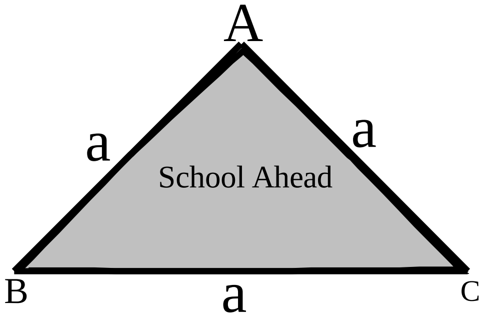
HERON'S FORMULA

NCERT Exercise – 12.1

1. A traffic signal board indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a' Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Sol.



Perimeter of the equilateral triangle $= 180\text{ cm}$

Let the side of equilateral triangle be a.

$\therefore 2s = (a + a + a) = 3a \Rightarrow s = \frac{3a}{2}$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

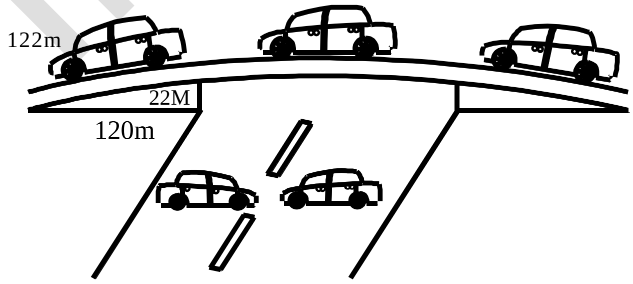
$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}a^2}{4}\text{cm}^2$

$\because$ Perimeter of signal board is 180 cm

$\therefore \text{Area} = \frac{\sqrt{3}}{4}(60)^2 = 900\sqrt{3}\text{cm}^2$

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122m, 22 m and 120 m (see Fig). The advertisements yield an earnings of `.5000 per m^2 per year. A company hired one of its walls for 3 months. How rent did it pay?



Sol. The sides of the triangular wall are

$a = 122m, b = 22m, c = 120m$

$s=\frac{a+b+c}{2}=\frac{122+22+120}{2}=\frac{264}{2}=132m$

Area of triangular wall $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{132(132-122)(132-22)(132-120)}$

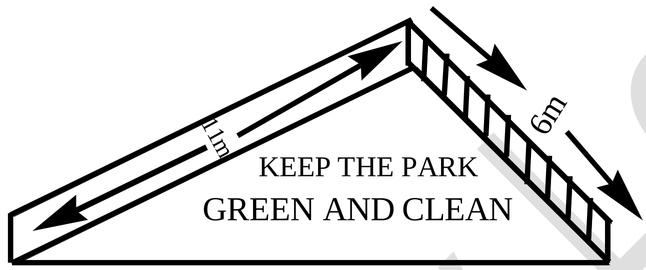
$=\sqrt{132\times10\times110\times12}=10\times11\times12=1320m^2$

Cost of hiring the walls for three months $=\text{Area}\times\text{Rate}\times\text{Time}$

$=1320\times5000\times\frac{1}{4}=330\times5000$ `= 1650000

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig). If the sides of the wall are 15 m, 11m and 6m, find the area painted in colour.

Sol.



The sides of the sliders are 15 m, 11m and 6m.

$s=\frac{a+b+c}{2}=\frac{15+11+6}{2}=16m$

Area of triangle slide

$=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{16(16-15)(16-11)(16-6)}$

$=\sqrt{16\times1\times5\times10}=\sqrt{800}=20\sqrt{2}m^2$

4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Sol. Let a, b and c be the sides of a triangle such that $a=18\,cm, b=10\,cm$ and

$a+b+c=42cm$.

