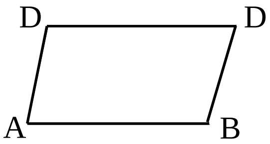
QUADRILATERALS

EXERCISE 8.1

1. The angles of quadrilateral are in the ratio $3:5:9:13$. Find all the angles of the quadrilateral.

Sol.



Let the angles be $(3x)°,(5x)°,(9x)°$ and $(13x)°$

Then, $3x+5x+9x+13x=360$

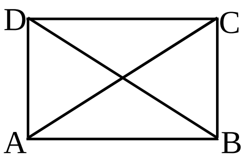
or $30x=360$

or $x=\frac{360}{30}=12$

$\therefore$ The angles are $(3\times12)°,(5\times12)°,(9\times12)°$ and $(13\times12)°$, i.e., $36°,60°,108°$ and $156°$.

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol.



Consider the triangles DAB and CBA,

$AD=BC$ [Opposite side of a parallelogram]

AB is common

$AC=BD$ [Given]

So, $\Delta DAB\cong\Delta CBA$ [SSS]

$\Rightarrow$ $\angle DAB=\angle CBA$ ...(1) [CPCT]

As ABCD is a parallelogram. $AD||BC$ and AB is transversal.

So, $\angle DAB+\angle CBA=180°$ [Sum of interior angles on the same side of transversal is $180°$.]

$\Rightarrow$ $2\angle DAB=180°$

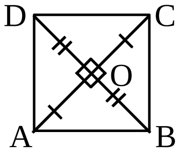
[using (1)]

$\Rightarrow$ $\angle DAB=90°$

As in parallelogram, $\angle ADB=90°$. Hence, the parallelogram is a rectangle.

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol.



Consider the triangle AOB and COD,

$AO=OC$ [Given]

$OB=OD$ [Given]

$\angle AOB=\angle COD$ [$90°$ each]

So, $\Delta AOB \cong \Delta COD$ [SAS]

$\Rightarrow$ $AB = CD$ ...(1)

Similarly, we can show that $BC = DA$ ...(2)

Consider triangles AOB and BOC.

$AO = OC$

[Given]

BO is common.

and $\angle AOB = \angle BOC$

[90° each]

so, $\Delta AOB \cong \Delta COB$ [SAS]

$\Rightarrow$ $AB = BC$ ...(3)

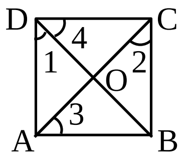
Hence from eqn No. (1),(2)&(3) we get

$AB = BC = CD = DA$

Hence, ABCD is a rhombus.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol.



Consider triangles DAB and CBA,

$AD = BC$ [Sides of a square]

AB is common.

$\angle DAB = \angle CBA$ [SAS]

$\Rightarrow$ $BD = AC$ [CPCT]

and $\angle 1 = \angle 2$ [CPCT] ...(1)

Proving as above we can show $\angle 3 = \angle 4$.

...(2)

Also, $\angle 2 = \angle 3$

...(3)

[$\because$ $AB = BC$, angles opposite to equal sides are equal.]

$\Rightarrow$ $\angle 1 = \angle 4$ [using (1),(2)&(3)]

Consider triangles AOD and COD,

[Sides of a square]

OD is commo [Prove above]

$\therefore$ $\Delta AOD \cong \Delta COD$ [SAS]

$\therefore$ $OA = OC$ ...(4) [CPCT]

Similarly, we can show that

$OB = OD$

and $\angle AOD = \angle COD$ ...(5) [CPCT]

Also, $\angle AOD + \angle COD = 180^\circ$

[Linear pair]

$\Rightarrow \quad 2\angle AOD = 180^\circ$

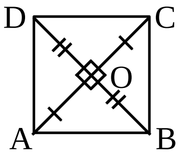
[Using (5)]

$\Rightarrow \quad \angle AOD = 90^\circ$

...(6)

Hence, diagonals are equal and bisect each other at right angles.

5. Show that if the diagonals of quadrilateral are equal and bisect each other at right angles, then it is a square.

Sol. 

