Infinity Learn by Sri Chaitanya Educational Institutions

NUMBER SYSTEM

EXERCISE 1.1

1. Is zero a rational number ? Can you write in the form of $\frac{p}{q}$, where p & q are integers and $q \neq 0$.

Sol. Yes zero (0) is a rational number. We can write zero (0) in the form of $\frac{p}{q}$ as

$$0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{-1} = \frac{0}{-2} = \ldots \text{ etc.}$$

2. Find six rational numbers between 3 and 4.

Sol. To find six (6) rational numbers between 3 and 4 we will multiply the numerator and denominator of $\frac{1}{3}$ & $\frac{4}{1}$ by $6+1=7$. So $\frac{1}{3}$ and $\frac{4}{1}$ will become $\frac{3\times 7}{1\times 7}$ and $\frac{4\times 7}{1\times 7}$ i.e., $\frac{21}{7}$ and $\frac{28}{7}$

So, 6 rational numbers between $3 \& 4$ i.e., between $\frac{21}{7}$ and $\frac{28}{7}$ are

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.

3. Find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. By same method as explained in Q.No.2 i.e., multiply numerator and denominator both of rational numbers $\frac{3}{5}$ and $\frac{4}{5}$ by one more than the number as many rational numbers are required to insert between $\frac{3}{4}$ and $\frac{4}{5}$ i.e., $5+1=6$

$$\Rightarrow \quad \frac{3}{4} \text{ and } \frac{4}{5} = \frac{3\times 6}{5\times 6} \text{ and } \frac{4\times 6}{5\times 6} \Rightarrow \frac{18}{30} \text{ and } \frac{24}{30}$$

Hence, required rational numbers are $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.
(ii) Every integer is a whole number.
(iii) Every rational number is a whole number.

Sol. (i) True, as the set of whole numbers contains all the natural numbers.
(ii) False, as $-1, -2, -3, \ldots$ are integers but they are not whole numbers.

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(iii) False, there are infinite numbers like $\frac{1}{2}, \frac{-3}{4}, \frac{1}{10}$ etc. Which are rational numbers but they are not whole numbers.

EXERCISE 1.2

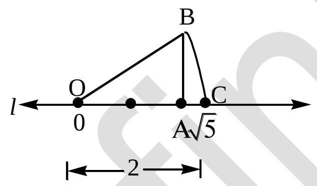
1. State whether the following statements are true or false. Justify your answer.
(i) Every irrational number is a real number.
(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
(iii) Every real number is an irrational number.

Sol. (i) We know that real number is either rational or irrational. So we can say that every irrational number is a real number. Hence, the given statement is true.
(ii) We know that real numbers can be represented on the number line. Thus, every point on the number line is of the form $\sqrt{m}$, where m is a natural number. Hence, the given statement is true.
(iii) We know that, rational number and irrational numbers taken together as known as real numbers. Hence, the given statement is false.

2. Are the square roots of all positive integers irrational ? If not, given an example of the square root of a number that is a rational number.

Sol. "The square root of all positive numbers are irrational", is not correct.
Since, square root of 25 i.e $\sqrt{25} = 5$, which is a rational number.
Hence, the given statement is not correct.

3. Show how $\sqrt{5}$ can be represented on number line.



Sol.

We shall now show how to represent $\sqrt{5}$ on the number line.
Draw a number line 1 and mark a point O, representing zero (0), on it. Let point A represents 2 as shown in the figure. Now, construct a right − angled ΔOAB, right angled at A such that
$OA = 2$ units and $AB = 1$ unit (see figure)
By Pythagoras theorem, we have $OB = \sqrt{OA^2 + AB^2} = \sqrt{4+1} = \sqrt{5}$
Draw an are with centre O and radius OB to cut the number line at C. Clearly
$OC = OB = \sqrt{5}.$
Thus, the point C represents the irrational number $\sqrt{5}$.

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EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$

(v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol. (i)

$$\begin{array}{l} 100)360(0.36 \\ \quad 300 \\ \hline \quad\; 600 \\ \quad\; 600 \\ \hline \qquad 0 \end{array}$$

Hence, decimal form of $\frac{36}{100} = 0.36$

It has terminating decimal expansion.

