AP EAMCET Mock Test – 1

INSTRUCTION:

- *The Entrance test is conducted for a duration of 3 hours and the question paper consists of a total of 160 questions comprising 80 questions in Mathematics, 40 questions in Physics, and 40 questions in Chemistry.*
- *All questions are having equal weightage.*
- *There is only one correct response for each question out of four responses given.*
- *There is no negative marking and No deduction from the total score will be made if no response is indicated for a question.*

MATHEMATICS

01. If [x] represents the greatest integer $\leq \mathbf{x}$, then the range of the real valued function $\mathbf{f}(\mathbf{x}) = \dfrac{\mathbf{1}}{\sqrt{[\mathbf{x}]^2 + [\mathbf{x}] - \mathbf{2}}}$ is

(1) $(-\infty, 0] \cup \left(\frac{1}{2}, \infty\right)$

(2) $\left(0, \frac{1}{2}\right]$

(3) $(-\infty, 0) \cup [2, \infty)$

(4) $(0, 2]$

Key: 2

Solution: Let [x] = y

$y^2 + y - 2 > 0$

$(y + 2)(y - 1) > 0$

$y < -2$ or $y > 1$

$[x] < -2$ or $[x] > 1$

$x \in (-\infty, -2) \cup [2, \infty)$

$$\text{Range } = \left(0, \frac{1}{2}\right]$$

02. Given, the function.

$\mathbf{f(x)=\frac{a^x + a^{-x}}{2}(a>2)}$, then

$\mathbf{f(x+y)+f(x-y)}$ is equal to

(1) f(x) – f(y)

(2) f(y)

(3) 2f(x) f(y)

(4) f(x) f(y)

Key: 3

Solution: Expansion

03. If the range of the function $\mathbf{f(x)=-3x-3}$ is {3, -6, -9, -18}, then which of the following elements is not in the domain of f?

(1) -1

(2) -2

(3) 1

(4) 2

Key: 1

Solution: $f(x)=-3x-3$

$f(x)=3 \Rightarrow x=-2$

$f(x)=-6 \Rightarrow x=1$

$f(x)=-9 \Rightarrow x=2$

$f(x) = -18 \Rightarrow x = 5$

Domain = {-2, 1, 2, 5}.

Hence – 1 cannot be in the domain of f

04. Let $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{k} \end{bmatrix}, \mathbf{k} \in \mathbf{R}$ and $\mathbf{A^3} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix}$. If d = 228, then b + c =

(1) 52

(2) 74

(3) 2

(4) 100

Key: 2

Solution: $A^3 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^3 = \begin{bmatrix} k & 1+k^2 \\ 1+k^2 & 2k+k^3 \end{bmatrix}$$

d = 228

$2k + k^3 = 228$

k = 6

$b + c = 2(1 + k^2) = 74$

05. Let M and N be two invertible square matrices over R of order 2 such that N is diagonal. Then MNM^{-1} is diagonal..........

(1) for all M

(2) only when M is a scalar matrix

(3) for all diagonal matrices M

(4) M must be a null matrix

Key: 3

Solution: Let $M=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $N=\begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix}$

$$M^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$MNM^{-1}=\frac{1}{ad-bc}\begin{bmatrix} adn_1-bcn_2 & abn_2-abn_1 \\ cdn_1-cdn_2 & adn_2-cdn_1 \end{bmatrix}$$

$abn_2 - abn_1 = 0 = cdn_1 - cdn_2$

If M is a diagonal matrix then

$b = 0, c = 0$

06. Let $\mathbf{A}=\begin{bmatrix} \mathbf{a} & \mathbf{3} & \mathbf{5} \\ \mathbf{5} & \mathbf{-1} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{-4} \end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix} \mathbf{b} & \mathbf{1} & \mathbf{4} \\ \mathbf{4} & \mathbf{c} & \mathbf{1} \\ \mathbf{-3} & \mathbf{1} & \mathbf{d} \end{bmatrix}$. If the trace of A is -4 and $\mathbf{AB}=\begin{bmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{17} \\ \mathbf{-3} & \mathbf{10} & \mathbf{25} \\ \mathbf{28} & \mathbf{-8} & \mathbf{3} \end{bmatrix}$ then a + b + c + d =

(1) 7

(2) - 1

(3) 3

(4) 1

Key: 3

Solution: a = 1, b = 2, c = -2, d = 3

07. Let the vectors $\overline{\mathbf{AB}} = \mathbf{2\bar{i}+2\bar{j}+\bar{k}}$ and $\overline{\mathbf{AC}}=\mathbf{2\bar{i}+4\bar{j}+4\bar{k}}$ be two sides of a triangle ABC. If G is the centroid of $\mathbf{\Delta ABC}$, then $\frac{\mathbf{27}}{\mathbf{7}}\left(\overline{\mathbf{AG}}\right)^{\mathbf{2}}+\mathbf{5}=$

(1) 25

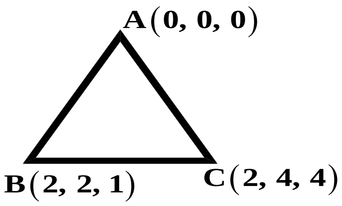
(2) 38

(3) 47

(4) 52

Key: 2

Solution:



$$G=\left(\frac{4}{3},\frac{6}{3},\frac{5}{3}\right)$$

$$\left|\overline{AG}\right|=\frac{\sqrt{77}}{3}$$

$$\frac{22}{7}\left(\overline{AG}\right)^{2}+5=38$$

08. If P is a point on the line parallel to the vector $\mathbf{2\bar{i}-3\bar{j}-6\bar{k}}$ and passing through the point A whose position vector $\mathbf{\bar{i}+2\bar{j}-2\bar{k}}$ and AP = 21, then the position vector of P can be

(1) $6\bar{i}-9\bar{j}-18\bar{k}$

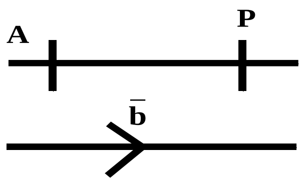
(2) $6\bar{i}+9\bar{j}-18\bar{k}$

(3) $-5\bar{i}+11\bar{j}+16\bar{k}$

(4) $5\bar{i}-11\bar{j}+16\bar{k}$

Key: 3

Solution:



$A(1, 2, -2)$ $\bar{b} = 2\bar{i} - 3\bar{j} - 6\bar{k}$

$|\overline{AP}| = 21$

$|\overline{AP}| = t\bar{b} \Rightarrow t = 7$ or -5 $\overline{OP}$ $7\bar{i} - 7\bar{j} - 2\bar{k}$ or $-5\bar{i} + 11\bar{j} + 16\bar{k}$

09. The set of real values of λ for which the vectors $\lambda\bar{\mathbf{i}} - 3\bar{\mathbf{j}} + 5\bar{\mathbf{k}}$ and $2\lambda\bar{\mathbf{i}} - \lambda\bar{\mathbf{j}} + \bar{\mathbf{k}}$ are perpendicular to each other is.

(1) $\{0, 1\}$

(2) $\{-2\}$

(3) $\{2, -1\}$

(4) ϕ

Key: 4

Solution:

$2\lambda^2 + 3\lambda + 5 = 0$

$\Delta < 0$

No real value s of λ

10. Three vectors $\bar{\mathbf{a}}$, $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ satisfy $\bar{\mathbf{a}} + \bar{\mathbf{b}} + \bar{\mathbf{c}} = \bar{\mathbf{0}}$. If $|\bar{\mathbf{a}}| = 3$, $|\bar{\mathbf{b}}| = 4$, $|\bar{\mathbf{c}}| = 2$, then $\bar{\mathbf{a}}.\bar{\mathbf{b}} + \bar{\mathbf{b}}.\bar{\mathbf{c}} + \bar{\mathbf{c}}.\bar{\mathbf{a}} + 2\left(|\bar{\mathbf{a}}| + |\bar{\mathbf{b}}| + |\bar{\mathbf{c}}|\right)$ is equal to

(1) $\frac{-7}{2}$

(2) $\frac{7}{2}$

(3) $\frac{-11}{2}$

(4) $\frac{11}{2}$

Key: 2

Solution: $\bar{a}+\bar{b}+\bar{c}=\bar{0}$ on squaring $\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=-\frac{1}{2}\left(a^{-2}+b^{-2}+c^{-2}\right)$

11. If $\bar{a}$ and $\bar{b}$ are two vectors such that $\bar{a}=2\bar{i}+2\bar{j}+p\bar{k}$, $|\bar{b}|=7$, $\bar{a}.\bar{b}=4$ and $|\bar{a}\times\bar{b}|=5\sqrt{17}$ then p =

(1) ± 5

(2) ± 6

(3) ± 1

(4) ± 3

Key: 3

Solution: Using $|\bar{a}\times\bar{b}|^2+(\bar{a}.\bar{b})^2=|\bar{a}|^2\,|\bar{b}|^2$

12. $p=2\hat{i}-3\hat{j}+k,\ q=\hat{i}+\hat{j}-k$. If the vectors $\bar{a}$ and $\bar{b}$ are the orthogonal projections of $\bar{p}$ on $\bar{q}$ and $\bar{q}$ on $\bar{p}$ respectively, then $\frac{\bar{a}\times\bar{b}}{\bar{a}.\bar{b}}=$

(1) $\frac{2\hat{i}+3\hat{j}+5k}{19\sqrt{2}}$

(2) $\frac{2\hat{i}+3\hat{j}+5k}{\sqrt{38}}$

(3) $\frac{2\hat{i}+3\hat{j}+5k}{2}$

(4) $\frac{3\hat{j}-2\hat{j}}{13}$

Key: 3

Solution: Using projection formula

13. If $\sin A = \frac{-7}{25}$, $\cos B = \frac{8}{17}$, **A** does not lie in the 3rd quadrant and B does not lie in the 1st quadrant, then

8 tan A – 5 cot B =

(1) 0

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) 1

Key: 2

Solution: $\sin A = \frac{-7}{25}, \cos B = \frac{8}{17}$

$\tan A = \frac{-7}{24}, \cot B = \frac{-8}{15}$

14. $\frac{1}{\sin 250^0} + \frac{\sqrt{3}}{\cos 290^0} =$

(1) $\frac{1}{\sqrt{3}}$

(2) 4

(3) $\frac{4}{\sqrt{3}}$

(4) 1

Key: 2

Solution: Simplify

15. If $\sinh x = \frac{-1}{2}$ then tanh 2x =

(1) $\frac{-\sqrt{5}}{2}$

(2) $-\sqrt{3}$

(3) $\frac{-\sqrt{5}}{3}$

(4) $\frac{-\sqrt{3}}{2}$

Key: 3

Solution: Using formula

16. Let $\mathbf{y=4\sin^2\theta - \cos 2\theta}$. If l and m are the minimum and maximum values of y respectively, then

(1) $lm=\frac{m}{l}$

(2) $lm=\frac{l}{m}$

(3) $l+m=\frac{l}{m}$

(4) $\frac{lm}{l-m}=l+m$

Key: 1

Solution: $y=4\sin^2\theta - \cos 2\theta$

$y=6\sin^2\theta-1$

$-1\le y\le 5$

$l=-1,\ m=5$

17. If $\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}+\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}=0$, then $\cot\theta_1 . \cot\theta_2 . \cot\theta_3 . \cot\theta_4 =$

(1) 1

(2) - 1

(3) 2

(4) 1/2

Key: 2

Solution: Using componendo and dividendo and transformation

18. In a triangle ABC, if $\frac{\mathbf{a}}{\mathbf{\tan A}} = \frac{\mathbf{b}}{\mathbf{\tan B}} = \frac{\mathbf{c}}{\mathbf{\tan C}}$ then $\mathbf{\cos^2 A + \cos^2 B + \cos^2 C =}$

(1) $\sqrt{2}$

(2) $\frac{3}{4}$

(3) $\frac{\sqrt{3}+1}{2}$

(4) $\frac{2\sqrt{3}-1}{2}$

Key: 2

Solution: $\frac{a}{\tan A} = \frac{b}{\tan B} = \frac{c}{\tan B}$

$\Rightarrow A = B = C = 60^0 \ \cos^2 A + \cos^2 B + \cos^2 C = \frac{3}{4}$

19. In a triangle ABC, AD and BE are medians. If AD = 4, $\underline{|\mathbf{DAB}} = \frac{\boldsymbol{\pi}}{\mathbf{6}}$ and $\underline{|\mathbf{ABE}} = \frac{\boldsymbol{\pi}}{\mathbf{3}}$ then the area of $\mathbf{\Delta ABC}$ is

(1) $\frac{14}{3\sqrt{3}}$

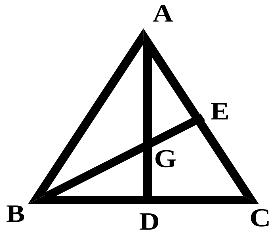
(2) $\frac{28}{3\sqrt{3}}$

(3) $\frac{11}{3\sqrt{3}}$

(4) $\frac{32}{3\sqrt{3}}$

Key: 4

Solution:



$AD = 4,\ \underline{|DAB} = \frac{\pi}{6},\ \underline{|ABE} = \frac{\pi}{3}$

$\underline{|AGB} = \frac{\pi}{2}$

$AG = \frac{2}{3}(AD) = \frac{8}{3}$

DG = 4 / 3

$BG = 8/3\sqrt{3}$

Area of $\Delta AGB = \frac{1}{2}(BG)(AG)$

$= \frac{32}{9\sqrt{3}}$

Area of $\Delta ABC = 3\left(\frac{32}{9\sqrt{3}}\right)$

$= \frac{32}{3\sqrt{3}}$

20. In a $\Delta\mathbf{ABC}$, with usual notation, if $r = r_1 - r_2 - r_3$, then 2R =

(1) a

(2) b + c

(3) c

(4) $c + a$

Key: 1

Solution: $r = r_1 - r_2 - r_3$

$r + r_2 + r_3 - r_1 = 0$

$\Rightarrow 4R\cos A = 0 \Rightarrow A = 90^0$

$A = 2R\sin A$

$\Rightarrow a = 2R$

21. If $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the roots of the quadratic equation $x^2 + 7x + 3 = 0$, then the equation having roots α and β is

(1) $3x^2 - x - 3 = 0$

(2) $11x^2 + 13x + 3 = 0$

(3) $13x^2 + 11x + 13 = 0$

(4) $11x^2 + 3x + 13 = 0$

Key: 2

Solution: Using formula

22. α, β are the roots of $x^2 - 10x - 8 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \in \mathbf{N}$, then the value of $\frac{a_{10} - 8a_8}{5a_9}$ is

(1) -3

(2) 3

(3) -2

(4) 2

Key: 4

Solution: $a_n = \alpha^n - \beta^n$

$x^2 - 10x - 8 = 0$

$$\frac{a_{10}-8a_8}{5a_9}=\frac{\left(\alpha^{10}-\beta^{10}\right)-8\left(\alpha^8-\beta^8\right)}{5a_9}$$

$$=\frac{\alpha^8\left(\alpha^2-8\right)-\beta^8\left(\beta^2-8\right)}{5a_9}$$

$$=\frac{\alpha^8\left(10\alpha\right)-\beta^8\left(10\beta\right)}{5a_9}$$

$$=\frac{10\left(\alpha^9-\beta^9\right)}{5\left(\alpha^9-\beta^9\right)}=2$$

23. Let $\mathbf{f}\left(\mathbf{x}\right)=\frac{\mathbf{6x^2-18x+21}}{\mathbf{6x^2-18x+17}}$. If *m* is the maximum value of f(x) and f(x) > $\mathbf{n}\,\forall\,\mathbf{x}\in\mathbf{R}$. The 14m – 7n =

(1) – 1

(2) 23

(3) 35

(4) 42

Key: 2

Solution: Let $y=\frac{6x^2-18x+21}{6x^2-18x+17}$

$\Delta \geq 0$

$1\leq y\leq\frac{15}{7}$

$m=\frac{15}{7},\ n=1$

14 m – 7n = 23

24. If α, β, γ are the roots of the equation $x^3 - 5x^2 - 2x + 24 = 0$ then $\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} =$

(1) 244

(2) $\frac{-1}{6}$

(3) 61

(4) $\frac{-61}{6}$

Key: 4

Solution: expand

25. Let the transformed equation of $2x^4 - 8x^3 + 3x^2 - 1 = 0$ so that the term containing the cubic power of x is absent be $2x^4 + bx^2 + cx + d = 0$. Then b =

(1) - 18

(2) - 15

(3) – 9

(4) - 16

Key: 3

Solution: Using Horner's Method

26. $\{x \in [0, 2\pi] /$ sin x + i cos 2x and cos x – i sin 2x are conjugate to each other$\}$ =

(1) $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi\right\}$

(2) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

(3) $\left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$

(4) ϕ

Key: 4

Solution: z_1 = sinx + i cos 2x

$z_2 = \cos x - i \sin 2x$

$z_1 = \overline{z_2}$ or $z_2 = \overline{z_1}$

No values of x

27. If $\mathbf{z = x + iy, x, y \in R, (x, y) \neq (0, -4)}$ and $\mathbf{Arg\left(\frac{2z-3}{z+4i}\right) = \frac{\pi}{4}}$, then the locus of z is

(1) $2x^2 + 2y^2 + 5x + 5y - 12 = 0$

(2) $2x^2 - 3xy + y^2 + 5x + y - 12 = 0$

(3) $2x^2 + 3xy + y^2 + 5x + y + 12 = 0$

(4) $2x^2 + 2y^2 - 11x + 7y - 12 = 0$

Key: 1

Solution: expand

28. If $\mathbf{1, \omega, \omega^2}$ are the cube roots of unity then the value of $\mathbf{(x+y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2}$ is

(1) $2x^2 . 3y^2$

(2) 4xy

(3) 6xy

(4) $2x^2 . 2y^2$

Key: 3

Solution: expand

29. If $\mathbf{(a+ib)^{\frac{1}{4}} = 2+3i}$, then 3b – 2a is equal to

(1) – 22

(2) – 122

(3) – 598

(4) –698

Key: 2

Solution: $a + ib = (2 + 3i)^4$

on expansion

a = - 119, b = - 120

30. If $|x|<1$, then the coefficient of x^5 in the expansion of $\frac{3x}{(x-2)(x+1)}$ is

(1) $\frac{33}{32}$

(2) $\frac{-33}{32}$

(3) $\frac{31}{32}$

(4) $\frac{-31}{32}$

Key: 2

Solution: $\frac{3x}{(x-2)(x+1)}=\frac{2}{x-2}+\frac{1}{x+1}$

Coefficient of x^5 is $\frac{-1}{32}-1=\frac{-33}{32}$

31. Number of positive divisors of 360 which are multiples of 3 is

(1) 16

(2) 15

(3) 24

(4) 23

Key: 1

Solution: Multiples of 3 are 3, 6, 9, 12, 15, 18, 24, 30, 36, 45, 60, 72, 90, 120, 180, 360.

32. If the middle term in the expansion of $(1 + x)^{2n}$ is the greatest term, then x lies in the interval.

(1) $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

(2) $\left(\frac{n+1}{n}, \frac{n}{n+1}\right)$

(3) (n – 2, n)

(4) (n – 1, n)

Key: 1

Solution: Using Middle term concept

33. If the total number of observations is 20, $\sum x_i = 1000$ and $\sum x_i^2 = 84000$, then the variance of the distribution is

(1) 1500

(2) 1600

(3) 1700

(4) 1800

Key: 3

Solution: Concept

34. Let N be the set of positive integers. The number of distinct triplets (x, y, z) satisfying x, y, z $\in$ N, x < y < z and x + y + z = 12 is

(1) 5

(2) 7

(3) 6

(4) 8

Key: 2

Solution: x + y + z = 12, x < y < z

$(x,y,z) \longrightarrow (1,2,9), (2,3,7), (3,4,5), (1,5,6), (2,4,6), (1,3,8), (1,4,7)$

35. The exponent of 6 in 72! is

(1) 34

(2) 70

(3) 17

(4) 35

Key: 1

Solution: Exponent of 2 in $\underline{|72}$ is

$$=\left[\frac{72}{2}\right]+\left[\frac{72}{4}\right]+\left[\frac{72}{8}\right]+\left[\frac{72}{16}\right]$$

$$+\left[\frac{72}{32}\right]+\left[\frac{72}{64}\right]+\left[\frac{72}{128}\right]+\ldots\ldots\ldots$$

$$= 36 + 18 + 9 + 4 + 2 + 1 + 0 = 66$$

Similarly exponent of 3 in $\underline{|72}$ is 34

Highest power of 6 in $\underline{|72}$ is 34

36. If the number of all possible permutations of the letters of the word MATHEMATICS in which the repeated letters are not together is 982(X), then X =

(1) 5040

(2) 14400

(3) 21600

(4) 86400

Key: 1

Solution: Total number of words $=\dfrac{\underline{|11}}{\underline{|2}\,\underline{|2}\,\underline{|2}}$,

Number of words in which all the letters are repeated together $=\underline{|8}\dfrac{\underline{|2}}{\underline{|2}}\,\dfrac{\underline{|2}}{\underline{|2}}\,\dfrac{\underline{|2}}{\underline{|2}}=\underline{|8}$.

Required words $=\dfrac{\underline{|11}}{\underline{|2}\,\underline{|2}\,\underline{|2}}-\underline{|8}$

$= 4949280$

$982x = 4949280$

$x = 5040$

37. Four cards are drawn at random from a pack of 52 playing cards. The probability of getting all four cards of the same suit is.

(1) $\frac{13}{270725}$

(2) $\frac{91}{190}$

(3) $\frac{178}{20825}$

(4) $\frac{44}{4165}$

Key: 4

Solution: Probability $=\frac{4\left({}^{13}C_4\right)}{{}^{52}C_4}$

$=\frac{44}{4165}$

38. ω is a complex cube root of unity. When an unbiased die is thrown 3 times, if β_1, β_2 and β_3 are the numbers appeared on the die, then the probability that β_1, β_2 and β_3 satisfy $\omega^{\beta_1}+\omega^{\beta_2}+\omega^{\beta_3}=0$ is

(1) $\frac{212}{513}$

(2) $\frac{1}{3}$

(3) $\frac{3}{5}$

(4) $\frac{2}{9}$

Key: 4

Solution: n(s) = 216

$\omega^{\beta_1}+\omega^{\beta_2}+\omega^{\beta_3}=0$

$(\beta_1, \beta_2, \beta_3)$

$(1, 2, 3) \longrightarrow$ 6 ways

$(2, 3, 4) \longrightarrow$ 6 ways

$(3, 4, 5) \longrightarrow$ 6ways

$(4, 5, 6) \longrightarrow$ 6 ways

$(1, 3, 5) \longrightarrow$ 6 ways

$(2, 4, 6) \longrightarrow$ 6 ways

$(1, 5, 6) \longrightarrow$ 6 ways

$(1, 2, 6) \longrightarrow$ 6 ways

48

$\text{Prob} = \frac{48}{216} = \frac{2}{9}$

39. The probabilities that A and B speak truth are $\frac{4}{5}$ and $\frac{3}{4}$ respectively. The probability that they contradict each other when asked to speak on a fact is.

(1) $\frac{1}{5}$

(2) $\frac{3}{20}$

(3) $\frac{4}{20}$

(4) $\frac{7}{20}$

Key: 4

Solution: $P(A)=\frac{4}{5}\quad P(B)=\frac{3}{4}$

$$P(\bar{A})=\frac{1}{5}\quad P(\bar{B})=\frac{1}{4}$$

$$Prob=\frac{4}{5}.\frac{1}{4}+\frac{3}{4}.\frac{1}{5}=\frac{7}{20}$$

40. Two persons A and B are throwing an unbiased six faced dice alternatively, with the condition that the person who throws 3 first wins the game. If A starts the game, then probabilities of A and B to win the game are respectively.

(1) $\frac{6}{11},\frac{5}{11}$

(2) $\frac{5}{11},\frac{6}{11}$

(3) $\frac{8}{11},\frac{3}{11}$

(4) $\frac{3}{11},\frac{8}{11}$

Key: 1

Solution: $p=\frac{1}{6},\ q=\frac{5}{6}$

$$P(A) = p + q^2p + q^4p + \ldots\ldots = \frac{p}{1-q^2}$$

$$=\frac{1/6}{1-25/36}=\frac{6}{11}$$

$$P(B)=1-\frac{6}{11}=\frac{5}{11}$$

41. A (5, 3), B (3, -2), C (2, -1) are the points and P is a point such that the area of the quadrilateral PABC is 10 square units then the locus of P is

(1) $(4x-3y-38)(4x-3y-2)=0$

(2) $(4x-3y-38)(4x-3y+2)=0$

(3) $(4x-3y+38)(4x-3y+2)=0$

(4) $(4x-3y+38)(4x-3y-2)=0$

Key: 2

Solution: Let P = (x, y), A = (5, 3) B(3, -2)

C (2, -1) Area of quadrilateral

PABC = 10

$$\frac{1}{2}\begin{vmatrix} x-3 & 3 \\ y+2 & 4 \end{vmatrix} = 10$$

$$\Rightarrow |4x-3y-18|=20$$

$$\Rightarrow 4x-3y-18=\pm 20$$

Locus of P is $(4x-3y-38)(4x-3y+2)=0$

42. If Q is the image of the point P(1, 1) with respect to the straight line $x+y+1=0$, then the length of the perpendicular drawn from Q to the line $\mathbf{3x-4y+3=0}$ is

(1) $\frac{5}{2}$

(2) 2

(3) 1

(4) $\frac{1}{2}$

Key: 3

Solution: Let Q (h. k) be the image of P (1, 1) $\Rightarrow \frac{h-1}{1}=\frac{k-1}{1}=\frac{-2(1+1+1)}{1+1}$