Consider triangles AOB and COD,

$AO = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [90° each]

$\Rightarrow \quad \Delta AOB \cong \Delta COD$ [SAS]

$\Rightarrow \quad AB = CD$

...(1)

Similarly, we can show that $BC = DA$

...(2)

Consider triangles AOB and BOC,

$AO = OC$ [Given]

OB is common.

and $\angle AOB = \angle BOC$ [90° each]

$\Rightarrow \quad \Delta AOB \cong \Delta COB$ [SAS]

$\Rightarrow \quad AB = BC$

...(3)

Hence from equation No. $(1),(2)\&(3)$ we get

$AB = BC = CD = DA$

Hence, ABCD is a rhombus.

Further, consider ΔDAB and ΔCBA

$AD = BC$ [Prove rhombus]

AB is common

$BD = AC$ [Given]

So, $\Delta DAB \cong \Delta CBA$ [SSS]

$\Rightarrow \quad \angle DAB = \angle CBA$

...(4)

Also, as $AD \| BC$ (opposite sides of a rhombus) and AB is transversal.

$\therefore \quad \angle DAB + \angle CBA = 180^\circ$

(Sum of interior angles on the same side of transversal is 180°)

$\Rightarrow \quad 2\angle DAB = 180^\circ$. [Using eqn no.4]

$\Rightarrow \quad \angle DAB = 90^\circ$.

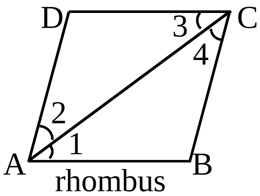
As in a rhombus one angle is $90°$. Hence, rhombus is a square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig.). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is as rhombus.

Sol.



rhombus

Given. A parallelogram ABCD in which diagonal AC bisects $\angle A$.

To prove. AC bisects $\angle C$. Proof. Since ABCD is a parallelogram, $\therefore AB \parallel DC$

$AB \parallel DC$ and transversal AC intersects AB and DC.

$\therefore \quad \angle 1 = \angle 3 \quad ...(1)$ [Alternate interior angles]

Again, $AD \parallel BC$ and AC intersects them.

$\therefore \quad \angle 2 = \angle 4 \quad ...(2)$

[Alternate interior angels]

But it is given that AC is the bisector of $\angle A$.

$\therefore \quad \angle 1 = \angle 2$
$...(3)$

From $(1), (2)$ and (3), we get

$\angle 3 = \angle 4$

Hence, AC bisects $\angle C$.

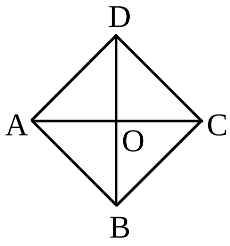
$\frac{1}{2}\angle A = \frac{1}{2}\angle C$ i.e., $\angle 2 = \angle 3$ so $AD = CD$

Similarly, $AB = BC$

Hence, ABCD is rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ and as well as $\angle D$.

Sol.



Consider triangles ADC and ABC, $AD = AB$ [Sides of a rhomb

AC is common. $CD = CB$ [Sides of a rhombus]

So, $\Delta ADC = \Delta ABC$ [SSS]

$\Rightarrow$ $\angle DAC = \angle BAC$ [CPCT...(1)]

and $\angle DCA = \angle BCA$ [CPCT...(2)]

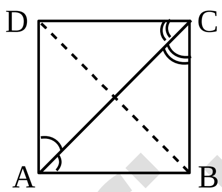
hence AC bisects $\angle A$ and $\angle C$.

Similarly, by taking BAD and BCD, we can show that BD bisects $\angle B$ and $\angle D$.

8. ABCD is a rectangle in with diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

(i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol.



(i) Consider triangles ADC and ABC,

$\angle DAC = \angle BAC$ [AC is bisector is $\angle A$]

$\angle DCA = \angle BCA$ [AC is bisector is $\angle C$]

AC is common.So, $\angle ADC = \angle ABC$ [ASA]

$\angle AD = \angle AB$ [CPCT]

As in rectangle ABCD, adjacent sides are equal. Hence ABCD is square.

