

## MHT CET Exam Pattern 2024

Particulars	Details
Mode of Examination	Online, Computer Based Test
Number of Sections	There will be two sections- Section 1- Physics and Chemistry Section 2- Mathematics
Duration of Exam	180 minutes (90 minutes for each section)
Type of Questions	Multiple Choice Questions
Total number of Questions	150 (50 questions for each subject)
Total Marks	200 marks (100 marks for each section)
Language of Examination	<ul style="list-style-type: none"> <li>Mathematics will be in the English language</li> <li>Physics and Chemistry will be in English/Urdu/Marathi</li> </ul>
Marking Scheme	Physics & Chemistry - 1 mark will be awarded for every correct answer. Mathematics - 2 marks will be awarded for every correct answer.
Negative Marking	<b>No negative marking</b>

PHYSICS

1. A point object is placed at a distance of 30cm from a convex mirror of focal length 30 cm. The image will form at
- A. Infinity
  - B. Focus
  - C. Pole
  - D. 15cm behind the mirror

Key: D

Solution:  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{30} = \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{30}$$

$v = 15\text{cm}$  the image will be formed behind the mirror.

2. A sonometer wire under a given tension T has a fundamental frequency of 400 Hz. When tension is decreased by 1 kgf, the fundamental frequency becomes 300 Hz. The value of T in kgf units is
- A. 16/7
  - B. 16/9
  - C. 25/16
  - D. 1.5

Key: A

Solution: for a sonometer

$$n \propto \sqrt{T} \quad \frac{4}{3} = \sqrt{\frac{T}{T-1}} \text{ solving } T = \frac{16}{7}$$

3. A body is projected vertically up with a velocity  $v$  and after some time it returns to the point from which it was projected. The average velocity and average speed of the body for the total time of flight are:

- A.  $\vec{v}/2$  and  $v/2$
- B. 0 and  $v/2$
- C. 0 and 0
- D.  $\vec{v}/2$  and 0

Key: B

Solution: Average velocity =  $\frac{\text{displacement}}{\text{time}} = 0$

$$\text{average speed} = \frac{2H}{T} = \frac{V}{2}$$

4. An accurate Celsius thermometer and a faulty Fahrenheit thermometer register  $60^\circ\text{C}$  and  $144^\circ\text{F}$  respectively, when placed in the same constant temperature enclosure. What is the error in Fahrenheit thermometer?

- A.  $1^\circ\text{F}$
- B.  $3^\circ\text{F}$
- C.  $-4^\circ\text{F}$
- D.  $2^\circ\text{F}$

Key: C

$$\text{Solution: } \frac{C}{100} = \frac{F-32}{180}$$

$$F = \frac{9C}{5} + 32 = \frac{9 \times 60}{5} + 32 = 140^\circ\text{F}$$

$$\Delta F = 140 - 144 = -4^\circ\text{F}$$

5. At time  $t = 0$ , activity of a radioactive substance is  $1600 \text{ Bq}$ , at  $t = 8\text{s}$  activity remains  $100 \text{ Bq}$ . Then the activity at  $t = 2 \text{ sec}$

- A.  $800 \text{ Bq}$
- B.  $600 \text{ Bq}$
- C.  $400 \text{ Bq}$
- D.  $1000 \text{ Bq}$

Key: A

$$\text{Solution: } A = A_0 \left(\frac{1}{2}\right)^n \text{ here } n \text{ is number of half lives.}$$

$$\text{Given, } A = \frac{A_0}{16} \therefore \frac{A_0}{16} = A_0 \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

*Four half lives are equivalent to 8s. Hence 2s is equal to one half life.*

*Therefore at  $t = 2\text{s}$ , activity*

$$A = \frac{1600}{2} \text{ Bq} = 800 \text{ Bq}$$

6. If  $E$  is the energy density of one mole of monoatomic in an ideal gas, then the pressure of the ideal gas is

A.  $P = 2/3 E$

B.  $P = 3/2 E$

C.  $P = 5/2 E$

D.  $P = 2/5 E$

Key: A

Solution: For Ideal mono atomic gas,

$$U = \frac{3RT}{2}$$

$$U = \frac{3PV}{2} (\because PV = RT)$$

$$P = \frac{2U}{3V} = \frac{2E}{3}$$

7. Two open organ pipes of fundamental frequencies  $n_1$  and  $n_2$  are joined in series. The fundamental frequency of the new pipe so obtained will be :

A.  $\frac{n_1 + n_2}{2}$

B.  $\sqrt{n_1^2 + n_2^2}$

C.  $\frac{n_1 n_2}{n_1 + n_2}$

D.  $(n_1 + n_2)$

Key: C

Solution: If frequency of an open pipe is  $n_1$ ,

$$n_1 = \frac{V}{2L_1}$$

$$L_1 = \text{length of the pipe}; L_1 = \frac{V}{2n_1}$$

$V$  is the velocity of sound. Similarly, if frequency is  $n_2$ ,

$$\text{Length of the open pipe, } L_2 = \frac{V}{2n_2}$$

If they are combined, frequency of the combined pipe,

$$n = \frac{V}{2(L_1 + L_2)} = \frac{V}{2\left(\frac{V}{2n_1} + \frac{V}{2n_2}\right)} = \frac{n_1 n_2}{n_1 + n_2}$$

