

Chapter 7: Alternating Current

Examples

7.1 A light bulb is rated at 100W for a 220 V supply. Find

a. the resistance of the bulb;

Solution (a) We are given $P = 100 \text{ W}$ and $V = 220 \text{ V}$. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220 \text{ V})^2}{100 \text{ W}} = 484 \Omega$$

b. the peak voltage of the source; and

Solution (b) The peak voltage of the source is

$$v_m = \sqrt{2}V = 311 \text{ V}$$

c. The rms current through the bulb.

Solution (c) Since, $P = I V$

$$I = \frac{P}{V} = \frac{100 \text{ W}}{220 \text{ V}} = 0.454 \text{ A}$$

7.2 A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solution The inductive reactance,

$$\begin{aligned}
 X_L &= 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} \Omega \\
 &= 7.85 \Omega
 \end{aligned}$$

The rms current in the circuit is

$$I = \frac{V}{X_L} = \frac{220 \text{ V}}{7.85 \Omega} = 28 \text{ A}$$

7.3 A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

Solution: When a dc source is connected to a capacitor, the capacitor charges, and no current flows in the circuit after charging, hence the lamp does not light. Even if C is decreased, nothing will change. The capacitor provides capacitive reactance ($1/C$) with an ac source, and current flows through the circuit. As a result, the lamp will light up. When C is reduced, reactance rises, and the bulb shines less brightly than before.

7.4 A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

Solution The capacitive reactance is

$$X_c = \frac{1}{2\pi\nu C} = \frac{1}{2\pi(50\text{Hz})(15.0 \times 10^{-6}\text{ F})} = 212\Omega$$

The rms current is

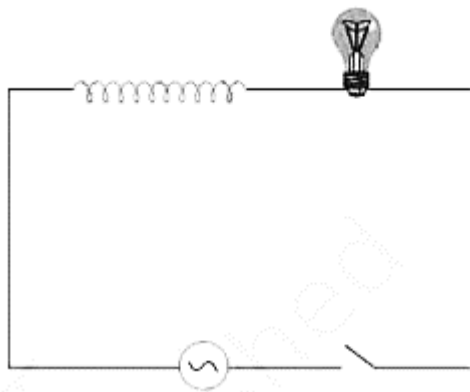
$$I = \frac{V}{X_c} = \frac{220\text{ V}}{212\Omega} = 1.04\text{ A}$$

The peak current is

$$i_m = \sqrt{2}I = (1.41)(1.04\text{ A}) = 1.47\text{ A}$$

This current oscillates between $+1.47\text{ A}$ and -1.47 A , and is ahead of the voltage by $\pi/2$. If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

7.5 A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig. 7.11.



The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases; (b) decreases; (c) is unchanged, as the iron rod is inserted. Give your answer with reasons.

Solution The magnetic field inside the coil magnetises the iron rod as it is introduced, increasing the magnetic field inside it. As a result, the coil's inductance rises. As a result, the coil's inductive reactance increases. As a result, the inductor receives a bigger portion of the provided ac voltage, leaving less voltage across the bulb. As a result, the light bulb's radiance fades.

7.6 A resistor of $200\ \Omega$ and a capacitor of $15.0\ \mu\text{F}$ are connected in series to a $220\ \text{V}$, $50\ \text{Hz}$ ac source.

(a) Calculate the current in the circuit;

Solution Given

$$R = 200\Omega, C = 15.0\ \mu\text{F} = 15.0 \times 10^{-6}\text{ F}$$

$$V = 220\text{ V}, \nu = 50\text{ Hz}$$

(a) In order to calculate the current, we need the impedance of the circuit. It is

$$\begin{aligned}
 &= \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi\nu C)^{-2}} \\
 &= \sqrt{(200\Omega)^2 + (2 \times 3.14 \times 50 \times 15.0 \times 10^{-6} \text{ F})^{-2}} \\
 &= \sqrt{(200\Omega)^2 + (212.3\Omega)^2} \\
 &= 291.67\Omega
 \end{aligned}$$

Therefore, the current in the circuit is

$$I = \frac{V}{Z} = \frac{220 \text{ V}}{291.5\Omega} = 0.755 \text{ A}$$

(b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Solution (b) Since the current is the same throughout the circuit, we have

$$V_R = IR = (0.755 \text{ A})(200\Omega) = 151 \text{ V}$$

$$V_C = IX_C = (0.755 \text{ A})(212.3\Omega) = 160.3 \text{ V}$$

The sum of the two voltages, V_R and V_C , is 311.3 V, which is greater than the 220 V source voltage. What is the best way to overcome this conundrum? The two voltages are not in phase, as you learned from the book. As a result, they can't be added together like regular numbers. The two voltages are ninety degrees out of phase. As a result, the Pythagorean theorem must be used to calculate the total of these voltages:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = 220 \text{ V}$$