$\therefore$ Q $=(h, k)=(-2, -2)$

$\perp^{r}$ distance from (-2, -2) to

3x – 4y + 3 = 0 is $\frac{|-6+8+3|}{\sqrt{9+16}}=1$

43. The vertex of an equilateral triangle is at (2, -1) and the side opposite to it has the equation x + y – 2 = 0 then the orthocentre of the triangle is

(1) $\left(\frac{1}{3}, \frac{1}{3}\right)$

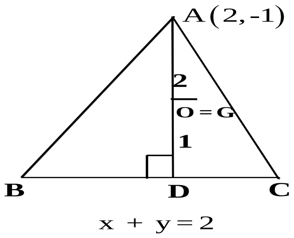
(2) (1, 1)

(3) $\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$

(4) $\left(\frac{7}{3}, \frac{-2}{3}\right)$

Key: 4

Solution:



In a equilateral triangle orthocentre = centriod Let D = (h, k)

$$\frac{h-2}{1}=\frac{k+1}{1}=\frac{-(2-1-2)}{2}$$

$$h-2=K+1=\frac{1}{2}$$

$$D=\left(\frac{5}{2},\frac{-1}{2}\right)$$

AO : OD = 2 : 1

$$O=\left(\frac{2\left(\frac{5}{2}\right)+1(2)}{2+1},\frac{2\left(\frac{-1}{2}\right)+1(-1)}{2+1}\right)$$

$$O=\left(\frac{7}{3},\frac{-2}{3}\right)$$

44. The distance between the point of concurrency of the two families of straight lines given by $\mathbf{x+(5\lambda+1)y+1-3\lambda=0}$ and $\mathbf{(5\mu+2)x-3y+3+6\mu=0}$ is

(1) 4

(2) $\frac{2\sqrt{2}}{5}$

(3) $\frac{\sqrt{2}}{5}$

(4) 6

Key: 2

Solution: Given $x+(5\lambda+1)y+1-3\lambda=0$ $(x+y+1)+\lambda(5y-3)=0$ ______ (1) $L_1+\lambda L_2=0$

from (1) P.C $P=\left(\frac{-8}{5},\frac{3}{5}\right)$

and $(5\mu+2)x-3y+3+6\mu=0$ $(2x-3y+3)+\mu(5x+6)=0$ ______ (2)

from (2) P.C $Q=\left(\frac{-6}{5},\frac{1}{5}\right)$

$$PQ=\sqrt{\left(\frac{-8}{5}+\frac{6}{5}\right)^2+\left(\frac{3}{5}-\frac{1}{5}\right)^2}$$

$$=\frac{2\sqrt{2}}{5}$$

45. If the line $2x - 3y + 4 = 0$ divides the line segment joining the points A (-2, 3) and B (3, -2) in the ratio m : n then the point which divides AB in the ratio – 4 m : 3n is

(1) (-17, 18)

(2) $\left(\frac{-59}{7},\frac{66}{7}\right)$

(3) (- 5, 6)

(4) $\left(\frac{-5}{7},\frac{12}{7}\right)$

Key:1

Solution: $L\equiv 2x-3y+4=0$

$L_{11}=L(-2, 3)=-9$

$L_{22}=2(3, -2)=16$

Ratio = $-L_{11} : L_{22}$ = 9 : 16

$$\frac{-4m}{3n} = \frac{-4}{3}\left(\frac{9}{16}\right) = \frac{-3}{4}$$

Apply section formula.

$$\text{Reqpt} = \left(\frac{3(3)-4(-2)}{3-4}, \frac{3(-2)-4(3)}{3-4}\right)$$

$= (-17, 18)$

46. If the slope of one of the lines represented by $\mathbf{15x^2 + 40xy + 3ky^2 = 0}$ is 3 then the angle between the pair of lines is

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{2}$

Key: 4

Solution: Given $15x^2 + 40xy + 3ky^2 = 0$ ___(1)

Slope of one of the line is 3

$\therefore\ y = 3x$

Put $y = 3x$ in equation (1)

Weget $K = -5$

$15x^2 + 40xy - 15y^2 = 0$

$a + b = 15 - 15 = 0$

$$\theta = \frac{\pi}{2}$$

47. If the lines joining the origin to the points of intersection of the line x + y = k and the curve $x^2 + y^2 - 2x - 4y + 2 = 0$ are at right angles then the sum of all the possible values of K is

(1) 0

(2) 1

(3) 3

(4) 5

Key:3

Solution: Given curve is $x^2 + y^2 - 2x - 4y + 2 = 0$ _____(1)

Line : $\frac{x+y}{K} = 1$_______(2)

Homogenising (1) with (2) $x^2 + y^2 - 2x\left(\frac{x+y}{k}\right) - 4y\left(\frac{x+y}{k}\right) + 2\left(\frac{x+y}{k}\right)^2 = 0$

Lines are $\perp^r \Rightarrow a + b = 0$

$$\left(1 - \frac{2}{k} + \frac{2}{k^2}\right) + \left(1 - \frac{4}{k} + \frac{2}{k^2}\right) = 0$$

$$\Rightarrow k^2 - 3k + 2 = 0$$

K = 1, 2

Sum = 1 + 2 = 3

48. In the tetrahedron ABCD, A = (1, 2, -3) and G = (-3, 4, 5) is centriod of tetrahedron. If P is the centriod of triangle BCD, then AP =

(1) $\frac{4\sqrt{21}}{3}$

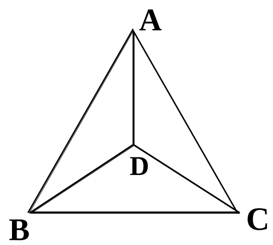
(2) $\frac{8\sqrt{21}}{3}$

(3) $4\sqrt{21}$

(4) $\frac{\sqrt{21}}{3}$

Key: 2

Solution:



A = (1, 2, -3) G = (-3, 4, 5)

$AG = \sqrt{84} = 2\sqrt{21}$

P is the centriod of ΔBCD

So, G divides $\overline{AP}$ in 3 : 1

Let AG = 3x, then GP = x

$3x = 2\sqrt{21}$

$x = \frac{2\sqrt{21}}{3}$

AP = AG + GP = 4x

$AP = \frac{8\sqrt{21}}{3}$

49. If the direction cosines of a line L are (ab, b, b) and the Angle between L and x – axis is $\frac{\pi}{3}$ then the value of $a^2 + b^2$ is equal to

(1) $\frac{24}{25}$

(2) $\frac{25}{24}$

(3) $\frac{5}{24}$

(4) $\frac{5}{11}$

Key: 2

Solution: D c's of L: (ab, b, b)

D c's of x – axis : (1, 0, 0)

$$\cos\frac{\pi}{3} = ab$$

$$\Rightarrow ab = \frac{1}{2} \text{ and } a^2b^2 + b^2 + b^2 = 1$$

$$\frac{1}{4} + 2b^2 = 1$$

$$\Rightarrow 2b^2 = \frac{3}{4} \Rightarrow b^2 = \frac{3}{8}$$

$$a^2 b^2 = \frac{1}{4}$$

$$a^2\left(\frac{3}{8}\right) = \frac{1}{4}$$

$$\Rightarrow\ a^2=\frac{2}{3}$$

$$\therefore\quad a^2+b^2=\frac{25}{24}$$

50. A plane containing two lines whose direction ratios are (-1, 2, 1) and (1, 3, 2) passing through the point (2, 1, K). If this plane also passes through the point (3, -1, 4) then K =

(1) 5

(2) 3

(3) 6

(4) – 3

Key: 1

Solution: equation of a plane is $\begin{vmatrix} x-3 & y+1 & z-4 \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}=0 \Rightarrow x+3y-5z+20=0$

$\therefore\ (2,1,K)$ lies on the plane

$2 + 3 – 5K + 20 = 0$

$25 – 5K = 0$

$K = 5$

51. $\displaystyle \mathop{\mathbf{Lt}}_{x\to 0}\frac{\mathbf{Tan\ 2x - 2Tan\,x}}{\mathbf{(1-cosx)(2^x-1)}}=$

(1) $\dfrac{2}{\log 2}$

(2) $\dfrac{1}{\log 4}$

(3) 4 log2

(4) $\frac{4}{\log 2}$

Key: 4

Solution: Given limit $= \underset{x\to 0}{Lt}\frac{\frac{2\tan x}{1-\tan^2 x}-2\tan x}{(1-\cos x)(2^x-1)} = \underset{x\to 0}{Lt}\frac{2\tan^3 x}{(1-\cos x)(1-\tan^2 x)(2^x-1)}$

Nr and Dr divided by x^3

$$= \underset{x\to 0}{Lt}\frac{2\frac{\tan^3 x}{x^3}}{\left(\frac{1-\cos x}{x^2}\right)(1-\tan^2 x)\frac{(2^x-1)}{x}}$$

$$= \frac{2}{\left(\frac{1}{2}\right)(1)\log 2}$$

$$= \frac{4}{\log 2}$$

52. $\underset{x\to -\infty}{\mathbf{Lt}}\left(\frac{\mathbf{6x^2 - \cos 3x}}{\mathbf{x^2 + 5}} - \frac{\mathbf{(5x^3 + 3)}}{\sqrt{\mathbf{x^6 + 2}}}\right) =$

(1) – 1

(2) 0

(3) 11

(4) 1

Key: 3

Solution: Given Limit $= \underset{x \to -\infty}{\text{Lt}} \left(\frac{x^6\left(6-\frac{\cos 3x}{x^2}\right)}{x^2\left(1+\frac{5}{x^2}\right)} - \frac{x^3\left(5+\frac{3}{x^3}\right)}{|x^3|\sqrt{1+\frac{2}{x^6}}} \right)$

$\therefore \quad x \longrightarrow -\infty \Rightarrow |x^3| = -x^3$ and $\frac{\cos 3x}{x^2} \longrightarrow 0$

$= \left(\frac{6-0}{1+0}\right) + \frac{(5+0)}{\sqrt{1+0}}$

$= 6 + 5$

$= 11$

53. If $\mathbf{f(x) = \log\left[e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right], (x^2 - 4 \neq 0)}$ then the value of $\mathbf{\frac{df}{dx}}$ at $x = 3$ is

(1) 1

(2) $\frac{8}{5}$

(3) 2

(4) $\frac{8e^3}{5}$

Key: 2

Solution: $f = \log e^x + \log\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}$

$f = x + \frac{3}{4}(\log(x-2) - \log(x+2))$

$\frac{df}{dx} = 1 + \frac{3}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$

Put x = 3

$$=1+\frac{3}{4}\left(1-\frac{1}{5}\right)$$

$$=1+\frac{3}{4}\left(\frac{4}{5}\right)$$

$$=\frac{8}{5}$$

54. If $\mathbf{8f}\left(\mathbf{x}\right)+\mathbf{6f}\left(\frac{\mathbf{1}}{\mathbf{x}}\right)=\mathbf{x}+\mathbf{5},\ \left(\mathbf{x}\neq\mathbf{0}\right)$ then $f^{1}(1)=$

(1) $\frac{-1}{14}$

(2) $\frac{1}{14}$

(3) $\frac{1}{2}$

(4) 0

Key: 3

Solution: Given $8f\left(x\right)+6f\left(\frac{1}{x}\right)=x+5$ ______(1) replace x by $\frac{1}{x}$

$$8f\left(\frac{1}{x}\right)+6f\left(x\right)=\frac{1}{x}+5\text{\_\_\_\_\_\_\_\_}(2)$$

Solve (1) and (2)

$$\text{weget } f\left(x\right)=\frac{1}{28}\left[8x-\frac{6}{x}+10\right]$$

$$f^{1}\left(x\right)=\frac{1}{28}\left(8+\frac{6}{x^{2}}\right)$$

$$f^1(1)=\frac{14}{28}=\frac{1}{2}$$

55. If $\mathbf{x^2+y^2=t+\frac{1}{t}}$ and $\mathbf{x^4+y^4=t^2+\frac{1}{t^2}}$ then $\mathbf{\frac{dy}{dx}}$ =

(1) $\frac{-x}{y}$

(2) $\frac{-y}{x}$

(3) $\frac{x^2}{y^2}$

(4) $\frac{y^2}{x^2}$

Key: 2

Solution: $x^2+y^2=t+\frac{1}{t}$

$$\left(x^2+y^2\right)^2=\left(t+\frac{1}{t}\right)^2$$

$$x^4+y^4+2x^2y^2=t^2+\frac{1}{t^2}+2$$

$$t^2+\frac{1}{t^2}+2x^2y^2=t^2+\frac{1}{t^2}+2$$

$$\Rightarrow\ 2x^2y^2=2$$

$$\Rightarrow\ xy=1$$

Diff wrt x

$$x\frac{dy}{dx}+y=0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

56. Equation of the tangent to the curve $y = x^3 + 3x^2 - 5$ and which is perpendicular to the line $2x - 6y + 1 = 0$ is