(ii) Consider triangles DAB and BCD,

$\angle DAC = \angle BAC$

[sides of a square]BD is common.

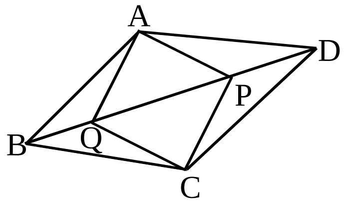
So $\angle DAB \cong \angle DCB$ [SSS]

So $\angle ADB \cong \angle CDB$ [CPCT]

and $\angle ABD \cong \angle CBD$ [CPCT]

$\Rightarrow$ BD bisects $\angle B$ and $\angle D$. [Using above result]

9.

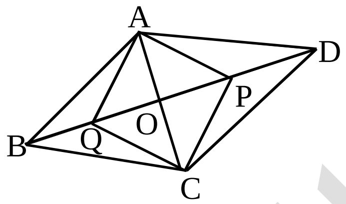


In parallelogram ABCD, two-point P and Q are taken on diagonal BD such that $DP = BQ$ (see Fig. 8.20). Show that:

(i) $\Delta APD \cong \Delta CQB$ (ii) $AP = CQ$ (iii) $\Delta AQB \cong \Delta CPD$ (iv) $AQ = CP$

(v) APCQ is a parallelogram

Sol.



ABCD is a parallelogram. P and Q are points on the diagonal BD such that $DP = BQ$.

Show that:

(i) $\Delta APD \cong \Delta CQB$ (ii) $AP = CQ$ (iii) $\Delta AQB \cong \Delta CPD$ (iv) $AQ = CP$

(v) APCQ is a parallelogram

Construction: Join AC to meet BD in O.

(i) $AD \parallel BC$ and BD is a transversal [ABCD is a || m]

$\therefore$ $\angle ADP = \angle CBQ$ $\ldots(1)$ [Alternate angles]

In Δs APD and CQB, we have:

$AD = BC$ [Opposite sides of a || gm]

$DP = BQ$ [Given]

$\angle ADP = \angle CBQ$ [From (1)]

$\therefore$ $\Delta APD \cong \Delta CQB$ [SAS]

(ii) $AP = CQ$ [CPCT in (1)]

(iii) $AB || CD$ and BD is a transversal. [ABCD is a || gm]

$\therefore$ $\angle ABQ = \angle CDP$ [Alternate angles]

In Δs AQB and CPD, we have:

$AB = DC$

$BQ = DP$

and $\angle AB = \angle CDP$

So, $\Delta AQB = \Delta CPD$ [SAS]

(iv) $AQ = CP$ CPCT in (iii)]

(v) $OA = OC$ [AC and BD bisect each other] ...(1)

and $OB = OD$

So, $OB - BQ = OD - DP$ [$BQ = DP$, given]

or $OQ = OP$ ...(2)

Thus, AC and PQ bisect each other [From (1) and (2)]

So, APCQ is a || gm.
Proved.

Alter:

We know that the diagonals of a parallelogram bisect each other. Therefore, AC and BD bisect each other at O.

$\therefore$ $OB = OD$

But $BQ = DP$

[Given]

$\therefore$ $OB - BQ = OD - DP$ or $OQ = OP$

Thus, in quadrilateral APCQ diagonals AC and PQ are such that $OQ = OP$ and $OA = OC$.

i.e., the diagonals AC and PQ bisect each other.

Hence, APCQ is a parallelogram, which prove the (v) part

(i) Δs APD and CQB, we have:

$AD = CB$ [Opp. sides of a || gm ABCD]

$AP = CQ$ [Opp. Sides of a || gm APCQ]

$DP = BQ$ [Given]

$\therefore$ By SSS criterion of congruence, we have:

$$\Delta APD \cong \Delta CQB$$

(ii) $AP = CQ$

[Corresponding part of congruent triangles]

(iii) In Δs AQB and CPD, we have:

$AB = CD$ [Opp. sides of a || gm ABCD]

$AQ = CP$ [Opp. sides of a || gm APCQ]

$BQ = DP$ [Given]

