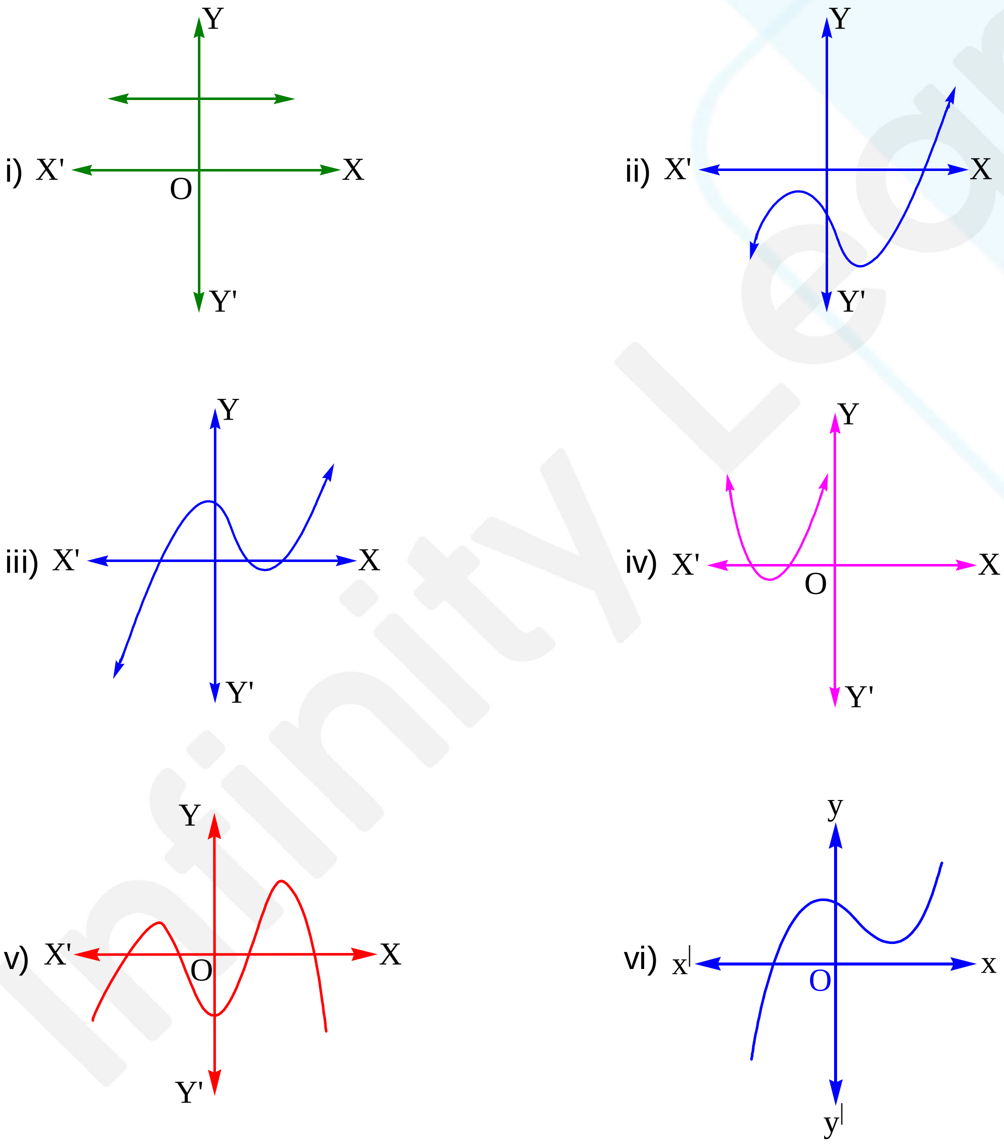
NCERT X CLASS MATHS QUESTIONS AND SOLUTIONS

2. POLYNOMIALS

NCERT EXERCISE : 2.1

1. The graphs of $y = p(x)$ are given in figure below, for some polynomials $p(x)$. Find the number of zeros of $p(x)$, in each case.



Sol. i) There is no zero as the graph does not intersect tge x-axis at any point.

ii) The number of zeros is 1 as the graph intersects the x-axis at one point only.

iii) The number of zeros is 3 as the graph intersects the x-axis at three points.

iv) The number of zeros is 2 as the graph intersects the x-axis at two points

v) The number of zeros is 4 as the graph intersects the x-axis at four points.

vi) The number of zeros is 3 as the graph intersects the x-axis at three points.