(1) $9x - 8y + 26 = 0$

(2) $2x + 9y + 20 = 0$

(3) $2x + 3y + 26 = 0$

(4) $3x + y + 6 = 0$

Key: 4

Solution: $y = x^3 + 3x^2 - 5$

$$\frac{dy}{dx} = 3x^2 + 6x = m_1$$

Slope of given line $m_2 = \frac{1}{3}$

$m_1 m_2 = -1$

$$\left(3x^2 + 6x\right)\frac{1}{3} = -1$$

$$\Rightarrow x^2 + 2x + 1 = 0$$

$$\Rightarrow x = -1, \quad y = -1 + 3 - 5 = -3$$

$P = (-1, -3)$

Equation of tangent at $P(-1, -3)$ is

$y + 3 = -3(x + 1)$

$3x + y + 6 = 0$

57. The angle between the curves $2x^2 + y^2 = 20$ and $4y^2 - x^2 = 8$ at a point where they intersect in the IVth Quadrant is

(1) $\frac{\pi}{2}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{3}$

Key: 1

Solution: Given curves are $2x^2 + y^2 = 20$ ______(1)

$4y^2 - x^2 = 8$ ___(2)

Point of intersection of (1), (2) is

$$P=\left(2\sqrt{2},\ -\sqrt{2}\right)$$

Slope of the tangent to both curves at P are $m_1 = 2\sqrt{2},\ m_2 = \frac{-\sqrt{2}}{4}$

$\therefore\ m_1 m_2 = -1$

$\therefore\ \theta = \frac{\pi}{2}$

58. There is an error of $\pm \mathbf{0.04cm}$ in the measurement of the diameter of a sphere. When the radius is 10cm, the percentage error in the volume of the sphere is

(1) ± 12

(2) ± 1.0

(3) ± 10.8

(4) ± 0.6

Key: 4

Solution: Given r = 10, $\delta r = \pm\ 0.02$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\delta V}{V} \times 100 = 3\frac{\delta r}{r} \times 100$$

$$= 3\frac{(\pm\ 0.02)}{10} \times 100$$

$$= \pm\ 0.6$$

59. The absolute maximum value of the function $\mathbf{f(x) = 2x^3 - 3x^2 - 36x + 9}$ defined on [-3, 3] is

(1) 36

(2) 53

(3) 63

(4) 72

Key: 2

Solution: $f(x) = 2x^3 - 3x^2 - 36x + 9$

$$f^1(x) = 6x^2 - 6x - 36$$

$$\text{If } f^1(x) = 0 \Rightarrow x^2 - x - 6 = 0$$

x = -2, 3

Maximum value = f(-2) = 53

60. The maximum area of a rectangle of perimeter 176 cm is

(1) 1854 cm^2

(2) 1916 cm^2

(3) 1936 cm^2

(4) 2110 cm^2

Key: 3

Solution: Given $2(x + y) = 176$

$x + y = 88$

Area of rectangle $S = xy$

If S is maximum, then $x = y = 44$

Maximum Area = 44 x 44 = 1936

61. If m_1, m_2 are the slopes of the tangents drawn from a point (1, -3) to the circle $\mathbf{x^2 + y^2 - 6x + 4y + 12 = 0}$ then $\mathbf{9\left(m_1^2 + m_2^2\right) =}$

(1) 16

(2) 25

(3) 4

(4) 1

Key: 1

Solution: Centre C = (3, -2), r = 1

Equation of tangent to circle in slope form is $y + 2 = m(x - 3) \pm 1\sqrt{1+m^2}$ its passes through (1, -3) weget

$3m^2 - 4m = 0$

$m = 0, \frac{4}{3}$

Let $m_1 = 0$, $m_2 = \frac{4}{3}$

$\therefore \quad 9\left(m_1^2+m_2^2\right)=16$

62. If 5x + 6y – 34 = 0 and 2x + y + c = 0 are conjugate lines with respect to the circle $\mathbf{x^2+y^2-8x-10y+25=0}$ then the point on the line 2x + y + c = 0 is

(1) (3, 3)

(2) (2, 4)

(3) (1, -5)

(4) (2, -2)

Key: 3

Solution: Condition for conjugate lines

$r^2\left(l_1 l_2+m_1 m_2\right)=\left(l_1 g+m_1 f-n_1\right)\left(l_2 g+m_2 f-n_2\right)$

63. Equation of circle whose radius is 5 and which touches the circle $\mathbf{x^2+y^2-2x-4y-20=0}$ at (5, 5) is

(1) $(x-9)^2+(y-8)^2=5$

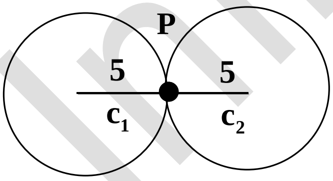
(2) $(x-9)^2+(y+8)^2=25$

(3) $x^2+y^2=25$

(4) $(x-9)^2+(y-8)^2=25$

Key: 4

Solution:



Centre of given circle $C_1=(1, 2)$ $r_1=5$ given $r_2=5$

P = mid point of c_1, c_2

$(5, 5)=\left(\frac{1+x}{2}, \frac{2+y}{2}\right)$ req circle is $\Rightarrow C_2=(9, 8)$

$\therefore \ (x-9)^2+(y-8)^2=25$

64. If (-1, -1) is the radical centre of the circles $\mathbf{x^2+y^2+2gx-4y+4=0}$, $\mathbf{x^2+y^2+6x+2fy+12=0}$ and $\mathbf{x^2+y^2+10y+20=0}$ then g – f =

(1) 0

(2) - 1

(3) 1

(4) 2

Key: 3

Solution: equation of radical axis of S = 0, $S^1 = 0$ is $S - S^1 = 0$

$\Rightarrow (2g-6)x-y(4+2f)-8=0$_______(1)

(1) passes through (-1, -1)

$\Rightarrow -(2g-6)+(4+2f)-8=0$

$\Rightarrow -2g+2f+2=0$

$\Rightarrow g-f=1$

65. The equation of the circle which passes through the origin and cuts orthogonally each of the circles $\mathbf{x^2+y^2-6x+8=0}$ and $\mathbf{x^2+y^2-2x-2y-7=0}$ is

(1) $3x^2+3y^2-8x-13y=0$

(2) $3x^2+3y^2-8x+29y=0$

(3) $3x^2+3y^2+8x+29y=0$

(4) $3x^2+3y^2-8x-29y=0$

Key: 2

Solution: equation of req circle is $\begin{vmatrix} x^2+y^2 & x & y \\ c_1 & -g_1 & -f_1 \\ c_2 & -g_2 & -f_2 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x^2+y^2 & x & y \\ 8 & 3 & 0 \\ -7 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 3\left(x^2+y^2\right) - 8x + 29y = 0$$

66. If one end of a focal chord of the parabola $\mathbf{y^2 = \frac{8}{a}x}$, $(\mathbf{a} > \mathbf{0})$ is at (1, 4), then the length of this focal chord is

(1) $\frac{25}{8}$

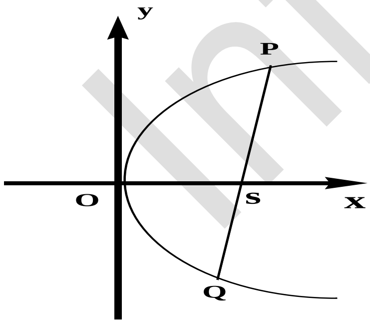
(2) $\frac{25}{2}$

(3) $\frac{25}{4}$

(4) 25

Key: 4

Solution:



$$y^2 = \frac{8}{a}x \_\_\_\_\_\_ (1)$$

(1) passes through (1, 4) $\Rightarrow$ $a=\frac{1}{2}$

$\therefore \quad y^2=16x$

$P(1, 4) = (at^2, 2at)$

$2at = 4$

$8t=4 \Rightarrow t=\frac{1}{2}$

Length of focal chord $PQ=a\left(t+\frac{1}{t}\right)^2$

$=4\left(\frac{1}{2}+2\right)^2$

= 25

67. The line y = 6x + 1 touch the parabola $y^2 = 24x$, the coordinates of a point P on this line from which the tangent to $y^2 = 24x$ is perpendicular to the line

y = 6x +1 is

(1) (-1, -5)

(2) (-2, -11)

(3) (-6, -35)

(4) (-7, -41)

Key: 3

Solution: The locus of the point of intersection of perpendicular tangents to a parabola is its directrix So, required point is the point of intersection of y = 6x + 1 and directrix x = -6

Hence P = (-6, -35)

68. If the eccentricity and the length of latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$ are $\frac{\sqrt{3}}{2}$ and 1 respectively then the sum of the lengths of major axis and minor axis of the ellipse is

(1) 6

(2) 3

(3) 10

(4) 8

Key: 1

Solution: We have $e=\frac{\sqrt{3}}{2}$

$LL^1 = 1$

$\frac{2b^2}{a}=1$

$\Rightarrow \frac{2}{a}a^2(1-e^2)=1$

$\Rightarrow 2a\left(1-\frac{3}{4}\right)=1$

$\Rightarrow a=2,\ b=1$

$\therefore\ AA^1 + BB^1 = 2a + 2b = 6$

69. If the line $2x + 5y = 12$ intersects the ellipse $4x^2 + 5y^2 = 20$ in two distinct points A and B then the midpoint of AB is

(1) (0, 1)

(2) (1, 2)

(3) (1, 0)

(4) (2, 1)

Key: 2

Solution: Let $(x_1\ y_1)$ be the mid point the chord equation of chord is $S_1 = S_{11}$

$4xx_1 + 5yy_1 = 4x_1^2 + 5y_1^2$ ______(1)

Given line is $2x + 5y = 12$ ____(2)

(1) and (2) are same

$$\frac{4x_1}{2} = \frac{5y_1}{5} = \frac{4x_1^2 + 5y_1^2}{12}$$

$$\therefore (x_1\ y_1) = (1, 2)$$

70. A hyperbola having its centre at the origin is passing through the point (5, 2) and has transverse axis of length 8 along the x – axis. Then eccentricity of the conjugate hyperbola is

(1) $\frac{\sqrt{13}}{3}$

(2) $\sqrt{\frac{13}{3}}$

(3) $\frac{\sqrt{13}}{2}$

(4) $\sqrt{\frac{13}{2}}$

Key: 3

Solution: Let equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ given $2a = 8$ $\Rightarrow a = 4$

It passes through (5, 2) weget $b = \frac{8}{3}$

$$\therefore\ e = \sqrt{\frac{b^2 + a^2}{b^2}} = \frac{\sqrt{13}}{2}$$

71. If $\int \frac{1+\cos 8x}{\tan 2x - \cot 2x} dx = f(x)\cos(g(x)) + c$ then $f\left(\frac{1}{4}\right) + g\left(\frac{1}{4}\right) =$

(1) 2

(2) $\frac{17}{8}$

(3) $\frac{15}{8}$

(4) $\frac{33}{16}$

Key: 4

Solution: $G.I = \int \frac{2\cos^2 4x}{\left(\frac{\sin^2 2x - \cos^2 2x}{\sin 2x \cos 2x}\right)} dx$

$$= \int \frac{\sin 4x \cos^2 4x}{-\cos 4x} dx$$

$$= \frac{-1}{2}\int \sin 8x \, dx = \frac{1}{16}\cos(8x) + c$$

$$\therefore f(x) = \frac{1}{16}, \; g(x) = 8x$$

$$f\left(\frac{1}{4}\right) + g\left(\frac{1}{4}\right) = \frac{1}{16} + 2 = \frac{33}{16}$$

72. If $\int \frac{\sqrt{1-x^4}}{x^7} dx = f(x)\left\{\sqrt{1-x^4}\right\}^n + c$ then $(f(x))^n$ is equal to

(1) $\frac{-1}{6x^6}$

(2) $\frac{-1}{216x^{18}}$

(3) $\frac{1}{36x^{12}}$

(4) $\frac{1}{216x^{18}}$

Key: 2

Solution: $I=\int \frac{\sqrt{1-x^4}}{x^7}dx$

$$I=\int x^{-5}\sqrt{x^{-4}-1}\ dx$$

$$=\frac{-1}{4}\int \sqrt{t}\ dx \qquad \text{Put } x^{-4}-1=t$$

$$=\frac{-1}{6}t^{3/2}+c \qquad -4x^{-5}\,dx = dt$$

$$=\frac{-1}{6}\left(x^{-4}-1\right)^{3/2}+c$$

$$=\frac{-1}{6x^6}\left(1-x^4\right)^{3/2}+c$$

$$f(x)=\frac{-1}{6x^6},\ n=3$$

$$\left(f(x)\right)^n=\left(\frac{-1}{6x^6}\right)^3=\frac{-1}{216\ x^{18}}$$

73. $\int e^x\left(\frac{x^2+1}{(x+1)^2}\right)dx =$

(1) $e^x\left(\frac{x-1}{x+1}\right)+c$

(2) $e^{-x}\left(\frac{x+1}{x-1}\right)+c$

(3) $e^{x}\left(\frac{x+1}{x^2+1}\right)+c$

(4) $e^{x}\left(\frac{1}{x+1}\right)+c$

Key: 1

Solution: $I=\int e^{x}\left(\frac{x^2-1+2}{(x+1)^2}\right)dx$

$=\int e^{x}\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)dx$

$=e^{x}\left(\frac{x-1}{x+1}\right)+c$

74. $\int \frac{\mathbf{1}}{\mathbf{1+\cos^2 x+2\sin x\cos x}}\mathbf{dx=}$

(1) $\text{Tan}^{-1}(\tan x+1)+c$

(2) $\frac{1}{2}\text{Tan}^{-1}(\tan x+1)+c$

(3) $\frac{1}{2}\text{Tan}^{-1}\left(\frac{1}{2}(\tan x+1)\right)+c$

(4) $\text{Tan}^{-1}\left(\frac{1}{2}(\tan x+1)\right)+c$

Key: 1

Solution: $I=\int\frac{1}{1+\cos^2 x+2\sin x\cos x}dx$

Nr and Dr divided by $\cos^2 x$

$$=\int \frac{\sec^2 x}{\sec^2 x+1+2\tan x}\,dx$$

$$=\int \frac{\sec^2 x}{1+(1+\tan x)^2}\,dx$$

$= \tan^{-1}(1 + \tan x) + c$

75. $\displaystyle\int_0^{\pi/2} \frac{\cos^3 x}{\sin x+\cos x}\,dx =$

(1) $\frac{\pi-1}{2}$

(2) $\frac{\pi+1}{4}$

(3) $\frac{\pi-1}{4}$

(4) $\frac{\pi-3}{4}$

Key:3

Solution: $I=\displaystyle\int_0^{\pi/2} \frac{\cos^3 x}{\sin x+\cos x}\,dx$ ______ (1)

use $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$$I=\int_0^{\pi/2} \frac{\sin^3 x}{\cos x+\sin x}\,dx \text{ ______ } (2)$$

