Chapter 9: Ray Optics and Optical Instruments

Exercise

1. A small candle 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Ans: Here given that size of candle which is denoted by $h = 2.5\text{ cm}$, image size denoted as h', object distance as $u = -27\text{ cm}$, and radius of curvature of concave mirror i.e. $R = -36\text{ cm}$

Now, Focal length of concave mirror $f = \frac{R}{2} = -18\text{ cm}$ whereas image distance $= v$

So, image distance obtained by using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{-18} - \frac{1}{-27} = \frac{-3+2}{54} = -\frac{1}{54}$$

$$v = -54\text{ cm}$$

Thus, screen placed 54 cm away from mirror to obtain sharp image. So, magnification of image is given as

$$m = \frac{h'}{h} = -\frac{v}{u}$$

$$h' = -\frac{v}{u} \times h = -\left(\frac{-54}{-27}\right) \times 2.5 = -5\text{ cm}$$

From this, height is 5 cm and negative sign indicates image is virtual and inverted. If candle is closer to mirror then screen moved away from mirror to obtain image.

2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Ans: Here Height of the needle $h_1 = 4.5\text{ cm}$, Object distance $u = -12\text{ cm}$, Focal length of the convex mirror $f = 15\text{ cm}$, Image distance $= v$

So, value of v obtained by using mirror formula:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$=\frac{1}{15}+\frac{1}{12}=\frac{4+5}{60}=\frac{9}{60}$$

$$v=\frac{60}{9}=6.7\text{ cm}$$

Thus, image of needle is 6.7 cm which is away from mirror.

Now, image size which is given by magnification formula:

$$m=\frac{h_2}{h_1}=-\frac{v}{u}$$

$$h_2=-\frac{v}{u}\times h_1=\frac{-6.7}{-12}\times 4.5=+2.5\text{ cm}$$

Thus, magnification image $m=\frac{h_2}{h_1}=\frac{2.5}{4.5}=0.56$

So, height of image is 2.5 cm where positive sign indicates image is erect, virtual, diminished and if needle is at some distant from mirror then image will also away from mirror and hence size gradually reduces.

3. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Ans: Let Actual depth of needle in water be $h_1=12.5\text{ cm}$, Apparent depth of needle in water $h_2=9.4\text{ cm}$, Refractive index of water $=\mu$

So, value of μ as follows:

$$\mu=\frac{h_1}{h_2}=\frac{12.5}{9.4}\approx 1.33$$

Thus, refractive index of water is $\mu'=1.63$

So, actual depth of needle remains same but its depth changes.

Now, let y represents new apparent depth of needle.

Thus, relation is

$$\mu' = \frac{h_1}{y}$$

$$y = \frac{h_1}{\mu'} = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

Thus, new apparent length is 7.67 cm which is less than h_2

So, microscope should be moved upward to focus needle again.

Hence distance through which microscope moved up $= 9.4 - 7.67 = 1.73 \text{ cm}$

4. Figures 9.34 (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.34(c)].

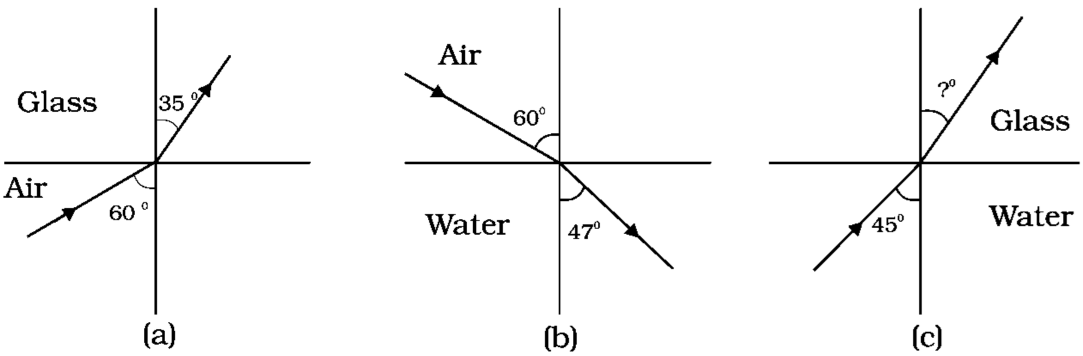


FIGURE 9.31

Ans: According to the give above figure glass to water interface:

Angle of incidence is $i = 60^\circ$ and angle of refraction is $r = 35^\circ$

According to snell's law, with respect to air, relative refractive index of glass is as follows:

$$\mu_g^a = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 35^\circ} = \frac{0.8660}{0.5736} = 1.51 \qquad \text{(i)}$$

Now, again according to figure air to water interface:

Angle of incidence is $i = 60^\circ$ and angle of refraction is $r = 47^\circ$

According to snell's law, with respect to air, relative refractive index of water is as follows:

$$\mu_g^a = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 47^\circ} = \frac{0.8660}{0.7314} = 1.184 \qquad \text{(ii)}$$

From (i) and (ii) we have $\mu_g^w = \frac{\mu_g^a}{\mu_w^a} = \frac{1.51}{1.184} = 1.275$

Angle of incidence $i = 45^\circ$ whereas angle of refraction $= r$

According to Snell's law, we have:

$$\frac{\sin i}{\sin r} = \mu_g^w$$

$$\frac{\sin 45^\circ}{\sin r} = 1.275$$

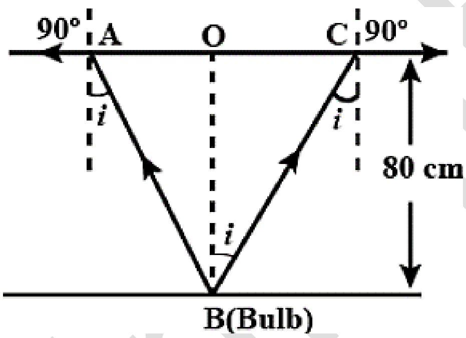
$$\sin r = \frac{1}{\sqrt{2}}$$

$$r = \sin^{-1}(0.5546) = 38.68^\circ$$

5. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Ans: Here given that Actual depth of the bulb in water is $d_1 = 80\text{ cm} = 0.8\text{ m}$ and Refractive index of water be $\mu = 1.33$

Now, represents given situation in the following figure:



