# POLYNOMIALS

## EXERCISE 2.1

**1.** Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer

**(i)** $4x^2-3x+7$ **(ii)** $y^2+\sqrt{2}$ **(iii)** $3\sqrt{t}+t\sqrt{2}$ **(iv)** $y+\frac{2}{y}$

**(v)** $x^{10}+y^3+t^{50}$

**Sol.** (i) We have, $4x^2-3x+7$

The given expression has single variable x.

The index of x is whole number, i.e., 2.

Hence, the given expression is polynomial in one variable.

(ii) In $y^2+\sqrt{2}$, the index of y is a whole number, i.e., 2. So it is polynomial in one variable y.

(iii) We have, $3\sqrt{t}+t\sqrt{2}=3t^{\frac{1}{2}}+\sqrt{2}t$, here the exponent of the first term is $\frac{1}{2}$, which is not a whole number. Therefore, it is not a polynomial.

(iv) We have, $y+\frac{2}{y}=y+2y^{-1}$, here the exponent of the second term is $-1$, which is not a whole number and so it is not a polynomial.

(v) We have, $x^{10}+y^3+t^{50}$. It is not a polynomial in one variable as three variables x, y, t occurs in it.

2. Write the coefficient of $x^2$ in each of the following:

(i) $2+x^2+x$ (ii) $2-x^2+x^3$ (iii) $\frac{\pi}{2}x^2+x$ (iv) $\sqrt{2}x-1$

Sol. (i) Coefficient of $x^2$; in $2+x^2+x$ is 1.

(ii) Coefficient of $x^2$; in $2-x^2+x^3$ is $-1$.

(iii) Coefficient of $x^2$; in $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

(iv) Coefficient of $x^2$; in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. Binomial of degree 35 may be taken as $8x^{35}-20x^3$.

Monomial of degree 100 may be taken as $20x^{100}$.

4. Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$ (ii) $4-y^2$ (iii) $5t-\sqrt{7}$ (iv) 3

Sol. (i) We have, $5x^3+4x^2+7x$, the highest power term is $5x^3$ and the exponent is 3. So, degree is 3.

(ii) We have, $4-y^2$. The highest power term is $-y^2$ and the exponent is 2. So, the degree is 2.

(iii) We have, $5t-\sqrt{7}$, the highest power term is 5t and the exponent is 1. So, the degree is 1.

(iv) We have, 3. The only term here is 3 which can be written as $3x^{\circ}$ and so the exponent is 0. Therefore, the degree is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2+x$ (ii) $x-x^3$ (iii) $y+y^2+4$ (iv) $1+x$

(v) 3t (vi) $r^2$ (vii) $7x^3$

Sol. (i) The highest degree of x in $x^2+x$ is 2. Hence, it is quadratic.

(ii) The highest degree of x in $x-x^3$ is 3. Hence, it is a cubic.

(iii) The highest degree of y in $y+y^3+4$ is 2. Hence, it is quadratic.

(iv) The highest degree of x in $1+x$ is 1. Hence, it is a linear polynomial.

(v) The highest degree of t in 3t is 1. Hence, it is a linear polynomial.

(vi) The highest degree of r in $r^2$ is 2. Hence, it is a quadratic polynomial.

(vii) The highest degree of x in $7x^3$ is 3. Hence, it is cubic polynomial.