Add (1) and (2)

$$2I=\int_0^{\pi/2} \frac{\cos^3 x+\sin^3 x}{\sin x+\cos x}\,dx$$

$$2I=\int_0^{\pi/2}(1-\sin x\ \cos x)\ dx$$

$$2I=\left(x+\frac{1}{4}\cos 2x\right)_0^{\frac{\pi}{2}}$$

$$2I=\left(\frac{\pi}{2}-\frac{1}{4}\right)-\left(0+\frac{1}{4}\right)$$

$$2I=\frac{\pi}{2}-\frac{1}{2}$$

$$I=\frac{\pi-1}{4}$$

76. $\mathop{\mathbf{Lt}}_{n\to\infty}\left(\frac{\mathbf{1+2\sqrt{2}+3\sqrt{3}+........+n\sqrt{n}}}{\mathbf{n^{5/2}}}\right)=$

(1) $\frac{5}{2}$

(2) $\frac{2}{5}$

(3) 0

(4) 1

Key:2

Solution: $\mathop{\text{Lt}}_{n\to\infty}\frac{1\sqrt{1}+2\sqrt{2}+3\sqrt{3}+..........+n\sqrt{n}}{n^2\sqrt{n}}$

$$=\mathop{\text{Lt}}_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r\sqrt{r}}{n\sqrt{n}}$$

$$=\int_0^1 x\sqrt{x}\ dx=\int_0^1 x^{3/2}\ dx=\frac{2}{5}$$

77. The area bounded by the curves $y = x^2$ and $\mathbf{y = \frac{2}{1 + x^2}}$ is

(1) $\pi - \frac{1}{3}$

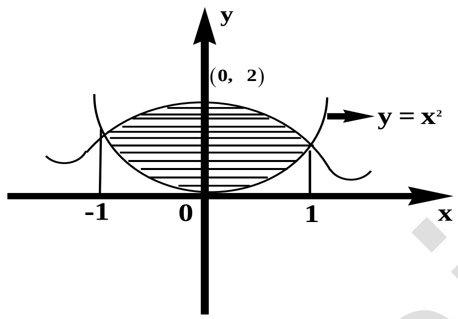
(2) $\pi - \frac{2}{3}$

(3) $\frac{2\pi - 1}{3}$

(4) $\frac{2\pi}{3}$

Key:2

Solution:



For point of intersection $x^2 = \frac{2}{1+x^2}$

$\Rightarrow \quad x^4 + x^2 - 2 = 0$

$\Rightarrow \quad (x^2 - 1)(x^2 + 2) = 0$

$x = \pm 1$

Area $= 2\int_0^1 \left(\frac{2}{1+x^2} - x^2\right) dx$

$$= 2\left[2\tan^{-1}x - \frac{x^3}{3}\right]_0^1$$

$$= \pi - \frac{2}{3}$$

78. The differential equation corresponding to the family of curves given by $ax^2 + by^2 = 1$, where a and b are arbitrary constants is

(1) $x\frac{d^2y}{dx^2} = \frac{dy}{dx}$

(2) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(3) $xy\frac{d^2y}{dx^2} + y\frac{dy}{dx} - x\frac{dy}{dx} = 0$

(4) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

Key:2

Solution: $ax^2 + by^2 = 1$

$2ax + 2byy_1 = 0$

$$\Rightarrow \frac{yy_1}{x} = \frac{-a}{b}$$

$$\Rightarrow \frac{x\left[yy_2 + y_1^2\right] - yy_1}{x^2} = 0$$

$$\Rightarrow xyy_2 + xy_1^2 - yy_1 = 0$$

79. The general solution of $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{x + \sin x \cos y + x\cos y + \sin x}$ is

(1) $\operatorname{Tan}\left(\frac{x}{2}\right)=\frac{y^2}{2}-\cos y+c$

(2) $\operatorname{Tan}\left(\frac{y}{2}\right)=\frac{x^2}{2}-\cos x+c$

(3) $\operatorname{Sec}^2\left(\frac{y}{2}\right)=\frac{x^2}{2}-\cos x+c$

(4) $\operatorname{Tan}\left(\frac{y}{2}\right)=\frac{x^2}{2}+\cos x+c$

Key:2

Solution: $\frac{dy}{dx}=(x+\sin x)(1+\cos y)$

$\int\frac{dy}{1+\cos y}=\int(x+\sin x)\,dx$

$\int\frac{1}{2}\sec^2(y/2)\,dy=\int(x+\sin x)\,dx$

$\Rightarrow \tan\left(\frac{y}{2}\right)=\frac{x^2}{2}-\cos x+c$

80. If y = f(x) is the solution of the differential equation $\mathbf{x\frac{dy}{dx}=x^2+3y,\ x>0,\ y(2)=4}$ then f(4) =

(1) 48

(2) 260

(3) 80

(4) 36

Key:1

Solution: $x\frac{dy}{dx}-3y=x^2$

$\Rightarrow \frac{dy}{dx}-\frac{3y}{x}=x$

$$\text{I.F} = e^{\int \frac{-3}{x}dx} = e^{-3\log x} = \frac{1}{x^3}$$

Solution is $y\left(\frac{1}{x^3}\right) = \int \frac{1}{x^2}\,dx$

$$\frac{y}{x^3} = \frac{-1}{x} + c \Rightarrow y = -x^2 + cx^3$$

If y(2) = 4

$\Rightarrow c = 1$

$\therefore \ y = x^3 - x^2$

PHYSICS

81. The dimensions of planck's constant are the same as that of

(1) Linear impulse

(2) Work

(3) Linear momentum

(4) Angular momentum

Key: 4

Solution: $E = h\vartheta$

$$h = \frac{E}{\vartheta} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$$

$$L = IW = ML^2T^{-1} = ML^2T^{-1}$$

82. The position of a particle as a function of time t is given by $x(t) = at + bt^2 - ct^3$ where a,b and c are constants when the particle attains zero acceleration, then its velocity will be

(1) $a+\frac{b^2}{4c}$

(2) $a+\frac{b^2}{3c}$

(3) $a+\frac{b^2}{c}$

(4) $a+\frac{b^2}{2c}$

Key: 2

Solution: $x(t)=at+bt^2-ct^3$

$$v=\frac{dx}{dt}=a+2bt-3ct^2$$

$$a=\frac{dv}{dt}=2b-6ct$$

$$a=0\Rightarrow t=\frac{2b}{6c}$$

$$V=a+2b\left(\frac{2b}{6c}\right)-3c\left(\frac{2b}{6c}\right)^2$$

$$=a+\frac{b^2}{3c}$$

83. If R and H represent horizontal range and maximum height of the projectile, then the angle of projection with the horizontal is

(1) $\tan^{-1}\left(\frac{H}{R}\right)$

(2) $\tan^{-1}\left(\frac{2H}{R}\right)$

(3) $\tan^{-1}\left(\frac{4H}{R}\right)$

(4) $\tan^{-1}\left(\frac{4R}{H}\right)$

Key: 3

Solution: $H = \frac{u^2 \sin^2\theta}{2g}$ $\quad R = \frac{u^2 \sin 2\theta}{g}$

$$(1) \div (2) \Rightarrow \frac{H}{R} = \frac{\tan\theta}{4}$$

$$\tan\theta = \frac{4H}{R}, \theta = \tan^{-1}\left(\frac{4H}{R}\right)$$

84. A constant retarding force of 50N is applied to a body of mass 10kg moving initially with a speed of $10ms^{-1}$. The body comes to rest after

(1) 2s

(2) 4s

(3) 6s

(4) 8s

Key: 1

Solution: $a = \frac{-50}{10} = -5ms^{-2}\left[F = ma\right]$

$$V = u + at, t = \frac{v-u}{a}$$

$$= \frac{0-10}{-5} = 2s$$

85. The coefficient of static friction between box and trains floor is 0.2. The maximum acceleration of the train in which a box lying on its floor will remain stationary is (g=10ms^{-2})

(1) $2ms^{-2}$

(2) $4ms^{-2}$

(3) $6ms^{-2}$

(4) $8ms^{-2}$

Key: 1

Solution: $a_{max} = \mu_s g = 0.2 \times 10 = 2ms^{-2}$

86. A spherical ball of mass m_1 collides head on with another ball of mass m_2 at rest. The collision is elastic. The fraction of kinetic energy lost by m_1 is

(1) $\frac{4m_1m_2}{(m_1+m_2)^2}$

(2) $\frac{m_1}{m_1+m_2}$

(3) $\frac{m_2}{m_1+m_2}$

(4) $\frac{m_1m_2}{(m_1+m_2)^2}$

Key: 1

Solution: $V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

Fraction of K.E lost by m_1

$$= \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2}{\frac{1}{2}m_1u_1^2} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

87. Two springs of spring constants $1000Nm^{-1}$ and $2000Nm^{-1}$ are stretched with same force. They will have potential energy in the ratio of

(1) 2: 1

(2) $2^2 : 1^2$

(3) $1 : 2$

(4) $1^2 : 2^2$

Key: 1

Solution: $U = \frac{1}{2}kx^2 = \frac{F^2}{2K}$

$$\frac{U_1}{U_2} = \frac{K_2}{K_1} = \frac{2000}{1000} = \frac{2}{1}$$

88. The moment of inertia of a solid sphere of mass M and radius R about a tangent to the sphere is

(1) $\frac{2}{5}MR^2$

(2) $\frac{6}{5}MR^2$

(3) $\frac{4}{5}MR^2$

(4) $\frac{7}{5}MR^2$

Key: 4

Solution: $I_{diametes}=\frac{2}{5}MR^2$

$I_{\tan gent}=I_{diametes}+MR^2$

$=\frac{7}{5}MR^2$

89. An automobile engine develops 100 KW power when rotating at a speed of 1800 rpm. The torque delivered by the engine is.