(ii)

$$\begin{array}{l} 11)100(0.090909... \\ \quad 99 \\ \hline \quad 100 \\ \quad 99 \\ \hline \quad 100 \\ \quad 99 \\ \hline \quad 100 \\ \quad 99 \\ \hline \qquad 1 \end{array}$$

Hence, decimal form of $\frac{1}{11} = 0.090909... = 0.\overline{09}$

It has non terminating and repeating decimal exapansion.

(iii) $4\frac{1}{8} = \frac{4 \times 8 + 1}{8} = \frac{32+1}{8} = \frac{33}{8}$

$$\begin{array}{l} 8)33.000(4.125 \\ \quad 32 \\ \hline \quad 10 \\ \quad\; 8 \\ \hline \quad 20 \\ \quad 16 \\ \hline \quad 40 \\ \quad 40 \\ \hline \qquad 0 \end{array}$$

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Hence, decimal form of $4\frac{1}{8} = 4.25$. It has terminating decimal expansion.

(iv)

13) 3.0000000 (0.23076923...
26

40
39

100
91

90
78

120
117

30
26

40
39

1

Hence, decimal form of $\frac{3}{13} = 0.23076923... = 0.\overline{230769}$

Also, it has non terminating repeating decimal expansion.

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(v)

11) 2.00000 (0.181818...
11

90
88

20
11

90
88

20
11

90
88

2

$\therefore \frac{2}{11} = 01818...,= 0.\overline{18}$, non-terminating and repeating decimal expansion.

(Vi)

```
400) 329.000 (0.8225
     3200
     ----
      900
      800
     ----
      1000
       800
      ----
       2000
       2000
       ----
          0
       ----
```

$\therefore \frac{329}{400} = 0.8225,$ terminating decimal.

In the above examples, rational numbers are expressed into decimal form and the division process comes to an end without leaving any remainder. Such type of decimals are called terminating decimals. Since, the number of digits after decimal is finite, so these numbers are also called finite decimal f orms.

2. You known that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division ? If so, have ?

Sol. Yes, we can predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division.

All of the above will have repeating decimals which are permutations of 1, 4, 2, 8, 5, 7

For example :

```
7)10 ( 0.1428571....
   7
  --
   30
   28
   --
    20  <--
    14
    --
     60
     56
     --
      40
      35
      --
       50
       49
       --
        10  <--
         7
        --
         3
        --
```

In case of $\frac{2}{7}$

We divide 2 by 7. On putting a decimal point in the quotient, 2 becomes 20. In the above long division we also get 20 as remainder after two steps. See the respective

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quotient (Here it is 2). In the division of 2 by 7, the first quotient is 2 and then 8571 …. .

Thus, $\frac{2}{7}=0.\overline{285714}$

To find $\frac{3}{7}$, we have to see the above long division when the remainder becomes 3 and see the respective quotient (here it is 4), then write the new quotient beginning from there.

Thus, $\frac{3}{7}=0.\overline{428571}$

Similarly, $\frac{4}{7}=0.\overline{571428}$

$\frac{5}{7}=0.\overline{714285}$

$\frac{6}{7}=0.\overline{857142}$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Sol. (i) Let $x=0.\overline{6}$ or $x=0.666...$ □ (i)

$10x=6.666...$ □ (ii) [On multiplying (i) by 10]

$\Rightarrow 9x=6$ [Subtracting (i) from (ii)]

$\therefore x=\frac{2}{3}$.

(i) Let $x=0.4\overline{7}$

$...(i)$

or $x=0.4777...$

Multiplying (i) by 10, we get

$10x=4.777...$

Again multiplying (ii) be 10,we get

□ (ii)

$100x=47.777...$

Subtracting equation (ii) from equation (iii), we get

$90x=43$

$\therefore \quad x = \frac{43}{90}.$

(iii) Let $x = 0.\overline{001}$

or $x = 0.001001 \ldots$ (i)

$\Rightarrow 1000x = 1.001001\ldots$ (ii)

[On multiplying (i) by 1000]

$\Rightarrow 999x = 1$ [On subtracting (i) from (ii)]

$\therefore \quad x = \frac{1}{999}.$

4. Express $0.99999\ldots$ in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmate, discuss why the answer makes sense.

Sol. Let $x = 0.99999\ldots$ (i)

Here is only one repeating digit (9) after decimal point so we multiply both sides of (1) by 10^1.