Here $i =$ angle of incidence and $r = 90^\circ =$ angle of refraction

As bulb is point source, so emergent light considered as radius of circle i.e. $R = \frac{AC}{2} = AO = OB$

Thus, by snell's law, we have:

$$\mu = \frac{\sin r}{\sin i}$$

$$1.33 = \frac{\sin 90^\circ}{\sin i}$$

$$i = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75^\circ$$

From figure, we have $\tan i = \frac{OC}{OB} = \frac{R}{d_1}$

So, $R = \tan 48.75^{\circ} \times 0.8 = 0.91\ m$

Area of surface of water is represents as $= \pi R^2 = \pi(0.91)2 = 2.61\ m^2$

Thus, area of surface of water through light is approximately $2.61\ m^2$

6. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40° . What is the refractive index of the material of the prism? The refracting angle of the prism is 60° . If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Ans: Here we have given that angle of minimum deviation $\delta_m = 40^{\circ}$, angle of prism $A = 60^{\circ}$, refractive index of water is $\mu = 1.33$, refractive index of material is μ'

Thus, angle of deviation which is related to refractive index μ' is as follows:

$$\mu' = \frac{\sin\frac{(A+\delta_m)}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(60^{\circ}+40^{\circ}\right)}{\sin\frac{60^{\circ}}{2}} = \frac{\sin 50^{\circ}}{\sin 30^{\circ}} = 1.532$$

Thus, refractive index of prism is 1.532

As prism is placed in water. So, let δ_m denotes the new angle of minimum deviation for that same prism. Now, refractive index of glass w.r.to water is as follows:

$$\mu_g^{\omega} = \frac{\mu'}{\mu} = \frac{\sin\frac{(A+\delta'_m)}{2}}{\sin\frac{A}{2}}$$

$$\sin\frac{(A+\delta'_m)}{2} = \frac{\mu'}{\mu}\sin\frac{A}{2}$$

$$\sin\frac{(A+\delta'_m)}{2} = \frac{1.532}{1.33}\times\sin\frac{60^{\circ}}{2} = 0.5759$$

$$\frac{(A+\delta'_m)}{2} = \sin^{-1}0.5759 = 35.16^{\circ}$$

$$60^{\circ}+\delta'_m = 70.32^{\circ}$$

$$\delta'_m = 70.32^{\circ}-60^{\circ} = 10.32^{\circ}$$

7. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Ans: Here given that Refractive index of glass $\mu = 1.55$, Focal length of the double-convex lens $f = 20\text{ cm}$

Let Radius of curvature of one face of lens be R_1

Radius of curvature of other face of lens be R_2

Radius of curvature of double-convex lens be R

So, $R_1 = R$, $R_2 = -R$

Thus, value of R is as follows:

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{20} = (1.55-1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\frac{1}{20} = 0.55\times\frac{2}{R}$$

$$R = 0.55\times2\times20 = 22\text{ cm}$$

8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

(a) a convex lens of focal length 20 cm

Ans: From given condition, object is virtual and image formed as real

So, object distance is $u = +12\text{ cm}$

Focal length $f = 20\text{ cm}$

From lens formula. We have:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

$$v = \frac{60}{8} = 7.5\text{ cm}$$

which is away from lens i.e. towards its right.

(b) a concave lens of focal length 16 cm

Ans: Here given that focal length $f=-16\text{ cm}$

Image distance is v

From lens formula, we have:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=-\frac{1}{16}+\frac{1}{12}=\frac{-3+4}{48}=\frac{1}{48}$$

$$v=48\text{ cm}$$

which is away from lens i.e. towards its right.

9. An object of size 3.0 cm **is placed** 14 cm **in front of a concave lens of focal length** 21 cm **. Describe the image produced by the lens. What happens if the object is moved further away from the lens?**

Ans: Here we have given that size $h_1=3\text{ cm}$, object distance $u=-14\text{ cm}$, focal length $f=-21\text{ cm}$, image distance $=v$

From lens formula, we have:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=-\frac{1}{21}-\frac{1}{14}=\frac{-2-3}{42}=\frac{-5}{42}$$

$$v=-\frac{42}{5}=-8.4\text{ cm}$$

Thus, image which is formed on other side is 8.4 cm away from it where this negative sign shows image is erect and virtual.

So, magnification is image is as follows:

$$m=\frac{\text{Image height }(h_2)}{\text{Object height }(h_1)}=\frac{v}{u}$$

$$h_2=\frac{-8.4}{-14}\times 3=0.6\times 3=1.8\text{ cm}$$

Hence, if object is moved away from lens then virtual image move towards focus of lens but not its beyond. So, size will decrease with increase in object distance.

10. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm ? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Ans: Here we have given that focal length of convex lens $f_1 = 30\text{ cm}$, focal length of concave lens $f_2 = -20\text{ cm}$, focal length of lenses $= f$

Thus, equivalent focal length of two lenses is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$
$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$
$$f = -60\text{ cm}$$

where negative sign indicates system of lenses which act as diverging lens.

11. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm . How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision 25 cm and (b) at infinity? What is the magnifying power of the microscope in each case?

Ans: Here given that focal length of objective lens $f_1 = 2.0\text{ cm}$, Focal length of eyepiece $f_2 = 6.25\text{ cm}$, distance between objective and eyepiece lens is $d = 15\text{ cm}$

(a) least distance of distinct vision is $d' = 25\text{ cm}$

Image distant for eyepiece $v_2 = -25\text{ cm}$

Object distance $= u_2$

From lens formula, we have

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$
$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$
$$= \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$
$$u_2 = -5\text{ cm}$$

Image distance for objective $v_1 = d + u_2 = 15 - 5 = 10\text{ cm}$

Object distance $= u_1$

Now, again from lens formula, we have:

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

$$\frac{1}{u_1}=\frac{1}{v_1}-\frac{1}{f_1}$$

$$=\frac{1}{10}-\frac{1}{2}=\frac{1-5}{10}=\frac{-4}{10}$$

$$u_1=-2.5\text{ cm}$$

Thus, magnitude is $|u_1|=2.5$ cm

So, magnifying power of microscope is given as

$$m=\frac{v_1}{|u_1|}\left(1+\frac{d'}{f_2}\right)=\frac{10}{2.5}\left(1+\frac{25}{6.25}\right)=4(1+4)=20$$

