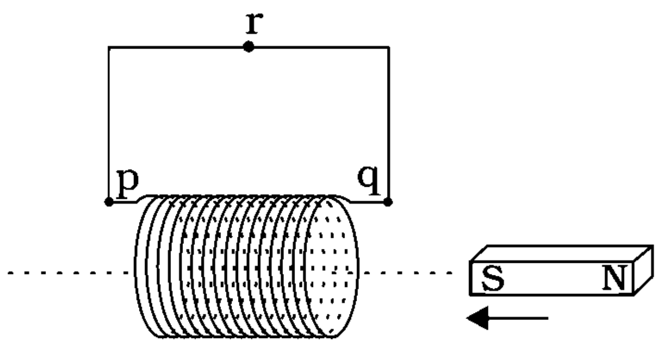
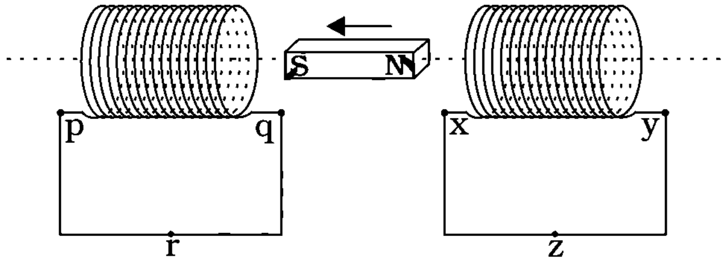
Chapter – 6: ELECTROMAGNETIC INDUCTION

Exercise

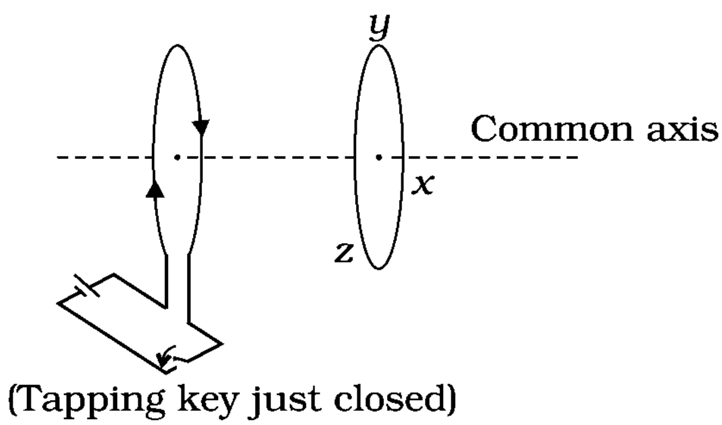
1 . Predict the direction of induced current in the situations described by the following Figs. 6. 18 (a) to (f).



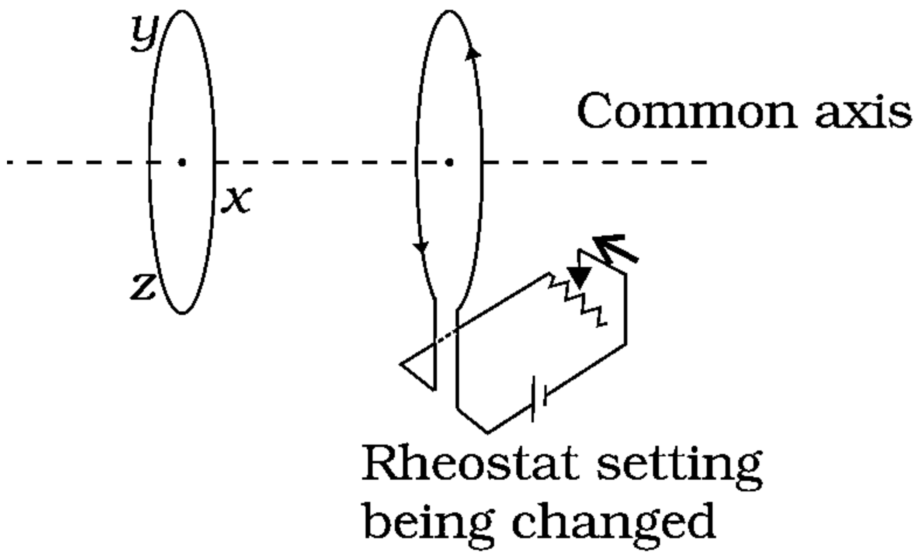
(a)