$\therefore c=42-a-b$

$\Rightarrow c=(42-18-10)cm=14cm$

Now, $s=\frac{1}{2}(a+b+c)=\frac{1}{2}\times42cm=21cm$

$\therefore s-a=(21-18)cm=3cm$

$s-b=(21-10)cm=11cm$

And $s-c=(21-14)cm=7cm$

$\therefore$ Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{21\times3\times11\times7}\text{cm}^2$

$=\sqrt{3\times7\times3\times11\times7}\text{cm}^2=\sqrt{3\times3\times7\times7\times11}\text{cm}^2$

$=3\times7\sqrt{11}\text{cm}^2=21\sqrt{11}\text{cm}^2$

5. Sides of a triangle are in the ratio of $12:17:25$ and its perimeter is 540 cm. Find its area.

Sol. Let the sides of the triangle be $a=25x, b=17x, c=12m$

Perimeter of $\Delta=25x+17x+12x=540\text{m}$

$\Rightarrow 54x=540\Rightarrow x=10$

$a=25\times10=250\text{m}, b=17\times10=170\text{m}, c=12\times10=120\text{m}$

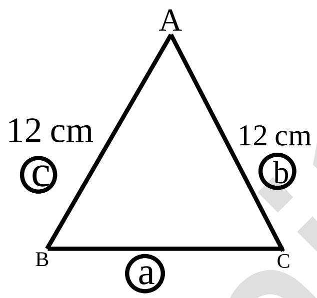
Now, $s=\dfrac{a+b+c}{2}=\dfrac{250+170+120}{2}=\dfrac{540}{2}=270\text{m}$

$\therefore \Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{270(270-250)(270-170)(270-120)\text{m}^2}$

$=\sqrt{270\times20\times100\times150\text{m}^2}=9000\text{m}^2$

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol.



Here $c=b=12\text{cm}$

And $a+b+c=\text{perimeter}=30\text{ cm}$

So, $a=(30-c-b)\text{cm}$

Or $a=(30-12-12)\text{cm}=6\text{cm}$

Now, $s=\dfrac{1}{2}\times30\text{cm}=15\text{cm}$

$\therefore\ s-a=(15-6)\text{cm}=9\text{cm}$

$s-b=(15-12)\text{cm}=3\text{cm}$

And $s-c=(15-12)\text{cm}=3\text{cm}$

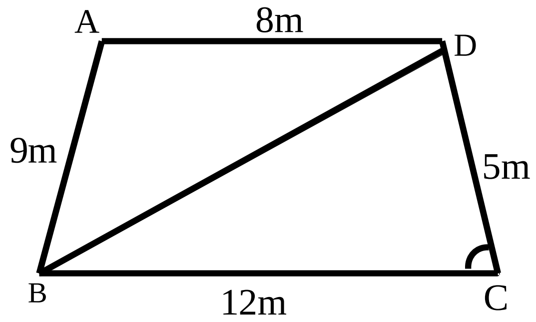
$\therefore$ Area of the triangle $=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{15\times9\times3\times3}\text{cm}^2$

$=\sqrt{5\times3\times3\times3\times3\times3}\text{cm}^2=3\times3\sqrt{5\times3}\text{cm}^2=9\sqrt{15}\text{cm}^2$

Exercise – 12.2

1. A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°, AB = 9m, BC = 12m, CD = 5m$ and $AD = 8m$. How much area does it occupy?

Sol.



Area of $\Delta BCD = \frac{1}{2} \times BC \times CD$

$= \left(\frac{1}{2} \times 12 \times 5\right) m^2$

$= 30m^2$

Using Pythagoras theorem, we have:

$BD^2 = BC^2 + CD^2 \Rightarrow BD^2 = 12^2 + 5^2$

So, $BD^2 = 144 + 25 \Rightarrow BD^2 = 169$

$\therefore BD = \sqrt{169}m = 13m$

For ΔABD:

Let $a = 13m$, $b = 8m$, and $c = 9m$

Now, $s = \frac{1}{2}(a + b + c) = \frac{1}{2}(13 + 8 + 9)m = \frac{1}{2} \times 30m = 15m$