## EXERCISE 2.2

1. Find the value of the polynomial: $5x-4x^2+3$ at

(i) $x=0$ (ii) $x=-1$ (iii) $x=2$

Sol. Let polynomial is $p(x)=5x-4x^2+3$

(i) Value of polynomial $p(x)$ at $x=0$

$$p(0)=5\times 0-4(0)^2+3=0-0+3=3$$

(ii) Value of polynomial at $x=-1$

$$p(-1)=-5-4(-1)^2+3=-6$$

(iii) Value of polynomial $x=2$

$$p(2)=5\times 2-4(2)^2+3=-3$$

2. Find $p(0), p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y)=y^2-y+1$ (ii) $p(t)=2+t+2t^2-t^3$

(iii) $p(x)=x^3$ (iv) $p(x)=(x-1)(x+1)$

Sol. (i) We have, $p(y)=y^2-y+1$

Putting $y=0$ in $(1)$, we get

$$p(0)=(0)^2-0+1=0-0+1=1,$$

Putting $y=1$ in $(1)$, we get

$$p(1)=(1)^2-1+1=1-1+1=1,$$

Putting $y=2$ in $(1)$, we get

$$p(2)=(2)^2-2+1=4-2+1=3$$

(ii) We have, $p(t)=2+t+t^2-t^3$

Putting $t=0$ in $(2)$, we get

$$p(0)=2+0+2(0)^2-(0)^3=2+0+0+0-0=2$$

Putting $t=1$ in $(2)$, we get

$$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=5-1=4$$

Putting $t=2$ in $(2)$, we get

$$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$$

(iii) We have, $p(x)=x^3$

Putting $x=0$ in $(3)$, we get

$$p(0)=(0)^3=0$$

Putting $x=1$ in $(3)$, we get

$$p(1)=(1)^3=1$$

Putting $x=2$ in $(3)$, we get

$$p(2)=(2)^3=8$$

(iv) We have $p(x)=(x-1)(x+1)$

Putting $x=0$ in $(4)$, we get

$$p(0)=(0-1)(0+1)=(-1)(1)=-1$$

Putting $x=1$ in $(4)$, we get

$$p(1)=(1-1)(1+1)=(0)(2)=0$$

Putting $x=2$ in $(4)$, we get

$$p(2)=(2-1)(2+1)=(1)(3)=3$$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$ (ii) $p(x)=5x-\pi, x=\frac{4}{5}$

(iii) $p(x)=x^2-1; x=1,-1$ (iv) $p(x)=(x+1)(x-2), x=-1,2$

(v) $p(x)=x^2, x=0$ (vi) $p(x)=lx+m, x=-\frac{m}{1}$

(vii) $p(x)=3x^2-1, x=\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ (viii) $p(x)=2x+1, x=\frac{1}{2}$

Sol. (i) We have, $p(x)=3x+1$

$$...(1)$$

Putting $x=-\frac{1}{3}$ in $(1)$, we get

$$p\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)+1=-1+1=0$$

Hence, $-\frac{1}{3}$ is a zero of $p(x)$.

(ii) We have, $p(x)=5x-4$

$$...(2)$$

Putting $x=\frac{4}{5}$ in $(2)$, we get

$$p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-4=4-4=0=0$$

Hence, $\frac{4}{5}$ is a zero of $p(x)$.

(iii) We have, $p(x)=x^2-1$

$$...(3)$$

Putting $x=1$, in $(3)$, we get

$$p(1)=(1)^2-1=1-1=0$$

Hence, 1 is a zero of $p(x)$.

Also putting $x=-1$ in $(3)$, we get

$$p(-1)=(-1)^2-1=1-1=0$$

Hence, 1 is zero of $p(x)$.

(iv) We have, $p(x)=(x+1)(x-2)$

$$...(4)$$

Putting $x=-1$, in $(4)$, we get

$$p(-1)=(-1+1)(-1-2)=(0)(-3)=0$$

Hence, $-1$ is a zero of $p(x)$.

Also putting $x=2$, in $(4)$, we get

$$p(2)=(2+1)(2-2)=(3)(0)=0$$

Hence, 2 is zero of $p(x)$.

(v) We have, $p(x)=x^2$

$$...(5)$$

Putting $x=0$, in $(5)$, we get

$$p(0)=(0)^2=0$$

Hence, 0 is a zero of $p(x)$.

(vi) We have, $p(x)=1x+m$

$$...(6)$$

Putting $x=-\frac{m}{l}$ in $(6)$, we get

$$p\left(-\frac{m}{l}\right)=l\left(\frac{-m}{l}\right)+m=-m+m=0$$

Hence, $\frac{-m}{l}$ is a zero of $p(x)$.

(vii) We have, $p(x)=3x^2-1$

$\ldots(7)$

Putting $x=-\frac{1}{\sqrt{3}}$ in $(7)$, we get

$$p\left(-\frac{1}{\sqrt{3}}\right)=3\left(-\frac{1}{\sqrt{3}}\right)^2-1=3\times\frac{1}{3}-1=1-1=0$$

Hence, $-\frac{1}{\sqrt{3}}$ is a zero of $p(x)$.