$10x = 9.9999\ldots$ (2)

Subtracting (1) from (2), we get

$10x - x = (9.9999\ldots) - (0.9999\ldots)$

$\Rightarrow \quad 9x = 9$

$\Rightarrow \quad x = 1$

Hence, $0.99999\ldots = 1$

Since $0.99999\ldots$ goes on forever. So, there is no gap between 1 and $0.99999\ldots$ and hence they are equal.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. Here, we have

17) 1.0000000000000000 (0.0588235294117647

85
150
136
140
136
40
34
60
51
90
85
50
34
160
153
70
68
20
17
30
17
130
119
110
102
80
68
120
119
1

Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

Hence, the maximum number of digits in the repeating block while computing $\frac{1}{17}$ are 16.

6. Look at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. Let the various such rational numbers be $\frac{1}{2}, \frac{1}{4}, \frac{7}{8}, \frac{37}{25}, \frac{8}{125}, \frac{17}{20}, \frac{31}{16}$ etc.

In all these cases we have to multiply the denominator q by such number so that it becomes 10 or a power of 10.

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$$\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}=0.5$$

$[\therefore 2\times5=10]$

$$\frac{1}{4}=\frac{1\times25}{4\times25}=\frac{25}{100}=0.25$$

$[\therefore 4\times25=100]$

$$\frac{7}{8}=\frac{7\times125}{8\times125}=\frac{875}{1000}=0.875$$

$[\therefore 8\times125=1000]$

$$\frac{37}{25}=\frac{37\times4}{25\times4}=\frac{148}{100}=1.48$$

$[\therefore 25\times4=100]$

$$\frac{8}{125}=\frac{8\times8}{125\times8}=\frac{64}{1000}=0.064$$

$[\therefore 125\times8=1000]$

$$\frac{17}{20}=\frac{17\times5}{20\times4}=\frac{85}{100}=0.85$$

$[\therefore 20\times5=100]$

$$\frac{31}{16}=\frac{31\times625}{16\times625}=\frac{19375}{10000}=1.9375$$

$[\therefore 16\times625=10000]$

Thus we have observed that those rational numbers whose denominators when multiplied by a suitable integer produce a power of 10 are expressible in the finite decimal form. But this can always be done only when denominator of the given rational number has either 2 or 5 or both of them as the only prime factors. Thus, we obtain the following property.

If the denominator of a rational number in standard form has no prime factors other than 2 or 5, then and only then it can be represented as a terminating decimal.

7. Write three numbers whose decimal expansions are non-terminating no-recurring.

Sol. Three numbers whose decimal representations are non − terminating and non − repeating are $\sqrt{2},\sqrt{3},$ and $\sqrt{5}$ or we can say 0.1001000100001 □., 0.202002000200002 □. and 0.003000300003 □.

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

So. We know that, $\frac{5}{7}=0.714$□ □ □ and $\frac{9}{11}=0.818$□ □.

We have to find out three irrational numbers between 0.714 □ □. and 0.818 □ □

The three irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ are :

0.72072007200072000072 □ □ □ □
0.76076007600076000076 □ □ □ □
0.79079007900079000079 □ □ □ □

9. Classify the following numbers are rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 □ (v) 1.101001000100001 □

Sol. (i) $\sqrt{23}$ is an irrational number as 23 is not a perfect square.

(ii) $\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5} = 3 \times 5$

$= 15$, which is a rational number.

Hence, $\sqrt{225}$ is a rational number.

(iii) 0.3796 is a terminating decimal and so, it is a rational number.

(iv) 7.478478 □ is non terminating but repeating, so, it is a rational number.