8. A particle moves according to the law  $a = -ky$  Find the velocity as a function of distance  $y$  where  $v_0$  is initial velocity and  $a$  is the acceleration

A.  $v^2 = v_0^2 - ky^2$

B.  $v^2 = v_0^2 - 2ky$

C.  $v^2 = v_0^2 - 2ky^2$

D.  $v^2 = v_0^2 + ky^2$

Key: A

$$\text{Solution: } a = \frac{dv}{dy} \frac{dy}{dt} = -ky$$

$$\Rightarrow \int_{v_0}^v v dv = - \int_0^y ky dy$$

$$\Rightarrow \frac{v^2 - v_0^2}{2} = -k \frac{y^2}{2}$$

$$\therefore v^2 = v_0^2 - ky^2$$

9. Eight small drops, each of radius  $r$  and having same charge  $q$  are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is

A. 8 : 1

B. 4 : 1

C. 2 : 1

D. 1 : 8

Key: B

$$\text{Solution: } V_{big} = n^{2/3} V_{small}$$

$$\Rightarrow \frac{V_{Big}}{V_{small}} = (8)^{2/3} = \frac{4}{1}$$

10. Two slits, 4 mm apart, are illuminated by a light of wavelength  $6000 \text{ \AA}$ . What will be the fringe width on a screen placed 2m from the slits?

- A. 0.12 mm  
B. 0.3 mm  
C. 3.0 mm  
D. 4.0 mm

Key: B

$$\lambda = 6000 \text{ \AA} = 6000 \times 10^{-10} \text{ m}$$

Solution: Given  $D = 2 \text{ m}$

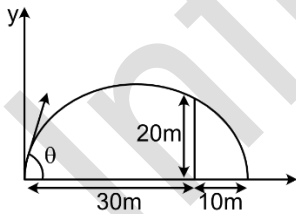
$$d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \beta = \frac{6000 \times 10^{-10} \times 2}{4 \times 10^{-3}}$$

$$\Rightarrow \beta = 0.3 \times 10^{-3} \text{ m} = 0.3 \text{ mm}$$

11. In the given diagram shown for a projectile, what is the angle of projection?



- A.  $\tan^{-1}(1)$   
B.  $\tan^{-1}\left(\frac{8}{3}\right)$



C.  $\tan^{-1}\left(\frac{4}{3}\right)$

D.  $\tan^{-1}\left(\frac{5}{3}\right)$

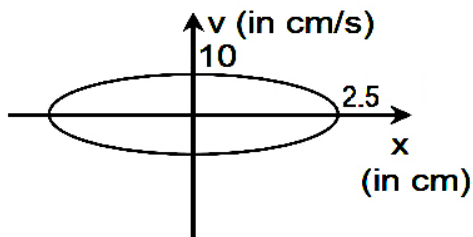
Key: B

Solution:  $y = x \tan \theta \left(1 - \frac{x}{R}\right) S$

$$20 = 30 \tan \theta \left(1 - \frac{30}{40}\right)$$

$$\theta = \tan^{-1}\left(\frac{8}{3}\right)$$

12. The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM. select the incorrect statement.



- A. the time period of the particle is 2s  
 B. the maximum acceleration will be  $40 \text{ cm/s}^2$   
 C. the velocity of particle is  $2\sqrt{21} \text{ cm/s}$  when it is at a distance 1 cm from the mean position.  
 D. Amplitude of SHM is 2.5 cm.

Key: A

Solution:  $V = \omega A$  and  $\omega = \frac{10}{2.5} = 4$

(1)  $T = \frac{2\pi}{\omega} = \frac{\pi}{2} = 1.57$

(2)  $a = \omega^2 A = 40$

(3)  $V = \omega \sqrt{A^2 - X^2} = 2\sqrt{21}$

(4)  $A = 2.5 \text{ cm}$

13. A certain metallic surface is illuminated with monochromatic light of wavelength,  $\lambda$ . The stopping potential for photoelectric current for this light is  $3V_0$ . If the same surface is illuminated with light of wavelength  $2\lambda$ , the stopping potential is  $V_0$ . The threshold wavelength for this surface for photoelectric effect is.

A.  $\frac{\pi}{4}$

B.  $\frac{\lambda}{6}$

C.  $6\lambda$

D.  $4\lambda$

Key: D

Solution: According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV_0$$

According to problem,

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + e(3V_0) \dots (1)$$

$$\frac{hc}{2\lambda} = \frac{hc}{\lambda_0} + eV_0 \dots (2)$$

Multiply equation (2) by 3, we get,

$$\frac{3hc}{2\lambda} = \frac{3hc}{\lambda_0} + 3eV_0 \dots (3)$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + e(3V_0) \dots (1)$$

Solve (1) and (3)

we get  $\lambda = 4\lambda_0$

14. A wheel having moment of inertia  $2 \text{ kg-m}^2$  about its geometrical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

A.  $\frac{\pi}{15} \text{ N-m}$

B.  $\frac{2\pi}{15} \text{ N-m}$

C.  $\frac{\pi}{18} \text{ N-m}$

D.  $\frac{\pi}{12} \text{ N-m}$

Key: A

Solution: We have,  $\omega_2 = \omega_1 + \alpha t$

$$\Rightarrow \omega_1 = \omega_2 - \alpha t$$

Here,  $\omega_1 = 0$

and  $\omega_2 = 60rpm$

$$= \frac{60 \times 2\pi}{60} \text{ rads}^{-1}$$

$$\Rightarrow 0 = 2\pi \times \frac{60}{60} - \alpha \cdot 60$$

or  $\alpha = \frac{2\pi}{60}$

$$\therefore \text{Torque, } \tau = I\alpha = 2 \times \frac{2\pi}{60} = \frac{\pi}{15} \text{ N-m}$$

