇨ $\tau = 30 \times 6 \times 1 \times 0.0201 \times \sin 60^\circ$

⇨ $\tau = 3.133\text{Nm}$

(b) It can be deduced from the relationship $\tau = nIBA\sin\theta$ that the magnitude of the applied torque is unaffected by the coil's shape. The area of the coil determines this**.** As a result, if the circular coil in the above scenario is substituted with a planar coil of some irregular shape that encloses the same area, the answer remains the same.

Additional Exercise

4.14: Two concentric circular coils X and Y of radii $16cm \& 10cm$, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of $16A$; coil Y has 25 turns and carries a current of $18A$. The sense of the current in X is anticlockwise, and clockwisein Y, for an observer looking at the coils facing west. Give the magnitudeand direction of the net magnetic field due to the coils at their centre.

Solution: Radius of coil X, $r_1 = 16\ cm = 0.16\ m$

Radius of coil Y, $r_2 = 10\ cm = 0.1\ m$

Number of turns of on coil X, $n_1 = 20$

Number of turns of on coil Y, $n_2 = 25$

Current in coil X, $I_1 = 16A$

Current in coil Y, $I_2 = 18A$

⇨ Magnetic field due to coil X at their centre: $B_1 = \dfrac{\mu_0 N_1 I_1}{2r_1}$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\mathrm{TmA^{-1}}$

so, $B_1 = \dfrac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16} = 4\pi \times 10^{-4} T$ (Towards East)

⇨ Magnetic field due to coil Y at their centre: $B_2 = \dfrac{\mu_0 N_2 I_2}{2r_2}$

where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}\,\mathrm{TmA^{-1}}$

so, $B_2 = \dfrac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.10} = 9\pi \times 10^{-4} T$ (Towards West)

Net magnetic field can be obtained as: $B = B_2 - B_1 = 9\pi \times 10^{-4} T - 4\pi \times 10^{-4} T$

⇨ $B = 5\pi \times 10^{-4} T$

⇨ $B = 5 \times 3.14 \times 10^{-4} T = 1.57 \times 10^{-3} T$ (Towards West)

4.15: A magnetic field of $100\ \mathrm{G}$ $\left(1\ \mathrm{G} = 10^{-4}\ \mathrm{T}\right)$ is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about $10^{-3}\ \mathrm{m}^2$. The maximum current carrying capacity of a given coil of wire is $15A$ and the number of turns per unit length that can be wound round a core is at most $1000\ \text{turns m}^{-1}$. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

Solution: Magnetic field strength, $B = 100\ G = 100 \times 10^{-4}\ T$

⇨ Number of turns per unit length, n = $\mathbf{1000\ turns\ m^{-1}}$

⇨ Current in the coil, I = $15\ A$

⇨ Permeability of free space, μ0 $= 4\pi \times 10^{-7}\,\mathrm{TmA^{-1}}$

Magnetic field is given by the relation, $B = \mu_0 n I$

⇨ $nI = \dfrac{B}{\mu_0} = \dfrac{100 \times 10^{-4}}{4\pi \times 10^{-7}} = 7957.74 \approx 8000\,\mathrm{Am^{-1}}$

If the coil's length is $50cm$, its radius is $4cm$, the number of turns is 400, and the current is $10A$, these quantities are not unique for the purpose.There is always the chance of certain limitations modifications.

4.16: For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by,

$$B=\frac{\mu_0 IR^2N}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$$

(a) Show that this reduces to the familiar result for field at the centre of thecoil.

(b) Consider two parallel co-axial circular coils of equal radius R, andnumber of turns N, carrying equal currents in the same direction, andseparated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R, and is given by, $B=0.72-\frac{\mu_0 BNI}{R}$ **, approximately.**

[Such an arrangement to produce a nearly uniform magnetic field over asmall region is known as Helmholtz coils.]

Solution: Radius of circular coil = R

Number of turns = N

Current in the coil = I

Magnetic field at a point on its axis at distance x is given by: $B=\frac{\mu_0 IR^2N}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$

where, $\mu_0 =$ Permeability of free space $=4\pi\times10^{-7}\,\mathrm{TmA^{-1}}$

(a) If the magnetic field at the centre of the coil is considered, then $x=0$

$$B=\frac{\mu_0 IR^2N}{2R^3}=\frac{\mu_0 IN}{2R}$$

This is the well-known result for the magnetic field at the coil's centre.