$s - a = (15 - 13) = 2$

$s - b = (15 - 8) = 7$

And $s - c = (15 - 9) = 6$

$\therefore$ Area of $\Delta ABD = \sqrt{15 \times 2 \times 7 \times 6m^2} = \sqrt{3 \times 5 \times 2 \times 7 \times 2 \times 3}m^2$

$= 2 \times 3\sqrt{35}m^2$

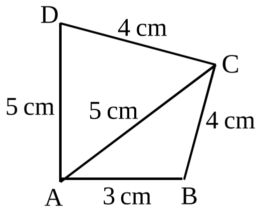
$= 6 \times 5.9m^2 (approx) = 35.4m^2 (approx)$

$\therefore$ Required area = Area of ΔABD + Area of ΔBCD

$= 35.4m^2 + 30m^2 = 65.4m^2$

2. Find the area of a quadrilateral ABCD in which $AB = 3\ cm, BC = 14\ cm, CD = 4\ cm, DA = 5\ cm$ and $AC = 5\ cm$.

Sol.



$\text{ar}(ABCD) = \text{ar}(ABC) + \text{ar}(ACD)$ ....(1)

For ΔABC,

$s = \dfrac{3+4+5}{2} = 6\text{cm}$

$\therefore \text{ar}(ABC) = \sqrt{6(6-3)(6-4)(6-5)}\text{cm}^2$

$= \sqrt{6\times3\times2\times1} = 6\text{ cm}^2$ ....(2)

For ΔACD,

$s = \dfrac{5+5+4}{2} = 7\text{cm}$

$\therefore \text{ar}(ACD) = \sqrt{7(7-5)(7-5)(7-4)}\text{cm}^2$

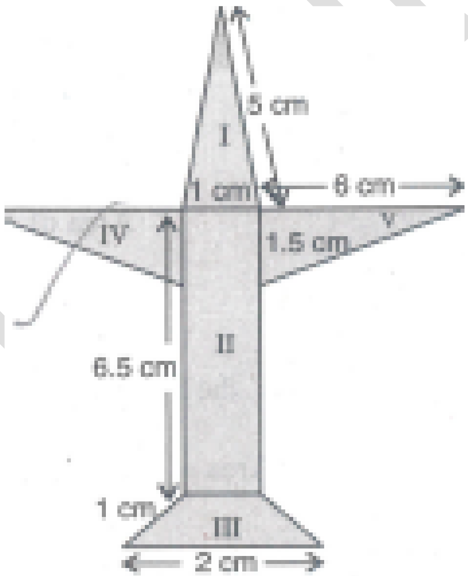
$= \sqrt{7\times2\times2\times3} = 2\sqrt{21}\text{cm}^2 = 9.17\text{cm}^2$ ....(3)

Substituting the value of areas from (2) and (3) in (1) we get

$\text{ar}(ABCD) = (6+9.17)\text{cm}^2 = 15.17\text{cm}^2$

3. Radha made a picture of an aeroplane with coloured paper as shown in figure given below. Find the total area of the paper used.

Sol.



There are five parts in the picture of an aeroplane

1. Sides of I part are 5 cm, 5 cm and 1 cm.

$s = \dfrac{a+b+c}{2} = \dfrac{5+5+1}{2} = 5.5\text{ cm}$.

Area of I part $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$=\sqrt{5.5\times0.5\times0.5\times4.5}$

$=\sqrt{6.1875}=2.4875\text{ cm}^2$

2. Sides of II rectangular parts are 6.5 cm and 1 cm.

Area $=$ length $\times$ breadth

$=6.5\times1=6.5\text{ cm}^2$

3. Length of non-parallel sides of trapezium (III part) are 1 cm and 2 cm.

$\therefore$ Area of III part $=$ Area of equilateral triangle $+$ area of parallelogram

$=\frac{\sqrt{3}}{4}a^2+\text{Base}\times\text{Height}$

$=\frac{\sqrt{3}}{4}(1)^2+1\times\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{4}=\frac{3\times1.732}{4}=\frac{5.196}{4}\text{ cm}^2$