(1) $\frac{10^2}{6\pi}Nm$

(2) $\frac{10^4}{6\pi}Nm$

(3) $\frac{10^6}{6\pi}Nm$

(4) $\frac{10^8}{6\pi}Nm$

Key: 2

Solution: $P=100KW=10^5W$

$w=1800rpm=1800\times\frac{2\pi}{60}=60\pi rads^{-1}$

$p=\tau\omega,\tau=\frac{p}{w}=\frac{105}{60\pi}=\frac{10^4}{6\pi}Nm$

90. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. It radius of the ball is R, then the fraction of total energy associated with its rotation will be

(1) $\frac{k^2+R^2}{R^2}$

(2) $\dfrac{K^2}{R^2}$

(3) $\dfrac{K^2}{K^2+R^2}$

(4) $\dfrac{R^2}{K^2+R^2}$

Key: 3

Solution: Total K.E= $(K.E)_{translation} + (K.E)_{rotation}$

$$=\frac{1}{2}MV^2+\frac{1}{2}IW^2$$

$$=\frac{1}{2}MV^2+\frac{1}{2}MK^2\frac{V^2}{R^2}=\frac{1}{2}MV^2\left(1+\frac{K^2}{R^2}\right)$$

$$\frac{(K.E)_{rot}}{(K.E)_{total}}=\frac{\frac{1}{2}MV^2\frac{K^2}{R^2}}{\frac{1}{2}MV^2\left(1+\frac{K^2}{R^2}\right)}=\frac{K^2}{K^2+R^2}$$

91. The escape speed of a body on the earth's surface is $11.2kms^{-1}$ A body is projected with thrice of this speed. The speed of the body when it escapes the gravitational pull of earth is

(1) $11.2kms^{-1}$

(2) $22.4\sqrt{2}\,kms^{-1}$

(3) $\dfrac{22.4}{\sqrt{2}}kms^{-1}$

(4) $22.4\sqrt{3}\,kms^{-1}$

Key: 2

Solution: $\frac{1}{2}mu^2-\frac{GMm}{R}=\frac{1}{2}mv^2-0$

$v^2-u^2=\frac{-2GM}{R}, v^2=u^2-v_e^2\left[V_e=\sqrt{\frac{2GM}{R}}\right]V=\sqrt{u^2-V_e^2}=\sqrt{(3Ve)^2-Ve^2}$

$=\sqrt{8}V_e=11.2\sqrt{8}=22.4\sqrt{2}Kms^{-1}$

92. How much pressure should be applied on a litre of water if it is to be compressed by 0.1% (bulk modulus of water = 2100 MPa)

(1) 2100 KPa

(2) 210 KPa

(3) 2100 MPa

(4) 210 MPa

Key: 1

Solution: $\frac{\Delta V}{V}=10^{-3}$ $B=\frac{p}{\Delta V/V}$

$P=B\frac{\Delta V}{V}=2100\times10^6\times10^{-3}$

$=2100KPa$

93. Surface tension of mercury is $0.465\ Nm^{-1}$. The excess pressure inside a mercury drop of diameter 6mm is

(1) 310 Pa

(2) 410 Pa

(3) 510 Pa

(4) 610 Pa

Key: 1

Solution: $D=6mm,$ $r=3\times10^{-3}m$

$$P=\frac{2s}{r}=\frac{2\times 0.465}{3\times 10^{-3}}=310Pa$$

94. Hydraulic brakes are based on

(1) Pascals law

(2) Toricellis law

(3) Newton's law

(4) Boyles law

Key: 1

Solution: pascal law

95. A rectangular body at 2000K has maximum intensity wavelength at λ_m. Its corresponding wavelength at 3000K is.

(1) $\frac{3}{2}\lambda_m$

(2) $\frac{2}{3}\lambda_m$

(3) $\frac{16}{81}\lambda_m$

(4) $\frac{81}{16}\lambda_m$

Key: 2

Solution: $\lambda_m \propto \frac{1}{T}$

When temperature becomes $\frac{3}{2}$ times λ_m becomes $\frac{2}{3}$ times

96. To increase the length of brass rod by 2% its temperature should increase by $\left(\alpha=0.00002\,^oC^{-1}\right)$

(1) 800^oC

(2) $900^{o}C$

(3) $1000^{o}C$

(4) $1100^{o}C$

Key: 3

Solution: $\Delta L = L \propto \Delta T,$ $\qquad \Delta T = \frac{\Delta l}{\propto L}$

$$\frac{\Delta L}{L} = \frac{2}{100}, \quad \Delta T = \frac{2}{100 \times 0.00002} = 1000^{o}C$$

97. A refrigerator with coefficient of performance $\frac{1}{3}$ releases 200 J of heat to a hot reservoir. Then the work done on the working substance is

(1) $\frac{100}{3}J$

(2) $100J$

(3) $\frac{200}{3}J$

(4) $150J$

Key: 4

Solution: $\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$

$$\frac{1}{3} = \frac{Q_2}{200 - Q_2} \quad 200 - Q_2 = 3Q_2$$

$$Q_2 = 50J$$

$$W = Q_1 - Q_2 = 200J - 50J = 150J$$

98. An ideal gas is having molar specific heat capacity at constant volume as $\frac{3}{2}R$. The molar specific heat capacity at constant pressure is.

(1) $\frac{1}{2}R$

(2) $\frac{5}{2}R$

(3) $\frac{7}{2}R$

(4) $\frac{9}{2}R$

Key: 2

Solution: $C_v = \frac{3}{2}R$

$$C_p - C_v = R$$

$$C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$$

99. At what temperature is the rms velocity of hydrogen molecule is equal to that of an oxygen molecule at 47°C

(1) 10 K

(2) 20 K

(3) 30 K

(4) 40 K

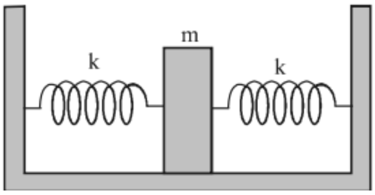
Key: 2

Solution: $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\sqrt{\frac{3RT}{2}}=\sqrt{\frac{3R(47+273)}{32}}$$

$$T=20K$$

100. Two ideal springs of spring constants K each are attached to a block of mass m and to fixed supports as shown. The time period of oscillation is



(1) $2\pi\sqrt{\frac{m}{K}}$

(2) $2\pi\sqrt{\frac{m}{2K}}$

(3) $2\pi\sqrt{\frac{2m}{K}}$

(4) $\pi\sqrt{\frac{m}{2K}}$

Key: 2

Solution: When mass m is displaced to right (small distance x)

$$F=F_1+F_2=-2kx$$

$$a=\frac{-2kx}{m}$$

Comparing with a = $-w^2x$

$$w=\sqrt{\frac{2k}{m}},T=\frac{2\pi}{w}\ T=2\pi\sqrt{\frac{m}{2K}}$$

101. An infinite line of charge with uniform line charge density of 1c/m is placed along the y-axis. A charge of 1c is placed on the X-axis at a distance of d =3m from the origin. At what distance r from the origin on the x-axis, the total electric field is zero (Assume 0 < r < d)

(1) 1m

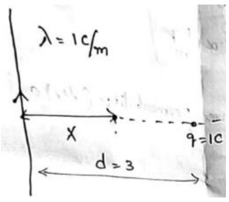
(2) 2 m

(3) 2.5 m

(4) 1.75m

Key: 2

Solution: For electric field zero at X



$$E_{linecharge} = E_{pointcharge}$$

$$\frac{2k\lambda}{x} = \frac{kq}{(3-x)^2}$$

$$\Rightarrow 2x^2 - 13x + 18 = 0$$

$$x_1 = 4.5(or)\boxed{x = 2m}$$

102. A hollow metal sphere of radius 15cm is charged such that potential on its surface is 20V, then the potential at the centre of sphere is

(1) 0 V

(2) 20 V

(3) 10 V

(4) 15 V

Key: 2

Solution: Potential at the centre = potential on the surface $\therefore \boxed{20V}$

103. The quantities that don't change when a resistor connected to a battery is heated due to the current are

a) drift speed b) resistivity

c) Resistance d) number of free electrons

(1) B and C

(2) D

(3) A

(4) A and D

Key: 2

Solution: Resistance, resistivity and drift velocity varies with relaxation time which is dependent on temperature

Number of free electrons in a conductor remain in variant even if temperature is changes.

104. In a potentiometer experiment the balancing length with a cell is 560 cm. When an external resistance of 10Ω is connected in parallel to the cell then the balancing length changes by 60cm. Find the internal resistance of a cell

(1) 1Ω

(2) 2Ω

(3) 1.2Ω

(4) 2.1Ω

Key: 3

Solution: $r = \left[\dfrac{l_1 - l_2}{l_2}\right]R$

$$= \left[\frac{560 - 500}{500}\right]10$$

$= \frac{6}{5}$

$\boxed{r = 1.2\Omega}$

105. A particle of mass "m" and charge "q" is moving in a cyclotson with magnetic field B. The frequency of the circular motion of the particle is proportional to

(1) $\frac{qB}{m}$

(2) $\frac{2m}{qB}$

(3) $\frac{mB}{q}$

(4) $\frac{mg}{B}$

Key: 1

Solution: In a cycloten, frequency of rotation of a charged practicle

$f = \frac{Bq}{2\pi m} \Rightarrow \boxed{f \alpha \frac{Bq}{m}}$

106. If relative permeability of iron is 5500, then its succeptibility is

(1) 5500×10^{7}

(2) 5500×10^{-7}

(3) 5501

(4) 5499

Key: 4

Solution: Relation between relative permeability (μ_r) and susceptibility (x_B) is

$\mu_r = 1 + x_B$

$\Rightarrow x_B = \mu_r - 1$

= 5500 – 1

$\boxed{x_B = 5499}$

107. An electric generator is based on

(1) Faraday's law of electromagnetic induction

(2) Motion of charged particles in an electromagnetic field

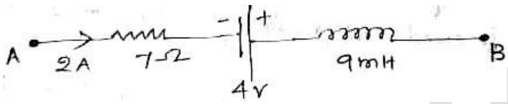
(3) Fission of uranium by slow neutrons

(4) Newton's laws of motion

Key: 1

Solution: conceptual

108. A branch of circuit is shown in the figure, if current is decreasing at the rate of $10^3 AS^{-1}$ then the potential difference between A and B is



(1) 1 V

(2) 5 V

(3) 10 V

(4) 2 V

Key: 1

Solution: $V_{AB} - V_R + V + V_L = 0$

$$V_{AB} - 14 + 4 + 9 = 0$$

$$\boxed{V_{AB} = 1V}$$

109. Microwaves are used in

(1) TV

(2) radio transmission

(3) Radar

(4) atmospheric research

Key: 3

Solution: Conceptual

110. If the image of an object is at the focal point "f" to the right side of a convex lens, the position of the object on the left of the lens is at

(1) f

(2) $2f$

(3) $< f$

(4) $\propto$

Key: 4

Solution: Conceptual

111. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe width recorded are $\beta_G, \beta_R, \beta_B$ respectively then

(1) $\beta_G > \beta_B > \beta_R$

(2) $\beta_B > \beta_G > \beta_R$

(3) $\beta_R > \beta_B > \beta_G$

(4) $\beta_R > \beta_G > \beta_B$

Key: 4

Solution: Conceptual

112. Both an electron and photon have same de-Broglie wavelength of 1.2A^o. The ratio of their energies is nearly.

(1) 1: 100

(2) 1: 10

(3) 1: 1000

(4) 1: 1

Key: 1

Solution: The debroglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{2mK_e}}$$

$$\boxed{K_e = \frac{h^2}{\lambda_e^2 .2m_e}}$$

$$\frac{K_e}{K_p} = \frac{h}{c\lambda_e \times 2me}$$

K.E of photon

$$\boxed{K_p = \frac{hc}{\lambda_p}}$$

$$= \frac{6.66\times 10^{-34}}{3\times 10^{8}\times 1.2\times 10^{-10}\times 2\times 9.1\times 10^{-31}}$$

$$\boxed{\frac{K_e}{K_p} \approx \frac{1}{100}}$$

113. As the quantum number increases, the difference in energy between consecutive energy levels

(1) Remains the same

(2) Increases

(3) Decreases

(4) sometimes increases and sometimes decreases

Key: 3

Solution: Conceptual

114. The half-life of ${}_{84}^{209}P_o$ is 103 years. The time taken for 100g sample of ${}_{84}^{209}P_o$ to decay to 3.125g is

(1) 3296 years

(2) $103\sqrt{2}$ years

(3) 1648 years

(4) 515 years

Key: 4

Solution: Half life of sample = 103 years

Initial amount $N_0 = 100g$

Final N = 3.125g

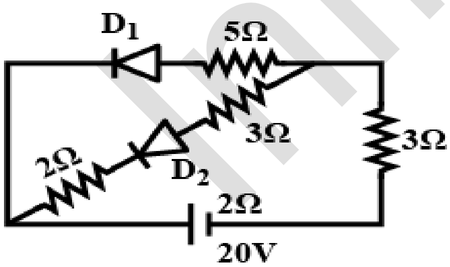
$$N = \frac{N_0}{2^n} \Rightarrow \frac{100}{3.125} = 2^n \Rightarrow 2^n = 32 \Rightarrow \boxed{n=5}$$

$$t = nT$$

$$t = 5(103)$$

$$\boxed{t = 515 years}$$

115. If the diodes are ideal in the circuit given below, then the current through cell is



(1) 4A

(2) 1.5 A

(3) 2A

(4) 3 A

Key: 3

Solution: $D_1 \rightarrow$ Reverse biased

Because of diodes are ideal, so voltage drop across D_2 is zero

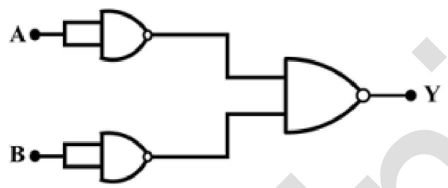
$$R_{eff} = 3+2+3+2 = 10\Omega$$

$$I = \frac{V}{R}$$

$$= \frac{20}{10} = 2A$$

$$\boxed{I = 2A}$$

116. Identify the logic operation performed by the following circuit



(1) OR

(2) AND

(3) NOT

(4) NAND

Key: 1

Solution: Conceptual

117. The maximum amplitude of an amplitude modulated wave is 16V, while the minimum amplitude is 4V the modulation index is

(1) 0.4

(2) 0.5

(3) 0.6

(4) 4

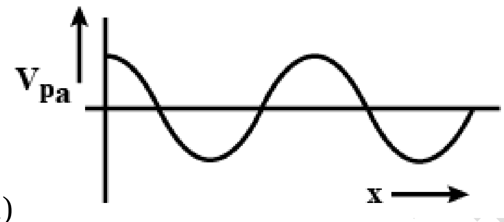
Key: 3

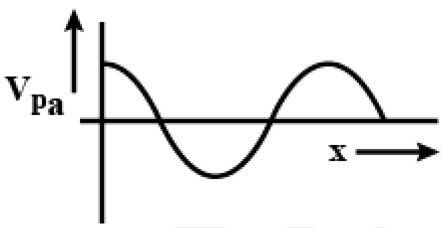
Solution: $m = \frac{A_m}{A_c} = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$

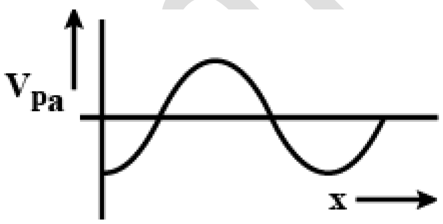
$$m = \frac{16-4}{16+4}$$

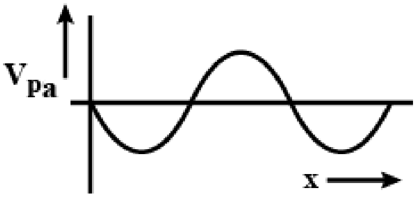
$$\boxed{m = 0.6}$$

118. A plane wave $y = a\sin(wt - kx)$ propagates through a stretched string. The particle velocity versus X graph at t = 0

(1) 

(2) 

(3) 

(4) 

Key: 2

Solution: Conceptual

119. A 50cm long solenoid has winding of 400 turns what current must pass through it to produce a magneti field of induction $4\pi \times 10^{-3} T$ at the centre?