$\therefore$ By SSS criterion of congruence, we have:

$$\Delta AQB = \Delta CPD$$

(iv) $AQ = CP$ [Corresponding parts of congruent triangles]

(v) In Δs AQC and PCA, we have

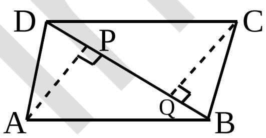
$$AQ = CP, CQ = AP \text{ and } AC = AC$$

$\therefore$ $\Delta AQC \cong \Delta PCA$ [By SSS Rule]

So, $\angle ACQ = \angle CAD$ [CPCT]

i.e., $QA \parallel CP$

Since $QA = CP$ and $QA \parallel CP$ i.e., one pair of opposite sides are equal and parallel.

10. 

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that

(i) $\Delta APB = \Delta CQD$ (ii) $AP = CQ$

Sol. (i) Since ABCD is a parallelogram, therefore $DC \parallel AB$.

Now, $DC \parallel AB$ and transversal BD intersects them at B and D.

$\therefore$ $\angle ABD = \angle BDC$ [Alternate interior angles]

Now, in Δs APB and CQD, we have:

$\angle ABP = \angle QDC$ [$\therefore \angle ABD = \angle BDC$]

$\angle APB = \angle CQD$ [Each $= 90°$] and

$AB = CD$ Opp. sides of a || gm]

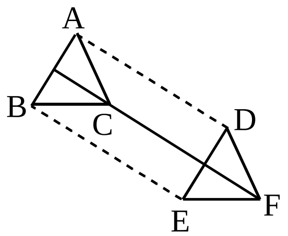
$\therefore$ By AAS criterion of congruence, we have:

$$\Delta APB \cong CQD$$

(ii) Since $\Delta APB \cong CQD$, therefore,

$\therefore$ $AP = CQ$

[$\therefore$ Corresponding parts of congruent triangles are equal]

11. 

In ΔABC and ΔDEF, $AB = DE, AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A and are joined to vertices D, E and F respectively (see Fig.). Show that

(i) quadrilateral ABED is a parallelogram (ii) quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$ (iv) quadrilateral ACFD is a parallelogram

(v) $AC = DF$ (vi) $\Delta ABC \cong \Delta DEF$.

Sol. Given: Two Δs ABC and DEF such that $AB = DE$ and $AB \parallel DE$. Also, $BC = EF$ and $BC \parallel EF$. To show that:

(i) quadrilateral ABED is a parallelogram (ii) quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$ (iv) quadrilateral ACFD is a parallelogram

(v) $AC = DF$ (vi) $\Delta ABC \cong \Delta DEF$.

(i) Consider the quadrilateral ABED.

We have: $AB = DE$ and $AB \parallel DE$

That is, one pair of opposite sides are equal and parallel.

So, ABED is a parallelogram.

(ii) Now, consider quadrilateral BEFC. We have:

$BC = EF$ and $BC \parallel EF$

That is, one pair of opposite sides are equal and parallel.

So, BEFC is a parallelogram.

(iii) Now, $AD = BE$ and $AD \parallel BE$ ...(1)

[$\therefore$ ABED is a || gm] and $CF = BE$ and $CF \parallel BE$ ...(2)

[$\therefore$ BEFE is a || gm]

From (1) and (2), we have:

$AD = CF$ and $AD \parallel CF$.

(iv) Since $AD = CF$ and $AD \parallel CF$, therefore one pair of opposite sides are equal and Parallel So, ACFD is a parallelogram.

(v) Since ACFD is a parallelogram.

$\therefore$ $AC = DF$ [Opp. sides of a || gm ACFE]

(vi) In Δs ABC and DEF, we have:

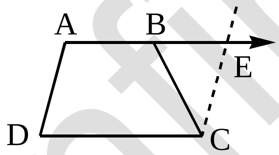
$AB = DE$ [Given]

$BC = EF$ [Given]

and, $CA = FD$ [Prove in (v)]

$\therefore$ By SSS criterion of congruence, we have:

$\Delta ABC \cong \Delta DEF$ Proved.