(b) Radii of two parallel co-axial circular coils = R

Number of turns on each coil = N

Current in both coils = I

Distance between both the coils = R,

Now, consider point Q at distance d from the centre. Then, one coil is at a distance of $\frac{R}{2}+d$ from point Q.

Magnetic field at point Q is given by: $B_1=\frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}+d\right)^2+R^2\right]^{\frac{3}{2}}}$

Now, the other coil is at a distance of $\frac{R}{2}-d$ from point Q.

Therefore, Magnetic field due to this coil is given as: $B_2 = \dfrac{\mu_0 NIR^2}{2\left[\left(\dfrac{R}{2}-d\right)^2 + R^2\right]^{\frac{3}{2}}}$

Net magnetic field: $B = B_1 + B_2$

$$B = \frac{\mu_0 IR^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{\frac{3}{2}} + \left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-\frac{3}{2}}\right]$$

$$B = \frac{\mu_0 IR^2}{2} \times \left(\frac{5R^2}{4}\right)^{\frac{3}{2}}\left[\left(1+\frac{4}{5}\frac{d^2}{R^2}-\frac{4}{5}\frac{d}{R}\right)^{\frac{3}{2}} + \left(1+\frac{4}{5}\frac{d^2}{R^2}+\frac{4}{5}\frac{d}{R}\right)^{\frac{3}{2}}\right]$$

For d<<R, neglecting the factor $\dfrac{d^2}{r^2}$, we get:

$$B \approx \frac{\mu_0 IR^2}{2} \times \left(\frac{5R^2}{4}\right)^{\frac{3}{2}} \times \left[\left(1-\frac{4d}{5R}\right)^{-\frac{3}{2}} + \left(1+\frac{4d}{5R}\right)^{\frac{-3}{2}}\right]$$

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 IN}{R} = 0.72\left(\frac{\mu_0 IN}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-point between the coils is uniform.

4.17: A toroid has a core (non-ferromagnetic) of inner radius $25cm$ and outer radius $26cm$, around which $3500\,turns$ of a wire are wound. If the current inthe wire is 11 A, what is the magnetic field (a) outside the toroid, (b) insidethe core of the toroid, and (c) in the empty space surrounded by the toroid.

Solution: Inner radius of the toroid, $r_1 = 25\ cm = 0.25\ m$

Outer radius of the toroid, $r_2 = 26\ cm = 0.26\ m$

Number of turns on the coil, $N = 3500 turns$

Current in the coil, $I = 11A$

(a) Magnetic field outside a toroid is zero. It is non-zero only inside the

core of a toroid.

(b) Magnetic field inside the core of a toroid is given by

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The relation, $B = \frac{\mu_0 NI}{l}$

Where, μ_0 = Permeability of free space $= 4\pi \times 10^{-7}\,\text{TmA}^{-1}$

$\Rightarrow \quad B = 2\pi\left[\frac{r_1 + r_2}{2}\right]$

$\Rightarrow \quad B = \pi(0.25 + 0.26)$

$\Rightarrow \quad B = \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi}$

$B \approx 3.0 \times 10^{-2}\,\text{T}$

(c) Magnetic field in the empty space surrounded by the toroid is zero.

4.18: Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Solution: (a) The particle's initial velocity is parallel or anti-parallel to the magnetic field. As a result, it goes along a straight route in the field with no deviation.

(b) Yes, the charged particle's end speed will be the same as its initial speed. Because magnetic force can only affect the direction of velocity, not the magnitude, this is the case.

(c) In a chamber with a uniform electric field in the North-South direction, an electron travelling from West to East enters. If the electric force acting on it is equal to and opposite to the magnetic field, the travelling electron will stay undeflected. The magnetic pull is oriented southward. Magnetic fields should be applied vertically downward, according to Fleming's left-hand rule.

4.19: An electron emitted by a heated cathode and accelerated through a potential difference of $2.0\ \text{kV}$, enters a region with uniform magnetic field of $0.15\ \text{T}$. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30° with the initial velocity.

Solution: Magnetic field: B = $0.15\ \text{T}$

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Charge on the electron, $e = 1.6 \times 10^{-19}\ C$

Mass of the electron, $m_e = 9.1 \times 10^{-31}\ kg$

Potential difference, $V = 2.0\ kV = 2 \times 10^3\ V$

Thus, kinetic energy of the electron = eV

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \quad \ldots(1)$$

Where, v = velocity of electron

(a) Magnetic force on the electron provides the required centripetal force of the electron.