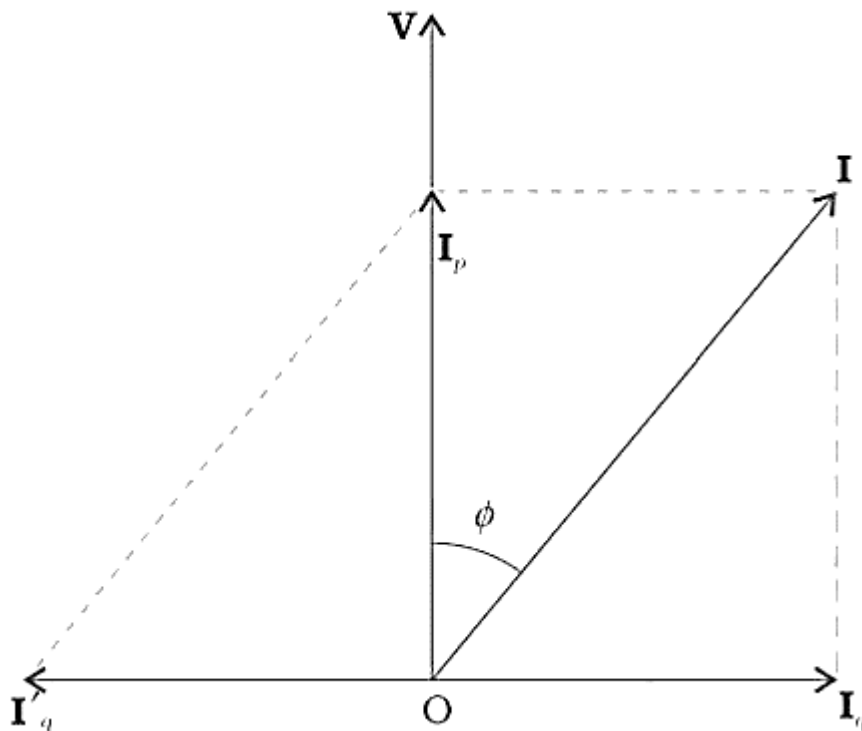
When the phase difference between two voltages is appropriately accounted for, the total voltage across the resistor and capacitor equals the source voltage.

7.7 (a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

Solution (a) We know that $P = I V \cos \phi$ where $\cos \phi$ is the power factor. To supply a given power at a given voltage, if $\cos \phi$ is small, we have to increase current accordingly. This, however, will result in significant transmission power loss ($I^2 R$).

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

(b) Suppose in a circuit, current I lags the voltage by an angle ϕ . Then power factor $\cos \phi = R/Z$. We can improve the power factor (tending to 1) by making Z tend to R . With the aid of a phasor diagram, (Fig. 7.17) let us comprehend how this can be achieved. Let us resolve I into two components. I_p along



Let's break I down into two parts. I_p is parallel to the applied voltage V , whereas I_q is perpendicular to it. As you learned in Section 7.7, I_q is known as the wattless component since there is no power loss associated to this component of current. Because it is in phase with the voltage and corresponds to power loss in the circuit, I_p is known as the power component.

This research shows that if we wish to enhance power factor, we need to replace the trailing wattless current I_q with an equivalent leading wattless current I'_q . This can be accomplished by connecting a capacitor of appropriate value in series with I_q and I'_q , resulting in P being essentially $I_p V$.

7.8 A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$, $L = 25.48 \text{ mH}$, and $C = 796\mu\text{F}$.

Find

(a) the impedance of the circuit;

Solution (a) To find the impedance of the circuit, we first calculate X_L and X_C .

$$\begin{aligned}
 X_L &= 2\pi\nu L = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} \Omega \\
 &= 8\Omega \\
 X_C &= \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\Omega
 \end{aligned}$$

Therefore,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8 - 4)^2} = 5\Omega$$

(b) the phase difference between the voltage across the source and the current;

Solution (b) Phase difference, $\phi = \tan^{-1} \frac{X_C - X_L}{R}$

$$= \tan^{-1} \left(\frac{4-8}{3} \right) = -53.1^\circ$$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) the power dissipated in the circuit; and

Solution (c) The power dissipated in the circuit is

$$\text{Now, } I = \frac{i_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{283}{5} \right) = 40 \text{ A}$$

$$\text{Therefore, } P = (40 \text{ A})^2 \times 3\Omega = 4800 \text{ W}$$

(d) the power factor.

$$\text{Solution (d) Power factor} = \cos \phi = \cos(-53.1^\circ) = 0.6$$

7.9 Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs?

Solution (a) The frequency at which the resonance occurs is

$$\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}} \\ &= 222.1 \text{ rad / sv}_r \\ &= \frac{\omega_0}{2\pi} = \frac{221.1}{2 \times 3.14} \text{ Hz} = 35.4 \text{ Hz} \end{aligned}$$

(b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

(b) The impedance Z at resonant condition is equal to the resistance:

$$Z = R = 3\Omega$$

The rms current at resonance is

$$= \frac{V}{Z} = \frac{V}{R} = \left(\frac{283}{\sqrt{2}} \right) \frac{1}{3} = 66.7 \text{ A}$$

The power dissipated at resonance is

$$P = I^2 \times R = (66.7)^2 \times 3 = 13.35 \text{ kW}$$

As you can see, the power wasted at resonance in this example is greater than the power dissipated in Example 7.8.

7.10 At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

Solution In ac circuits, the metal detector works on the principle of resonance. You are travelling through a coil of several turns as you walk through a metal detector. The coil is connected to a capacitor that has been calibrated to achieve resonance in the circuit. The impedance of the circuit changes when you walk through with metal in your pocket, resulting in a large shift in current in the circuit. This difference in current is sensed, and the electronic circuitry generates an alarm sound.

7.11 Show that in the free oscillations of an LC circuit, the sum of energies stored in the capacitor and the inductor is constant in time.

Solution Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L . As you have studied in Section 7.8, this LC circuit will sustain an oscillation with frequency

$$\omega \left(= 2\pi\nu = \frac{1}{\sqrt{LC}} \right)$$

At an instant t , charge q on the capacitor and the current i are given by:

$$q(t) = q_0 \cos \omega t$$

$$i(t) = -q_0 \omega \sin \omega t$$

Energy stored in the capacitor at time t is

$$U_E = \frac{1}{2} CV^2 = \frac{1}{2} \frac{q^2}{C} = \frac{q_0^2}{2C} \cos^2(\omega t)$$

Energy stored in the inductor at time t is

$$\begin{aligned} U_M &= \frac{1}{2} Li^2 \\ &= \frac{1}{2} Lq_0^2 \omega^2 \sin^2(\omega t) \\ &= \frac{q_0^2}{2C} \sin^2(\omega t) \quad (\because \omega = 1/\sqrt{LC}) \end{aligned}$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) = \frac{q_0^2}{2C}$$

This sum is constant in time as q_0 and C , both are time-independent.