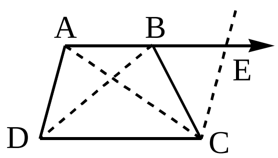
12. 

ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig.). Show that

(i) $\angle A = \angle B$ (ii) $\angle C = \angle D$ (iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal $AC =$ diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Sol. 

Given: ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$.

To show that: (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$ (iv) diagonal $AC =$ diagonal BD.

Construction: Produce AB and draw a line $CE \parallel DA$. Also, join AC and BD.

(i) Since $AD \parallel CE$ and transversal AE cuts them at A and E respectively, therefore

$$\angle A + \angle E = 180° \quad ...(1)$$

Since $AB \parallel CD$ and $AD \parallel CE$, therefore AECD is a parallelogram.

So, $AD = CE$

$\therefore$ $BC = CE$ [$\therefore AD = BC$ (Given)]

Thus, in ΔBCE, we have:

$$BC = CE$$

$\therefore$ $\angle CBE = \angle CEB$

So, $180° - \angle B = \angle E$

$\therefore$ $180° - \angle E = \angle B$

From (1) and (2), we get

$$\angle A = \angle B$$

(ii) Since $\angle A = \angle B$, therefore $\angle BAD = \angle ABC$

$\therefore$ $180° - \angle BAD = 180° - \angle ABC$

So, $\angle ADC = \angle BCD$

or $\angle D = \angle C$, i.e., $\angle C = \angle D$

(iii) In Δs ABC and BAD, we have:

$BC = AD$ [Given]

$AB = BA$ [Common]

$\angle B = \angle A$ [Shown above]

$\therefore$ By SAS criterion of congruence, we have:

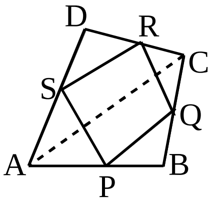
$$\Delta ABC \cong \Delta BAD$$

(iv) Since $\Delta ABC \cong \Delta BAD$, therefore

$AC = BD$ [Corresponding parts of congruent triangles are equal]

EXERCISE 8.2

1.



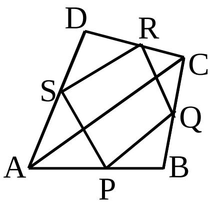
ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig.). AC is a diagonal. So that:

(i) $SR \parallel AC$ and $SE = \frac{1}{2}AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

Sol.



Given: A quadrilateral ABCD in which P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA.

Also, AC is its diagonal.

To show:

(i) $SR \parallel AC$ and $SE = \frac{1}{2}AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

(i) In ΔACD, we have

S is the mid-point of AD and R is the mid-point of CD.

Therefore, $SR \parallel AC$ and $SR = \frac{1}{2}AC$ [Mid-point theorem]

(ii) In ΔABC, we have:

P is the mid-point of AD and R is the mid-point of CD.

Therefore, $PQ \parallel AC$

and $PQ = \frac{1}{2}AC$ [Mid-point theorem]

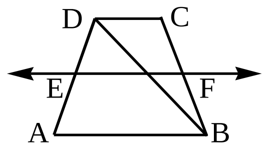
Thus, we have shown that:

$$\left.\begin{array}{l} PQ \parallel AC \\ SR \parallel AC \end{array}\right\} \Rightarrow PQ \parallel SR$$

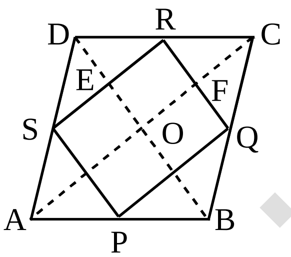
Also, $$\left.\begin{array}{l} PQ = \frac{1}{2}AC \\ SR = \frac{1}{2}AC \end{array}\right\} \Rightarrow PQ = SR$$

(iii) Since $PQ = SR$ and $PQ \parallel SR$, therefore one pair of opposite sides are equal and parallel.

$\therefore$ PQRS is a parallelogram.

2. 

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. 

Given. □ ABCD is a rhombus, P, Q, R and S are the mid-points of AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.

To Prove. □ PQRS is a rectangle.

Construction. Join AC and BD.