EXERCISE : 2.2

1. Find the zeroes of the quadratic polynomials and verify a relationship between zeroes and its coefficients.

i) $x^2 - 2x - 8$ ii) $4s^2 - 4s + 1$ iii) $6x^2 - 3 - 7x$ iv) $4u^2 + 8u$

v) $t^2 - 15$ vi) $3x^2 - x - 4$

Sol. i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$

So, the value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$ i.e., when $x = 4$ or $x = -2$.

So, the zeroes of $x^2 - 2x - 8$ are 4, – 2.

Sum of the zeroes $= 4 - 2 = 2 = \dfrac{-(-2)}{1} = \dfrac{-\text{coefficient of } x}{\text{coefficient of } x^2} = 2$

Product of the zeroes $= 4(-2) = -8 = \dfrac{-8}{1} = \dfrac{\text{constant term}}{\text{coefficient of } x^2} = -8$

ii) $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1$

$= 2s(2s-1) - 1(2s-1)$

$= (2s-1)(2s-1) = (2s-1)^2$

So, the value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, or $s = \dfrac{1}{2}$

Zeroes of the polynomial are $\dfrac{1}{2}, \dfrac{1}{2}$

Sum of the zeroes $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = -\left(\dfrac{-4}{4}\right) = \dfrac{-\text{coefficient of } s}{\text{coefficient of } s^2} = 1$

Product of the zeroes $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\text{constant term}}{\text{coefficient of } s^2} = \dfrac{1}{4}$

iii) We have : $6x^2 - 3 - 7x = 6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is 0, when the value of $(3x+1)(2x-3)$ is 0, i.e.,

when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., when $x = -\dfrac{1}{3}$ or $x = \dfrac{3}{2}$

$\therefore$ The zeroes of $6x^2 - 3 - 7x$ are $-\dfrac{1}{3}$ and $\dfrac{3}{2}$

Therefore, sum of the zeroes $= \dfrac{-1}{3} + \dfrac{3}{2} = \dfrac{7}{6} = \dfrac{-(-7)}{6} = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \dfrac{7}{6}$

and product of zeroes = $\left(\frac{-1}{3}\right)\left(\frac{3}{2}\right)=\frac{-3}{6}=\frac{\text{constant term}}{\text{coefficient of } x^2}$

iv) We have : $4u^2 + 8u = 4u(u + 2)$
The value of $4u^2 + 8u$ is 0, when the value of $4u(u+2) = 0$, i.e., when $u = 0$ or $u + 2 = 0$, i.e., when $u = 0$ or $u = -2$
$\therefore$ The zeroes of $4u^2 + 8u$ are 0 and -2

Therefore, sum of the zeroes $= 0+(-2)=-2=\frac{-8}{4}=\frac{-\text{Coefficient of } u}{\text{Coefficient of } u^2}=-2$

and product of zeroes $=(0)(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}=0$

v) We have $t^2-15=\left(t-\sqrt{15}\right)\left(t+\sqrt{15}\right)$

The value of $t^2 - 15$ is 0, when the value of $\left(t-\sqrt{15}\right)\left(t+\sqrt{15}\right)$ is 0, i.e., when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when $t-\sqrt{15}$ or $t+\sqrt{15}$

$\therefore$ The zeroes of t^2-15 are $\sqrt{15}$ and $-\sqrt{15}$

Therefore, sum of the zeroes $=\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-\text{Coefficient of } t}{\text{Coefficient of } t^2}=0$

and product of the zeroes $=\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } t^2}=-15$

vi) We have : $3x^2 - x - 4 = 3x^2 + 3x - 4x - 4 = 3x(x + 1) - 4(x + 1) = (x + 1)(3x - 4)$
The value of $3x^2 - x - 4$ is 0, when the value of $(x + 1)(3x - 4)$ is 0, i.e., when $x + 1 = 0$ or

$3x - 4 = 0$, i.e., when $x = -1$ or $x=\frac{4}{3}$

$\therefore$ The zeroes of $3x^2 - x - 4$ are -1 and $\frac{4}{3}$

Therefore, sum of the zeroes $=-1+\frac{4}{3}=\frac{-3+4}{3}=\frac{1}{3}=\frac{-1(-1)}{3}=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\frac{1}{3}$

and product of the zeroes : $=(-1)\left(\frac{4}{3}\right)=\frac{\text{Constant term}}{\text{Coefficient of } x^2}=\frac{-4}{3}$

2. Find a quadratic polynomial whose sum of zeroes and product of zeroes are respectively.

i) $\frac{1}{4},-1$ ii) $\sqrt{2},\frac{1}{3}$ iii) $0,\sqrt{5}$ iv) 1, 1 v) $-\frac{1}{4},\frac{1}{4}$
vi) 4, 1