$=1.299\text{ cm}^2$

4. Sides of IV part (rt angled triangle) are 6 cm, 1.5 cm^2

Area of IV part $=\frac{1}{2}\times6\times1.5=3\times1.5=4.5\text{cm}^2$

5. Area of V part $=$ Area of IV part $=4.5\text{ cm}^2$

$\therefore$ Area of the picture of aeroplane

$=$ Area of I $+$ Area of II $+$ Area of III $+$ Area of IV $+$ Area of V

$=(2.4875+6.5+1.299+4.5+4.5)\text{ cm}^2$

$=19.2865\text{ cm}^2=19.3\text{ cm}^2$

4. A triangle and a parallelogram have the same base and the same area. If the side of the triangle are 26 cm, 28 cm and the parallelogram stand on the base 28 cm, find the height of the parallelogram

Sol. Let its sides be a ,b and c such that $a=26\text{ cm}, b=26\text{ cm}, b=28\text{ cm, and } c=30$. Then,

$s=\frac{1}{2}(26+28+30)\text{ cm}$

$=\frac{1}{2}\times84\text{cm}=42\text{cm}$

Now, $s-a=(42-26)\text{ cm}=16\text{ cm}$

$s-b=(42-28)\text{ cm}=14\text{ cm}$

And $s-c=(42-30)\text{ cm}=12\text{ cm}$

$\therefore$ Area of the triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{42\times16\times14\times12}\text{ cm}^2$

$=\sqrt{2\times3\times7\times4\times4\times2\times7\times2\times2\times3}\text{ cm}^2$

$=\sqrt{2\times2\times2\times2\times3\times3\times4\times4\times7\times7}\text{ cm}^2$

$=(2\times2\times3\times4\times7)\ \text{cm}^2=336\ \text{cm}^2$

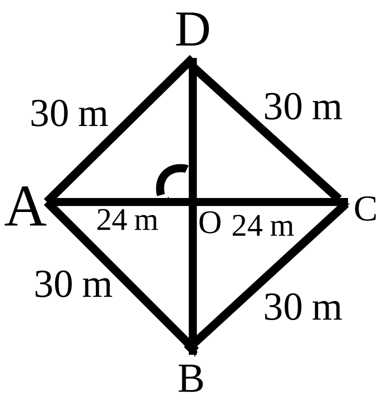
For the parallelogram:

$\text{Area}=\text{Base}\times\text{Height}$

$\therefore\ \text{Height}=\dfrac{\text{Area}}{\text{Base}}=\left(\dfrac{336}{28}\right)\text{cm}$ $\left[\begin{array}{l}\therefore \text{Area of }\|\text{gm}=\text{Area of }\Delta(\text{given})\\ \therefore \text{Area of }\|\text{gm}=336\ \text{cm}^2\text{and its}\\ \text{base}=28\ \text{cm}\end{array}\right]$

5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Sol.



We know that the diagonals of the rhombus bisect each other at right angles. Using Pythagoras theorem, we have:

$\text{OD}=\sqrt{\text{AD}^2-\text{AO}^2}=\sqrt{30^2-24^2}\ \text{m}$

$=\sqrt{(30+24)(30-24)}\ \text{m}$

$=\sqrt{54\times6}\ \text{m}$

$=\sqrt{9\times6\times6}\ \text{m}$

$=(3\times6)\ \text{m}=18\ \text{m}$

Area of $\Delta\text{AOD}=\left(\dfrac{1}{2}\times24\times18\right)\text{m}^2$

$=216\ \text{m}^2$

$\therefore$ Area of rhombus $=4\times\Delta\text{AOD}$

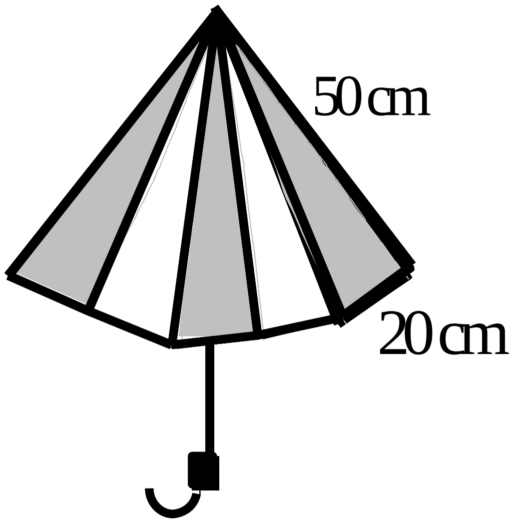
$=(4\times216)\ \text{m}^2=864\ \text{m}^2$

$\therefore$ Grass area for 18 cows $=864\ \text{m}^2$

Grass area for 1 cow $=\left(\dfrac{864}{18}\right)\text{m}^2=48\text{m}^2$.

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours each piece measuring 20 cm, 50 cm and 50cm. How much cloth of each colour is required for the umbrella?

Sol.



The sides of a triangular piece are 20 cm, 50 cm, and 50 cm.

$s = \frac{a+b+c}{2} = \frac{20+50+50}{2} = 60 \text{ cm}$

Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}$

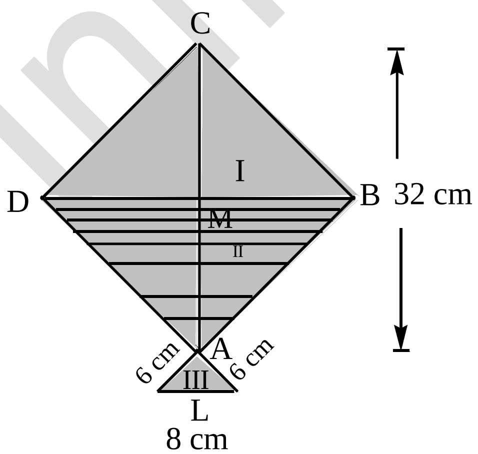
$= \sqrt{60 \times 40 \times 10 \times 10} = \sqrt{240000}$

$= 200\sqrt{6} \text{ cm}^2$

Area of cloth of each colour for five triangular pieces $= 5 \times 200\sqrt{6} = 1000\sqrt{6} \text{ cm}^2$

7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used it?

Sol.



ABCD is a square such that $AC = BD = 32$ cm and AEF

Is an isosceles triangle in which $AE = AF = 6$ cm

For the area of shades, I and II

Clearly, from the figure,

Area of shade $\mathrm{I} = \text{Area of shade II} = \text{Area of } \Delta\mathrm{CDB}$

$$= \frac{1}{2} \times \mathrm{DB} \times \mathrm{CM} = \frac{1}{2} \times 32 \times 16 \text{sq cm}$$

$$= 256 \text{ cm}^2$$

Since areas of shades I and II are equal. Therefore, area of shade

For the area of shade III (i.e., $\Delta\mathrm{AEF}$)

$$\mathrm{EL} = \mathrm{LF} = \frac{1}{2}\mathrm{EL} = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$$

and $\mathrm{AE} = 6 \text{ cm}(\text{given})$

$$\therefore \mathrm{AL} = \sqrt{\mathrm{AE}^2 - \mathrm{EL}^2} = \sqrt{36 - 16} \text{ cm} = \sqrt{20} \text{ cm } 2\sqrt{5} \text{ cm}$$