Exercises

7.1: A 100 ohm resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit?

Answer:

Resistance of the resistor, $R = 100$ ohm

Supply voltage, $V = 220V$

Frequency, $\nu = 50Hz$

The rms value of the current in the circuit is given as

$$I = v \div r = 220 \div 100 = 2.20A$$

(b) What is the net power consumed over a full cycle?

Answer:

(b) The net power consumed over a full cycle is given as:

$$P = VI = 220 \times 2.2 = 484W$$

7.2.(a) The peak voltage of an ac supply is 300V. What is the rms voltage?

Answer:

(a) Peak voltage of the ac supply, $V_o = 300V$

Rms voltage is given as:

$$V = V_o \sqrt{2} = 300 \sqrt{2} = 212.1V$$

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

(b) The rms value of current is given as:

$$I = 10A$$

Now, peak current is given as: $I_o = \sqrt{2}I = \sqrt{2} \times 10 = 14.1A$

7.3: A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.3:

Inductance of inductor, $L = 44mH = 44 \times 10^{-3}H$

Supply voltage, $V = 220V$

Frequency, $\nu = 50 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu$

Inductive reactance, $X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 44 \times 10^{-3} \Omega$

rms value of current is given as:

$$I = \frac{V}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}} = 15.92 \text{ A}$$

Hence, the rms value of current in the circuit is 15.92 A.

7.4: A $60 \mu\text{F}$ capacitor is connected to a 110 V , 60 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.4:

Capacitance of capacitor, $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Supply voltage, $V = 110 \text{ V}$

Frequency, $\nu = 60 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}} \Omega$$

rms value of current is given as:

$$I = \frac{V}{X_C} = \frac{110}{\frac{1}{2\pi \times 60 \times 60 \times 10^{-6}}} = 2.49 \text{ A}$$

Hence, the rms value of current is 2.49 A.

7.5: In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Answer 7.5: In the inductive circuit, Rms value of current,

In the inductive circuit

Rms, value of current $I = 15.92 \text{ A}$

Rms value of voltage, $V = 220 \text{ V}$

Hence, the net power absorbed can be obtained by the relation,

$$P = VI \cos \Phi$$

Where,

Φ = Phase difference between V and I .

For a pure inductive circuit, the phase difference between alternating voltage and current is 90° i.e., $\Phi = 90^\circ$.

Hence, $P=0$ i.e., the net power is zero.

In the capacitive circuit,

Rms value of current, $I=2.49\text{ A}$

Rms value of voltage, $V=110\text{ V}$

Hence, the net power absorbed can be obtained as:

$$P=VI\cos\Phi$$

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90° i.e., $\Phi=90^\circ$.

Hence, $P=0$ i.e., the net power is zero.

7.6: Obtain the resonant frequency ωr of a series LCR circuit with $L=2.0\text{ H}$, $C=32\ \mu\text{F}$ and $R=10\ \Omega$. What is the Q-value of this circuit?

Answer 7.6: Inductance, $L=2.0\text{ H}$

Capacitance, $C=32\ \mu\text{F}=32\times 10^{-6}\text{ F}$

Resistance, $R=10\ \Omega$

Resonant frequency is given by the relation,

$$\begin{aligned}\omega r &= 1\sqrt{LC} = 1\sqrt{2\times 32\times 10^{-6}} \\ &= 18\times 10^{-3} = 125\text{ rad / s.}\end{aligned}$$

Now, Q-value of the circuit is given as:

$$Q = \frac{1}{R}\sqrt{LC} = \frac{1}{10}\sqrt{2\times 32\times 10^{-6}} = 110\times 4\times 10^{-3} = 25$$

Hence, the Q-Value of this circuit is 25.

7.7: A charged $30\ \mu\text{F}$ capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Answer 7.7:

Capacitance, $C=30\ \mu\text{F}=30\times 10^{-6}\text{ F}$ Inductance, $L=27\text{ mH}=27\times 10^{-3}\text{ H}$

Angular frequency is given as:

$$\omega r = 1\sqrt{LC}$$

$$=1\sqrt{27\times 10^{-3}\times 30\times 10^{-6}}=19\times 10^{-4}=1.11\times 10^3 \text{ rad / s}$$

Hence, the angular frequency of free oscillations of the circuit is $1.11\times 10^3 \text{ rad / s}$.

7.8: Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC . What is the total energy stored in the circuit initially? What is the total energy at later time?

Answer 7.8: Capacitance of the capacitor, $C=30\mu F=30\times 10^{-6} F$

Inductance of the inductor, $L=27 \text{ mH}=27\times 10^{-3} H$

Charge on the capacitor, $Q=6 \text{ mC}=6\times 10^{-3} C$

Total energy stored in the capacitor can be calculated as:

$$E=12Q^2 C=12(6\times 10^{-3})^2\times 30\times 10^{-6}=610=0.6 J$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

7.9: A series LCR circuit with $R=20\Omega$, $L=1.5 H$ and $C=35\mu F$ is connected to a variable frequency $200 V$ ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Answer 7.9: At resonance, the frequency of the supply power equals the natural frequency of the given LCR circuit.

Resistance, $R=20\Omega$

Inductance, $L=1.5 H$

Capacitance, $C=35\mu F=30\times 10^{-6} F$

AC supply voltage to the LCR circuit, $V=200V$

Impedance of the circuit is given by the relation,

$$Z=\sqrt{R^2+(XL-XC)^2}$$

At resonance, $XL=XC$

$$\therefore Z=R=20\Omega$$

Current in the circuit can be calculated as:

$$I=V\div Z=200\div 20=10 A$$

Hence, the average power transferred to the circuit in one complete cycle:

$$VI=200\times 10=2000W.$$

7.10: A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of $200 \mu H$, what must be the range of its variable capacitor?