Also putting $x=\frac{2}{\sqrt{3}}$ in $(7)$, we get

$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\times\frac{4}{3}-1=4-1=3\neq 0$$

Hence, $\frac{2}{\sqrt{3}}$ is not a zero of $p(x)$.

(viii) We have, $p(x)=2x+1$

$\ldots(8)$

Putting $x=\frac{1}{2}$ in $(8)$, we get

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)+1=1+1=2\neq 0$$

Hence, $\frac{1}{2}$ is not a zero of $p(x)$.

4. Find the zero of the polynomials in each of the following cases:

(i) $p(x)=x+5$ (ii) $p(x)=x-5$ (iii) $p(x)=2x+5$ (iv) $p(x)=3x-2$

(v) $p(x)=3x$ (vi) $p(x)=ax, a\neq 0$

(vii) $p(x)=cx+d, c\neq 0$, c, d are real numbers.

Sol. (i) For zero, $p(x)=0\Rightarrow x+5=0$

$\Rightarrow x=-5$ is zero of the polynomials $p(x)$.

(ii) For zero, $p(x)=0\Rightarrow x-5=0$

$\Rightarrow x=5$ is a zero of the polynomials $p(x)$.

(iii) For zero, $p(x)=0\Rightarrow 2x+5=0$

$\Rightarrow x = --\frac{5}{2}$ is a zero of the polynomials $p(x)$.

(iv) For zero, $p(x)=0 \Rightarrow 3x-2=0$

$\Rightarrow x = \frac{2}{3}$ is a zero of the polynomials $p(x)$.

(v) For zero, $p(x)=0 \Rightarrow 3x=0$

$\Rightarrow x=0$ is a zero of the polynomials $p(x)$.

(vi) For zero, $p(x)=0 \Rightarrow ax=0$

$\Rightarrow x=0$ , $\Rightarrow x=0 \;\; a \neq 0$

Therefore, $x=0$ is a zero of the polynomials $p(x)$.

For zero, $p(x)=0 \Rightarrow cx+d=0$

$$\Rightarrow \qquad x = \frac{-d}{c}, (c \neq 0)$$

Therefore, $x = \frac{-d}{c}$ is a zero of the polynomials $p(x)$.

## EXERCISE 2.3

1. Find the remainder when $x^3+3x^2+3x+1$ is divided by

(i) $x+1$ (ii) $x-\frac{1}{2}$ (iii) x (iv) $x+\pi$

(v) $5+2x$

Sol. We have, $p(x)=x^3+3x^2+3x+1$

(i) If $p(x)$ is divided by $x+1$, the remainder will be $p(-1)$.

So,
$$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$$
$$=-1+3-3+1=0.$$

(ii) If $p(x)$ is divided by $\left(x-\frac{1}{2}\right)$, the remainder will be $p\left(\frac{1}{2}\right)$.

So, $p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$
$$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$$
$$=\frac{1+6+12+8}{8}=\frac{27}{8}.$$

(iii) If $p(x)$ is divided by x, then the remainder will be $p(0)$

$$\therefore \qquad p(0)=0+0+0+1=1$$

(iv) If $p(x)$ is divided by $x+\pi$, then the remainder will be $p(-\pi)$

So,
$$p(-\pi)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$$
$$=-\pi^3+3\pi^2-3\pi+1$$

(v) If $p(x)$ is divided by $5+2x$, then the remainder will be $p\left(-\frac{5}{2}\right)$

So,
$$p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(\frac{-5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$
$$=-\frac{125}{8}+\frac{75}{4}+\frac{15}{2}+1$$
$$=\frac{-125+150-60+8}{8}=\frac{-27}{8}.$$

2. Find the remainder when $x^3-ax^2+6x-a$ is divided by $x-a$.

Sol. If $p(x)=x^3-ax^2+6x-a$ is divided by $(x-a)$, then the remainder will be $p(a)$

So, $p(a)=a^3-a^3+6a-a=5a$.

3. Check whether $7+3x$ is a factor of $3x^3+7x$.

Sol. If $7+3x$ is a factor of $3x^3+7x$, then $p\left(-\frac{7}{3}\right)=0$

So, $\Rightarrow$
$$p\left(-\frac{7}{3}\right)=3\times\left(\frac{-7}{3}\right)^3+7\times\frac{(-7)}{3}$$
$$=3\times\frac{-343}{27}-\frac{49}{3}=\frac{-343}{9}=-\frac{49}{3}$$
$$=\frac{-343-147}{9}=\frac{-490}{9}\neq 0.$$

As remainder is not zero, so $(7+3x)$ is not a factor of $3x^3+7x$.