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

i) Here, $\alpha+\beta=\frac{1}{4}$ and $\alpha,\beta=-1$

Thus the polynomial formed = x^2 – (Sum of zeroes)x + Product of zeroes = $x^2-\left(\frac{1}{4}\right)x-1$

$$=x^2-\frac{x}{4}-1$$

The other polynomials are $k\left(x^2-\frac{x}{4}-1\right)$

If k = 4, then the polynomial is $4x^2 - x - 4$

ii) Here, $\alpha+\beta=\sqrt{2}$ $\qquad \alpha\beta=\frac{1}{3}$

Thus the polynomial formed

= x^2 – (Sum of zeroes) x + Product of zeroes

$=x^2-\left(\sqrt{2}\right)x+\frac{1}{3}$ or $x^2-\sqrt{2}x+\frac{1}{3}$

Other polynomials are $k\left(x^2-\sqrt{2}x+\frac{1}{3}\right)$

If k = 3, then the polynomial is $3x^2-3\sqrt{2}x+1$

iii) Here, $\alpha+\beta=0$ and $\alpha.\beta=\sqrt{5}$

Thus the polynomial formed = x^2 – (Sum of zeroes) x + Product of zeroes

$$=x^2-(0)x+\sqrt{5}=x^2+\sqrt{5}$$

iv) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$\alpha+\beta=1=\frac{-(-1)}{1}=\frac{-b}{a}$

$\alpha\beta=1=\frac{1}{1}=\frac{c}{a}$

If a = 1, then b = –1 and c = 1

$\therefore$ One quadratic polynomial which satisfy the given conditions is $x^2 - x + 1$

v) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then

$$\alpha+\beta=-\frac{1}{4}=\frac{-1}{4}=\frac{-b}{a}$$

and $\alpha\beta=\frac{1}{4}=\frac{c}{a}$

If a = 4, then b = 1 and c = 1

$\therefore$ One quadratic polynomial which satisfy the given conditions is $4x^2 + x + 1$.

vi) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then.

$$\alpha+\beta=4=\frac{-(-4)}{1}=\frac{-b}{a}$$

and $\alpha\beta=1=\frac{1}{1}=\frac{c}{a}$

If a = 1, then b = –4 and c = 1

$\therefore$ One quadratic polynomial which satisfy the given conditions is $x^2 - 4x + 1$.

EXERCISE - 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each given of the following:

i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Sol. i) Here, dividend and divisor are both in standard forms. So, we have :

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & \underset{-}{x^3} \quad\quad \underset{+}{-\,2x} \\ \hline & -3x^2 + 7x - 3 \\ & \underset{+}{-\,3x^2} \quad\quad \underset{-}{+\,6} \\ \hline & 7x - 9 \\ \hline \end{array}$$

$\therefore$ The quotient is $x - 3$ and the remainder is $7x - 9$

ii) Here, the dividend is already in the standard form and the divisor is not in the standard form. It can be written as $x^2 - x + 1$.

We have :

$$\begin{array}{r|l} & x^2 + x - 3 \\ \hline x^2 - x + 1 & x^4 - 3x^2 + 4x + 5 \\ & \underline{x^4 - x^3 + x^2} \\ & \quad (-)\ (+)\ (-) \\ & x^3 - 4x^2 + 4x \\ & \underline{x^3 - x^2 + x} \\ & \quad (-)\ (+)\ (-) \\ & -3x^2 + 3x + 5 \\ & \underline{-3x^2 + 3x - 3} \\ & \quad (+)\ (-)\ (+) \\ & 8 \end{array}$$

$\therefore$ The quotient is $x^2 + x - 3$ and the remainder is 8.

iii) We have divisor $\left[-x^2 + 2\right]$ and dividend : $x^4 - 5x - 6$

$$\begin{array}{r|l} & -x^2 - 2 \\ \hline -x^2 + 2 & x^4 - 5x + 6 \\ & \underline{x^4 - 2x^2} \\ & \quad (-)\ (+) \\ & 2x^2 - 5x + 6 \\ & \underline{2x^2 - 4} \\ & \quad (-)\ (+) \\ & -5x + 10 \end{array}$$

$\therefore$ The quotient is $-x^2 - 2$ and the remainder is $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial :

i) $t^2 - 3; 2t^4 + 3t^3 - 2t^2 - 9t - 12$

ii) $x^2 + 3x + 1 ; 3x^4 + 5x^3 - 7x^2 + 2x + 2$

iii) $x^3 - 3x + 1 ; x^5 - 4x^4 + x^2 + 3x + 1$

Sol. i) Let us divide $2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$

We have :