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

Answer:

The range of frequency (ν) of a radio is $800 \text{ kHz to } 1200 \text{ kHz}$.

Lower tuning frequency, $\nu_1 = 800 \text{ kHz} = 800 \times 10^3 \text{ Hz}$

Upper tuning frequency, $\nu_2 = 1200 \text{ kHz} = 1200 \times 10^3 \text{ Hz}$

Effective inductance of circuit $L = 200 \mu H = 200 \times 10^{-6} \text{ H}$

Capacitance of variable capacitor for ν_1 is given as: $C_1 = \frac{1}{\omega_1^2 L}$

Where,

$\omega_1 =$ Angular frequency for capacitor C_1

$$= 2\pi\nu_1$$

$$= 2\pi \times 800 \times 10^3 \text{ rad / s}$$

$$\therefore C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 1.9809 \times 10^{-10} \text{ F} = 198 \text{ pF}$$

Capacitance of variable capacitor for ν_2 is given as:

$$C_2 = \frac{1}{\omega_2^2 L}$$

Where,

$\omega_2 =$ Angular frequency for capacitor C_2

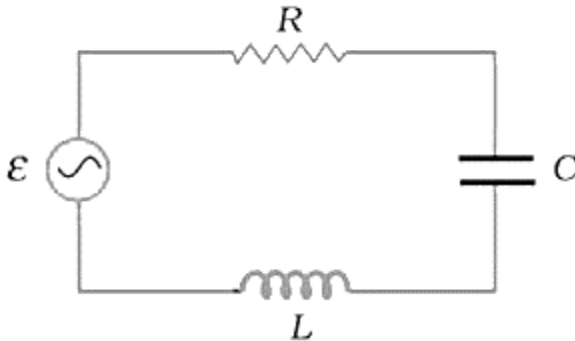
$$= 2\pi\nu_2 = 2\pi \times 1200 \times 10^3 \text{ rad / s}$$

$$\therefore C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 0.8804 \times 10^{-10} \text{ F} = 88 \text{ pF}$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

7.11: Figure shows a series LCR circuit connected to a variable frequency 230 V source.
 $L=5.0\text{ H}$, $C=80\mu\text{F}$, $R=40\Omega$



(a) Determine the source frequency which drives the circuit in resonance.

Answer 7.11:

Inductance of the inductor, $L=5.0\text{ H}$

Capacitance of the capacitor, $C=80\mu\text{F}=80\times 10^{-6}\text{ F}$

Resistance of the resistor, $R=40\Omega$

Potential of the variable voltage source, $V=230\text{ V}$

(a) Resonance angular frequency is given as:

$$\omega r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50\text{ rad/s}$$

As a result, for a source frequency of, 50 rad/s the circuit will achieve resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(b) Impedance of the circuit is given by the relation:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C \Rightarrow Z = R = 40\Omega$

Amplitude of the current at the resonating frequency is given as: $I_o = \frac{V_o}{Z}$

Where,

$$V_o = \text{Peak voltage} = \sqrt{2}V$$

$$\therefore I_o = \frac{\sqrt{2}V}{Z} = \frac{\sqrt{2} \times 230}{40} = 8.13 \text{ A}$$

As a result, the circuit's impedance is 40 at resonance, and the current amplitude is 8.13 A.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

(c) Rms potential drop across the inductor,

$$(VL)_{rms} = I \times \omega r L$$

Where,

$$I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} = \frac{23}{4} \text{ A}$$

$$\therefore (VL)_{rms} = \frac{23}{4} \times \left(\frac{1}{50} \times 5 \right) = 1437.5 \text{ V}$$

Potential drop across the capacitor: \therefore

$$(VC)_{rms} = I \times \frac{1}{\omega r C} = \frac{23}{4} \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5 \text{ V}$$

Potential drop across the resistor:

$$(VR)_{rms} = IR = \frac{23}{4} \times 40 = 230 \text{ V}$$

Potential drop across the LC combination:

$$VLC = I(XL - XC)$$

At resonance,

ADDITIONAL EXERCISES

7.12: An LC circuit contains a 20 mH inductor and a 50 μ F capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t=0$.

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

Answer: Inductance of the inductor, $L = 20 \text{ mH} = 20 \times 10^{-3} \text{ H}$

Capacitance of the capacitor, $C = 50 \mu\text{F} = 50 \times 10^{-6} \text{ F}$

Initial charge on the capacitor, $Q = 10 \text{ mC} = 10 \times 10^{-3} \text{ C}$

(a) Total energy stored initially in the circuit is given as:

$$E = \frac{1}{2} \frac{Q^2}{C} = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1\text{J}$$

As a result, because there is no resistor in the LC circuit, the complete energy stored in it will be conserved.

(b) What is the natural frequency of the circuit?

(b) Natural frequency of the circuit is given by the relation,

$$\begin{aligned} \nu &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} \\ &= \frac{10^3}{2\pi} = 159.24\text{Hz} \end{aligned}$$

Natural angular frequency,

$$\begin{aligned} \omega_r &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = \frac{1}{\sqrt{10^{-6}}} = 10^3 \text{ rad / s} \end{aligned}$$

As a result, the circuit's inherent frequency is 10^3 rad / s

(c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)?

(c) (i) For time period $\left(T = \frac{1}{\nu} = \frac{1}{159.24} = 6.28\text{ms} \right)$, total charge on the capacitor at time t ,

$$Q' = Q \cos \frac{2\pi}{T} t$$

For energy stored is electrical, we can write $Q' = Q$.