## EXERCISE 2.4

1. Determine which of the following polynomials has $(x+1)$ is a factor:

(i) $x^3+x^2+x+1$
(ii) $x^4+x^3+x^2+x+1$
(iii) $x^4+3x^3+3x^2+x+1$
(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Sol. (i) If $(x+1)$ is a factor of $p(x)=x^3+x^2+x+1$, the $p(-1)=0$

Now, $p(-1)=-1+1-1+1=0$.

Hence, $(x+1)$ is a factor.

(ii) If $(x+1)$ is a factor of $p(x)=x^4+x^3+x^2+x+1$, then $p(-1)=0$.

Now, $p(-1)=1-1+1-1+1\neq 0$

Hence, $(x+1)$ not a factor.

(iii) If $(x+1)$ is a factor of $p(x)=x^4+3x^3+3x^2+x+1$, then $p(-1)=0$.

Now, $p(-1)=1-3+3-1+1=1\neq 0$.

Hence, $(x+1)$ is not a factor.

(iv) If $(x+1)$ is a factor of $p(x)=x^3+x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$, then, $p(-1)=0$.

Now, $p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$=2\sqrt{2}\neq 0$.

Hence, $(x+1)$ is not a factor.

2. Using the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1, g(x)=x+1$ (ii) $p(x)=x^3+3x^2+3x+1,\ g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6,\ g(x)=x-3$

Sol. (i) As given that $g(x)$ is a factor of $p(x)$, so $p(-1)=0$

As, $p(-1)=-2+1+2-1=0$.

Hence, $g(x)$ is a factor of $p(x)$.

(ii) As given that $g(x)$ is a factor of $p(x)$, so $p(-2)=0$.

As, $p(-2)=-8+12-6+1=-14+13=-1\neq 0$.

Hence, $g(x)$ is not a factor of $p(x)$.

(iii) As given that $g(x)$ is a factor of $p(x)$, so $p(3)=0$.

As, $p(3)=27-36+3+6=36-36=0$.

Hence, $g(x)$ is a factor of $p(x)$.

3. Find the value of k, if $x-1$ is a factor of $p(x)$ in each the following cases:

(i) $p(x)=x^2+x+k$ (ii) $p(x)=2x^2+kx+\sqrt{2}$

(iii) $p(x)=kx^2-\sqrt{2}x+1$ (iv) $p(x)=kx^2-3x+k$

Sol. (i) As given that $(x-1)$ is a factor of $p(x)$, so $p(1)=0$.

As, $p(1)=0\Rightarrow 1+1+k\Rightarrow k=-2$.

(ii) As given that $(x-1)$ is a factor of $p(x)$, so $p(1)=0$

As, $p(1)=0\Rightarrow 2+k+\sqrt{2}=0\Rightarrow k=-(2+\sqrt{2})$.

(iii) As given that $(x-1)$ is a factor of $p(x)$, so $p(1)=0$.

$p(1)=0\Rightarrow k-\sqrt{2}+1=0\Rightarrow k=\sqrt{2}-1$.

(iv) As given that $(x-1)$ is a factor of $p(x)$, so $p(1)=0$.

As, $p(1)=0\Rightarrow k-3+k=0\Rightarrow k=\frac{3}{2}$.

4. Factorize:

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$ (iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$

(ii) $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$

(iii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3) = (3x - 2)(2x + 3)$

(iv) $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$.

5. Factorize:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$ (iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Sol. (i) Let $p(x) = x^3 - 2x^2 - x + 2$

Possible factors of 2 are $\pm 1, \pm 2$.

We notice that $p(1) = 1 - 2 - 1 + 2 = 0$

$\Rightarrow x = 1$ is a zero polynomial $p(x) \Rightarrow (x - 1)$ is a factor of $p(x)$.