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad\quad - 6t^2 \\
 & - \qquad\qquad + \\
\hline
 & \qquad 3t^3 + 4t^2 - 9t \\
 & \qquad 3t^3 \qquad\quad - 9t \\
 & \qquad - \qquad\qquad + \\
\hline
 & \qquad\qquad 4t^2 \qquad - 12 \\
 & \qquad\qquad 4t^2 \qquad - 12 \\
 & \qquad\qquad - \qquad\quad + \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

Since the remainder is 0, therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

ii) Let us divide $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \qquad - \qquad - \\
\hline
 & \qquad -4x^3 - 10x^2 + 2x \\
 & \qquad -4x^3 - 12x^2 - 4x \\
 & \qquad + \qquad + \qquad + \\
\hline
 & \qquad\qquad 2x^2 + 6x + 2 \\
 & \qquad\qquad 2x^2 + 6x + 2 \\
 & \qquad\qquad - \qquad - \qquad - \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

Since the remainder is 0, therefore, $x^2 + 3x + 1$ is a facotro of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

iii) Let us divide $x^5 - 4x^3 + x^2 + 3x + 1$ by $x^3 - 3x + 1$. We get,

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \qquad + \qquad - \\
\hline
 & \qquad -x^3 \qquad + 3x + 1 \\
 & \qquad -x^3 \qquad + 3x - 1 \\
 & \qquad + \qquad\quad - \qquad + \\
\hline
 & \qquad\qquad\qquad 2 \\
\hline
\end{array}
$$

Here, remainder is $2(\neq 0)$. Therefore, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, $x=\sqrt{\frac{5}{3}}$, $x=-\sqrt{\frac{5}{3}}$

$\Rightarrow\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=x^2-\frac{5}{3}$ or $3x^2 - 5$ is a factor of the given polynomial. Now, we apply the division algorithm to the given polynomial and $3x^2 - 5$.

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline 3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \quad\quad - 5x^2 \\ & - \quad\quad\quad + \\ \hline & \quad 6x^3 + 3x^2 - 10x - 5 \\ & \quad 6x^3 \quad\quad - 10x \\ & \quad - \quad\quad\quad + \\ \hline & \quad\quad 3x^2 - 5 \\ & \quad\quad 3x^2 - 5 \\ & \quad\quad - \quad + \\ \hline & \quad\quad\quad 0 \end{array}$$

First term of quotient is $\frac{3x^4}{3x^2}=x^2$

Second term of quotient is $\frac{6x^3}{3x^2}=2x$

Third term of quotient is $\frac{3x^2}{3x^2}=1$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1) + 0 = (3x^2 - 5)(x + 1)^2$
Quotient : $x^2 + 2x + 1 = (x + 1)^2$
Zeroes of $(x + 1)^2$ are $-1, -1$.

Hence , all its zeroes are $\sqrt{\frac{5}{3}},-\sqrt{\frac{5}{3}},-1,-1$

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

$p(x) = x^3 - 3x^2 + x + 2$
$q(x) = x - 2$ and $r(x) = -2x + 4$

Sol. By division Algorithm, we know that

$p(x) = q(x) \times g(x) + r(x)$

Therefore, $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$
$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2) \times g(x)$

$\Rightarrow g(x)=\frac{x^3-3x^2+3x-2}{x-2}$

On dividing $x^3 - 3x^2 + 3x - 2$ by $x - 2$, we get g(x)

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

First term of $q(x) = \dfrac{x^3}{x} = x^2$

Second term of $q(x) = \dfrac{-x^3}{x} = -x$

Third term of $q(x) = \dfrac{x}{x} = 1$

Hence, $g(x) = x^2 - x + 1$.

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and
i) deg p(x) = deg q(x) ii) deg q(x) = deg r(x) iii) deg q(x) = 0

Sol. i) Let $q(x) = 3x^2 + 2x + 6$, degree of q(x) = 2
$p(x) = 12x^2 + 8x + 24$, degree of p(x) = 2
Here, deg p(x) = deg q(x)

ii) $p(x) = x^5 + 2x^4 + 3x^3 + 5x^2 + 2$
$q(x) = x^2 + x + 1$, degree of q(x) = 2
$g(x) = x^3 + x^2 + x + 1$
$r(x) = 2x^2 - 2x + 1$, degree of r(x) = 2
Here, deg q(x) = deg r(x)

iii) Let $p(x) = 2x^4 + 8x^3 + 6x^2 + 4x + 12$
$q(x) = 2$, degree of q(x) = 0
$g(x) = x^4 + 4x^3 + 3x^2 + 2x + 1$
$r(x) = 10$,
Here, deg q(x) = 0