Area of shade $\mathrm{III} = \frac{1}{2} \times \mathrm{EF} \times \mathrm{AL} = \frac{1}{2} \times 8 \times 2\sqrt{5} \text{sq cm}$

$$= 8\sqrt{5} \text{ sq cm}(\text{approx})$$

$$= 8 \times 2.24 \text{ sq cm}(\text{approx})$$

$$= 17.92 \text{ sq cm}(\text{approx})$$

Alter: Let $a = 6 \text{cm}, b = 6 \text{ cm}$ and $c = 8 \text{ cm}$

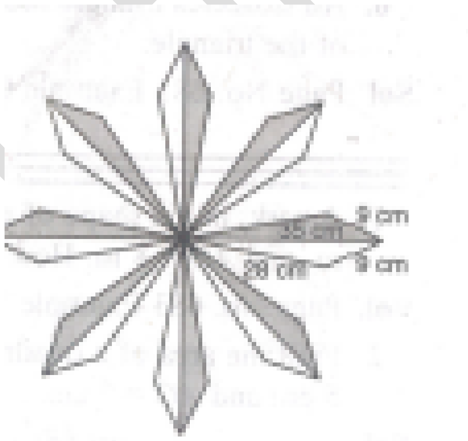
So, $s = \frac{a+b+c}{2} = \frac{6+6+8}{2} \text{ cm} = 10 \text{ cm}$

Area of shade $\mathrm{III} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10 \times 4 \times 4 \times 2} \text{ cm}^2 = 17.92 \text{ cm}^2$

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being

9 cm, 28 cm and 35 cm (see Fig).. Find the cost of polishing the tiles at the rate of 50p per cm^2.

Sol.



Here the sides of one tile are 28 cm, 35 cm and 9 cm.

$\therefore$ Perimeter of triangular tile $= 28 + 35 + 9 = 72 \text{ cm}$

$$\Rightarrow s = \frac{72}{2} = 36 \text{ cm}$$

Area of one tile $=\sqrt{36(36-28)(36-35)(36-9)}$

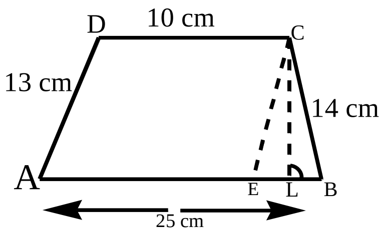
$=\sqrt{36\times 8\times 1\times 27}=\sqrt{7776}=88.2\ \text{cm}^2$

Area of 16 tiles $=16\times 88.2=1411.2\ \text{cm}^2$

Cost of polishing $=₹\left(\frac{1}{2}\times 1411.2\right)=₹705.60$

9. A field in the shape of a trapezium whose parallel sides are 25 m and 10m. The non- parallel sides are 14 m and 13m. Find the area of the field.

Sol.



From C, draw $CE \parallel DA$. Clearly, ADCE is a parallelogram having $AD \parallel CE$ and $AD \parallel AE$

Such that $AD = 13\ \text{m}$ and $CD = 10\ \text{m}$

$\therefore AE = 10\ \text{m}$

And $CE = AD = 13\ \text{m}$

Also $BE = AB - AE = (25-10)\ \text{m} = 15\text{m}$

In ΔBCE, we have:

$BC = 14\text{m}, CE = 13\ \text{m}$ and $BE = 15\ \text{m}$. Then,

$2s = (14+13+15)\ \text{m} = 42\ \text{m}$

So, $s = 21\ \text{m}$

$\therefore \text{Area}(\Delta BCE) = \sqrt{21(21-14)(21-13)(21-15)} = \sqrt{21\times 7\times 8\times 6}$

$=\sqrt{7\times 3\times 7\times 4\times 2\times 2\times 3} = 7\times 3\times 4\text{m}^2 = 84\text{m}^2$

Also, $\text{Area}(\Delta BCE) = \frac{1}{2}\times BE\times CL$

So, $84 = \frac{1}{2}\times 15\times CL$

$\Rightarrow CL = \frac{168}{15} = \frac{56}{5}\text{m}$

i.e., Height of the trapezium (CL) $= \frac{56}{5}\text{m}$

$\therefore \text{Area of trapezium} = \frac{1}{2}(AB+CD)\times CL = \frac{1}{2}\times(25+10)\times\frac{56}{5}\text{m}^2$

$= \frac{1}{2}\times 35\times\frac{56}{5}\text{m}^2 = 7\times 28\text{m}^2 = 196\text{m}^2$