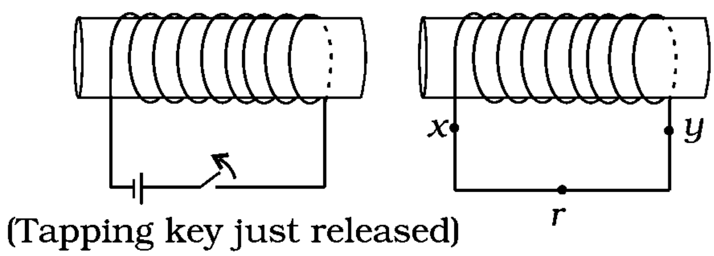
(b)



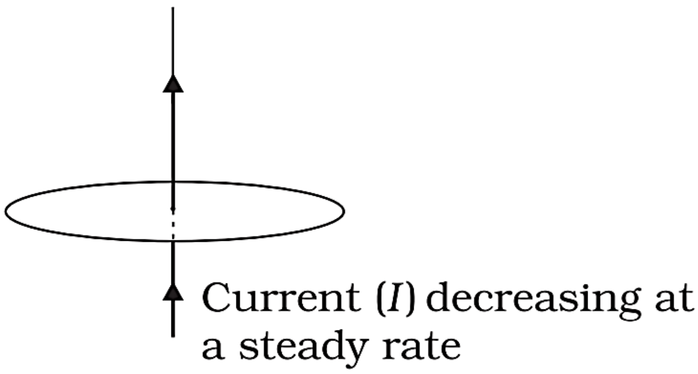
(c)


y
x
z
Common axis
Rheostat setting
being changed

(d)


x
y
r
(Tapping key just released)

(e)


Current (I) decreasing at
a steady rate

(f)

Ans: In a closed loop, Lenz's law determines the direction of the induced current. When the North pole of a bar magnet is moved towards or away from a closed loop, the induced current is shown in the following pairs of figures.

The direction of the induced current in the aforementioned scenarios can be predicted using Lenz's rule as follows:

(a) The induced current flows in the direction of qrpq.

(b) The induced current flows in the direction of prqp.

(c) The induced current flows in the direction of yzxy.

(d) The induced current flows in the direction of zyxz.

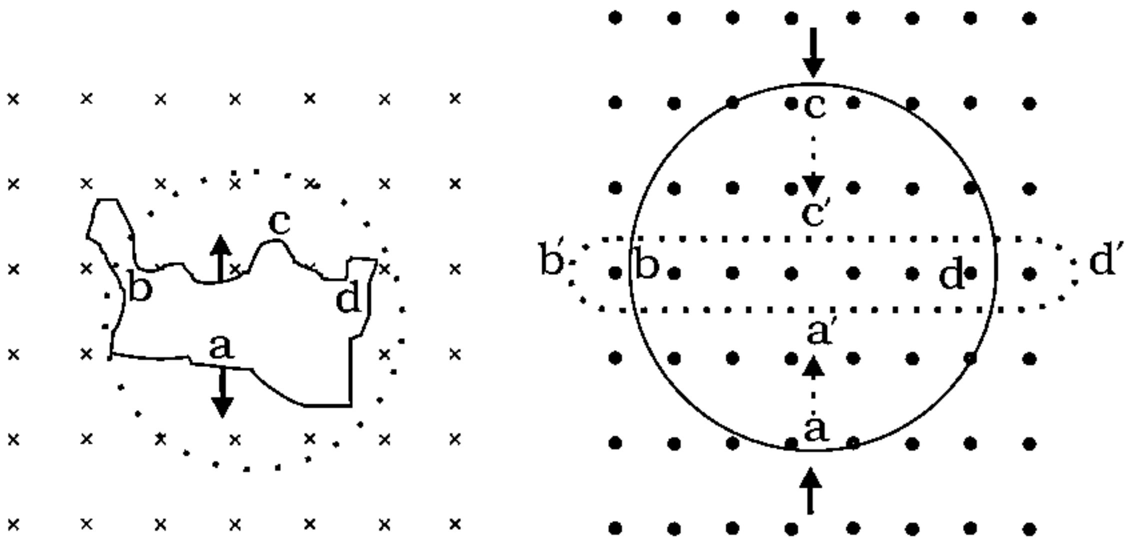
(e) The induced current flows in the direction of xryx.