(b) here final image is at infinity

Image distance for eyepiece $v_2=\infty$

Object distance $=u_2$

From lens formula, we have

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

$$\frac{1}{\infty}-\frac{1}{u_2}=\frac{1}{6.25}$$

$$u_2=-6.25\text{ cm}$$

Image distance for objective lens is given as $v_1=d+u_2=15-6.25=8.75$ cm

Object distance $=u_1$

From lens formula, we have:

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

$$\frac{1}{u_1}=\frac{1}{v_1}-\frac{1}{f_1}$$

$$=\frac{1}{8.75}-\frac{1}{2.0}=\frac{2-8.75}{17.5}$$

$$u_1=-\frac{17.5}{6.75}=-2.59\text{ cm}$$

Magnitude $|u_1|=2.59$ cm

Magnifying power of compound microscope is as follows:

$$m = \frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right)$$

$$= \frac{8.75}{2.59}\times\frac{25}{6.25} = 13.51$$

12. A person with a normal near point 25 cm using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope,

Ans: Here we have given that Focal length of the objective lens $f_o = 8\text{ mm} = 0.8\text{ cm}$

Focal length of the eyepiece is $f_e = 2.5\text{ cm}$

Object distance for the objective lens is $u_o = -9.0\text{ mm} = -0.9\text{ cm}$

Least distance of distant vision is $d = 25\text{ cm}$

Image distance for the eyepiece is $v_e = -d = -25\text{ cm}$

Object distance for the eyepiece is equal to u_e

From lens formula, we have

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{-25} - \frac{1}{2.5} = \frac{-1-10}{25} = \frac{-11}{25}$$

$$u_e = -\frac{25}{11} = -2.27\text{ cm}$$

Here, also we can obtain value of image distance for v_o from lens formula:

$$\frac{1}{v_0} - \frac{1}{u_\circ} = \frac{1}{f_\circ}$$

$$\frac{1}{v_\circ} = \frac{1}{f_\circ} + \frac{1}{u_\circ}$$

$$= \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.72} = \frac{0.1}{0.72}$$

$$v_\circ = 7.2\text{ cm}$$

Distance between objective and eyepiece lens be

$|u_e| + v_\circ = 2.27 + 7.2 = 9.47\text{ cm}$

So, magnifying power of microscope is as follows:

$$\frac{v_e}{|u_o|}\left(1+\frac{d}{f_e}\right) = \frac{7.2}{0.9}\left(1+\frac{25}{2.5}\right) = 8(1+10) = 88$$

13. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Ans: Here given that Focal length of objective lens be $f_o = 144\text{ cm}$

Focal length of eyepiece be $f_e = 6.0\text{ cm}$

Thus, magnifying power of the telescope is as follows:

$$m = \frac{f_o}{f_e} = \frac{144}{6} = 24$$

So, separation between objective and eyepiece as:

$f_o + f_e = 144 + 6 = 150\text{ cm}$

14. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48\times106\text{ m}$, and the radius of lunar orbit is $3.8\times108\text{ m}$

Ans: We have given that focal length of objective be $f_o = 15\text{ m} = 15\times102\text{ m}$

Focal length of eyepiece be $f_e = 1.0\text{ cm}$

(a) Angular magnification of refracting telescope be given as

$$\alpha = \frac{f_o}{f_e} = \frac{15\times10^2}{1.0} = 1500$$

(b) We have given that diameter of moon be $d = 3.48\times106\text{ m}$ and radius is $r_o = 3.8\times108\text{ m}$

Now, let d' represents diameter of image of moon which is formed by objective lens.

So, angle subtended by diameter of moon which is equal to angle which subtended by image.

$$\frac{d}{r_0} = \frac{d'}{f_o}$$

$$\frac{3.48\times10^{6}}{3.8\times10^{8}}=\frac{d'}{15}$$

$$d'=\frac{3.48}{3.8}\times10^{-2}\times15=13.74\times10^{-2}\ \text{m}=13.74\ \text{cm}$$

15. Use the mirror equation to deduce that:

(a) an object placed between f, $2f$ of a concave mirror produces a real image beyond $2f$

Ans: For concave mirror, focal length should be negative i.e. $f<0$

Accordingly, when object is on left side then object distance is negative i.e. $u<0$

For image distance, we have lens formula as follows:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \qquad \text{(i)}$$

So, object lies between f, $2f$

$$2f<u<f$$

$$\frac{1}{2f}>\frac{1}{u}>\frac{1}{f}$$

$$-\frac{1}{2f}<-\frac{1}{u}<-\frac{1}{f}$$

$$\frac{1}{f}-\frac{1}{2f}<\frac{1}{f}-\frac{1}{u}<0 \qquad \text{(ii)}$$

From (i), we have:

$$\frac{1}{2f}<\frac{1}{v}<0$$

Therefore, $\frac{1}{v}$ is negative so, v is negative

$$\frac{1}{2f}<\frac{1}{v}$$

$$2f>v$$

$$-v>-2f$$

Hence, image lies beyond $2f$

(b) a convex mirror always produces a virtual image independent of the location of the object.

Ans: Here, for convex mirror, focal length is positive i.e. $f>0$

Accordingly, when object is on left side then object distance is negative i.e. $u<0$

So, for image distance, from mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

From (ii), we have

$$\frac{1}{v}<0$$
$$v>0$$

Hence, image is on back side.

Thus, convex mirror produces virtual image

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

Ans: Here focal length is positive i.e. $f>0$

Accordingly, when object is on left side then object distance is negative i.e. $u<0$

By mirror formula, we have:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

But $u<0$

So, $\frac{1}{v}>\frac{1}{f}$

$$v<f$$

Thus,image is diminishing in size and located between focus and pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

Ans: Here focal length is negative i.e. $f<0$

Accordingly, when object is on left side then object distance is negative i.e. $u<0$

So, we have

$$f>u>0$$
$$\frac{1}{f}<\frac{1}{u}<0$$
$$\frac{1}{f}-\frac{1}{u}<0$$

From mirror formula, we have:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$
$$\frac{1}{v}<0$$
$$v>0$$

So, image is on right side and its virtual image.

So, for $u<0$, $v>0$

$$\frac{1}{u}>\frac{1}{v}$$
$$v>u$$

Thus, magnification $m=\frac{v}{u}>1$

16. A small pin fixed on a table top is viewed from above from a distance of 50 cm . By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass $=1.5$. Does the answer depend on the location of the slab?