Proof. From Δs SRD and PQB,

$$DS = QB \quad \left(\frac{1}{2}AD = \frac{1}{2}BC\right)$$

$$DR = PB \quad \left(\frac{1}{2}DC = \frac{1}{2}AB\right)$$

$$\angle SDR = \angle PBQ$$

(Opp. sides $\angle s$ of parallelogram are equal)

$\therefore$ $$\Delta SRD \cong \Delta QPB$$

(By SAS)

then $SR = PQ$

(Corresponding parts of congruent triangles are equal)

In Δs RCQ and ASP,

$$RC = AP \qquad \left(\frac{1}{2}CD = \frac{1}{2}AB\right)$$

$$CQ = AS \qquad \left(\frac{1}{2}BC = \frac{1}{2}AD\right)$$

$$\angle SAP = \angle RCQ$$

(Opp. $\angle s$ of parallelogram are equal)

$\therefore$ $\Delta RCQ \cong \Delta PAS$

(By SAS)

then $RQ = SP$

(Corresponding parts of congruent triangles are equal)

Since both the pairs of opposite sides of quadrilateral PQRS are equal, hence quadrilateral PQRS is a parallelogram.

In $\Delta CDB,$ $\therefore$ R and Q are the mid-points of Dc and CB respectively,

$\therefore$ $RQ \parallel DB$

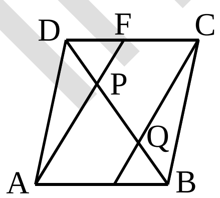
Similarly, $RE \parallel AC$

$\therefore$ OFRE is a parallelogram.

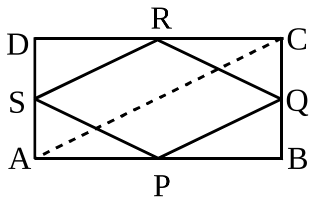
Then $\angle E = \angle EOR = 90°$

($\therefore$ Opp. $\angle s$ of rhombus are equal and diagonals of a rhombus intersect at $90°$)

Thus, quadrilateral PQRS is a rectangle.

3. 

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS in a rhombus.

Sol. 

Given. A rectangle ABCD in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA Respectively.

To Prove. PQRS is a rhombus.

Construction. Join AC.

Proof. In ΔABC, P and Q are the mid-points AB and BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC \quad ...(1)$

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2}AC \quad ...(2)$

From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$

$\Rightarrow$ PQRS is a parallelogram $...(3)$

$\therefore$ ABCD is a rectangle.

$\Rightarrow \quad AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$

$\Rightarrow \quad AS = BQ \quad ...(4)$

In Δs APS and BPQ,

$AP = BP$ [$\therefore$ P is the mid-point of AB]

$\angle PAS = \angle PBQ$ [Each equal to $90°$] and,

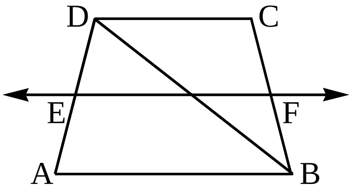
$AS = BQ$ [From (4)]

$\Delta APS \cong \Delta BPQ$ (By SAS)

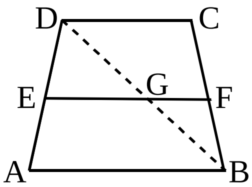
$\Rightarrow \quad PS = PQ \quad ...(5)$

[$\therefore$ Corresponding parts of congruent triangle are equal]

From (3) and (5), we obtain that PQRS is a parallelogram such that $PS = PQ$ i.e. two adjacent. Hence, PQRS is a rhombus.

4. 

ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is draw through E parallel to AB intersecting BC at F (see Fig.) Show that F is the mid-point of BC.

Sol. 

Given. In trapezium ABCD, $AB \parallel DC$.

E is the mid-point of AD and $EF \parallel AB$.

To show. F is the mid-point of BC.

Construction. Join DB. Let it intersect EF and G.

In ΔDAB, E is the mid-point of AD
[Given]

and $EG \parallel AB$ $[\because EG \parallel AB]$

$\therefore$ By converse of mid-point theorem, G is the mid-point of DB.