(f) No current is produced because the field lines are parallel to the closed loop plane.

2. Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6. 19:

(a) A wire of irregular shape turning into a circular shape

(b) A circular loop being deformed into a narrow straight wire



(a) (b)

Ans: Angular frequency, $\omega = 400$ rad/s

Magnetic field strength, B = 0.5 T

One end of the rod has zero linear velocity, while the other end has a linear velocity of

$l\omega$.

$v = \frac{l\omega+0}{2} = \frac{l\omega}{2}$

The rod's average linear velocity, the induced emf between the centre and the ring, and so on.is= $e = Blv = Bl\left(\frac{l\omega}{2}\right) = \frac{Bl^{2\omega}}{2}$

$\frac{0.5\times(1^2)\times 400}{2}$=100

As a result, the emf between the centre and the ring is 100 V.

3. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans: Number of turns on the solenoid = 15 $turns/cm = 1500\ turns/m$

Number of turns per unit length, $n = 1500\ turns$

The solenoid has a small loop of area, $A = 2.0\ cm2 = 2 \times 10 - 4\ m2$

Current carried by the solenoid changes from 2 $A\ to\ 4\ A$.

Change in current in the solenoid,

$di = 4 - 2 = 2\ A$

Change in time, dt = 0.1 s

Given by faradays that In solenoid,induced emf is

$$e = \frac{d\emptyset}{dt}$$

$\emptyset$ =Induced flux

= BA ... (ii)

B = Magnetic field = μ0

μ0 = Permeability of free space = 4π × 10 − 7 H/m

hence from equation {i}

$e = \frac{d}{dt}$BA

$\mu An\left(\frac{di}{dt}\right)$

$= 4 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$

=7.54× 10^{-6}V

Hence,7.54× 10^{-6}V is the induced voltage in the loop.

4. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s–1 in a direction normal to the

(a) longer side,

(b) shorter side of the loop? For how long does the induced voltage last in each case?

Ans: In the rectangular wire,

Length, l = 8 $cm = 0.08\ m$

Width ,b = 2 $cm = 0.02\ m$

Area of the loop is,

A = lb = $0.08 \times 0.02 = 16 \times 10 - 4\ m2$

Magnetic field strength, B = 0.3 T

Velocity of the loop, $v = 1\ cm/s = 0.01\ m/s$

(a) developed emf in the loop is: e

= Blv = $0.3 \times 0.08 \times 0.01 = 2.4 \times 10 - 4\ V$

Time taken to travel along the width is = distance travelled/velocity=b/v

$$= \frac{0.02}{0.01} = 2\ secs$$

Hence, the induced voltage is $2.4 \times 10 - 4\ V$ for 2 s.

(b) Emf developed,

e = Bbv = $0.3 \times 0.02 \times 0.01 = 0.6 \times 10 - 4\ V$

Time taken to travel along the length is = distance travelled/velocity=l/v

$$= \frac{0.08}{0.01} = 8\ secs$$

Hence, the induced voltage is $0.6 \times 10 - 4\ V$ for 8 s.

6. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s–1 about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans: Maximum induced emf = 0.603 V

Average induced emf = 0 V

Maximum current in the coil = $0.0603\ A$

Average loss in power = $0.018\ W$

Radius of the circular coil is ,r = $8\ cm = 0.08\ m$

Area of the coil, $A = \pi r$

$$2 = \pi \times (0.08)2\ m^2$$

Number of turns, $N = 20$

Angular speed, $\omega = 50\ rad/s$

Magnetic field strength, B = $3 \times 10-2\,T$

Resistance of the loop, $R = 10\,\Omega$

Maximum induced emf is :

$$e = N\omega AB$$

$$= 20 \times 50 \times \pi \times (0.08)2 \times 3 \times 10-2$$

$$= 0.603\,V$$

maximum emf induced is $0.603\,V$ in the coil.

the average emf induced in the coil is zero over a full cycle.

Maximum current is :

$$I = \frac{e}{R}$$

$$= \frac{0.603}{10} = 0.0603A$$

Due to joules law of heating,

Average loss in power:

$$p = \frac{el}{2}$$

$$0.603 \times \frac{0.0603}{2} = 0.018W$$

The current created in the coil creates a torque that opposes the coil's spin. The rotor is a third-party component. To maintain the coil revolving consistently, it must generate a torque to counteract this torque. As a result, the external rotor is the source of dissipated power.

7. A horizontal straight wire 10 m long extending from east to west is falling with a speed of $5.0\ m\ s-1$, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^4 Wb m^2

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Ans: Length of the wire i.e. l = $10\ m$

speed of the wire, $v = 5.0\ m/s$

Magnetic field strength is , $B = 0.3 \times 10-4$

a. in the wire,iinduced e.m.f. is , e = Blv

$$= 0.3 \times 10^{-3} \times 5 \times 10$$

$$= 1.5 \times 10^{-3}\, V$$

b. Using Fleming's right hand rule, it's possible to deduce that the induced emf is travelling from West to East.

c. The higher potential is at eastern end.

8. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

Ans: current at initial point, $I1 = 5.0\, A$

Current at final point , $I2 = 0.0\, A$

Change In Current Is,

$$dl = I1 + I2 = 5A$$

Time taken for the change is $t = 0.1\, s$

Average emf, e = $200\, V$

For average emf, we have the following relationship for self-inductance (L) of the coil:

e=$L\frac{dl}{dt}$

L=$\frac{e}{\frac{di}{dt}} = \frac{200}{\frac{5}{0.1}} = 4$H

Hence,4H is the self induction of the coil.

9.A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Ans: Mutual inductance, μ = $1.5\, H$

Current at initial point, $I1 = 0\, A$

Current at final point, $I2 = 20\, A$

Change in current,

$$dI = I2 - I1 = 20 - 0 = 20A$$

Time taken, $t = 0.5\, s$

Induced emf,=$d = \frac{d\emptyset}{dt}$

Change in flux is $d\emptyset$ in the coil.

Emf is linked with mutual inductance in the following way:

$$e = \mu\frac{dI}{dt}$$

From equation 1 and 2 ,

$$\frac{d\emptyset}{dt} = \mu\frac{dI}{dt}$$

$$d\emptyset = 1.5 \times 20 = 30Wb$$

So, $30Wb$ is the change in flux.

10. A jet plane is travelling towards west at a speed of $1800\ km/h$. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of $5 \times 10-4$ T and the dip angle is $30°$.

Ans: Speed of the jet plane is given, $v = 1800\ km/h = 500\ m/s$

Wing span of jet plane is $l = 25\ m$

Earth's magnetic field strength is , $B = 5.0 \times 10-4\ T$

Angle of dip is $\delta = 30°$

The Earth's magnetic field's vertical component,

$$BV = B \sin$$

$$= 5 \times 10-4 \sin 30°$$

$$= 2.5 \times 10-4\ T$$

Voltage difference between the ends of the wing is:

$e = (BV) \times l \times v$

$= 2.5 \times 10-4 \times 25 \times 500 = 3.125\ V$

Hence, $3.125\ V$ is the voltage difference developed between the ends of the wings .