Ans: Here, we have given that actual depth $d=15\text{ cm}$

Apparent dept $=d'$

Refractive index $\mu=1.5$

Ration of actual to apparent depth is equal to refractive index which is given as follows:

$$\mu=\frac{d}{d'}$$
$$d'=\frac{d}{\mu}=\frac{15}{1.5}=10\text{ cm}$$

So, distance raised when pin appears $= d^{'} - d = 15 - 10 = 5$ cm

Thus, for small angle of incidence, distance does not depend on location of slab.

17. (a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68 . The outer covering of the pipe is made of a material of refractive index 1.44 . What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?

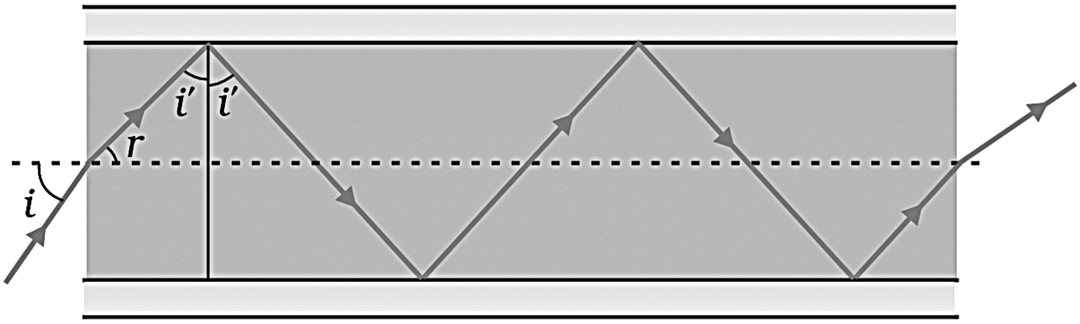


FIGURE 9.32

Ans: (a) Here given that refractive index of glass fibre is $\mu_1 = 1.68$

Refractive index of outer covering is $\mu_2 = 1.44$

Angle of incidence be i , angle of refraction be r , angle of incidence at interface be i'

So, Refractive index of inner minus outer core is as follows:

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{\sin i^{'}}$$

$$\sin i^{'} = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68} = 0.8571$$

$$i^{'} = 59^{\circ}$$

Now, for critical angle, total internal reflection taken place when $i > i^{'}$

Maximum angle of reflection is $r_{max} = 90^{\circ} - i^{'} = 90^{\circ} - 59^{\circ} = 31^{\circ}$

Let i_{max} denoted as maximum angle of incidence

So, $\mu_1 = \dfrac{\sin i_{max}}{\sin r_{max}}$

$\sin i_{max} = \mu_1 \sin r_{max}$

$= 1.68 \sin 31^{\circ}$

$= 1.68 \times 0.5150$

$= 0.8652$

$\sin i_{max} = \sin^{-1} 0.8652 \approx 60^{\circ}$

Hence, all rays incident at angles $0<i<60^{\circ}$ which suffer total internal reflection.

(b) Assume outer covering is not present then refractive index of outer pipe is equal to refractive index of air

So, for angle of incidence $i = 90^{\circ}$

Thus, from snell's law, we have:

$$\frac{\sin i}{\sin r} = \mu_2 = 1.68$$

$$\sin r = \frac{\sin 90^{\circ}}{1.68} = \frac{1}{1.68}$$

$$r = \sin^{-1}(0.5952) = 36.5^{\circ}$$

$$i' = 90^{\circ} - 36.5^{\circ} = 53.5^{\circ}$$

As $i' > r$, hence all incident rays will suffer TIR

18. Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

Ans: Yes because Plane and convex mirrors also produces real images. If object is virtual or we can say if light rays converging at a point behind a plane mirror or we can say on a convex mirror then reflected to a point on a screen which placed in front of mirror. So, real image will be formed.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

Ans: No because when light rays diverge then virtual image is formed. So, convex lens of the eye causes divergent rays to converge at retina. Accordingly, virtual image serves as object for the lens which produce a real image.

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

Ans: Here, diver is in the water whereas fisherman is on land or we can say in air. As Water is denser medium than air, so the diver is viewing the fisherman which indicates the light rays that are travelling from denser medium to rarer medium. Thus, refracted rays will move away from the normal which implies fisherman will appear to be taller.

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

Ans: Yes and apparent depth decreases because apparent depth of a tank of water changes when viewed obliquely as light bends on travelling from one medium to another. So, apparent depth of the tank when viewed obliquely is less than the near-normal viewing.

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Ans: Yes because refractive index of diamond which is 2.42 more than that of ordinary glass which is 1.5. Thus, critical angle for diamond is less than as compared to glass. Thus, diamond cutter uses a large angle of incidence which ensures that light which is entering in the diamond is totally reflected from its faces. So, that is reason for sparkling effect of a diamond.

19. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Ans: Here we have given that distance between object and image is $d = 3\text{ m}$

Let Maximum focal length of the convex lens be f_{max}

Now, For real images, maximum focal length is as follows:

$$f_{max} = \frac{d}{4} = \frac{3}{4} = 0.75\text{ m}$$

Thus, maximum possible focal length of the convex lens is 0.75 m

20. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Ans: Here given that Distance between screen and object is $D = 90\text{ cm}$

Distance between two locations of the convex lens be $d = 20\text{ cm}$

Focal length of the lens denoted as f

Now, Focal length which is related to d, D is as follows:

$$f = \frac{D^2-d^2}{4D}$$

$$= \frac{(90)^2-(20)^2}{4\times90} = \frac{770}{36} = 21.39\text{ cm}$$

21. (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

Ans: Here, we have focal length of convex lens be $f_1 = 30\text{ cm}$, focal length of concave lens be $f_2 = -20\text{ cm}$, distance between these lenses be $d = 8.0\text{ cm}$

So, when parallel beam of light incident on convex lens, we have according to lens formula i.e.

$\frac{1}{v_1}-\frac{1}{u_1} = \frac{1}{f_1}$ where u_1 = object distance which is infinity and v_1 = image distance

Now,

$$\frac{1}{v_1} = \frac{1}{30}-\frac{1}{\infty} = \frac{1}{30}$$
$$v_1 = 30\text{ cm}$$

So, image act as virtual for concave lens.