Hence, the electron traces a circular path of radius r.

Magnetic force on the electron is given by the relation, B ev

Centripetal Force $= \dfrac{mv^2}{r}$

So, $Bev = \dfrac{mv^2}{r}$

$$r = \frac{mv}{Be}. \qquad \ldots(2)$$

From equations (1) and (2), we get:

$$r = \frac{m}{Be}\left[\frac{2eV}{m}\right]^{\frac{1}{2}} = \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left(\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^3}{9.1 \times 10^{-31}}\right)^{\frac{1}{2}}$$

⇨ $r = 100.55 \times 10^{-5}$

⇨ $r = 1.01 \times 10^{-3}\text{m}$

⇨ $r = 1\text{mm}$

Therefore, the electron has a circular trajectory of radius 1.0 mm normal to themagnetic field.

(b) When the field makes an angle θ of 30° with initial velocity, the initial velocity will be, $v_1 = v\sin\theta$

From equation (2), we can write the expression for new radius as: $r_{1.} = \dfrac{mv_1}{Be} = \dfrac{mv\sin\theta}{Be}$

$$r_1 = \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left[\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}\right]^{\frac{1}{2}}\times\sin30^\circ$$

$$r_1 = 0.5\text{mm}$$

Therefore, the electron has a helical trajectory of radius 0.5 mm along themagnetic field direction.

4.20: A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of $0.75\ \text{T}$. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through $15\ \text{kV}$ enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9.0\times10^{-5}\ \text{V m}^{-1}$, make a simple guess as to what the beam contains. Why is the answer not unique?

Solution: Magnetic field, B = $\mathbf{0.75\ T}$

- Accelerating voltage, $V = 15\ kV = 15\times10^{3}\ V$
- Electrostatic field, E = $\mathbf{9.0\times10^{-5}\ V\ m^{-1}}$
- Mass of the electron = m
- Charge of the electron = e
- Velocity of the electron = v
- Kinetic energy of the electron = eV,

$$\frac{1}{2}mv^2 = eV$$

$$\frac{e}{m} = \frac{v^2}{2V} \quad \ldots(1)$$

We can infer that the electric field is balancing the magnetic field because the particle is undeflected by electric and magnetic fields.

$$\therefore eE = evB$$

$$v = \frac{E}{B} \ldots(2)$$

Putting equation (2) in equation (1), we get: $\frac{e}{m} = \frac{1}{2}\frac{\left(\frac{E}{B}\right)^2}{V} = \frac{E^2}{2VB^2}$

$$= \frac{\left(9.0\times10^{5}\right)^2}{2\times15000\times(0.75)^2} = 4.8\times10^{7}\,\text{C/kg}$$

This value of specific charge e/m is equal to the value of deuteron ordeuterium ions.

This is not a unique answer. Other possible answers are He^{++}, Li^{++}, etc.

4.20: A straight horizontal conducting rod of length $0.45\ \mathrm{m}$ and mass $60\ \mathrm{g}$ is suspended by two vertical wires at its ends. A current of $5.0\mathrm{A}$ is set up in the rod through the wires. (a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? (b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) $\mathrm{g}=9.8\mathrm{m/s}^2$

Solution: Length of the rod, l = 0.45 m

Mass suspended by the wires, $m=60\ g=60\times10^{-3}\ kg$

Acceleration due to gravity, $\mathrm{g}=9.8\mathrm{m/s}^2$

Current in the rod: I = 5 A

(a) Magnetic field $\left(\vec{B}\right)$ is equal and opposite to the weight of the wire i.e.,

$$\therefore\ B=\frac{mg}{Il}=\frac{60\times10^{-3}\times9.8}{5\times0.45}=0.26\mathrm{T}$$

A horizontal magnetic field of $0.26T$ normal to the length of the conductorshould be set up in order to get zero tension in the wire. The magnetic field should be strong enough to produce an upward magnetic force according to Fleming's left hand rule.