Let us divide $p(x)$ by $(x - 1)$

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
x - 1 & x^3 - 2x^2 - x + 2 \\
 & x^3 - x^2 \\
 & - \quad + \\
 & -x^2 - x \\
 & -x^2 + x \\
 & + \quad - \\
 & -2x + 2 \\
 & -2x + 2 \\
 & + \quad - \\
 & 0
\end{array}
$$

So, $p(x) = x^3 - 2x^2 - x + 2$

$= (x - 1)(x^2 - x - 2) = (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x + 1)(x - 2)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

Possible factors of 5 are $\pm 1, \pm 5$

We notice that $p(-1) = -1 - 3 + 9 - 5 = 0$

$\Rightarrow x = -1$ is zero of polynomial $p(x)$.

$\Rightarrow (x+1)$ is a factor of $p(x)$.

Let us divide $p(x)$ by $(x+1)$.

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x - 1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -4x^2 - 9x \\
 & -4x^2 + 4x \\
 & + \quad + \\
\hline
 & -5x - 5 \\
 & -5x - 5 \\
 & + \quad + \\
\hline
 & 0
\end{array}
$$

So, $p(x) = x^3 - 3x^2 - 9x - 5$

$= (x+1)(x^2 - 4x - 5)$

$= (x+1)(x^2 - 5x + x - 5)$

$= (x+1)\{x(x-5) + 1(x-5)\}$

$= (x+1)(x+1)(x-5)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

Possible factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$.

We notice that $p(-1) = -1 + 13 - 32 + 20 = 0$

$\Rightarrow$ $x = -1$ is a zero of $p(x)$

$\Rightarrow$ $(x+1)$ is a factor of $p(x)$.

Let us divide $p(x)$ by $(x+1)$

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x + 1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

So, $p(x) = x^3 + 13x^2 + 32 + 20$

$=(x+1)(x^2+12x+20)$

$=(x+1)(x^2+10x+2x+20)$

$=(x+1)\{x(x+10)+2(x+10)\}$

$=(x+1)(x+2)(x+10)$

(iv) Let $p(y)=2y^3+y^2-2y-1$

$=y^2(2y+1)-1(2y+1)=(y^2-1)(2y+1)$

$=(y-1)(y+1)(2y-1)$

## EXERCISE 2.5

1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Sol. (i) $(x+4)(x+10)=x^2+(4+10)x+(4\times10)$ [Using $(x+a)(x+b)$ $=x^2+x(a+b)+ab$ ]

$=x^2+14x+40$.

(ii) $(x+8)(x-10)=x^2+(8-10)x-8\times10$

[Using: $(x+a)(x-b)=x^2+x(a-b)-ab$ ]

$=x^2-2x-80$.

(iii) $(3x+4)(3x-5)=(3x)^2+(4-5)(3x)-4\times5$

[Using $(x+a)(x-b)=x^2+x(a-b)-ab$ ]

$=9x^2-3x-20$.

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=(y^2)^2-\left(\frac{3}{2}\right)^2$

[by using identity $(x+y)(x-y)=x^2-y^2$ ]

$=y^4-\frac{9}{4}$.

(v) $(3-2x)(3+2x)=(3)^2-(2x)^2=9-4x^2$

[Using $(x+y)(x-y)=x^2-y^2$ ]

2. Evaluate the following products without multiplying directly:

(i) $103\times107$ (ii) $95\times96$ (iii) $104\times96$

Sol. (i) $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)\times100+3\times7$

[Using $(x+a)(x+b)=x^2+(a+b)x+ab$ ]

$= 10000 + 1000 + 21 = 11021$.

(ii) $95 \times 96 = (90+5)(90+6)$

$= (90)^2 + (5+6) \times 90 + 5 \times 6$

[Using $(x+a)(x+b) = x^2 + x(a+b) + ab$ ]

$= 8100 + 990 + 30 = 9120$.

(iii) $104 \times 96 = (100+4)(100-4) = (100)^2 - (4)^2$

[Using $(a+b)(a+b) = a^2 - b^2$ ]

$= 10000 - 16 = 9984$.

3. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$ (iii) $x^2 - \frac{y^2}{100}$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2 = (3x+y)^2$

[Using $a^2 + b^2 + 2ab = (a+b)^2$ ]

$= (3x+y)(3x+y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y-1)^2$

[Using $a^2 - 2ab + b^2 = (a-b)^2$ ]

$= (2y-1)(2y-1)$

(iii) $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

[Using $a^2 - b^2 = (a-b)(a+b)$ ]

4. Expand each of the following using suitable identities:

(i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$ (iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$ (iv) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Sol. (i) $(x+2y+4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(x)(4z)$

[Using $(a+b+c)^2 = a^2 + b + c^2 + 2ab + 2bc + 2ca$ ]

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$.