From lens formula, we have:

$\frac{1}{v_2}-\frac{1}{u_2} = \frac{1}{f_2}$ where object distance is $u_2 = (30-d) = 30-8 = 22\text{ cm}$ and image distance is v_2 =

We have,

$$\frac{1}{v_2} = \frac{1}{22}-\frac{1}{20} = \frac{10-11}{220} = \frac{-1}{220}$$
$$v_2 = -220\text{ cm}$$

Thus, parallel beam diverge at a point i.e. $\left(220-\frac{d}{2} = 220-4 = 216\text{ cm}\right)$ from centre

Now, when parallel beam is incident from left on concave lens.

Fromm lens formula, we have:

$$\frac{1}{v_2}-\frac{1}{u_2} = \frac{1}{f_2}$$
$$\frac{1}{v_2} = \frac{1}{f_2}+\frac{1}{u_2}$$

where object distance is $u_2 = -\infty$ and image distance is v_2

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$
$$v_2 = -20 \text{ cm}$$

Thus, image act as real. Again applying lens formula, we have

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

where object distance is $u_1 = -(20+d) = -(20+8) = -28 \text{ cm}$ and image distance is v_1

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-28} = \frac{14-15}{420} = \frac{-1}{420}$$
$$v_2 = -420 \text{ cm}$$

Thus, parallel beam diverge from a point i.e. $420-4 = 416 \text{ cm}$ from left.

Hence, answer depend on side of combination where parallel beam is incident.

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm . Determine the magnification produced by the two-lens system, and the size of the image.

Ans: Here, it is given that height $h_1 = 1.5 \text{ cm}$, object distant from side convex lens is $u_1 = -40 \text{ cm}$

$$|u_1| = 40 \text{ cm}$$

Now, from lens formula, we have:

$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$ where image distance is v_1 i.e.

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{4-3}{120} = \frac{1}{120}$$
$$v_1 = 120 \text{ cm}$$

Magnification is $m = \frac{v_1}{|u_1|} = \frac{120}{40} = 3$

Now, image of convex lens act as object for concave lens

From lens formula:

$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$ where object distance is $u_2 = (120-8) = 112 \text{ cm}$ and image distance is v_2 i.e.

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-112+20}{2240} = \frac{-92}{2240}$$

$$v_2 = \frac{-2240}{92} \text{ cm}$$

Magnification is $m' = \left|\frac{v_2}{u_2}\right| = \frac{2240}{92} \times \frac{1}{112} = \frac{20}{92}$

Thus, magnification which is produced by combination of two lenses which is calculated as

$$m \times m' = 3 \times \frac{20}{92} = \frac{60}{92} = 0.652$$

So, magnification of combination is as follows:

$$\frac{h_2}{h_1} = 0.652$$

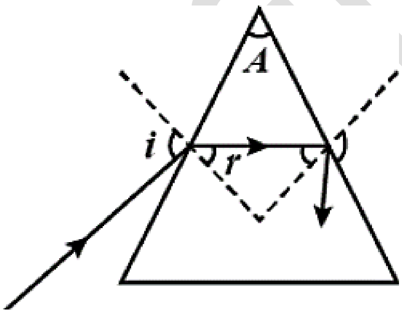
$$h_2 = 0.652 \times h_1$$

where object size is $h_1 = 1.5$ cm and size of image is h_2

So, $h_2 = 0.652 \times 1.5 = 0.98$ cm

22. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524

Ans: The incident, refracted and emergent rays which is associated with glass prism ABC which is shown in following figure:



It is given that angle of prism A which is equal to 60° and refractive index is $\mu = 1.524$

Here, incident angle denoted as i_1 and refracted angle denoted as r_1 and angle of incidence at AC $r_2 = 90^\circ$

For AC, From snell's law, it follows

$$\frac{\sin e}{\sin r_2} = \mu$$

$$\sin r_2 = \frac{1}{\mu} \times \sin 90^\circ = \frac{1}{1.524} = 0.6562$$

$$r_2 = \sin^{-1} 0.6562 \approx 41^\circ$$

$$r_1 = A - r_2 = 60 - 41 = 19^\circ$$

From figure, angle $A = r_1 + r_2$

From snell's law, relation is as follows:

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\sin i_1 = \mu \sin r_1 = 1.524 \times \sin 19^\circ = 0.496$$

$$i_1 = 29.75^\circ$$

23. A card sheet divided into squares each of size $1\ \text{mm}^2$ is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

Ans: Here it is given that area of square A which is equal to $1\ \text{mm}^2$

Object distance is $u = -9$ cm

Focal length $f = 10$ cm

From lens formula, we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{9}$$

$$\frac{1}{v} = -\frac{1}{90}$$

$$v = -90 \text{ cm}$$

Magnification is $m = \frac{v}{u} = \frac{-90}{-9} = 10$

Now area of each square $= (10)2\ A = 102 \times 1 = 100\ \text{mm}^2 = 1\ \text{cm}^2$

(b) What is the angular magnification (magnifying power) of the lens?

Ans: Magnifying power of lens is $\frac{d}{|u|} = \frac{25}{9} = 2.8$

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Ans: Here, magnification in (a) is not same as (b)

So, magnification magnitude is $\left(\left|\frac{v}{u}\right|\right)$ and magnifying power is $\left(\frac{d}{|u|}\right)$

Thus, two quantities will equal when image is at near (25 cm)

24. (a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?

Ans: Here, maximum possible magnification obtained when image is at near point $d = 25\text{ cm}$

Image distance $v = -d = -25\text{ cm}$

Focal length $f = 10\text{ cm}$

Object distance $= u$

From lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{10} = \frac{-2-5}{50} = -\frac{7}{50}$$

$$u = -\frac{50}{7} = -7.14\text{ cm}$$

(b) What is the magnification in this case?

Ans: Magnification $= \left|\frac{v}{u}\right| = \frac{25}{\frac{50}{7}} = 3.5$

(c) Is the magnification equal to the magnifying power in this case? Explain.