Hence, it can be inferred that the energy stored in the capacitor is completely electrical at time,

$$t = 0, \frac{T}{2}, T, \frac{3T}{2}, \dots$$

(ii) Completely magnetic (i.e., stored in the inductor)?

(ii) Magnetic energy is the maximum when electrical energy, Q' is equal to 0.

Hence, it can be inferred that the energy stored in the capacitor is completely magnetic at time,

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots$$

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(d) Q^1 = Charge on the capacitor when total energy is equally shared between the capacitor and the inductor at time t .

When total energy is equally shared between the inductor and capacitor, the energy stored in the

$$\text{capacitor} = \frac{1}{2} \text{ (maximum energy)}$$

$$\Rightarrow \frac{1}{2} \frac{(Q^1)^2}{C} = \frac{1}{2} \left(\frac{1}{2C} \frac{Q^2}{C} \right) = \frac{1Q^2}{4C}$$

$$Q^1 = \frac{Q}{\sqrt{2}}$$

$$\text{But } Q^1 = Q \cos \frac{2\pi}{T} t$$

$$\frac{Q}{\sqrt{2}} = Q \cos \frac{2\pi}{T} t$$

$$\cos \frac{2\pi}{T} t = \frac{1}{\sqrt{2}} = \cos(2n+1) \frac{\pi}{4} \quad \text{where } n = 0, 1, 2, \dots$$

$$t = (2n+1) \frac{T}{8} \text{ aligned}$$

As a result, total energy is shared equally between the inductor and the capacity at any given point in time.

$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \dots$$

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

(e) When a resistor is placed in the circuit, the complete original energy is dissipated as heat energy. The LC oscillation is dampened by the resistance.

7.13: A coil of inductance 0.50 H and resistance 100Ω is connected to a $240 \text{ V}, 50 \text{ Hz}$ ac supply.

(a) What is the maximum current in the coil?

Answer: Inductance of the inductor, $L=0.50 \text{ H}$

Resistance of the resistor, $R=100 \Omega$

Potential of the supply voltage, $V=240 \text{ V}$

Frequency of the supply, $\nu=50 \text{ Hz}$

(a) Peak voltage is given as:

$$\begin{aligned}
 V_0 &= \sqrt{2}V \\
 &= \sqrt{2} \times 240 = 339.41 \text{ V}
 \end{aligned}$$

Angular frequency of the supply,

$$\begin{aligned}\omega &= 2\pi\nu \\ &= 2\pi \times 50 = 100\pi \text{ rad/s}\end{aligned}$$

Maximum current in the circuit is given as:

$$\begin{aligned}I_0 &= \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \\ &= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82 \text{ A}\end{aligned}$$

(b) What is the time lag between the voltage maximum and the current maximum?

(b) Equation for voltage is given as:

$$V = V_0 \cos \omega t$$

Equation for current is given as:

$$I = I_0 \cos(\omega t - \Phi)$$

Where,

Φ = Phase difference between voltage and current At time, $t = 0$.

$V = V_0$ (voltage is maximum)

For $\omega t - \Phi = 0$ i.e., at time $t = \frac{\phi}{\omega}$

$I = I_0$ (current is maximum)

As a result, there is a temporal lag between the highest voltage and the maximum current. $\frac{\phi}{\omega}$

Now, phase angle Φ is given by the relation,

$$\tan \phi = \frac{\omega L}{R}$$

$$= \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$

$$\phi = 57.5^\circ = \frac{57.5\pi}{180} \text{ rad}$$

$$\omega t = \frac{57.5\pi}{180}$$

$$t = \frac{57.5}{180 \times 2\pi \times 50}$$

$$= 3.19 \times 10^{-3} \text{ s}$$

$$= 3.2 \text{ ms}$$

As a result, the time difference between maximum voltage and maximum current is 3.2 milliseconds.

7.14: Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Answer: Inductance of the inductor, $L=0.5 \text{ Hz}$

Resistance of the resistor, $R=100 \Omega$

Potential of the supply voltages, $V=240 \text{ V}$

Frequency of the supply, $\nu=10 \text{ kHz}=10^4 \text{ Hz}$

Angular frequency, $\omega=2\pi\nu=2\pi \times 10^4 \text{ rad/s}$

(a) Peak voltage, $V_0 = \sqrt{2} \times V = 240\sqrt{2} \text{ V}$

Maximum current,

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 \times (0.50)^2}} = 1.1 \times 10^{-2} \text{ A}$$

(b) For phase difference ϕ , we have the relation:

$$\tan \phi = \frac{\omega L}{R}$$

$$= \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$$

$$\phi = 89.82^\circ = \frac{89.82\pi}{180} \text{ rad}$$

$$\omega t = \frac{89.82\pi}{180}$$

$$t = \frac{89.82\pi}{180 \times 2\pi \times 10^4} = 25 \mu\text{s}$$

I_0 is very little in this situation, as can be seen. As a result, the inductor acts as an open circuit at high frequencies.

In a dc circuit, after a steady state is achieved, $\omega = 0$. As a result, inductor L acts as a pure conducting item.

7.15: A $100 \mu\text{F}$ capacitor in series with a 40Ω resistance is connected to a 110 V , 60 Hz supply.

(a) What is the maximum current in the circuit?

Answer: Capacitance of the capacitor, $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$

Resistance of the resistor, $R = 40 \Omega$

Supply voltage, $V = 110 \text{ V}$

(a) Frequency of oscillations, $\nu = 60 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 \text{ rad/s}$

For a RC circuit, we have the relation for impedance as:

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}$

Maximum current is given as:

$$I_0 = \frac{V_0}{Z}$$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\begin{aligned}
 &= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}} \\
 &= \frac{110\sqrt{2}}{\sqrt{1600 + \frac{10^8}{(120\pi)^2}}} = 3.24 \text{ A}
 \end{aligned}$$

(b) What is the time lag between the current maximum and the voltage maximum?