(1) 10.5A

(2) 12.5A

(3) 25.0 A

(4) 20.0 A

Key: 2

Solution: Length of solenoid L = 50cm

No.of turns N = 400

$$B = 4\pi \times 10^{-3} T$$

$$n = \frac{N}{L} = \frac{400 \times 100}{50} = 800\, turns / m$$

$$B = \mu_o ni$$

$$i = \frac{B}{\mu_o n}$$

$$= \frac{4\pi \times 10^{-3}}{4\pi \times 10^{-7} \times 800}$$

$$\boxed{i = 12.5A}$$

120. A cylindrical metallic wire is stretched to increase its length if the resistance of the wire is increased by 4% then the percentage increase in its length is

(1) 4%

(2) 8%

(3) 1%

(4) 2%

Key: 4

Solution: Area x length = constant

$$\Rightarrow \frac{dA}{A}+\frac{dl}{l}=0$$

$$\frac{dA}{A}=-\frac{dl}{l}$$

$$R=\frac{\rho L}{A}$$

$$\Rightarrow \frac{dR}{R}\times 100=\left[\frac{d\rho}{\rho}+\frac{dl}{L}-\frac{dA}{A}\right]\times 100$$

$$=\left[0+\frac{dl}{l}+\frac{dl}{l}\right]\times 100$$

$$=\frac{2dl}{l}$$

$$4=2\frac{dl}{l}\Rightarrow \boxed{\frac{dl}{l}=2\%}$$

CHEMISTRY

121. The number of orbitals associated with Quantum numbers n=5, ms = +1/2 is

(1) 25

(2) 11

(3)15

(4) 50

Key: 1

Solution: No orbitals = n^2

122. The Shortest wavelength of H – atom in the Lyman series is λ_1. The longest wavelength in the Balmer series of He^+ is.

(1) $\frac{9\lambda_1}{5}$

(2) $\frac{5\lambda_1}{9}$

(3) $\frac{27\lambda_1}{5}$

(4) $\frac{36\lambda_1}{5}$

Key: 1

Solution: $\frac{1}{\lambda_{(B)}} = R_4 z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda_B} = R_H 4\left[\frac{1}{4} - \frac{1}{9}\right]$$

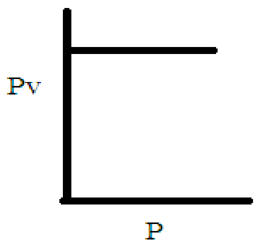
$$\frac{1}{\lambda_B} = R \times \cancel{4} \times \frac{5}{\cancel{36}\ 9} = \frac{5R}{9}$$

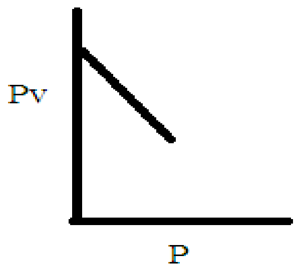
$$\lambda B = \frac{9}{5R}$$

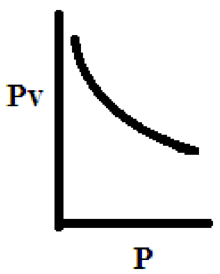
$$\frac{\lambda_B}{\lambda L} = \frac{9}{5R}\ \lambda_L = \lambda_1$$

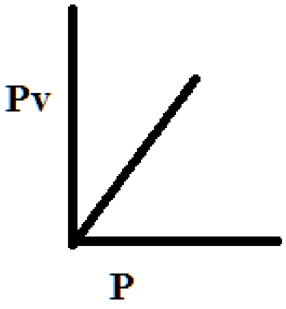
$$\lambda_B = \frac{9}{5}\lambda_1$$

123. Which one of the following is the correct pv vs p plot at constant temperature for an ideal gas.

(1) 

(2) 

(3) 

(4) 

Key: 1

Solution: Conceptual

124. The Number of significant figures in 0.00340 is ______

(1) 2

(2) 3

(3) 5

(4) 6

Key: 2

Solution: Conceptual

125. For Which of the following reactions ΔH is Equals to ΔU ?

(1) $2\underset{(g)}{NO_2} \rightarrow \underset{(g)}{N_2O_4}$

(2) $2\underset{(g)}{HI} \rightarrow \underset{(g)}{H_2} + \underset{(g)}{I_2}$

(3) $2\underset{(g)}{SO_2} + \underset{(g)}{O_2} \rightarrow 2\underset{(g)}{SO_3}$

(4) $\underset{(g)}{N_2} + 3\underset{(g)}{H_2} \rightarrow 2\underset{(g)}{NH_3}$

Key: 2

Solution: $\Delta H = \Delta U + \Delta nRT$

126. For the reaction

$Fe_2N_{(s)}+\frac{3}{2}\underset{(g)}{H_2}\rightleftharpoons 2\underset{(s)}{Fe}+\underset{(g)}{NH_3}$

1) $k_C=k_P(RT)$

2) $k_C=k_P(RT)^{-1/2}$

3) $k_C=k_P(RT)^{1/2}$

4) $k_C=k_P(RT)^{3/2}$

Key: 2

Solution: $k_p=k_c(RT)^{\Delta n}$

127. In which of the following reactions, increase in the volume at constant temperature does not affect the number of moles at equilibrium?

1) $2NH_{3(g)}\rightarrow\underset{(g)}{N_2}+3\underset{(g)}{H_2}$

$2\underset{(g)}{C}+\frac{1}{2}\underset{(g)}{o_2}\rightarrow\underset{(g)}{CO}$

3) $\underset{(g)}{H_2}+\underset{(g)}{o_2}\rightarrow\underset{(g)}{H_2O_2}$

4) None of these

Key: 4

Solution: W: f $\Delta n=0$ increase in volume at constant temperature does not effect the number of moles at equilibrium.

128. The solubility of $Ca(OH)_2$ in water is [Given: the solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}]

1) 1.77×10^{-6}

2) 1.11×10^{-6}

3) 1.11×10^{-2}

4) 1.77×10^{-2}

Key: 3

Solution: $Ca(OH)_2 = AB_2\,Thre$

$K_{sp} = 4s^3$

$$\left(\frac{5.5\times10^{-6}}{4}\right)^{1/3} = 5 \Rightarrow 1.11\times10^{-2}$$

129. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are respectively

1) 1:2:4

2) 4:2:1

3) 8:1:6

4) 4:2:3

Key: 1

Solution: Simple curve $\rightarrow 8\times\frac{1}{8}=1$

$BCC \rightarrow 8\times\frac{1}{8}+1=2$

$FCC \rightarrow 8\times\frac{1}{8}+6\times\frac{1}{2}=4$

130. Freezing point of 4% aqueous solution of X is equals to freezing point of 12% aqueous solution of 'y'. If molecular weight of 'X' is A, then molecular weight of Y is ___

1) 2A

2) 3A

3) A

4) 4A

Key: 2

Solution: $\frac{\%x}{Gmw} = \frac{\%y}{Gmw}$

$$\frac{4}{x}=\frac{12}{y}\left(X=A\right)$$

$$y=\frac{12A}{4}=3A$$

131. A solution of $Ni\left(NO_3\right)_2$ is electrolysed between platinum electrodes using 0.1 faraday electricity. How many moles of Ni will be deposited at the cathode?

1)0.10

2) 0.15

3)0.20

4) 0.05

Key: 4

Solution: At cathode

$$Ni^{+2}+2e^{-}\rightarrow Ni$$

2F required $\rightarrow$ 1 mole of Ni

0.1 F passed $\rightarrow$?

$$=\frac{0.1}{2}=0.05\,moles$$

132. For the reaction $2\underset{(g)}{H_2}+2\underset{(g)}{NO}\rightarrow\underset{(g)}{N_2}+2\underset{(g)}{H_2O}$ the observed rate expression is, rate = $k_f\left[NO\right]^2\left[H_2\right]$. the rate expression for the reverse reaction is

1) $kb\left[N_2\right]\left[H_2O\right]^2/\left[H_2\right]$

2) $kb\left[N_2\right]\left[H_2O\right]$

3) $kb\left[N_2\right]\left[H_2O\right]^2$

4) $kb\left[N_2\right]\left[H_2O\right]^2/\left[NO\right]$

Key: 1

Solution: $r_b = k_b \frac{[N_2][H_2O]^2}{[H_2]}$

because in forward reaction only one mole of H_2 isconsumed

133. In Freundlich adsorption isotherm at moderate pressure the extent of adsorption (x/m) is directly proportional to p^x. The value of x is

1) zero

2) 1

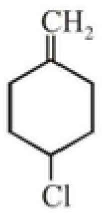
3) 1/n

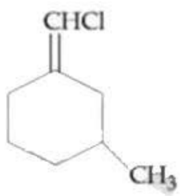
4) ∞

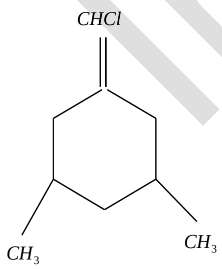
Key: 3

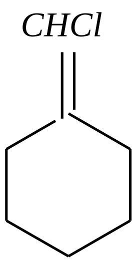
Solution: Conceptual

134. Among the following compounds geometrical isomerism is exhibited by

1) 

2) 

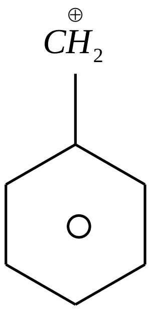
3) 

4) 

Key: 2

Solution: Conceptual

135. The order of stability of the following carbocations is $\underset{I}{CH_2=CH-\overset{\oplus}{CH_2}}$, $\underset{II}{CH_3-CH-\overset{\oplus}{CH_2}}$,



III

1) $III > I > II$

2) $III > II > I$

3) $II > III > I$

4) $I > II > III$

Key: 1

Solution: Benzyl Carboration > Allyl $C\oplus >$

$3^0 C\oplus 2^0 C\oplus > 1^0 C\oplus$ stability order of carboration.

136. Excess of Isobutane on reaction with Br_2 in the presence of light at 125^0C gives which one of the following as the major product?

1) $CH_3-\underset{\substack{| \\ CH_2-Br}}{CH}-CH_2-Br$

2) $CH_3-\underset{\substack{| \\ CH_3}}{CH}-CH_2-Br$

3) $CH_3-\overset{\substack{Br \\ |}}{\underset{\substack{| \\ CH_3}}{C}}-CH_2-Br$

4) $CH_3-\overset{\substack{CH_3 \\ |}}{\underset{\substack{| \\ CH_3}}{C}}-Br$

Key: 4

Solution: Follows maronikoff's rule

137. The stereo Isomers that are formed by electrophillic addition of bromine to trans – but – 2ene is / are

1) 2 identical mesomers

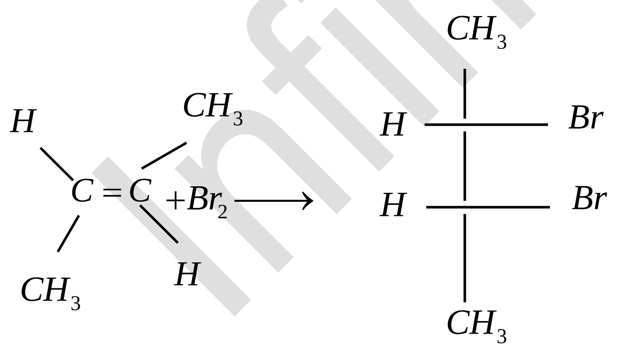
2) 2 Ennontiomers

3) 2 Ennontimers and 2 mesomers

4) 1 recemic and 2 Enatiomers

Key: 1

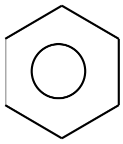
Solution:

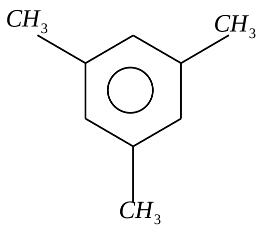


Trans   Trans addition   Meso Compound

138. For the given reaction $\underset{\substack{| \\ CH_3}}{CH}=CH-Br \xrightarrow[ii) \text{Red hot Iron tube } 873k]{i) NaNH_2} \underset{\text{major product}}{(A)}$. What is 'A'.?