Ans: Magnifying power $= \frac{d}{u} = \frac{25}{\frac{50}{7}} = 3.5$

25. What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of $6.25\ mm^2$? Would you be able to see the squares distinctly with your eyes very close to the magnifier? [Note: Exercises 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

Ans: Here, Area of the virtual image of each square be $A = 6.25\ mm^2$

Area of each square be $A_o = 1\ mm^2$

Thus, linear magnification of the object be as follows:

$$m = \sqrt{\frac{A}{A_0}} = \sqrt{\frac{6.25}{1}} = 2.5$$

But $m = \dfrac{\text{Image distance (v)}}{\text{Object distance (u)}}$

So, $v = m\,u = 2.5\,u$ (i)

Focal length $f = 10\ cm$

From lens formula we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{2.5\,u} - \frac{1}{u} = \frac{1}{u}\left(\frac{1}{2.5} - \frac{1}{1}\right) = \frac{1}{u}\left(\frac{1-2.5}{2.5}\right)$$

$$u = -\frac{1.5\times10}{2.5} = -6\ cm$$

$$v = 2.5\,u = 2.5\times6 = -15\ cm$$

Hence, virtual image is at $15\ cm$ which is less than nearest point i.e. $25\ cm$ so, can not be seen distinctively by eyes.

26. Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

Ans: As image size is bigger than object then angular size of image is equal to angular size of the object. So, magnifying glass helps see the objects which is placed closer than least distance of distinct vision which is $25\ cm$. Thus, closer object causes a large angular size and magnifying glass provides angular magnification. Thus, without magnification, object cannot be placed closer to the eye whereas with magnification, object can be placed much closer to the eye.

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

Ans: Yes because angular magnification changes. When the distance between eye and magnifying glass is increased then angular magnification decreases. So, angle which subtends at the eye is fewer less than the angle subtended at the lens. Thus, Image distance does not have any effect on angular magnification.

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

Ans: Here, focal length of convex lens cannot be decreased by greater amount because for making lenses very small focal lengths which is not easy. So, spherical and chromatic aberrations are produced by convex lens which have very small focal length.

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

Ans: Here, angular magnification which produced by eyepiece of a compound microscope which is given as $[(\frac{25}{f_e})+1]$ where Focal length of eyepiece is f_e which represents that it is small then angular magnification of eyepiece will be large.

Thus, angular magnification of objective lens of a compound microscope is as follow:

$\frac{1}{(|u_o|f_o)}$ where object distance denoted as u_o and focal length be f_o

Now, magnification is large when $u_o > f_o$. So, in case of microscope, object is kept close to the objective lens. Thus, object distance is very small. As u_o is small then f_o will be smaller. Hence, f_e , f_o are both small in the given condition.

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Ans: In this, when we place our eyes too close to the eyepiece of a compound microscope then we are unable to collect refracted light where field of view decreases substantially. Thus, clarity of the image gets blurred. So, best position of the eye for viewing from compound microscope is eye-ring which is attached to the eyepiece. Hence, precise location of the eye depends on separation between objective lens and eyepiece.

27. An angular magnification (magnifying power) of $30\ X$ is desired using an objective of focal length $1.25\ cm$ and an eyepiece of focal length $5\ cm$. How will you set up the

Ans: It is given that Focal length of the objective lens be $f_o = 1.25$ cm

Focal length of the eyepiece $f_e = 5$ cm

Least distance of distinct vision $d = 25$ cm

Angular magnification of the compound microscope $= 30$ X

Total magnifying power of the compound microscope $m = 30$

angular magnification of the eyepiece is as follows:

$$m_e = \left(1+\frac{d}{f_e}\right) = \left(1+\frac{25}{5}\right) = 6$$

Angular momentum denoted as m_o is as follows $m_o m_e = m$

$$m_o = \frac{m}{m_e} = \frac{30}{6} = 5$$

We have

$$m_o = \frac{\text{Image distance for the objective lens } (v_o)}{\text{Object distance for the objective lens } (-u_o)}$$

$$5 = \frac{v_o}{-u_o}$$

$$v_o = -5\,u_o$$

From lens formula, we have:

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_o}$$

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = \frac{-6}{5u_o}$$

$$u_0 = \frac{-6}{5} \times 1.25 = -1.5 \text{ cm}$$

$$v_o = -5\,u_o = -5 \times (-1.5) = 7.5 \text{ cm}$$

So, object placed at 1.5 cm away from objective lens

From lens formula we have:

$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ where image distance is $v_e = -d = -25$ cm , object distance is u_e i.e.

$$\frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}$$

$$=\frac{-1}{25}-\frac{1}{5}=-\frac{6}{25}$$

$$u_e=-4.17\text{ cm}$$

Thus, separation between objective lens and eye-piece is $|u_e|+|v_o|=4.17+7.5=11.67\text{ cm}$

28. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

Ans: Given that focal length of objective lens be $f_o=140\text{ cm}$

And focal length of eye-piece be $f_e=5\text{ cm}$

Least distance $d=25\text{ cm}$

When telescope is in normal adjustment then its magnifying power is as follows:

$$m=\frac{f_o}{f_e}=\frac{140}{5}=28$$

(b) the final image is formed at the least distance of distinct vision 25 cm

Ans: Let image formed at d then magnifying power is as follows:

$$\frac{f_o}{f_e}\left[1+\frac{f_e}{d}\right]=\frac{140}{5}\left[1+\frac{5}{25}\right]=28[1+0.2]=28\times1.2=33.6$$

29. (a) For the telescope described in Exercise 9.34$ (a), what is the separation between the objective lens and the eyepiece?

Ans: accordingly, $f_o=140\text{ cm}$

$f_e=5\text{ cm}$

So, separation between the objective lens and the eyepiece $f_o+f_e=140+5=145\text{ cm}$

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

Ans: Let $h_1=100\text{ m}$ be the height of tower

Also, distance is $u = 3\text{ km} = 3000\text{ m}$

So, the angle subtended by tower is as follows:

$$\theta = \frac{h_1}{u} = \frac{100}{3000} = \frac{1}{30}\text{ rad}$$

The angle subtended by objective lens is as follows:

$\theta = \frac{h_2}{f_0} = \frac{h_2}{140}$ rad where height of tower formed by objective lens is given as h_2 i.e.

$$\frac{1}{30} = \frac{h_2}{140}$$

$$h_2 = \frac{140}{30} = 4.7\text{ cm}$$

(c) What is the height of the final image of the tower if it is formed at 25 cm

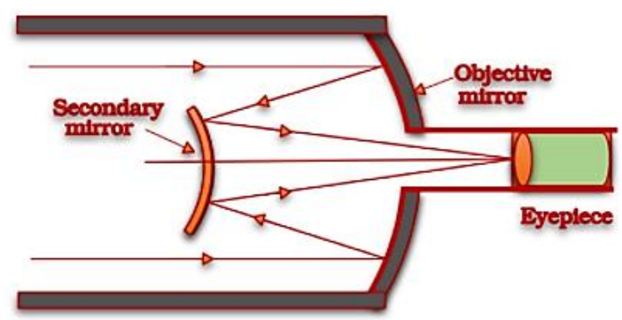
Ans: Here given that image formed at distance is $d = 25\text{ cm}$

Magnification $m = 1+\frac{d}{f_e} = 1+\frac{25}{5} = 1+5 = 6$

Height $= m\,h_2 = 6\times4.7 = 28.2\text{ cm}$

30. A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

Ans: Here, figure shows Cassegrain telescope which consists concave mirror and a convex mirror.