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle of Φ . This angle is given by the relation:

$$\begin{aligned}
 \therefore \tan \phi &= \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR} \\
 &= \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635 \\
 \phi &= \tan^{-1}(0.6635) = 33.56^\circ \\
 &= \frac{33.56\pi}{180} \text{ rad} \\
 \therefore \text{Time lag} &= \frac{\phi}{\omega} \\
 &= \frac{33.56\pi}{180 \times 120\pi} = 1.55 \times 10^{-3} \text{ s} = 1.55 \text{ ms}
 \end{aligned}$$

7.16: Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110V, 12kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Answer: Capacitance of the capacitor, $C = 100 \mu F = 100 \times 10^{-6} F$

Resistance of the resistor, $R = 40 \Omega$

Supply voltage, $V = 110V$

Frequency of the supply, $\nu = 12 \text{ kHz} = 12 \times 10^3 \text{ Hz}$

Angular Frequency, $\omega = 2\pi\nu = 2 \times \pi \times 12 \times 10^3 = 24\pi \times 10^3 \text{ rad / s}$

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2} \text{ V}$

Maximum current,

$$\begin{aligned}
 I_0 &= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \\
 &= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10^3 \times 100 \times 10^{-6})^2}}} \\
 &= \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9 \text{ A}
 \end{aligned}$$

For an RC circuit, the voltage lags behind the current by a phase angle of Φ given as:

$$\begin{aligned}
 \tan \phi &= \frac{\omega C}{R} = \frac{1}{\omega CR} \\
 &= \frac{1}{24\pi \times 10^3 \times 100 \times 10^{-6} \times 40} \\
 \tan \phi &= \frac{1}{96\pi} \\
 \therefore \phi &\approx 0.2^\circ \\
 &= \frac{0.2\pi}{180} \text{ rad} \\
 \therefore \text{Time lag} &= \frac{\phi}{\omega} \\
 &= \frac{0.2\pi}{180 \times 24\pi \times 10^3} = 1.55 \times 10^{-3} \text{ s} = 0.04 \mu\text{s}
 \end{aligned}$$

As a result, at high frequencies, Φ tends to become zero. Capacitor C works as a conductor at high frequencies. After the steady state is reached in a dc circuit, $\phi = 0$. As a result, capacitor C is an open circuit.

7.17: Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L , C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Answer: An inductor (L), a capacitor (C), and a resistor (R) is connected in parallel with each other in a circuit where, $L=5.0 \text{ H}$

$$C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$$

$$R = 40 \Omega$$

Potential of the voltage source, $V = 230 \text{ V}$

Impedance (Z) of the given parallel LCR circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where, ω = Angular frequency

$$\frac{1}{\omega L} - \omega C = 0$$

At resonance, $\therefore \omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad/s}$$

Hence, the magnitude of Z is the maximum at 50 rad/s.

As a result, the total current is minimum. rms current flowing through inductor L is given as:

$$I_L = \frac{V}{\omega L} = \frac{230}{50 \times 5} = 0.92 \text{ A}$$

rms current flowing through capacitor C is given as:

$$I_C = \frac{V}{\frac{1}{\omega C}} = \omega CV$$

$$= 50 \times 80 \times 10^{-6} \times 230 = 0.92 \text{ A}$$

rms current flowing through resistor R is given as:

$$I_R = \frac{V}{R} = \frac{230}{40} = 5.75 \text{ A}$$

7.18: A circuit containing a 80mH inductor and a 60μF capacitor in series is connected to a 230V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

Answer: Inductance, $L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$

Capacitance, $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Supply voltage, $V = 230 \text{ V}$

Frequency, $\nu = 50 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu = 100\pi \text{ rad/s}$

Peak voltage, $V_0 =$

(a) Maximum current is given as:

Hence, rms value of current,

$$\begin{aligned}
 I_0 &= \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)} \\
 &= \frac{230\sqrt{3}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)} \\
 &= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63 \text{ A}
 \end{aligned}$$

The negative sign appears because $\omega L < \frac{1}{\omega C}$.

Amplitude of maximum current,

$$\begin{aligned}
 |I_0| &= 11.63 \text{ A} \\
 I &= \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.22 \text{ A}
 \end{aligned}$$

(b) Obtain the rms values of potential drops across each element.

(b) Potential difference across the inductor,

$$\begin{aligned}
 V_L &= I \times \omega L \\
 &= 8.22 \times 100\pi \times 80 \times 10^{-3} \\
 &= 206.61 \text{ V}
 \end{aligned}$$

Potential difference across the capacitor,

$$\begin{aligned}
 V_c &= I \times \frac{1}{\omega C} \\
 &= 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 436.3 \text{ V}
 \end{aligned}$$

(c) What is the average power transferred to the inductor?

(c) Average power consumed by the inductor is zero as actual voltage leads the current by $\pi/2$.

(d) What is the average power transferred to the capacitor?

(d) Because voltage lags current by $\pi/2$, the capacitor's average power consumption is 0.

(e) What is the total average power absorbed by the circuit?

[‘Average’ implies ‘averaged over one cycle’.]

(e) Total power absorbed is 0 (averaged over one cycle).