Additional exercise

11:Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of $0.3\ T$ at the rate of $0.02\ T\ s-1$. If the cut is joined and the loop has a resistance of $1.6\ \Omega$ how much power is dissipated by the loop as heat? What is the source of this power?

Ans: 8 cm and 2 cm are the Sides of the rectangular loop

So, the area of the rectangular wire loop is

A = length × width

$= 8 \times 2 = 16\ cm^2$

$= 16 \times 10-4\ m^2$

Initial value of the magnetic field is given by,

$$B' = 0.3T$$

Rate of decrease in the magnetic field is $\frac{dB}{dt}$ = 0.02 T/s

Emf developed in the loop is:

e = $\frac{d\emptyset}{dt}$

Change in flux is d∅=AB

$$e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$= 16 \times 10^{-4} \times 0.02$$

$$= 0.32 \times 10^{-4} V$$

Resistance of the loop is R = 1.6 Ω

$$i = \frac{e}{R}$$

$$= \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} A$$

Power = P=$i^2 r$

$$(2 \times 10^{-5})1^2 \times 1.6$$

$$6.4 \times 10^{-10} W$$

An external agent is responsible for modifying the magnetic field over time, which is the source of this heat loss.

12.A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s−1 in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10 − 3 T cm−1 along the negative x-direction (that is it increases by 10 − 3 T cm−1 as one moves in the negative x-direction), and it is decreasing in time at the rate of 10 − 3 T s−1. Determine the direction and magnitude of the induced current in the loop

if its resistance is 4.50 mΩ.

Ans: Side of the square loop is s = 12 cm = 0.12 m

Area = 0.12 × 0.12 = 0.0144 $m2$

Velocity = 8 cm/s = 0.08 m/s

along negative x-direction ,Gradient of the magnetic field is :

$$\frac{dB}{dx} = 10^{-3} T cm^{-1} = 10^{-1} T m^{-1}$$

Rate of decrease=$\frac{dB}{dt} = 10^{-3}Ts^{-1}$

Change in magnetic flux =

$$\frac{d\emptyset}{dt} = A \times \frac{dB}{dx} \times v$$

$$=144 \times 10^{-4}m^2 \times 10^{-1} \times 0.08$$

$$= 11.52 \times 10^{-5}\ Tm^2s$$

Induced current-$i = \frac{e}{R}$

$$= \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$

$$i = 2.8 \times 10^{-2}A$$

So, the direction of the induced current is as similar as that there is an increase in the flux

through the loop along positive z-direction.

13.it is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of $area\ 2\ cm2\ with\ 25$ closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer Induced current, connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is $0.50\ \Omega$. Estimate the field strength of magnet.

Ans: Area of the coil, $A = 2\ cm2 = 2 \times 10 - 4\ m2$

Number of turns, $N = 25$

Total charge in the coil, $Q = 7.5\ mC = 7.5 \times 10 - 3\ C$

Total resistance of the coil and galvanometer, $R = 0.50\ \Omega$

Induced current =
$i = \frac{e}{R}$……………………………………1

Induced emf is given as:

$e = -N\frac{d\emptyset}{dt}$………………………………..2

d∅ is change is flux.