(ii) $(2x-y+z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy - 2zy + 4xz$.

(iii)

$(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$.

(iv) $(3a-7b-c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(3a)(-c)$

$$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.$$

(v)

$$(-2x+5y-3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-2x)(-3z)$$

$$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz.$$

(vi)

$$\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2\left(\frac{1}{4}a\right)(1)$$

$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a.$$

5. Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ (ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$$

[Using: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ ]

$$= (2x + 3y - 4z)^2.$$

$$= (2x + 3y - 4z)(2x + 3y - 4z).$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$$= \left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)(y) + 2(y)\left(2\sqrt{2}z\right) + 2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$$

[Using: same identity used in (i) Part]

$$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$$

$$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2x} + y + 2\sqrt{2}z\right)$$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$ (iii) $\left[\frac{3}{2}x + 1\right]^3$ (iv) $\left[x - \frac{2}{3}y\right]^3$

Sol. (i) $(2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)$

[Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ ]

$$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

[Using $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$ ]

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times1\left(\frac{3}{2}x+1\right)$

[Using $(a+b)^3 = a^3+b^3+3ab(a+b)$]

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x-\frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

[Using $(a-b)^3 = a^3-b^3-3ab(a-b)$]

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$.

7. Evaluate the following using suitable identities:

(i) $(99)^2$ (ii) $(102)^3$ (iii) $(998)^3$

Sol. (i) $(99)^3 = (100-1)^3$

$= (100)^3 - (1)^3 - 3\times100\times1(100-1)$

[Using $(a-b)^3 = a^3-b^3-3ab(a-b)$]

$= 1000000 - 1 - 30000 + 300 = 970299$.

(ii) $(102)^3 = (100+2)^3$

$= (100)^3 + (2)^3 + 3\times100\times2(100+2)$

[Using $(a+b)^3 = a^3+b^3+3ab(a+b)$]

$= 1000000 + 8 + 60000 + 1200 = 1061208$.

(iii) $(998)^3 = (100-2)^3$

$= (1000)^3 - (2)^3 - 3\times1000\times2(1000-2)$

[Using $(a-b)^3 = a^3-b^3-3ab(a-b)$]

$= 1000000000 - 8 - 6000000 + 12000 = 994011992$.

8. Factorize each of the following:

(i) $8x^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$

$= (2a+b)^3$

[Using $x^3 + y^3 + 3xy(x+y) = (x+y)^3$]

$$=(2a+b)(2a+b)(2a+b).$$

(ii) $8a^3-b^3-12a^2b+6ab^2=(2a)^3-(b)^3-3(2a)^2b+3(2a)b^2$

$$=(2a-b)^3$$

[Using $x^3-y^3-3x^2y+3xy^2=(x-y)^3$]

$$=(2a-b)(2a-b)(2a-b).$$

(iii) $27-125a^3-135a+225a^2=(3)^3-(5a)^3-3\times(3)^2\times(5a)+3(3)(5a)^2$

$$=(3-5a)^3$$

[Using $x^3-y^3-3x^2y+3xy^2=(x-y)$]

$$=(3-5a)(3-5a)(3-5a)$$

(iv) $64a^3-27b^3-144a^2b+108ab^2=(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$$=(4a-3b)^3$$

[Using $x^3-y^3-3x^2y+3xy^2=(x-y)^3$]

$$(4a-3b)(4a-3b)(4a-3b).$$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p=(3p)^3-\left(\frac{1}{6}\right)^3-3\times(3p)^2\times\frac{1}{6}+3\times(3p)\times\left(\frac{1}{6}\right)^2$

$$=\left(3p-\frac{1}{6}\right)^3$$

[Using $x^3-y^3-3x^2y+3xy^2=(x-y)^3$]

$$=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right).$$

9. Verify

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$ (ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Sol. (i) We have to verify that:

$$x^3+y^3=(x+y)(x^2-xy+y^2)$$

So, on taking RHS we have: $(x+y)(x^2-xy+y^2)$

$$=x\times\left[x^2-xy+y^2\right]+y(x^2-xy+y^2)$$

$$=x^3-x^2y+xy^2+yx^2-xy^2+y^3$$

$x^3+y^3$ (LHS) Hence verified.