EXERCISE - 2.4 (OPTIONAL)

1. Verify that the number given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case :

i) $2x^3 + x^2 - 5x + 2; \dfrac{1}{2}, 1, -2$ ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol. i) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get
a = 2, b = 1, c = –5 and d = 2

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{1+1-10+8}{4} = \frac{0}{4} = 0$$

$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$
$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 2(-8) + 4 + 10 + 2 = -16 + 16 = 0$

$\therefore \frac{1}{2}$, 1 and −2 are the zeroes of $2x^3 + x^2 - 5x + 2$

So, $\alpha = \frac{1}{2}, \beta = 1$ and $\gamma = -2$

Therefore, $\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = \frac{1+2-4}{2} = \frac{-1}{2} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \left(\frac{1}{2}\right)(1) + (1)(-2) + (-2)\left(\frac{1}{2}\right) = \frac{1}{2} - 2 - 1 = \frac{1-4-2}{2} = \frac{-5}{2} = \frac{c}{a}$

and $\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -1 = \frac{-2}{2} = \frac{-d}{a}$

ii) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

a = 1, b = –4, c = 5 and d = – 2

$p(2) = (2)^3 - 4(2)^3 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0$

$\therefore$ 2, 1 and 1 are the zeroes of $x^3 - 4x^2 + 5x - 2$

So, $\alpha = 2, \beta = 1 \, and \, \gamma = 1$

Therefore, $\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5 = \frac{5}{1} = \frac{c}{a}$

and $\alpha\beta\gamma = (2)(1)(1) = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$.

2. Find a cubic polynomial with the sum, sum of the products of its zeroes taken two at a time and the product of its zeroes as 2, – 7, – 14 respectively.

Sol. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$, and its zeroes be α, β and γ.

Then, $\alpha + \beta + \gamma = 2 = \frac{-(-2)}{1} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7 = \frac{-7}{1} = \frac{c}{a}$

and $\alpha\beta\gamma = -14 = \frac{-14}{1} = \frac{-d}{a}$

If a = 1, then b = – 2, c = – 7 and d = 14

So, one cubic polynomial which satisfy the given conditions will be $x^3 - 7x + 14$

3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, find a and b.

Sol. Since $(a - b)$, a and $(a + b)$ are the zeroes of the polynomial $x^3 - 3x^2 + x + 1$, therefore

$$(a-b)+a+(a+b)=\frac{-(-3)}{1}=3$$

So, $3a = 3 \Rightarrow a = 1$

$$(a-b)a+a(a+b)+(a+b)(a-b)=\frac{1}{1}=1$$

$$\Rightarrow a^2-ab+a^2+ab+a^2-b^2=1 \Rightarrow 3a^2-b^2=1$$

So, $3(1)^2-b^2=1 \Rightarrow 3-b^2=1$

$\Rightarrow b^2=2$ or $b=\pm\sqrt{2}$

Hence, $a = 1$ and $b=\pm\sqrt{2}$

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2\pm\sqrt{3}$, find other zeroes

Sol. We have : $2\pm\sqrt{3}$ are two zeroes of the polynomial

$p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Let $x=2\pm\sqrt{3}$. So, $x-2=\pm\sqrt{3}$

Squaring, we get

$x^2 - 4x + 4 = 3$, i,e., $x^2 - 4x + 1 = 0$

Let us divide p(x) by $x^2 - 4x + 1$ to obtain other zeroes.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore$ $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35$

$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)$

$= (x^2 - 4x + 1)(x + 5)(x - 7)$

So, $(x + 5)$ and $(x - 7)$ are other factors of p(x)

$\therefore$ -5 and 7 are other zeroes of the given polynomial

5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to $x + a$, find k and a.

Sol. Let us divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 25x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8 - k)x^2 + (4k - 25)x + 10 \\
 & (8 - k)x^2 - 2(8 - k)x + (8 - k)k \\
 & - \quad + \quad - \\
\hline
 & (2k - 9)x - (8 - k)k + 10 \\
\hline
\end{array}
$$

$\therefore$ Remainder = $(2k - 9)x - (8 - k)k + 10$

But the remainder is given as $x + a$

On comparing their coefficients, we have

$2k - 9 = 1 \Rightarrow 2k = 10 \Rightarrow k = 5$

and $-(8 - k)k + 10 = a$

So, $a = -(8 - 5)5 + 10$

$= -3 \times 5 + 10 = -15 + 10 = -5$