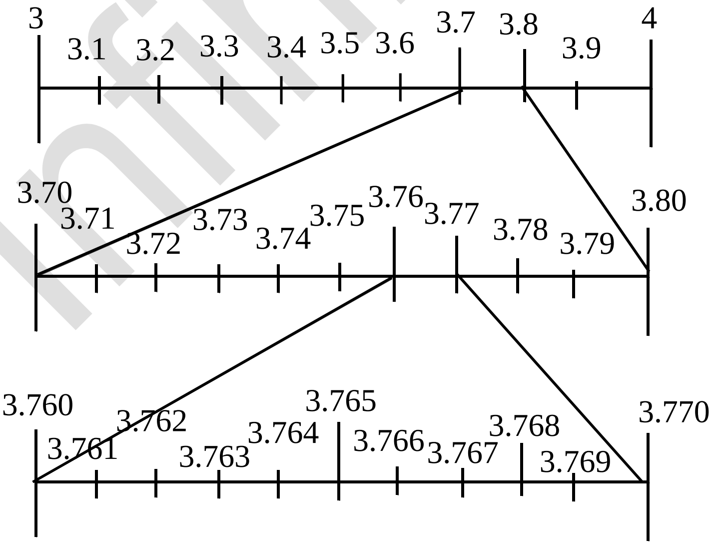
(v) 1.101001000100001 □ is non terminating and non repeating so, it is an irrational number.

EXERCISE 1.4

1. Visualize 3.765 on the number line.

Sol. This number lies between 3 and 4. The distance between 3 and 4 is divided into 10 equal parts. Then the first mark to the right of 3 will represent 3.1 and second 3.2 and so on. Now, 3.765 lies between 3.7 and 3.8. We divide the distance between 3.7 and 3.8 into 10 equal parts

3.76 will be on the right of 3.7 at the sixth mark, and 3.77 will be on the right of 3.7 at the 7th mark and 3.765 will lie between 3.76 and 3.77 and so on.



To mark 3.765, we have to use magnify glass.

2. Visualize $4.\overline{26}$ on the number line, upto 4 decimal places.

Sol. We have, $4.\overline{26} = 4.2626$

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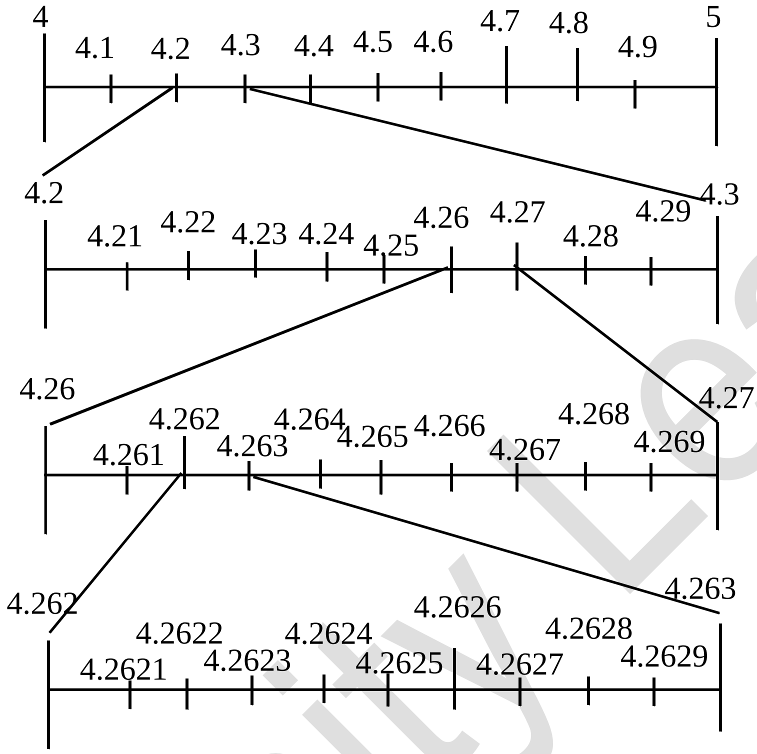
This number lies between 4 and 5. The distance between 4 and 5 is divided into 10 equal parts. Then the first mark to the right of 4 will represent 4.1 and second 4.2 and so on. Now, 4.2626 lies between 4.2 and 4.3. We divide the distance between 4.2 and 4.3 into 10 equal parts. 4.2626 lies between 4.26 and 4.27. Again we divide the distance between 4.26 and 4.27 into equal parts. The number 4.2626 lies between 4.262 and 4.263. The distance between 4.262 and 4.263 is again divided into 10 equal parts. Sixth mark from right to the 4.262 is 4.2626.



EXERCISE 1.5

1. Classify the following numbers as rational or irrational :

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Sol. (i) $2-\sqrt{5}$ is an irrational number because 2 is rational and $\sqrt{5}$ is irrational and we know that the

difference of a rational and an irrational is always irrational.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3$, which is clearly irrational.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$ is clearly rational.