(ii) We have: $x^3-y^3=(x-y)(x^2+xy+y^2)$

On taking RHS we have $(x-y)(x^2+xy+y^2)$

$$=x\times(x^2+xy+y^2)-y\times(x^2+xy+y^2)$$

$$=x^3+x^2y+xy^2-yx^2-xy^2-y^3$$

$= x^3 - y^3$ (LHS) Hence verified.

10. Factorize each of the following:

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343m^3$

Sol. (i) Consider $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$$= (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$$

[Using $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$$= (3y + 5z)(9y^2 - 15yz + 25z^2)$$

(ii) Consider $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$$= (4m - 7n)\{(4m)^2 + (4m)(7n) + (7n)^2\}$$

[Using $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$$= (4m - 7n)(16m^2 + 28mn + 49n^2)$$

11. Factorize: $27x^3 + y^3 + z^3 - 9xyz.$

Sol. Consider $27x^3 + y^3 + z^3 - 9xyz.$

$$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$$

[Using $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= (3x + y + z)\{(3x)^2 + y^2 + z^2 - (3x)y - yz - (3x)z\}$$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz).$$

12. Verify that:

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$$

Sol. Consider identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Multiply second bracket by 2 and divide outside by $2$ we get

$$= \frac{1}{2} \times 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= \frac{1}{2}(x + y + z)\{(x^2 + y^2 - 2xy) + (y^2 + z^2 + 2yz) + (z^2 + x^2 - 2xz)\}$$

$$= \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (z - x)^2\}.$$

[Using $a^2 - 2ab + b^2 = (a - b)^2$]

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol. We have $x + y + z = 0$

$\Rightarrow \quad x+y=-z$

$\ldots(1)$

On cubing sides, we get $(x+y)^3=(-z)^3$

$\Rightarrow \quad x^3+y^3+3xy(x+y)=-z^3$

Now put $(-z)$ in place of $(x+y)$ in equation 2

$\ldots(2)$

We get $x^3+y^3+3xy\times(-z)=-z^3$

$\Rightarrow x^3+y^3-3xyz=-z^3$

$\Rightarrow x^3+y^3+z^3=3xyz$ Proved

14. Without actually calculating the cubes, find the values of each of the following.

(i) $(-12)^3+(7)^3+(5)^3$ (ii) $(28)^3+(-5)^3+(-13)^3$

Sol. (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12,\ b=7,\ c=5$

So, value of $a+b+c=-12+7+5=-12+12=0$

We know that if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Hence using this fact, we find that

$(-12)^3+(7)^3+(5)^3=3\times(-12)\times(7)(5)=-1260$

(ii) $(28)^3+(-5)^3+(-13)^3$

Same as in part $(i)$

We get $a=28, b=b=-15, c=-13$

So $a+b+c=28+(-15)+(-13)=0$

Using identity that if

$a+b+c=0$ then $a^3+b^3+c^3=3abc$

We get

$(28)^3+(-15)^3+(-13)^3=3\times28\times(-15)(-13)=16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2-35a+12$ (ii) Area: $35y^2+13y-12$

Sol. (i) Area $=25a^2-35a+12=25a^2-20a-15a+12$

$=5a(5a-4)-3(5a-4)=(5a-3)(5a-4)$.

Hence, possible expressions for length and breadth are $(5a-3)$ and $(5a-4)$.

(ii) Area $=35y^2+13y-12=35y^2+28y-15y-12$

$=7y(5y+4)-3(5y+4)=(7y-3)(5y+4)$

Hence possible expressions for length and breadth are $(7y-3)$ and $(5y+4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2 - 12x$ (ii) Volume: $12ky^2 + 8ky - 20k$

Sol. (i) Volume $= 3x^2 - 12x = 3x(x-4)$

Possible dimensions are 3, x and $(x-4)$.

(ii) Volume $= 12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$

$$= 4k(3y^2 + 5y - 3y - 5)$$

$$= 4k\{y(3y+5) - 1(3y+5)\}$$

$$= 4k(y-1)(3y+5).$$

Hence, possible dimensions are $4k, (y-1)$ and $(3y+5)$