15. Force acts on a 20g particle in such a way that the position of the particle as a function of time is given by  $x = 2t - 3t^2 + 2t^3$  where x is in metre and t in second. The work done in first three seconds is

A. 5.28 J

B. 28.8 J

C. 14.4 J

D. 7.2 J

Key: C

Solution:  $x = 2t - 3t^2 + 2t^3$

$$V = 2 - 6t + 6t^2$$

Initial velocity  $V_1 = 2\text{m/s}$

velocity at  $t = 3$  sec,  $V_2 = 38\text{m/s}$

Using work energy theorem,

$$W = \Delta k = \frac{1}{2} \left( \frac{20}{1000} \right) (38^2 - 2^2) = 14.4 \text{ J}$$

16. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation  $AD = C \ln(BD)$  holds true. Then which of the combination is not a meaningful quantity?

A.  $AD + C$

B.  $A + \frac{C}{D}$

C.  $\frac{A}{B} - C$

D.  $\frac{A^2 - AC}{D}$

Key: D

Solution:  $[BD]$  dimensionless;  $[AD] = [C]$

$$\Rightarrow [A^2] \neq [AC]$$

17. A small object of uniform density rolls up a curved surface with an initial velocity '  $v$  '. It reaches upto a maximum height of  $\frac{3v^2}{4g}$  with respect to the initial position. The object is

A. hollow sphere

B. disc

C. ring

D. solid sphere

Key: B

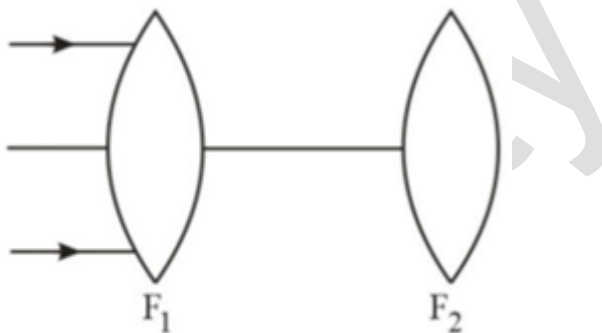
Solution: When the body rolls up along the inclined plane, at the highest point, all its rotational KE will be converted to gravitational PE. Hence, height attained,  $h = \frac{3v^2}{4g}$  where  $v$  is the velocity of centre of mass of the rolling body. Applying law of conservation of energy, total KE = mgh

$$\frac{1}{2}mv^2 \left( 1 + \frac{K^2}{R^2} \right) = mgh \Rightarrow 1 + \frac{K^2}{R^2} = 2gh = 2g \frac{3v^2}{4g}$$

$$\frac{K^2}{R^2} = \frac{1}{2}$$

It could be a disc or solid cylinder

18. A parallel beam of light is incident on a system of two convex lenses of focal length  $f_1 = 20$  cm and  $f_2 = 10$  cm



What should be the distance between the two lenses so that the rays after refraction from both the lenses pass undeviated?

- A. 60 cm
- B. 30 cm
- C. 90 cm

D. 40 cm

Key: B

Solution:  $\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

when  $f_{eff} = \infty$  the rays will travel with out deviation as a parallel beam.

$$\frac{1}{\infty} = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{d}{f_1 f_2}$$

$$d = f_1 + f_2$$

19. The rise in the water level in a capillary tube of radius 0.07 cm when dipped vertically in a beaker containing water of surface tension  $0.07 \text{ Nm}^{-1}$  is ( $g = 10 \text{ ms}^{-2}$ )

A. 2 cm

B. 4 cm

C. 1.5 cm

D. 3 cm

Key: A

Solution: Rise of a liquid in a capillary tube  $h = \frac{2S \cos \theta}{r \rho g}$

Here,  $r = 0.07 \text{ cm} = 0.07 \times 10^{-2} \text{ m}$

For water,  $S = 0.07 \text{ Nm}^{-1}$ ,  $\rho = 10^3 \text{ kgm}^{-3}$

Angle of contact  $\theta = 0^\circ$

$$\begin{aligned} \therefore h &= \frac{2 \times (0.07 \text{ Nm}^{-1}) \times 1}{(0.07 \times 10^{-2} \text{ m})(10^3 \text{ kgm}^{-3})(10 \text{ ms}^{-2})} \\ &= 2 \times 10^{-2} \text{ m} = 2 \text{ cm} \end{aligned}$$

20. A force  $F = 20 + 10y$  acts on a particle in  $y$  direction where  $F$  is in newton and  $y$  in meter. Work done by this force to move the particle from  $y = 0$  to  $y = 1$  m is

A. 25 J

B. 20 J

C. 30 J

D. 5 J

Key: A

Solution: Work done by variable force is

$$W = \int_{y_i}^{y_f} F dy$$

Here,  $y_i = 0, y_f = 1 \text{ m}$

$$\therefore W \int_0^1 (20 + 10y) dy = \left[ 20y + \frac{10y^2}{2} \right]_0^1 = 25 \text{ J}$$