From equations (1) and (2), we get

I=$\frac{-N\frac{d\emptyset}{dt}}{R}$

Idt=$-\frac{N}{R}d\emptyset$

Where,

B = Magnetic field strength

Integrating equation (3) on both sides,we get,

Initial flux through the coil is ,$\emptyset^i$ = BA

$$\int Idt = \frac{-N}{R}\int_{\emptyset^i}^{\emptyset^F} d\emptyset$$

$$Q = \int Idt$$

Total charge is=$Q = \frac{-N}{R}(\emptyset^f - \emptyset^i) = \frac{-N}{R}(-\emptyset^i) = +\frac{N}{R}(\emptyset^i)$

Q=$\frac{NBA}{R}$

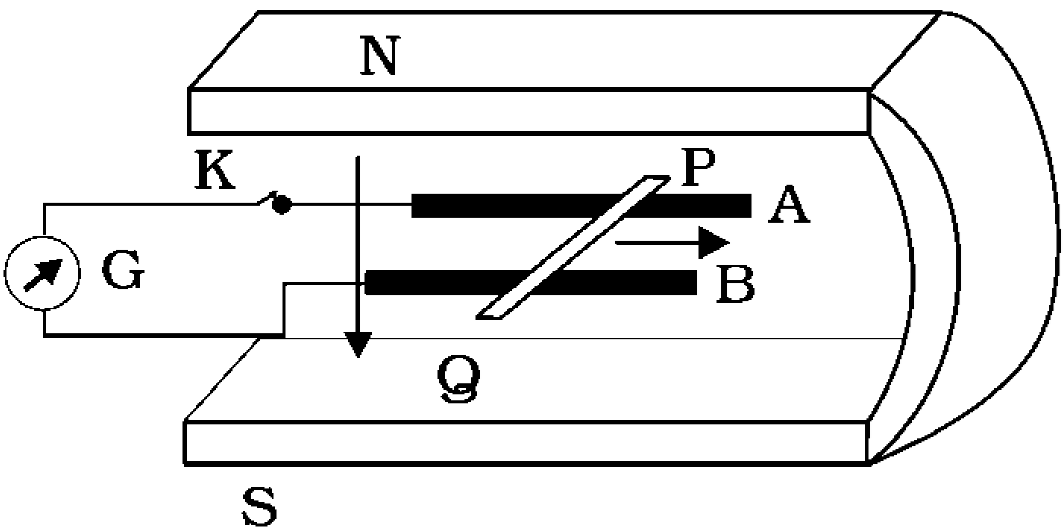
$B = \frac{QR}{NA}$

$= \frac{7.5\times10^{-3}\times0.5}{25\times2\times10^{-4}} = 0.75T$

$0.75T$ is the field strength of the magnet.

14. Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s−1 in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod.

Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 $cm\ s-1$) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit?

What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Ans: Length of the rod is given , l = 15 $cm = 0.15\ m$

Magnetic field strength is given, $B = 0.50\ T$

Resistance of the closed loop is given $R = 9\ m\Omega = 9 \times 10-3\ \Omega$

(a) Induced emf = 9 mV; polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed, $v = 12\ cm/s = 0.12\ m/s$

Induced emf is given as:

$$e = Bvl$$

$$= 0.5 \times 0.12 \times 0.15$$

$$= 9 \times 10-3\ v$$

$$= 9\ mV$$

The induced emf's polarity is such that end P exhibits positive ends while end Q shows negative ends.

(b)Yes, while key K is closed, the continuous flow of current maintains extra charge.

Excess charge builds up at both ends of the rods when key K is opened.

(c)When key K is closed, the continuous flow of current maintains the surplus charge.. Due to the excess charge of opposing nature at both ends of the rod, the magnetic force is neutralised by the electric force setup.

(d) Retarding force, F = IBl

I is Current flowing through the rod.

$\frac{e}{R} = \frac{9\times10^{-3}}{9\times10^{-3}}$ =1A

F=$1 \times 0.5 \times 0.15$

$F = 75 \times 10^{-3} N$

(e) 9 mW; when key K is open no power is expended.

Speed of the rod is v = 12 $cm/s = 0.12\ m/s$

so the power is $P = Fv$

$= 75 \times 10^{-3} \times 0.12$

$= 9 \times 10^{-3} W$

No power is expended when the key K is open .

(f) 9 mW; power is supplied by a third party.