1) $\underset{\substack{|\\ CH_3}}{CH} = CH - NH_2$

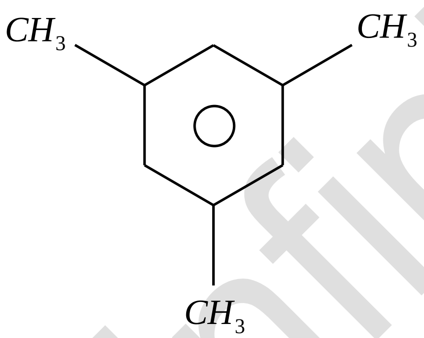
2) 

3) 

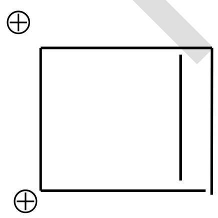
4) $CH_3 - CH_2CH_2NH_2$

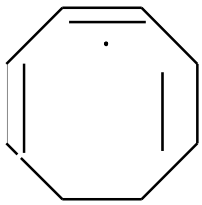
Key: 3

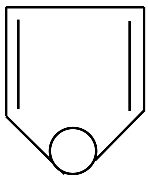
Solution: $CH_3 - CH = CH - Br \xrightarrow{NaNH_2} \begin{array}{c} CH_3 - C \equiv CH \\ \downarrow \\ \text{red hot Fe Tube} \end{array}$

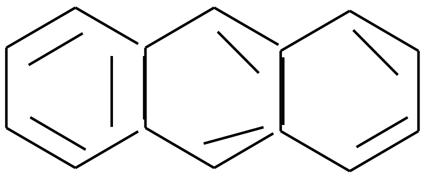


139. Which one of the following compounds is non – Aromatic?

1) 

2) 

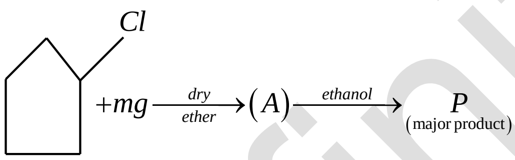
3) 

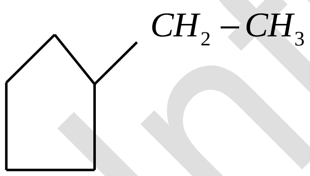
4) 

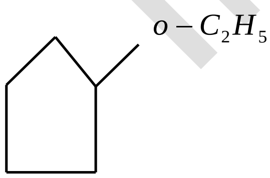
Key: 2

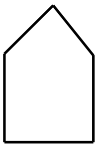
Solution: Conceptual

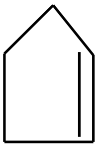
140. In the following sequence of reactions the‘P’ is



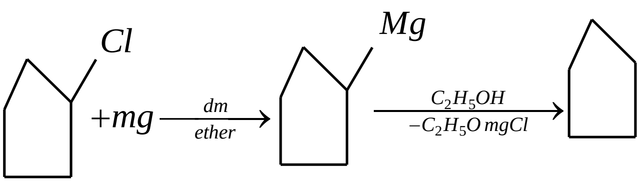
1) 

2) 

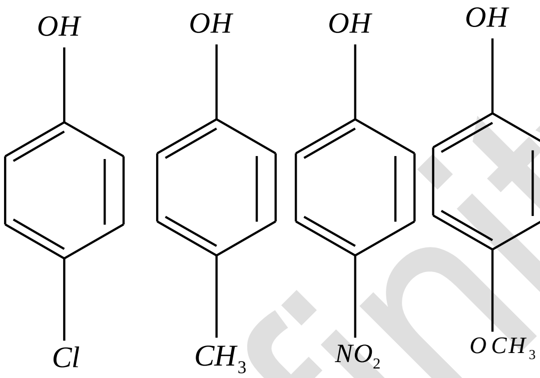
3) 

4) 

Key: 3

Solution: 

141. Arrange the following compounds in order of the decreasing acidity



(I) (II) (III) (IV)

1) $IV > III > I > II$

2) $II > IV > I > III$

3) $I > II > III > IV$

4) $III > I > II > IV$

Key: 4

Solution: EWG increases Acidic strength increases

142. HBr react with $CH_2 = CH - OCH_3$ under anhydrous condition at room temperature to give

1) CH_3CHO and CH_3Br

2) $Br-CH_2CHO$ and CH_3OH

3) $Br-CH_2-CH_2-OCH_3$

4) $CH_3-\underset{\underset{Br}{|}}{CH}-OCH_3$

Key: 4

Solution: $CH_2=CH-OCH_3+HBr \longrightarrow CH_3-\underset{\underset{Br}{|}}{CH}-OCH_3$

Electrophillic addition reaction

143. R – CN $\xrightarrow[(ii)H_2O]{(i)DIBAL-H} R-Y$ consider the above reaction and identify 'Y'.

1) $-CONH_2$

2) $-CHO$

3) $-COOH$

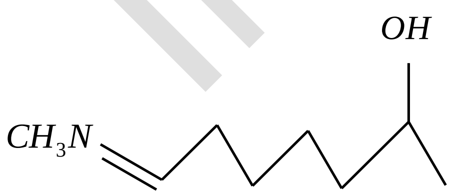
4) $-CH_2NH_2$

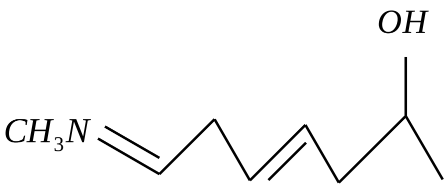
Key: 2

Solution: DIBAL – H

CN, Esters converted into aldehyde

144. The Major product of the following reaction is 

1) 

2) 

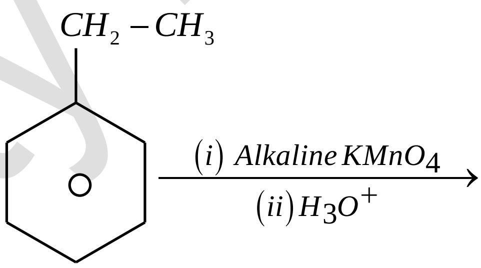
3) 

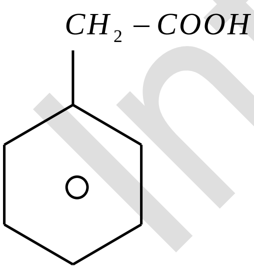
4) 

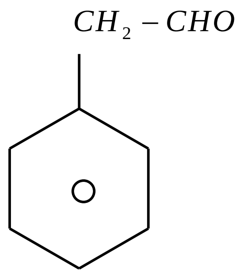
Key: 4

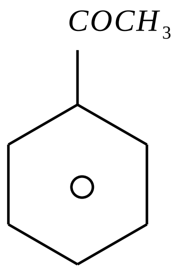
Solution: $NaBH_4$ is reduction carbonyl compounds and Imines

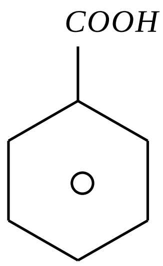
145. The major product of the following reaction is



1) 

2) 

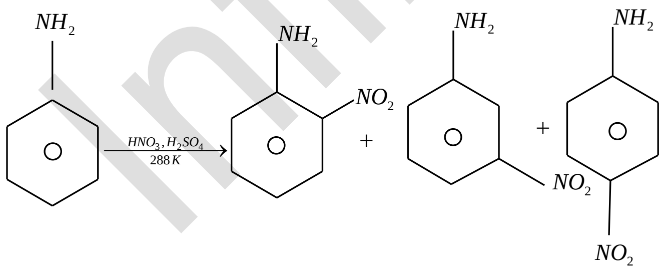
3) 

4) 

Key: 4

Solution: Conceptual

146.



(A) (B) (C)

consider the given reaction percentage yield of

1) $C > A > B$

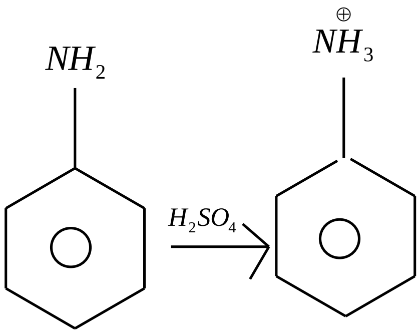
2) $C > B > A$

3) $A > C > B$

4) $B > C > A$

Key: 2

Solution:



($\overset{\oplus}{N}H_3$ is a EWG so meta and para product are major)

147. Which Among the following is not a polyester.

1) Glyptal

2) PHBV

3) Novolac

4) Dacran

Key: 3

Solution: Conceptual

148. Number of sterocentres present in Linear and Cyclic structures of glucose are respectively

1) 4 and 4

2) 4 and 5

3)5 and 5

4) 5 and 4

Key: 2

Solution: Conceptual

149. The Functions of Antihistamine are

1) Antiallergic and Anti depresent

2) Antacid and Antiallergic

3) Anlagesic and antacid

4) Antiallergic and Analagesic

Key: 2

Solution: Conceptual

150. Which pair of oxides is Acidic in nature?

1) CaO, SiO_2

2) B_2O_3, SiO_2

3) B_2O_3, CaO

4) N_2O, BaO

Key: 2

Solution: Conceptual

151. Which refining process is generally used in the purification of low melting metals?

1) Chromatographic method

2) electrolysis

3)zone refining

4) Liquation

Key: 4

Solution: Conceptual

152. The total number of Isotopes of hydrogen and number of radioactive Isotopes Among them, respectively are

1) 2 and 1

2) 3 and 2

3) 2 and 0

4) 3 and 1

Key: 4

Solution: Conceptual

153. On combustion of Li, Na and K in Excess of air, the major oxides formed respectively are

1) $Li_2O, Na_2O \, and \, K_2O_2$

2) Li_2O, Na_2O, K_2O

3) $Li_2O, Na_2O_2 \, and \, KO_2$

4) $Li_2O, Na_2O_2 \, and \, K_2O_2$

Key: 3

Solution: Li $\rightarrow$ Monoxide

Na $\rightarrow$ Monoxide, peroxide

K $\rightarrow$ Monoxide, Peroxide, Superoxide Li_2O, Na_2O_2, KO_2 *Major*

154. Number of paramagnetic oxides Among the following given oxides is _____ $Li_2O, CaO, Na_2O_2, KO_2$, $MgO, and \, K_2O$

1) 1

2) 3

3) 0

4) 2

Key: 1

Solution: Conceptual

155. The hybridisation of the atomic orbitals of Nitrogen in NO_2^-, NO_2^+ and $NH_4^{\oplus}$ respectively are

1) $sp^3, sp \, and \, sp^2$

2) $sp^2, sp \, and \, sp^3$

3) $sp^3, sp^2 \, and \, sp$

4) $sp, sp^2 \, and \, sp^3$

Key: 2

Solution: Conceptual

156. The Bond Order and the magnetic characteristic of CN^- are

1) 3, dia magnatic

2) 3, paramagnetic

3) 2 ½ , paramagnetic

4) 2 ½ diamagnetic

Key: 1

Solution: Conceptual

157. The set having ions which are coloured and paramagnetic both is

1) Cu^{+}, Zn^{+2}, mn^{+4}

2) $Ni^{+2}, Mn^{+7}, Hg^{+2}$

3) Cu^{+2}, Cr^{+3}, Sc^{+}

4) Sc^{+3}, V^{+5}, Ti^{+4}

Key: 3

Solution: Conceptual

158. The pair that has similar atomic radii is

1)Mn and Re

2) Ti and Hf

3) Sc and Ni

4) Mo and w

Key: 4

Solution: Conceptual

159. The complex that can show fac – and mer – isomers is

1) $\left[Co(NH_3)_3(NO_2)_3\right]$

2) $\left[Pt(NH_3)_2Cl_2\right]$

3) $\left[Co(NH_3)_4Cl_2\right]^{+}$

4) $\left[CoCl_2(en)_2\right]$

Key: 1

Solution: $\left[ma_3b_9\right]$TypeShow Fac – and Mer Isomerism

160. Reaction of Ammonia with excess Cl_2 gives

1) NH_4Cl and N_2

2) $NH_4Cl\ and\ HCl$

3) $NCl_3\ and\ NH_4Cl$

4) $NCl_3\ and\ HCl$

Key: 4

Solution: $NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$