21. How much water should be filled in a container 21 cm in height, so that it appears half filled when viewed from the top of the container (given that  $a\mu_w = \frac{4}{3}$ )

- A. 8.0 cm
- B. 10.5 cm
- C. 12.0 cm
- D. 15.7 cm

Key: C

Solution: To see the container half-filled from top, water should be filled up to height  $x$  so that bottom of the container should appear to be raised upto height  $(21-x)$ .

$$\mu = \frac{h}{h'} \Rightarrow \frac{4}{3} = \frac{x}{21-x} \Rightarrow x = 12\text{cm}$$

22. A bob of a simple pendulum of mass 40gm with a positive charge  $4 \times 10^{-6} \text{ C}$  is oscillating with time period  $T_1$ . An electric field of intensity  $3.6 \times 10^4 \text{ N/C}$  is applied vertically upwards, and now the time period is  $T_2$ . The value of  $T_2/T_1$  is ( $g = 10 \text{ m/s}^2$ )

- A. 0.16
- B. 0.64
- C. 1.25
- D. 0.8

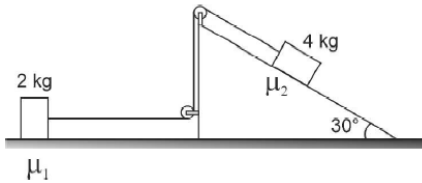
Key: C

$$T_1 \propto \frac{1}{\sqrt{g}}; T_2 \propto \frac{1}{\sqrt{g'}}; g = 10 \text{ and } g' = g - \frac{Eq}{m}$$

$$\text{Solution: } = 10 - \frac{3.6 \times 10^4 \times 4 \times 10^{-6}}{40 \times 10^{-3}} = 10 - 3.6 = 6.4$$

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_1}} \Rightarrow \sqrt{\frac{10}{6.4}} = \sqrt{\frac{100}{64}} = \frac{10}{8} = 1.25$$

23. If the tension in the string in figure is 16 N and the acceleration of each block is  $0.5 \text{ ms}^{-2}$ , find the friction coefficients at the two contacts with the blocks.



- A.  $\mu_1 = 0.75, \mu_2 = 0.06$   
 B.  $\mu_1 = 0.06, \mu_2 = 0.75$   
 C.  $\mu_1 = 0.85, \mu_2 = 0.04$   
 D.  $\mu_1 = 0.85, \mu_2 = 0.002$

Key: A

$$m_1 = 2kg \text{ and } m_2 = 4kg$$

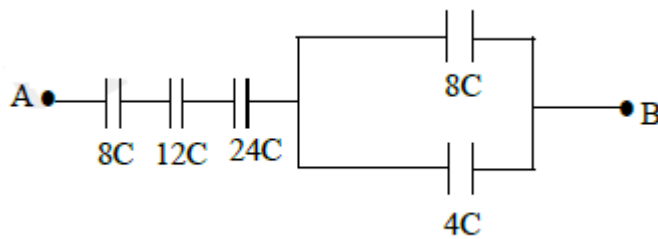
$$T - \mu_1 m_1 g = m_1 a$$

Solution :  $\Rightarrow \mu_1 = \frac{15}{19.6} = 0.75$

$$m_2 g \sin \theta - \mu_2 m_2 g \cos \theta - T = m_2 a$$

$$\Rightarrow \mu_2 = \frac{1}{17.32} = 0.06$$

24. Find equivalent capacitance between A and B?



A. 2C

B. 1C

C. 3C

D. 4C

Key: C

Solution: The effective capacitance of parallel combination will be 12C, this is in series combination with 8C, 12C, 24C.

So, the effective capacitance is given by

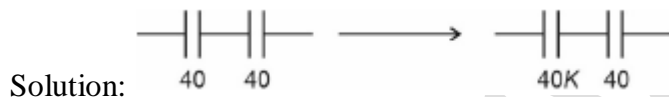
$$\frac{1}{C} = \frac{1}{12} + \frac{1}{24} + \frac{1}{12} + \frac{1}{8} = \frac{2+1+2+3}{24}$$

$$C = \frac{24}{8} = 3C$$

25. Two capacitors, each having capacitance  $40 \mu F$  are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant  $K$  such that the equivalence capacitance of the system became  $24 \mu F$ . The value of  $K$  will be :

- A. 1.5
- B. 2.5
- C. 1.2
- D. 3

Key: A



$$\frac{40K \times 40}{40K + 40} = 24$$

$$40K = 24(K + 1)$$

$$40K = 24K + 24$$

$$16K = 24$$

$$K = \frac{24}{16} = \frac{3}{2} = 1.5$$

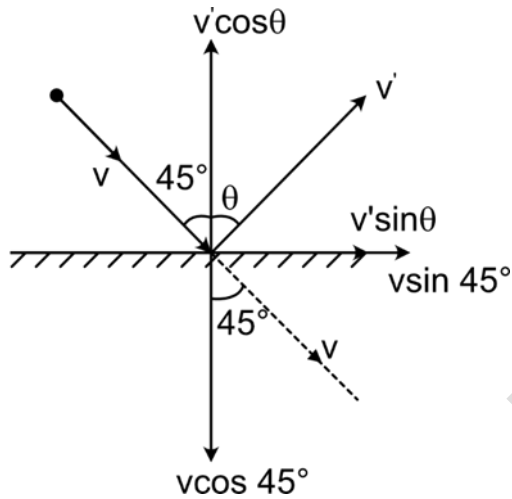
26. A ball of mass 500 gm moving with speed 2 m/s collide to a floor making an angle of  $45^\circ$  with vertical. If coefficient of restitution is  $1/\sqrt{3}$ , then the angle made by the ball with normal, after collision is