7.19: Suppose the circuit in Exercise 7.18 has a resistance of 15Ω . Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Answer : Average power transferred to the resistor = $788.44W$

Average power transferred to the capacitor = $0W$

Total power absorbed by the circuit = $788.44W$

Inductance of inductor, $L = 80mH = 80 \times 10^{-3} H$

Capacitance of capacitor, $C = 60\mu F = 60 \times 10^{-6} F$

Resistance of resistor, $R = 15 \Omega$

Potential of voltage supply, $V = 230V$

Frequency of signal, $\nu = 50 Hz$

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100\pi \text{ rad / s}$

The elements are linked in a sequential order. As a result, the circuit's impedance is:

Current flowing in the circuit,

$$\begin{aligned}
 Z &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\
 &= \sqrt{(15)^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{(100\pi \times 60 \times 10^{-6})}\right)^2} \\
 &= \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728 \Omega \\
 I &= \frac{V}{Z} = \frac{230}{31.728} = 7.25 \text{ A}
 \end{aligned}$$

Average power transferred to resistance is given as:

$$\begin{aligned}
 P_R &= I^2 R \\
 &= (7.25)^2 \times 15 = 788.44W
 \end{aligned}$$

Average power transferred to capacitor,

$$P_C = \text{Average power transferred to inductor, } P_L = 0$$

Total power absorbed by the circuit:

$$\begin{aligned}
 &= P_R + P_C + P_L \\
 &= 788.44 + 0 + 0 = 788.44W
 \end{aligned}$$

As a result, the circuit's overall power consumption is $788.44W$

Question 7.20: A series LCR circuit with $L=0.12\text{ H}$, $C=480\text{ nF}$, $R=23\Omega$ is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

Answer: Inductance, $L = 0.12\text{ H}$

Capacitance, $C = 480$

$\text{nF} = 480 \times 10^{-9}\text{ F}$

Resistance, $R = 23\Omega$

Supply voltage, $V = 230\text{ V}$

Peak voltage is given as: $V_0 = \sqrt{2} \times 230 = 325.22\text{ V}$

(a) Current flowing in the circuit is given by the relation,

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Where,

I_0 = maximum at resonance

At resonance, we have

$$\omega_R L - \frac{1}{\omega_R C} = 0$$

Where,

ω_R = Resonance angular frequency

$$\begin{aligned} \therefore \omega_R &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.67\text{ rad/s} \end{aligned}$$

$$\therefore \text{Resonant frequency, } \nu_R = \frac{\omega_R}{2\pi} = \frac{4166.67}{2 \times 3.14} = 663.48\text{ Hz}$$

$$\text{And, maximum current } (I_0)_{\text{Max}} = \frac{V_0}{R} = \frac{325.22}{23} = 14.14\text{ A}$$

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(b) The circuit's maximum average power absorbed is provided as:

$$\begin{aligned}
 (P_{av})_{\text{Max}} &= \frac{1}{2} (I_0)_{\text{Max}}^2 R \\
 &= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3 \text{ W}
 \end{aligned}$$

Hence, resonant frequency (ν_R) is 663.48 Hz.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(c) At resonance frequency, the power delivered to the circuit is half that.

Frequencies at which power transferred is half,

$$\begin{aligned}
 &= \omega_R \pm \Delta\omega \\
 &= 2\pi(\nu_R \pm \Delta\nu)
 \end{aligned}$$

Where,

$$\begin{aligned}
 \Delta\omega &= \frac{R}{2L} \\
 &= \frac{23}{2 \times 0.12} = 95.83 \text{ rad/s}
 \end{aligned}$$

Hence, change in frequency,

$$\begin{aligned}
 \Delta\nu &= \frac{1}{2\pi} \Delta\omega = \frac{95.83}{2\pi} = 15.26 \text{ Hz} \\
 \therefore \nu_R + \Delta\nu &= 663.48 + 15.26 = 678.74 \text{ Hz} \\
 \text{And, } \nu_R - \Delta\nu &= 663.48 - 15.26 = 648.22 \text{ Hz}
 \end{aligned}$$

Hence, at 648.22 Hz and 678.74 Hz frequencies, the power transferred is half.

At these frequencies, current amplitude can be given as:

$$\begin{aligned}
 I' &= \frac{1}{\sqrt{2}} \times (I_0)_{\text{Max}} \\
 &= \frac{14.14}{\sqrt{2}} = 10 \text{ A}
 \end{aligned}$$

(d) What is the Q-factor of the given circuit?

(d) The Q-factor of a circuit can be calculated using the formula

$$\begin{aligned}
 Q &= \frac{\omega_R L}{R} \\
 &= \frac{4166.67 \times 0.12}{23} = 21.74
 \end{aligned}$$

Hence, the Q-factor of the given circuit is 21.74

7.21: Obtain the resonant frequency and Q-factor of a series LCR circuit with $L=3.0\text{ H}$, $C=27\ \mu\text{F}$, and $R=7.4\ \Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Answer: Inductance, $L=3.0\text{ H}$

Capacitance, $C=27\ \mu\text{F}=27\times 10^{-6}\text{ F}$

Resistance, $R=7.4\ \Omega$

For the given LCR series circuit, the angular frequency of the source at resonance is given as:

$$\begin{aligned}\omega_r &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{10^3}{9} = 111.11 \text{ rads}^{-1}\end{aligned}$$

Q-factor of the series:

$$\begin{aligned}Q &= \frac{\omega_r L}{R} \\ &= \frac{111.11 \times 3}{7.4} = 45.0446\end{aligned}$$

Reduce the resonance's 'whole width at half maximum' to improve its sharpness by a factor of 2

without changing ω_r , we need to reduce R to half i.e., Resistance = $\frac{R}{2} = \frac{7.4}{2} = 3.7\ \Omega$

7.22: Answer the following questions:

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

Answer: (a) In the case of rms voltage, the assertion is correct. The applied voltage in any ac circuit is equal to the average sum of the instantaneous voltages across the circuit's series elements. However, because voltages across distinct elements may not be in phase, this is not true for rms voltage.

(b) A capacitor is used in the primary circuit of an induction coil.