(b) If the direction of the current is revered, then the force due to magnetic field and the weight of the wire acts in a vertically downward direction. Total tension in the wire

$$=\mathrm{BIl}+\mathrm{mg}$$

$$=0.26\times5\times0.45+\left(60\times10^{-3}\right)\times9.8$$

$$=1.176\mathrm{N}$$

4.22: The wires which connect the battery of an automobile to its starting motor carry a current of $300A$ (for a short time). What is the force per unit length between the wires if they are $70cm$ long and $1.5cm$ apart? Is the force attractive or repulsive?

Solution: Current in both wires, I = 300 A

Distance between the wires, $r=1.5\ cm=0.015\ m$

Length of the two wires, $l=70\ cm=0.7\ m$

Force between the two wires is given by the relation, $F=\dfrac{\mu_0 I^2}{2\pi r}$

where, $\mu_0=$ Permeability of free space $=4\pi\times10^{-7}\mathrm{TmA}^{-1}$

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$$\therefore \quad F=\frac{4\pi\times10^{-1}\times(300)^2}{2\pi\times0.015}=1.2\text{N}/\text{m}$$

Because the current in the wires is in the opposite direction, a repulsive force exists between them.

4.23: A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of

6.0 cm?

Solution: (a) Magnetic field strength, B = **1.5 T**

Radius of the cylindrical region, $r=10\ cm=0.1\ m$

Current in the wire passing through the cylindrical region, I = **7.0 A**

If the wire crosses the axis, the diameter of the cylindrical region is the length of the wire.

Thus, $l = 2r = 0.2$ m

Angle between magnetic field and current, $\theta = 90^\circ$

Magnetic force acting on the wire is given by the relation, $F=BIl\sin\theta$

$F=1.5\times7\times0.2\times\sin90^\circ=2.1\text{N}$

Therefore, a force of 2.1 N acts on the wire in a vertically downward direction.

(b) New length of the wire after turning it to the Northeast-Northwestdirection can be given as:

$l_1=\dfrac{l}{\sin\theta}$

Angle between magnetic field and current, $\theta=45°$

Force on the wire, $F=BIl_1\sin\theta$

$=1.5\times7\times0.2$

$F=2.1\text{N}$

Therefore, a force of $2.1\ N$ acts vertically downward on the wire. This is independent of angle θ because $l\ sin\theta$ is fixed.

(c) The wire is lowered from the axis by distance, $d\ =\ 6.0\ cm$

Let l_2 be the new length of the wire. $\therefore\left(\frac{l_2}{2}\right)^2=4(d+r)=4(10+6)=4(16)$

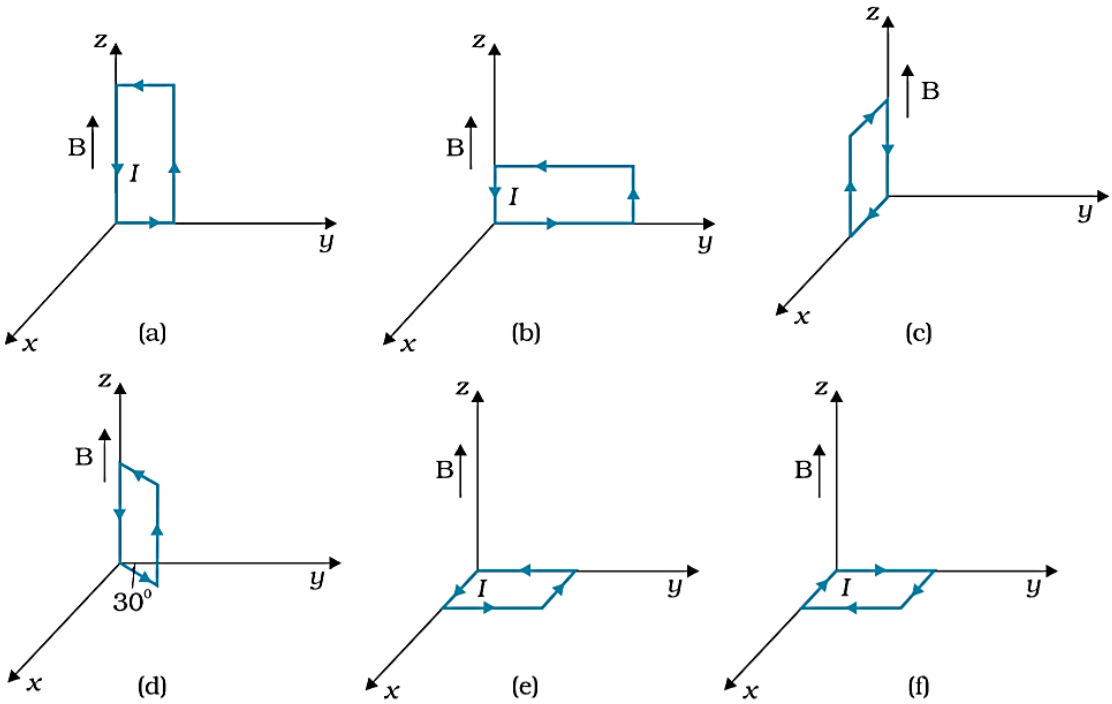
$\therefore\ l_2 = 8\times 2 = 16\text{cm} = 0.16\text{m}$