- A.  $60^\circ$
- B.  $30^\circ$

C.  $45^\circ$

D.  $37^\circ$

Key: A



Solution:

Horizontal velocity remains constant.

$$v \sin 45^\circ = v' \sin \theta \text{ -----(1)}$$

perpendicular to the plane

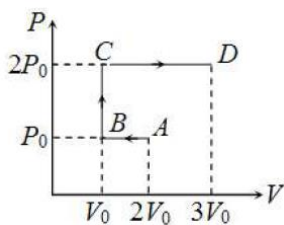
$$v \cos 45^\circ = v' \cos \theta \text{ -----(2)}$$

dividing 1 by 2

$$\frac{1}{e} = \tan \theta = \sqrt{3}$$

$$\theta = 60^\circ$$

27. P - V diagram of an ideal gas is as shown in figure. Work done by gas in process ABCD is.



A.  $4P_0V_0$

B.  $2P_0V_0$

C.  $3P_0V_0$

D.  $P_0V_0$

Key: C

Solution: Workdone is given by area under the curve.

Therefore,

$$W_{AB} = -P_0V_0, W_{ac} = 0 \text{ and } W_{CD} = 4P_0V_0$$

$$\Rightarrow W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$$

28. When the hydrogen atom emits a photon in going from  $n=5$  to  $n=1$  state, its recoil speed is nearly

A.  $10^{-4} m/s$

B.  $2 \times 10^{-2} m/s$

C.  $4 m/s$

D.  $8 \times 10^{-2} m/s$

Key: C

Solution: Energy of photon  $E = 13.6 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\Rightarrow E = 13.6 \left( \frac{1}{1^2} - \frac{1}{5^2} \right) = 13.05 eV$$

Momentum of photon ( $mv$ ) = momentum of hydrogen atom

$$mv = \frac{h\nu}{C}$$

$$\Rightarrow v = \frac{h\nu}{mC} = \frac{E}{mC} = \frac{13.05 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times 3 \times 10^8} = 4 \text{ m/s}$$

29. A straight thin uniform rod of length  $4L$  and mass  $4M$  is bent into a square. Its M.I about one side is

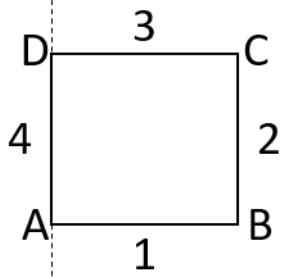
A.  $\frac{5}{3} ML^2$

B.  $\frac{7}{6} ML^2$

C.  $\frac{ML^2}{48}$

D.  $\frac{ML^2}{3}$

Key: A



Solution:

$$I = I_1 + I_2 + I_3 + I_4$$

$$= \frac{ML^2}{3} + ML^2 + \frac{ML^2}{3} + 0$$

$$= \frac{5ML^2}{3}$$

30. The ratio of escape velocity at earth ( $v_e$ ) to the escape velocity at a planet ( $v_p$ ) whose radius and mean density are twice as that of earth is:
- A. 1:4
  - B. 1: $\sqrt{2}$
  - C. 1:2
  - D. 1: $2\sqrt{2}$

Key: D

Solution: Escape velocity from any planet,



$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \frac{4}{3} \pi R^3 \rho}$$

[Where M is the mass and R is the radius of the planet.]

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \frac{4}{3} \pi R^3 \rho}$$

$$V_e \propto R$$

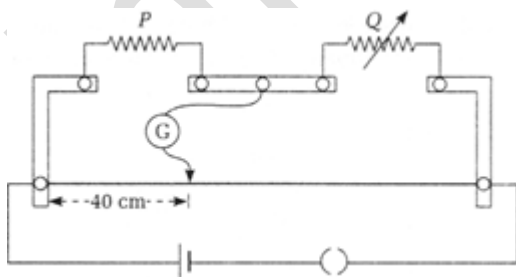
$$V_e \propto \sqrt{\rho}$$

Escape velocity ratio,

$$\frac{v_1}{v_2} = \frac{R_1}{R_2} \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\frac{v_e}{v_p} = \frac{R}{2R} \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

31. In a metre bridge [Fig], the gaps are closed by two resistances P and Q and the balance point is obtained



at 40 cm. When Q is shunted by resistance of  $10 \Omega$ , the balance point shifts to 50 cm. The values of P and Q are

A.  $(10/3)\Omega, 5\Omega$

B.  $20\Omega, 30\Omega$

C.  $10\Omega, 15\Omega$

D.  $5\Omega, (15/2)\Omega$

Key: A

Solution: For the balance of metre bridge

$$\frac{P}{Q} = \frac{R}{S} = \frac{40}{60} = \frac{2}{3}$$

or  $P = \frac{2}{3}Q \dots\dots(1)$

When Q is shunted by a resistance of  $10 \Omega$  then net resistance in this gap should be a parallel combination of Q and  $10 \Omega$  resistance i.e.,  $10Q / (10 + Q)$ .