Power = $I^2 r = (1)2 \times 9 \times 10 - 3 = 9\ mW$

An external agent is the source of this power.

(g) Null

Because the rod motion does not cut across the field lines, no emf is induced in the coil in this scenario.

15. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans: Length, l = 30 cm = 0.3 m

Area of cross-section is $A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$

Number of turns in the solenoid is N = 500

Current, I = 2.5 A

time, $t = 10^{-3}\ s$

average emf=

$e = \frac{d\emptyset}{dt}$ ……………………………………………………………………………..1

d∅=change in flux

NAB………………………………………………………………………….2

The magnetic field strength is denoted by B .

$= \mu o \frac{NI}{l}$

Permeability of free space = $4\pi \times 10 - 7\ T\ m\ A - 1$

From equation 1,3 and 2 :

$e = \frac{\mu o N^2 IA}{It} = \frac{4\pi\times10^{-7}\times(500)^2\times2.5\times2.5\times10^{-4}}{0.3\times10^{-3}} = 6.5\text{V}$

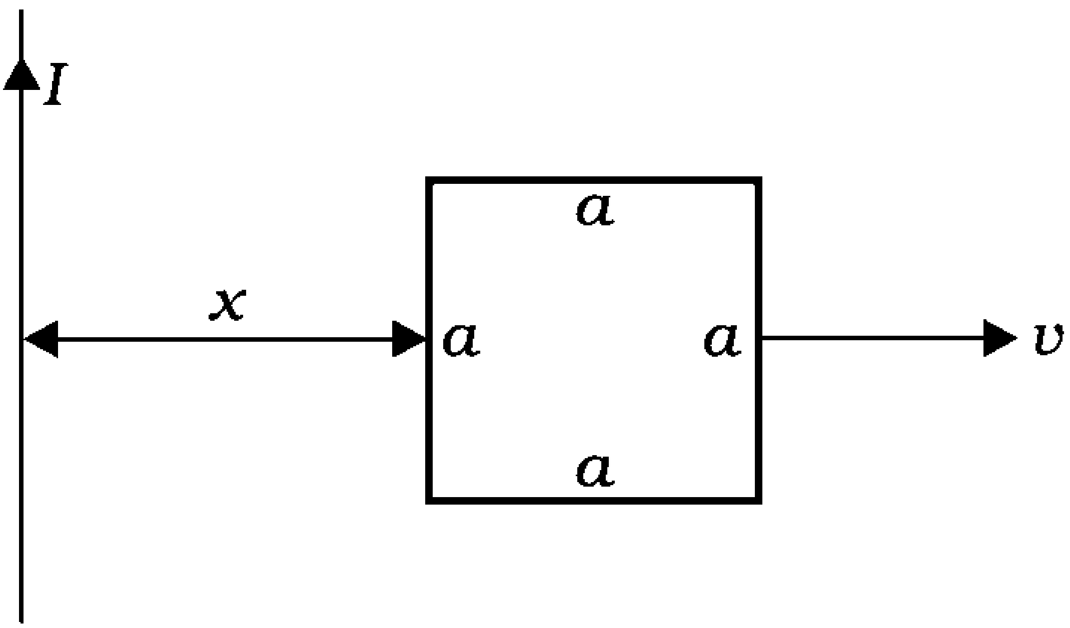
As a result, the solenoid's average back emf is 6.5 V.

16.(a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.

(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, $v = 10\ m/s$.

Calculate the induced emf in the loop at the instant when $x = 0.2\ m$.

Take $a = 0.1$ m and assume that the loop has a large resistance.



Ans: (a) Insert a small element dy in the loop, y distance from the long straight wire.

magnetic flux=$dy, d\emptyset$ =BdA

B is the magnetic field at y.

$= \frac{\mu o I}{2\pi y}$

I is the current in the wire.