Magnetic force exerted on the wire, $F_2 = BIl_2 = 1.5\times 7\times 0.16$

$F_2 = 1.68\text{N}$

As a result, a force of 1.68 N operates on the wire in a vertically downward direction.

4.24: A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides $10cm$ and $5cm$ carries a current of 12A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?



Solution: Magnetic field: $B = 3000\ G = 3000\times 10^{-4}\ T = 0.3\ T$

Length of the loop, $l =\ 10\ cm$

Width of the loop, $b\ =\ 5\ cm$

Area of the loop, $A = l\times b = 10\times 5\ = 50cm^2 = 50\times 10^{-4} m^2$

Current in the loop, $I\ =\ 12\ A$

Now, taking the anti-clockwise direction of the current as positive and vise-versa:

(a) Torque, $\vec{\tau} = I\vec{A}\times\vec{B}$
(b) A is normal to the y-z plane, while B is directed along the z-axis, as can be seen in the diagram. $\therefore\ \tau = 12\times\left(50\times 10^{-1}\right)\hat{i}\times 0.3\hat{k} = -1.8\times 10^{-2}\ \hat{j}\text{Nm}$

The torque is $1.8\times10^{-2}\,\text{Nm}$ along the negative y-direction. The force on the loop is zero because the angle between A and B is zero.

(c) This case is similar to case (a). Hence, the answer is the same as (a).

(d) Torque, $\vec{\tau} = I\vec{A}\times\vec{B}$

As a result, it can be seen that A is normal to the x-z plane and B is directed along the z-axis in the diagram. $\therefore \tau = -12\times\left(50\times10^{-4}\right)\hat{j}\times0.3\hat{k} = -1.8\times10^{-2}\hat{i}\text{Nm}$

The torque is $1.8\times10^{-2}\,\text{Nm}$ along the negative x direction and the force is zero.

(e) Magnitude of torque is given as:

$$|\tau| = IAB$$

$$= 12\times50\times10^{-4}\times0.3$$

$$= 1.8\times10^{-2}\,\text{Nm}$$

Therefore, The torque is $1.8\times10^{-2}\,\text{Nm}$ at an angle of 240° with positive x direction. The force is zero.

(f) Torque, $\vec{\tau} = I\vec{A}\times\vec{B}$

$$\tau = \left(50\times10^{-4}\times12\right)\hat{k}\times0.3\hat{k} = 0$$

Therefore, the torque is zero. The force is also zero.

(g) Torque, $\vec{\tau} = I\vec{A}\times\vec{B}$

⇨ $\tau = \left(50\times10^{-4}\times12\right)\hat{k}\times0.3\hat{k} = 0$

Therefore, the torque is zero. The force is also zero.

The direction of is the same in example (e), and the angle between them is zero. If they are displaced, they return to their original position. As a result, the equilibrium is stable.

Whereas, in case (f), the direction of is opposite. The angle between them is $180°$. If disturbed, it does not come back to its original position.Hence, its equilibrium is unstable.

4.25: A circular coil of 20 turns and radius $10cm$ is placed in a uniform magnetic field of $0.10T$ normal to the plane of the coil. If the current in the coil is $5.0A$, what is the

(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area $10^{-5}\,\text{m}^2$, and the free electron density in copper is given to be about $10^{29}\ \text{m}^{-3}$.)