Now the balance shifts to 50 cm i.e.,  $R = 50\text{cm}$  and  $S = 50 \text{cm}$ . In this case

$$\frac{P}{[10Q / (10 + Q)]} = \frac{50}{50} = 1 \dots\dots(2)$$

Substituting the value of P from eq. (1) in eq. (2), we get

$$\frac{(2/3)Q}{[10Q / (10 + Q)]} = 1$$

Solving we get  $Q = 5\Omega$

From eq. (1),  $P = \frac{2}{3} \times 5 = \frac{10}{3} \Omega$

32. The co-ordinates of centre of mass of particles of mass 10, 20 and 30 gm are (1, 1, 1) cm. The position co-ordinates of mass 40 gm which when added to the system, the position of combined centre of mass be at (0, 0, 0) are,

A. (3/2, 3/2, 3/2)

B. (-3/2, -3/2, -3/2)

C. (3/4, 3/4, 3/4)

D. (-3/4, -3/4, -3/4)

Key: B

Solution: in x direction

$$x_{cm} = \frac{10(x_1) + 20(x_2) + 30(x_3) + 60(x_4)}{100} = 0$$

$$\Rightarrow 60 + 40(x_4) = 0$$

$$\Rightarrow x_4 = \frac{-6}{4} = \frac{-3}{2}$$

similarly  $y_4 = \frac{-3}{2}$

$$z_4 = \frac{-3}{2}$$

$$\therefore \left( \frac{-3}{2}, \frac{-3}{2}, \frac{-3}{2} \right) \text{ should be position of } 40 \text{ gm}$$

$$x_{cm} = \frac{10(x_1) + 20(x_2) + 30(x_3)}{60} = 1 \text{ finally}$$

$$\Rightarrow 10(x_1) + 20(x_2) + 30(x_3) = 60$$

33. A current carrying straight conductor is placed in a uniform magnetic field and force experienced by the conductor is 6N. When the conductor is rotated through an angle of 90° in the plane containing the rod and the field, force experienced by the rod is 8 N. Then maximum possible force experienced by the conductor when placed in the same magnetic field is

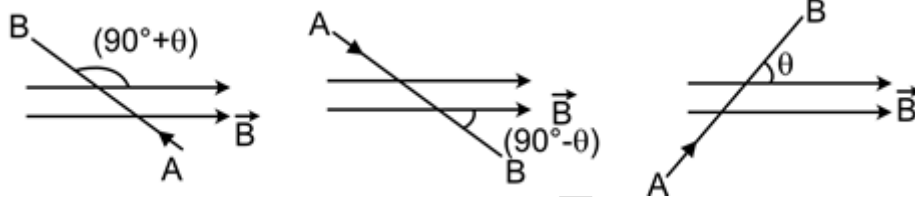
A. 14 N

B. 16 N

C. 10 N

D. 12 N

Key: C



Solution:

$$F_1 = ilB \sin \theta \quad \text{and} \quad F_2 = ilB \sin(90^\circ \pm \theta) = ilB \cos \theta$$

$$\therefore F_{\max} = ilB = \sqrt{F_1^2 + F_2^2} = \sqrt{6^2 + 8^2} \text{ N} = 10 \text{ N}$$

34. An Electromagnetic wave is propagating in x direction. Magnetic field in space is given by  $\vec{B} = 2 \times 10^{-8} \hat{k} T$ . What will be the value and direction of electric field (in  $Vm^{-1}$ )?

A.  $0.6 \hat{j}$

B.  $6 \hat{j}$

C.  $0.6 \hat{k}$

D.  $6 \hat{k}$

Key: B

Solution:  $\hat{E} = \hat{B} \times \hat{v} = \hat{k} \times \hat{i} = \hat{j}$

$$E = Bc = 2 \times 10^{-8} \times 3 \times 10^8 = 6Vm^{-1}$$

35. An earth satellite is moved from one stable circular orbit to a farther stable circular orbit, which one of the following quantities increase

- A. Gravitational force
- B. Gravitational P.E
- C. Linear orbital speed
- D. Centripetal acceleration.

Key: B

Solution:  $U = \frac{-GMm}{r}$  If r increases then U also increase

36. A galvanometer of resistance, G is shunted by a resistance S ohm. To keep the main current in the circuit unchanged, the resistance to be put in series with the galvanometer is

A.  $\frac{S^2}{(S+G)}$

B.  $\frac{SG}{(S+G)}$

C.  $\frac{G^2}{(S+G)}$

D.  $\frac{G}{(S+G)}$

Key: C

Solution:  $\frac{GS}{G+S} + R_s = G \Rightarrow R_s = \frac{G^2}{G+S}$

37. When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 V AC is applied across the same solenoid the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are

A.  $200\Omega$  and  $0.55\text{H}$

B.  $100\Omega$  and  $0.86\text{H}$

C.  $200\Omega$  and  $1.0\text{H}$

D.  $1100\Omega$  and  $0.93\text{H}$

Key: A

Solution: For DC:  $-i = \frac{V}{R}$

$\Rightarrow 1 = \frac{100}{R} \Rightarrow R = 100\Omega$

For AC:  $-i = \frac{V}{Z}$

$\Rightarrow 0.5 = \frac{100}{Z} \Rightarrow Z = 200\Omega$

$$Z = \sqrt{R^2 + x_L^2}$$

$$\Rightarrow (200)^2 = (100)^2 + 4\pi^2(50)^2 L^2$$

$$\Rightarrow L = 0.55H$$

38. A wire in the form of a circular loop of one turn carrying a current produces a magnetic field  $B$  at the centre. If the same wire is looped into a coil of two turns and carries the same current, the new value of magnetic induction at the centre is

- A.  $5B$
- B.  $3B$
- C.  $2B$
- D.  $4B$

Key: D

Solution:  $B' = n^2 B = (2)^2 B = 4B$

39. A circular coil of area  $8\text{m}^2$  and number of turns 20 is placed in a magnetic field of  $2\text{T}$  with its plane perpendicular to it. It is rotated with an angular velocity of  $20\text{ rev/s}$  about its natural axis. The emf induced is.