Answer: (b) The capacitor is charged with a high induced voltage. In the primary circuit of an induction coil, a capacitor is used. This is because, in order to avoid sparks, a high induced voltage is utilised to charge the capacitor when the circuit is disrupted.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.

Answer: (c) Because the impedance of an inductor (L) is insignificant for dc signals, but the impedance of a capacitor (C) is quite high, the dc signal will appear across capacitor C. (almost

infinite). As a result, a dc signal is seen across C. The impedance of L is high for a high-frequency ac signal and very low for a low-frequency ac signal. As a result, a high-frequency ac signal develops across L.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

Answer: (d) The lamp will glow weakly if an iron core is inserted in the choke coil (which is in series with a lamp attached to the ac line). Because the choke coil and the iron core raise the circuit's resistance, this is the case.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Answer: (e) The lamp will glow weakly if an iron core is placed into the choke coil (which is in series with a lamp attached to the ac line). The choke coil and the iron core raise the circuit's resistance.

7.23: A power transmission line feeds input power at 2300 V to a stepdown transformer with its primary windings having 4000 turns . What should be the number of turns in the secondary in order to get output power at 230 V ?

Answer: Input voltage, $V_1 = 2300$

Number of turns in primary coil, $n_1 = 4000$

Output voltage, $V_2 = 230V$

Number of turns in secondary coil $= n_2$

Voltage is related to the number of turns as:

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

$$\frac{2300}{230} = \frac{4000}{n_2}$$

$$n_2 = \frac{4000 \times 230}{2300} = 400$$

Hence, there are 400 turns in the second winding.

7.24: At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100 \text{ m}^3 \text{ s}^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8 \text{ m s}^{-2}$).

Answer : Height of water pressure head, $h = 300 \text{ m}$

Volume of water flow per second, $V = 100 \text{ m}^3 / \text{s}$

Efficiency of turbine generator, $n=60\%=0.6$

Acceleration due to gravity, $g=9.8\text{ m/s}^2$

Density of water, $\rho=103\text{ kg/m}^3$

Electric power available from the plant $=\eta \times h \rho g V$

$$=0.6 \times 300 \times 103 \times 9.8 \times 100$$

$$=176.4 \times 10^6 \text{ W} = 176.4 \text{ MW}$$

7.25: A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000/220 V step-down transformer at a sub-station in the town.

(a) Estimate the line power loss in the form of heat.

Answer : Total electric power required, $P=800\text{ kW}=800 \times 10^3 \text{ W}$

Supply voltage, $V=220\text{ V}$

Voltage at which electric plant is generating power, $V'=440\text{ V}$

Distance between the town and power generating station, $d=15\text{ km}$

Resistance of the two wire lines carrying power $=0.5\ \Omega / \text{ km}$

Total resistance of the wires, $R=(15+15)0.5=15\ \Omega$

A step-down transformer of rating 4000–220V is used in the sub-station.

Input voltage, $V_1=4000\text{ V}$

Output voltage, $V_2=220\text{ V}$

rms current in the wire lines is given as:

$$\begin{aligned}
 I &= \frac{P}{V_1} \\
 &= \frac{800 \times 10^3}{4000} = 200 \text{ A}
 \end{aligned}$$

(a) Line power loss $= I^2 R$

$$= (200)^2 \times 15$$

$$= 600 \times 10^3 \text{ W}$$

$$= 600 \text{ kW}$$

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(b) Assume that the current leakage causes no significant power loss:

The plant's total electricity output = $800\text{ kW} + 600\text{ kW} = 1400\text{ kW}$

(c) Characterise the step up transformer at the plant.

(c) Voltage drop in the power line = $IR = 200 \times 15 = 3000\text{ V}$

As a result, the overall voltage transmitted by the plant = $3000 + 4000 = 7000\text{ V}$

Also, the power generated is 440 V .

As a result, the step-up transformer at the power plant is rated at $440\text{ V} - 7000\text{ V}$.

7.26: Do the same exercise as above with the replacement of the earlier transformer by a $40,000 - 220\text{ V}$ step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Answer 7.26: The rating of a step-down transformer is $40000\text{ V} - 220\text{ V}$.

Input voltage, $V_1 = 40000\text{ V}$

Output voltage, $V_2 = 220\text{ V}$

Total amount of electricity required, $P = 800\text{ kW} = 800 \times 10^3\text{ W}$

Source potential, $V = 220\text{ V}$

Voltage at which the electric plant generates power, $V' = 440\text{ V}$

Distance between the town and power generating station, $d = 15\text{ km}$

Resistance of the two wire lines carrying power = $0.5\ \Omega / \text{km}$

Total resistance of the wire lines,

$$R = (15 + 15) 0.5 = 15\ \Omega$$

$$P = V_1 I$$

Rms current in the wire line is given as:

$$I = \frac{P}{V_1}$$

$$= \frac{800 \times 10^3}{40000} = 20\text{ A}$$

(a) Line power loss

$$= I^2 R$$

$$= (20)^2 \times 15$$

$$= 6 \text{ kW}$$

(b) Assuming that the power loss due to current leakage is low.

$$\text{As a result, the plant's power output} = 800 \text{ kW} + 6 \text{ kW} = 806 \text{ kW}$$

(c) Drop in voltage in the power line $= IR = 20 \times 15 = 300 \text{ V}$

As a result, the voltage transmitted by the power plant is $= 300 + 40000 = 40300 \text{ V}$. The facility generates electricity at a voltage of

440V.

As a result, the plant's step-up transformer must be rated at 440V-40300V. As a result, transmission power loss Equals

$$\frac{600}{1400} \times 100 = 42.8\%$$

In the previous exercise, the power loss due to the same reason is

$$\frac{6}{806} \times 100 = 0.744\%$$

High voltage transmissions are preferred for this purpose since the power loss is lower.