(iv) $\frac{1}{\sqrt{2}}$ is an irrational number because numerator 1is a rational number and

denominator $\sqrt{2}$ is an

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irrational so by rule that rational upon irrational is always irrational number) $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π is an irrational number, because 2π is product of 2 and π .(Product of a rational number and an irrational number is always irrational number so 2π which is product of 2 and π is an irrational number).

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$ (iii) $(\sqrt{5}+\sqrt{2})^2$ iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol. (i) $(3+\sqrt{3})(2+\sqrt{2})=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})=(3)^2-(\sqrt{3})^2=9-3=6$

(iii) $(\sqrt{5}+\sqrt{2})^2=(\sqrt{5})^2+(\sqrt{2})^2+2\times\sqrt{5}\times\sqrt{2}=5+2+2\sqrt{10}\quad=7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2=5-2=3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. We know that the ratio of circumference of a circle to length of its corresponding diameter is known as π (pie).

i.e., $\pi=\frac{c}{d}$

Now, read this to understand why 'π' is an irrational number.

If we take circumference of a circle as an integer (natural number) then we find length of its corresponding diameter comes in decimal [not an integer (natural number)] so ratio of $\frac{c}{d}$ is not in the form of $\frac{p}{q}$, i.e., p and q both are not integers. Similarly if we take diameter of circle as an integer (natural number) then circumference of its corresponding circle comes out to be in the decimal form so again ratio $\frac{c}{d}=\pi$ does not comes in the form of $\frac{p}{q}$ because diameter, i.e., 'q' is an integer and 'p' is in decimal form. So it is clear that for any circle if length of diameter is natural number then circumference is the decimal form or if circumference is natural number then diameter is a decimal number. So we can say that the ratio $\frac{\pi}{d}$ can never come in the

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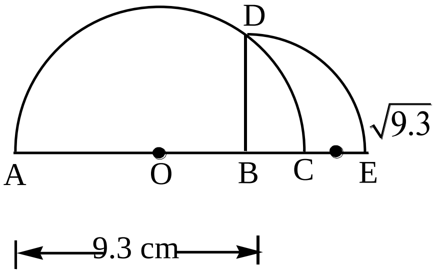
form of $\frac{p}{q}$ where p and q both integer and $q \neq 0$. So it is clear that the ratio $\frac{c}{d}$, i.e., π can never be rational hence 'π' is always an irrational number.

4. Represent $\sqrt{9.3}$ on the number line.



Sol. Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that $AB = 9.3$ units. From B, mark a distance of 1 units and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semi-circle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semi$-$circle at D. Then $BD = \sqrt{9.3}$. To represent $\sqrt{9.3}$ on the number line. Let us treat the line BC as the number line, with B as 0 and C as 1. Draw an arc with centre B and radius BD, which intersects the number line at E. Then, E represents $\sqrt{9.3}$.

5. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$.

EXERCISE 1.6

1. Find (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Sol. (i) $(64)^{\frac{1}{2}} = (2\times2\times2\times2\times2\times2)^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}} = (2)^{6\times\frac{1}{2}} = (2)^3 = 8$

(ii) $(32)^{\frac{1}{5}} = (2\times2\times2\times2\times2)^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2^1 = 2$

(iii) $(125)^{\frac{1}{3}} = (5\times5\times5)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}} = (5)^{3\times\frac{1}{3}} = 5^1 = 5$

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2. Find (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Sol. (i) $9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}} = (3)^{2\times\frac{3}{2}} = (3)^3 = 27$.

(ii) $32^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = (2)^2 = 4$.

(iii) $16^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}} = (2)^{4\times\frac{3}{4}} = (2)^3 = 8$.

(iv) $(125)^{\frac{-1}{3}} = \left(5^3\right)^{\frac{-1}{3}} = (5)^{3\times\left(\frac{-1}{3}\right)} = (5)^{-1} = \frac{1}{5}$.

3. Simplify: (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$.

Sol. (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{13}{15}}$.

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1}{\left(3^3\right)^7} = \frac{1}{3^{3\times7}} = \frac{1}{3^{21}} = 3^{-21}$.

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7.8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$.

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