Distance between objective and secondary mirror is denoted as $d = 20\text{ mm}$

Radius of objective mirror is $R_1 = 220\text{ mm}$

Thus, focal length of objective mirror is $f_1 = \frac{R_1}{2} = 110\text{ mm}$

Radius of secondary mirror is $R_2 = 140$ mm

Thus, focal length of secondary mirror is $f_2 = \frac{R_2}{2} = \frac{140}{2} = 70$ mm

So, image is at infinity and act as virtual object for secondary mirror.

Thus, it implies $u = f_1 - d = 110 - 20 = 90$ mm

From mirror formula for secondary mirror, we have

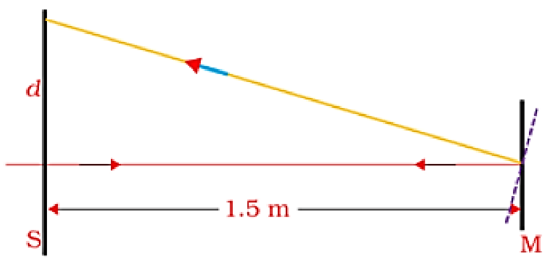
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

$$\frac{1}{v} = \frac{1}{f_2} - \frac{1}{u}$$

$$= \frac{1}{70} - \frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$v = \frac{630}{2} = 315 \text{ mm}$$

31. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



Ans: Here given that angle of deflection is $\theta = 3.5^\circ$ and distance is 1.5 m

So, reflected rays deflected twice angle of defection which implies $2\theta = 7.0^\circ$

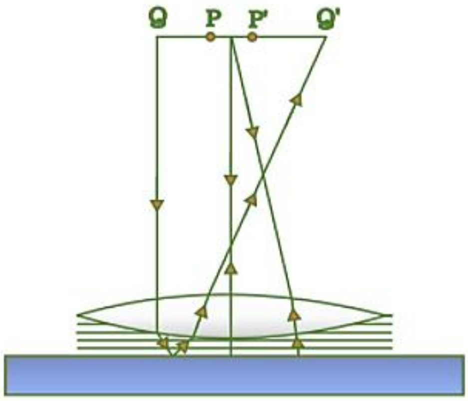
Thus, displacement is given as

$$\tan 2\theta = \frac{d}{1.5}$$

$$d = 1.5 \times \tan 7^\circ = 0.184 \text{ m} = 18.4 \text{ cm}$$

32. Figure 9.37 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle

from the lens is measured to be 45.0 cm . The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm . What is the refractive index of the liquid?



Ans: Let focal length of convex lens be $f_1 = 30$ cm

Here, liquid acts as mirror

So, focal length of liquid is f_2

Focal length of both system i.e. convex lens and liquid be $f = 45$ cm

The equivalent focal length is as follows:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$= \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$f_2 = -90 \text{ cm}$$

Now, let refractive index be μ_1 and radius of curvature for one surface be R then radius for other surface be $-R$

We can find R by using $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R} + \frac{1}{-R}\right)$

$$\frac{1}{30} = (1.5-1)\left(\frac{2}{R}\right)$$

$$R = \frac{30}{0.5 \times 2} = 30 \text{ cm}$$

Now, let μ_2 be refractive index

Radius of curvature on side of plane mirror be infinity and on the side of lens $R = -30$ cm

So, value of μ_2 be as follows:

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right]$$

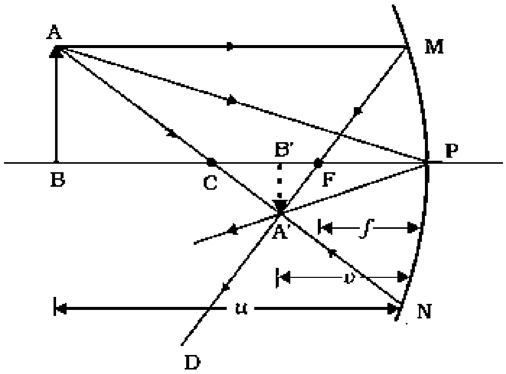
$$\frac{-1}{90} = (\mu_2 - 1)\left[\frac{1}{+30} - 0\right]$$

$$\mu_2 - 1 = \frac{1}{3}$$

$$\mu_2 = \frac{4}{3} = 1.33$$

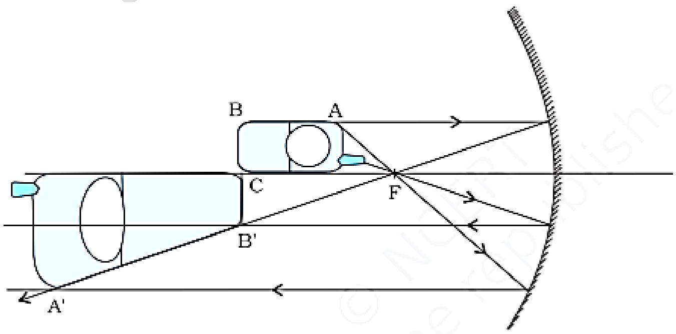
Examples

1. Suppose that the lower half of the concave mirror's reflecting surface in Fig. is covered with an opaque (non-reflective) material. What effect will this have on the image of an object placed in front of the mirror?



Ans: here, image will not show half object but from laws of reflection to be true at all points of remaining part of mirror, so image will be of whole object. Thus, since area has bee reduced then intensity of image will be low.