- A.  $400\text{V}$
- B.  $800\pi\text{V}$
- C. zero
- D.  $400\pi\text{V}$

Key: C

Solution: About natural axis emf induced in it is equal to zero. Because there is no change of flux.

40. A fluid is flowing through a horizontal pipe of varying cross-section, with speed  $v \text{ ms}^{-1}$  at a point where the pressure is  $P$  Pascal. At another point where pressure is  $\frac{P}{2}$  Pascal its speed is  $V \text{ ms}^{-1}$ . If the density of the fluid is  $\rho \text{ kg m}^{-3}$  and the flow is streamline, then  $V$  is equal to :

A.  $\sqrt{\frac{P}{\rho} + v}$

B.  $\sqrt{\frac{2P}{\rho} + v^2}$

C.  $\sqrt{\frac{P}{2\rho} + v^2}$

D.  $\sqrt{\frac{P}{\rho} + v^2}$

Key: D

Solution: Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

For horizontal pipe,  $h_1 = 0$  and  $h_2 = 0$  and taking  $P_1 = P, P_2 = \frac{P}{2}$ , we get



$$\Rightarrow P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2 \Rightarrow \frac{P}{2} + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2 \Rightarrow V = \sqrt{v^2 + \frac{P}{\rho}}$$

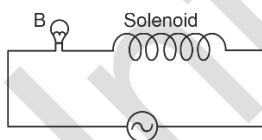
41. A magnetic field in a certain region is given by  $\vec{B} = (40\hat{i} - 18\hat{k})\text{G}$ . How much flux passes through a  $5.0\text{cm}^2$  area loop if the loop lies flat on  $xy$ -plane?

- A.  $-900\text{ nwb}$
- B.  $-700\text{ nwb}$
- C.  $-200\text{ nwb}$
- D.  $-800\text{ nwb}$

Key: A

Solution:  $\vec{B} = 40\hat{i} - 18\hat{k}; \vec{A} = 5 \times 10^{-4}\hat{k}$   
 $\phi = \vec{B} \cdot \vec{A} = -900\text{ nwb}$

42. A bulb connected in series with an air-cored solenoid is lit by an a.c. source. If a soft iron core is introduced in the solenoid, then.

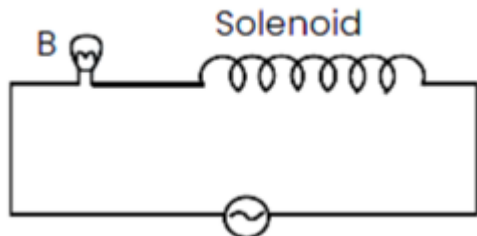


- A. The bulb will glow brighter.
- B. There is no change in glow of bulb.

C. The bulb will become dimmer.

D. The bulb stops glowing.

Key: C



Solution:

For the given circuit diagram, current is

$$i = \frac{E_{(rms)}}{\sqrt{R^2 + L^2\omega^2}}$$

where  $R$  is the resistance and  $\omega$  is the angular frequency of AC supply.

Self-inductance of the solenoid,

$$L = \frac{\mu_0 N^2 A}{\ell}$$

where  $N$  is the number of turns,  $A$  is area, and  $\ell$  is length of the solenoid. If a soft iron core is introduced,

$$L' = \frac{\mu_0 N^2 A \mu}{\ell}$$

So,  $L$  increases  $i$  decreases, and bulb will become dimmer.

43. If the terminal speed of a sphere of gold ( $density = 19.5 \text{ kg/m}^3$ ) is  $0.2 \text{ m/s}$  in a viscous liquid ( $density = 1.5 \text{ kg/m}^3$ ), find the terminal speed of a sphere of silver ( $density = 10.5 \text{ kg/m}^3$ ) of the same size in the same liquid

- A. 0.4 m/s
- B. 0.133 m/s
- C. 0.1 m/s
- D. 0.2 m/s

Key: C

Solution: Terminal velocity,  $v_T = \frac{2r^2(d_1 - d_2)g}{9\eta}$

$$v_T 0.2 = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_T = 0.2 \times \frac{9}{18}$$

$$\therefore v_T = 0.1 \text{ m/s}$$

44. A metal rope of density  $5000 \text{ kgm}^{-3}$  has a breaking stress  $9.8 \times 10^8 \text{ Nm}^{-2}$ . This rope is used to measure the depth of the sea. The depth of the sea that can be measured without breaking is
- A.  $10 \times 10^3 \text{ m}$
  - B.  $20 \times 10^3 \text{ m}$
  - C.  $25 \times 10^3 \text{ m}$
  - D.  $40 \times 10^3 \text{ m}$