$\mu o = Permeability\ of\ free\ space = 4\pi \times 10 - 7$

$d\emptyset = \frac{\mu o\ Ia\ dy}{2\pi r\ \ y}$

$d\emptyset = \frac{\mu o\ Ia}{2\pi r} \int \frac{dy}{y}$

let v be x to a+x

$d\emptyset = \frac{\mu o\ Ia}{2\pi r}\int_{x}^{a+x}\frac{dy}{y}$

$= \frac{\mu o\ Ia}{2\pi r}[\log_e y]_x^{a+x}$

$= \frac{\mu o\ Ia}{2\pi r}\log_e\left(\frac{a+x}{x}\right)$

mutual inductance=∅ =MI

$MI = \frac{\mu o\ Ia}{2\pi r}\log_e\left(\frac{a}{x}+1\right)$

$M = \frac{\mu o}{2\pi}\log_e\left(\frac{a}{x}+1\right)$

$M = \frac{\mu o I}{2\pi X}av$

(b)Emf induced in the loop, e = B'av

according to the question,

$e = \frac{4\pi\times10^{-7}\times50\times0.1\times10}{2\pi\times0.2}$

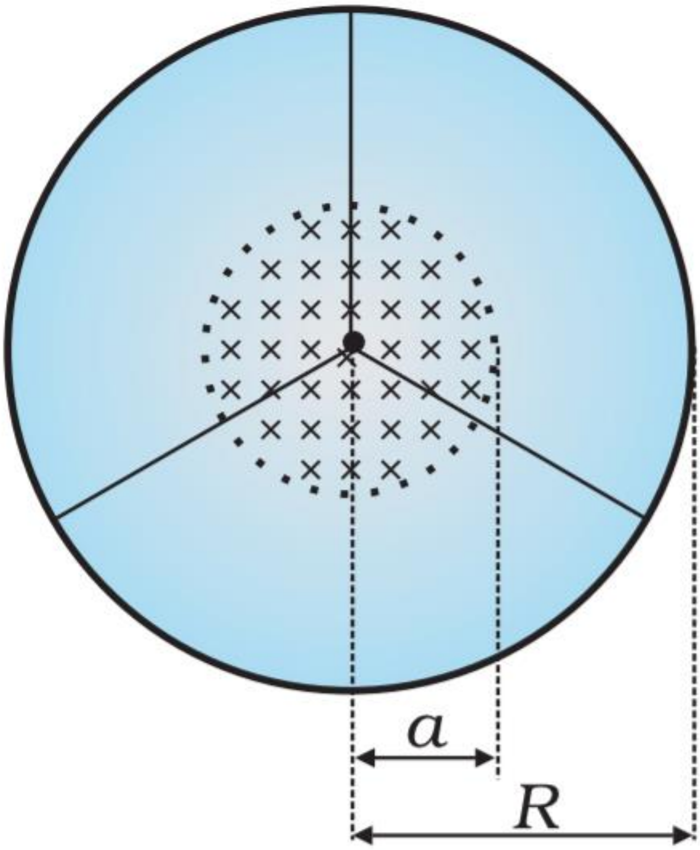
$e = 5\times10^{-5}V$

17.A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis ($\boldsymbol{Fig. 6.22}$). A uniform magnetic field extends over a circular region within the rim. It is given by,

$\boldsymbol{B = -B0\ k\ (r \le a;\ a < R)}$

= 0 (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?



Ans: r is the Distance of the point within the wheel

Mass of the wheel is M

Radius of the wheel is R

magnetic field, $\overline{B} = -Bo\hat{k}$

the magnetic force is balanced by the centripetal force at distance r, i.e.,

$$BQv = \frac{Mv^2}{r}$$

$$B2\pi R\lambda = \frac{Mv}{r}$$

$$v = \frac{B2\pi R\lambda r^2}{M}$$

angular velocity in this case is, $\omega = \frac{v}{R} = \frac{B2\pi\lambda r^2}{MR}$

For $r \leq and\ a < R,$

$$\omega = \frac{2Bo\ a^2\lambda}{MR}\hat{k}$$