2. A mobile phone lies along the principal axis of a concave mirror, as shown in Fig. . Show by suitable diagram, the formation of its image. Explain why the magnification is not uniform. Will the distortion of image depend on the location of the phone with respect to the mirror?



Ans: here, diagram shows the information of image of the phone which is on the plane perpendicular to principal axis on that same plane and same size which represents by $B'C = BC$

So, image is distorted.

3. Example 9.3 An object is placed at (i) $10\ \text{cm}$ (ii) $5\ \text{cm}$ in front of a concave mirror of radius of curvature $15\ \text{cm}$. Find the position, nature, and magnification of the image in each case.

Ans: Here, focal length $f = \frac{-15}{2} = -7.5\ \text{cm}$

(i) Object distance be $u = -10\ \text{cm}$

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{-7.5}$$

$$v = \frac{10 \times 7.5}{-2.5} = -30\ \text{cm}$$

Image be $30\ \text{cm}$ from the mirror on the same side

Thus, magnification $m = -\frac{v}{u} = -\frac{(-30)}{(-10)} = -3$

So, image is magnified, real and inverted.

(ii) Object distance be $u = -5\ \text{cm}$

$$\frac{1}{v} + \frac{1}{-5} = \frac{1}{-7.5}$$

$$v = \frac{5 \times 7.5}{(7.5-5)} = 15\ \text{cm}$$

Image be $15\ \text{cm}$ from the mirror on the same side

Thus, magnification $m = -\frac{v}{u} = -\frac{(15)}{(-5)} = 3$

So, image is magnified, virtual and erect

4. Example 9.4 Suppose while sitting in a parked car, you notice a jogger approaching towards you in the side view mirror of $R = 2\ \text{m}$. If the jogger is running at a speed of $5\ \text{m s}^{-1}$, how fast the image of the jogger appear to move when the jogger is (a) $39\ \text{m}$ (b) $29\ \text{m}$ (c) $19\ \text{m}$ and (d) $9\ \text{m}$ away.

Ans: According to mirror equation we have: $v = \frac{fu}{u-f}$

For convex mirror, as $R = 2\text{ m}, f = 1\text{ m}$. Then for $u = -39\text{ m}$, $v = \frac{(-39)\times 1}{-39-1} = \frac{39}{40}\text{ m}$

As jogger moves at constant speed of 5 m s^{-1}. Thus after one second position of image be (for $u = -39+5 = -34$) is $(34/35)\text{m}$

So, shift in one second be $\frac{39}{40}-\frac{34}{35} = \frac{1365-1360}{1400} = \frac{5}{1400} = \frac{1}{280}\text{ m}$

Thus, average speed between 39 m and 34 m from mirror is $(\frac{1}{280})\text{ms}^{-1}$

In the following manner, for $u = -29\text{ m}, -19\text{ m}, -9\text{ m}$ the speed with which image appears to move is $\frac{1}{150}\text{ m s}^{-1}$, $\frac{1}{60}\text{ m s}^{-1}$, $\frac{1}{10}\text{ m s}^{-1}$ respectively.

As jogger moves with constant speed then speed increases substantially and move closer to mirror.

5. The earth takes 24 h to rotate once about its axis. How much time does the sun take to shift by 1° when viewed from the earth?

Ans: Here, Time taken for 360° is equal to 24 h

Thus, Time taken for 1° is equal to $\frac{24}{360}\text{ h} = 4\text{ min}$

6. Light from a point source in air falls on a spherical glass surface ($n = 1.5$ and radius of curvature $= 20\text{ cm}$. The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Ans: Here $u = -100\text{ cm}, v = ?, R = +20\text{ cm}, n_1 = 1, n_2 = 1.5$

Thus We have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20}$$
$$v = +100\text{ cm}$$

Hence, image formed at distance of 100 cm from the glass surface which is in direction of incident light.

7. A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

Ans: Here refractive index of liquid is equal to 1.47 in order to make the lens disappear which means $n_1 = n_2$. So, it implies $\frac{1}{f} = 0$, $f \to \infty$. Thus, lens in liquid will act like plane sheet of glass. No because liquid is not water which could be glycerine.

8. (i) If $f = 0.5\text{ m}$ for a glass lens, what is the power of the lens?

Ans: Here power is equal to $+2$ dioptre

(ii) The radii of curvature of the faces of a double convex lens are 10 cm, 15 cm. Its focal length is 12 cm. What is the refractive index of glass?

Ans: According to given conditions, we have

$f = +12\text{ cm}$, $R_1 = +10\text{ cm}$, $R_2 = -15\text{ cm}$

Here refractive index is unity (let)

From lens formula, we have :

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

which implies $n = 1.5$

(iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water $= 1.33$, refractive index for air-glass $= 1.5$.)

Ans: Here, for glass lens which is in air $n_2 = 1.5$, $n_1 = 1$, $f = +20\text{ cm}$

From lens formula, we have:

$$\frac{1}{20} = 0.5\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{(i)}$$

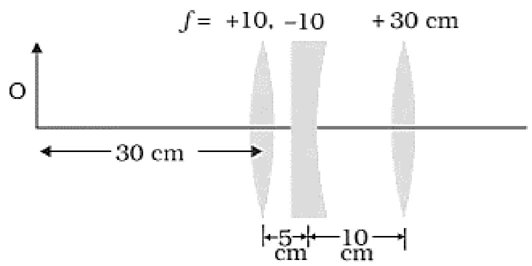
Now, for same glass lens in water $n_2 = 1.5$, $n_1 = 1.33$

$$\text{Then } \frac{1.33}{f} = (1.5-1.33)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{(ii)}$$

From (i) and (ii) we get:

$f = +78.2\text{ cm}$

9. Find the position of the image formed by the lens combination given in the figure



Ans: Image by first lens be

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$v_1 = 15 \text{ cm}$$

Thus it serves as object for second lens image which is at distance $(15\text{-}5)\text{ cm} = 10 \text{ cm}$ which is on the right of second lens.

Although this image is real and serves as virtual for second lens which implies rays come from it for second lens.

$$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10}$$

$$v_2 = \infty$$

Thus, virtual image is formed at infinite distance to the left of second lens which act for third lens,

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

$$\frac{1}{v_3} = \frac{1}{\infty} + \frac{1}{30}$$

$$v_3 = 30 \text{ cm}$$

Hence, final image is formed 30 cm right of third lens.