In ΔBCD, G is the mid-point of BD and [Show]

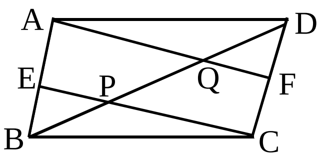
$GF \parallel DC$ $[\because AB \parallel DC, EF \parallel AB \Rightarrow DC \parallel EF]$

$\therefore$ By converse of mid-point theorem, F is the mid-point of BC.

5. In a parallelogram ABCD, E and F are the mid=-points of sides AB and CD respectively

(see Fig.).

Show that the line segments AF and EC trisect the diagonal BD.

Sol. 

Given. ABCD is a parallelogram. E and F are the mid-point of the sides AB and CD respectively.

To prove. $BP = PQ = QD$

Proof. $AE \parallel CF$ (Given)

and $AE = CF$ $\left(\because \frac{1}{2}AB = \frac{1}{2}CD\right)$

$\therefore$ AECF is a parallelogram

$\Rightarrow EC \parallel AF$ ...(1)

In ΔDPC,

F is the mid-point of CD [Given]

and $FQ \parallel CP$. [From (1)]

$\therefore$ Q is the mid-point of PD.

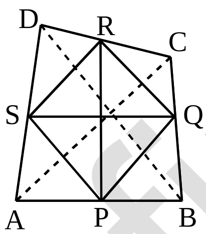
or $PQ = QD$ ...(2) Similarly, in ABQ,

$BP = PQ$ ...(3)

From (2) and (3), $BP = PQ = QD$. Proved.

6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol.



Given. In a quad. ABCD, P, Q, R and S are respectively the mid-points of AB, BC, CD and DA. PR and QS intersect each other at O.

To show. $OP = OR, OQ = OS$

Construction. Join PQ, QR, RS. SP, AC and BD.

In ΔABC, P and Q are mid-points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(1)

Similarly, we can show that

$RS \parallel AC$ and $RS = \frac{1}{2}AC$ ...(2)

$\therefore$ From (1) and (2) $PQ \| SR$ and $PQ = SR$

Thus, a pair of opposite sides of a quadrilateral PQRS are parallel and equal.

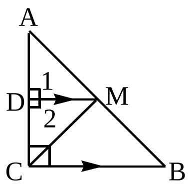
$\therefore$ Quadrilateral PQRS is a parallelogram.

Since the diagonals of a parallelogram bisects each other, therefore diagonals PR and QS of $\|$gm PQRS, i.e., the line segments joining the mid-points of opposite sides of quadrilateral ABCD bisect each other.

7. ABC is triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC (ii) $MD \perp AC$ (iii) $CM = MA = \frac{1}{2}AB$

Sol.



Given. In ΔABC, $\angle C = 90°$, M is midpoint of hypotenuse AB.

To prove. (i) D is midpoint of AC (ii) $MD \perp AC$ (iii) $CM = AM = \frac{1}{2}AB$

Proof. (i) In ΔABC, M is the midpoint of side AB and MD is parallel to BC, so by converse of mid-point theorem D is midpoint of AC.

(ii) ΔABC, $\angle C = 90°$

(Given)

MD is parallel to BC and AC is transversal so $\angle 1 = \angle ACB$ [Corresponding angles]

But $\angle ACB = 90°$ (Given)

so $\Rightarrow \angle 1 = \angle 2 = 90°$

as $\angle 1 = \angle 2 = 90°$, so MD is perpendicular to AC.

(iii) In $\Delta AMD \,\&\, \Delta CMD$, $AD = DC$

(Proved in (i) part as D is midpoint of AC by converse of mid-point theorem)

$\angle 1 = \angle 2$ ($90°$ each)

$DM = DM$ (Common)

So, by SAS and $\Delta AMD \cong \Delta CMD$

$\Rightarrow$ $AM = CM$ (by CPCT) ...(1)

But $AM = BM = \frac{1}{2}AB$ (as M is given midpoint of AB) $\quad ...(2)$

$\Rightarrow$ From equation $(1) \& (2)$

$$AM = BM = CM = \frac{1}{2}AB$$