- ⇨ **Solution:** Number of turns on the circular coil, n = 20
- ⇨ Radius of the coil, $r = 10\ cm = 0.1\ m$
- ⇨ Magnetic field strength, $B = 0.10\ T$
- ⇨ Current in the coil, $I = 5.0\ A$
- ⇨ The total torque on the coil is zero because the field is uniform.
- ⇨ The total force on the coil is zero because the field is uniform.
- ⇨ Cross-sectional area of copper coil, $A = 10^{-5}\,m^2$
- ⇨ Number of free electrons per cubic meter in copper, N = $\mathbf{10^{29}\ m^{-3}}$

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⇨ Charge on the electron, $e = 1.6\times10^{-19}\ C$

Magnetic force, $F = Bev_d$

Where, v_d = Drift velocity of electrons

$$v_d = \frac{I}{\text{NeA}}$$

$$\therefore\ F = \frac{BeI}{NeA} = \frac{0.10\times5.0}{10^{29}\times10^{-5}} = 5\times10^{-25}\,\text{N}$$

Therefore, the average force on each electron is $5\times10^{-25}\,\text{N}$

4.26: A solenoid $60cm$ long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (nearits centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to theaxis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) inthe windings of the solenoid can support the weight of the wire? $\mathbf{g = 9.8\ ms^{-2}}$

Solution: Length of the solenoid, $L = 60\ cm = 0.6\ m$

Radius of the solenoid, $r = 4.0\ cm = 0.04\ m$

It is given that there are $\mathbf{3}$ layers of windings of **300 turns** each.

Total number of turns, $n = 3 \times 300 = 900$

Length of the wire, $l = 2\ cm = 0.02\ m$

Mass of the wire, $m = 2.5\ g = 2.5 \times 10-3\ kg$

Current flowing through the wire, $i = 6\ A$

Acceleration due to gravity, $\mathbf{g = 9.8\ ms^{-2}}$

Magnetic field produced inside the solenoid, $B = \frac{\mu_0 nI}{L}$

where, $\mu_0 =$ Permeability of free space $= 4\pi\times10^{-7}\,\text{TmA}^{-1}$

I = Current flowing through the windings of the solenoid

Magnetic force: $F = Bil = \frac{\mu_0 nI}{L} il$

The force on the wire is equal to the weight of the wire.

$$\therefore\ mg = \frac{\mu_0 nIil}{L}$$

$$\Rightarrow \quad I = \frac{mgL}{\mu_0 nil} = \frac{2.5\times10^{-3}\times9.8\times0.6}{4\pi\times10^{-7}\times900\times0.02\times6} = 108\text{A}$$

As a result, the current flowing through the solenoid is $108A$.

4.27: A galvanometer coil has a resistance of $12\ \Omega$ and the metre shows full scale deflection for a current of $3\ \text{mA}$. How will you convert the metre into a voltmeter of range 0 to $18\ \text{V}$?

Solution: Resistance of the galvanometer coil, G = $\mathbf{12\ \Omega}$

Current for which there is full scale deflection, $= 3\ mA = 3\times10^{-3}\ A$

Range of the voltmeter is 0, which needs to be convertedto $18V$.

$V = 18\ V$

To transform the galvanometer into a voltmeter, put a resistor of resistance R in series with it.

Resistance: $R = \frac{V}{I_g} - \text{G}$

$$R = \frac{18}{3\times10^{-3}} - 12 = 6000 - 12 = 5988\Omega$$

So, a resistor of resistance 5988Ω is to be connected in series with the

galvanometer.

4.28: A galvanometer coil has a resistance of $15\ \Omega$ and the metre shows full scale deflection for a current of $4\ \text{mA}$. How will you convert the metre into an ammeter of range 0 to $6\ \text{A}$?

Solution: Resistance: G = $\mathbf{15\ \Omega}$

Current for which the galvanometer shows full scale deflection, $= 4mA = 4\times10^{-3}\ A$

Range of the ammeter is 0, which needs to be converted to $6A$.

Current, $I = 6\ A$

To transform the galvanometer into an ammeter, a shunt resistor of resistance S must be connected in parallel with it.

The value of S is given as: $S = \frac{I_g G}{I - I_g} = \frac{4\times10^{-3}\times15}{6-4\times10^{-3}}$

$$S = \frac{6\times10^{-2}}{6-0.004} = \frac{0.06}{5.996}$$

$S \approx 0.01\Omega = 10\text{m}\Omega$

Therefore, a shunt resistor is to be connected in parallel with the galvanometer.

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