Key: C

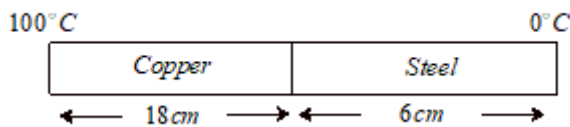
Solution: The tension in the string will be the apparent weight of the rope,

$$F = Mg - B = (\rho - \rho_w) A l g$$

$$\Rightarrow l = \frac{\text{Breaking stress}}{(\rho_{\text{wire}} - \rho_{\text{water}})g} = \frac{9.8 \times 10^8}{(5000 - 1000)9.8} \text{ m}$$

$$= \frac{1}{4} \times 10^5 \text{ m} = 25 \times 10^3 \text{ m}$$

45. The coefficient of thermal conductivity of copper is 9 times that of steel. In the composite cylindrical bar shown in the figure. What will be the temperature at the junction of the copper and steel.



- A. 75°C  
B. 67°C  
C. 33°C  
D. 25°C

Key: A

Solution: Rate of heat flow for both the rods are same.

$$\frac{Q}{t} = KA \left( \frac{\theta_1 - \theta_2}{l} \right) = \text{constant}$$

$$\Rightarrow \frac{9KA(100^\circ - \theta)}{18} = \frac{KA(\theta - 0^\circ)}{6}$$

$$\Rightarrow 300^\circ - 3\theta = \theta$$

$$\Rightarrow \theta = 75^\circ \text{C}$$

46. The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has the current sensitivity of 10 divisions per mA and the voltage sensitivity of 2 divisions per mV. How the galvanometer be designed to read i) 6A, per division and ii) 1V, per division ?
- A.  $40\mu\Omega$  is connected in parallel,  $1000\Omega$  is connected in series
- B.  $20\mu\Omega$  is connected in parallel,  $9995\Omega$  is connected in series
- C.  $8.3\mu\Omega$  is connected in parallel,  $9995\Omega$  is connected in series
- D.  $8.3 \times 10^{-5}\Omega$  is connected in parallel,  $9995\Omega$  is connected in series

Key: D

Solution:  $N = 150; i_g = 0.1\text{mA} / \text{division}$

$$V_g = 0.5\text{mV} / \text{division} ; G = \frac{V_g}{i_g} = 5\Omega$$

(i) shunt required to read  $i = 6\text{A}/\text{division}$  is  $S = \frac{Gi_g}{i - i_g} = 8.3 \times 10^{-5}\Omega$  ( in parallel)

(ii) Resistance required to read the voltage  $V = 1\text{V} / \text{division}$  is  $= G\left(\left(\frac{V}{V_g}\right) - 1\right) = 9995\Omega$  (in series)

47. A transverse wave is represented by the equation,  $y = y_0 \sin \frac{2\pi}{\lambda}(vt - x)$ . For what value of  $\lambda$ , the maximum particle velocity is equal to two times the wave velocity ?
- A.  $\lambda = 2\pi y_0$
- B.  $\lambda = \pi y_0 / 3$
- C.  $\lambda = \pi y_0 / 2$
- D.  $\lambda = \pi y_0$

Key: D

Solution: On comparing the given equation with standard equation  $y = a \sin \frac{2\pi}{\lambda}(vt - x)$

It is clear that wave speed,  $(v)_{\text{wave}} = v$ , and ,

maximum particle velocity,  $(v_{\text{max}})_{\text{particle}} = a\omega = y_0 \times \frac{2\pi v}{\lambda}$

$$\therefore (v_{\text{max}})_{\text{particle}} = 2(v)_{\text{wave}} \Rightarrow \lambda = \pi y_0$$

48. The vibrations of a string fixed at both ends are represented  $y = 16 \sin \frac{\pi x}{15} \cos 96\pi t$  where x and y are in cm and t in seconds. Then the phase difference between the points at  $x = 13$  cm and  $x = 16$  cm (in radian) is

- A.  $\frac{\pi}{5}$
- B.  $\pi$
- C. zero
- D.  $\frac{2\pi}{5}$

Key: B

Solution:  $k = \frac{2\pi}{\lambda} = \frac{\pi}{15} \therefore \lambda = 30\text{cm}$

The distance between two successive nodes is

$$\frac{\lambda}{2} = 15\text{cm}$$

The two points of  $x = 13$  cm and  $x = 16$  cm are in adjacent loops. They vibrate in opposite phase. i.e. phase difference is  $\pi$  radian.

49. A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has.

- A. three nodes and three antinodes
- B. three nodes and four antinodes
- C. four nodes and three antinodes
- D. four nodes and four antinodes

Key: D

Solution: It is known that when closed pipe vibrates in fundamental mode, there are one node and one antinode.

When it vibrates in first overtone, there are two nodes and two antinodes.

When it vibrates in second overtone, there are three nodes and three antinodes.

When it vibrates in third overtone, there are four nodes and four antinodes.

50. The normal density of a material is  $\rho$  and its bulk modulus of elasticity is  $K$ . The magnitude of increase in density of material, when a pressure  $P$  is applied uniformly on all sides, will be:

A.  $\frac{PK}{\rho}$

B.  $\frac{\rho K}{P}$

C.  $\frac{\rho P}{K}$

D.  $\frac{K}{\rho P}$

Key: C

Solution: We know that,  $K = -V \frac{dP}{dV}$

from the question as mass remains constant,

$$\rho = \frac{M}{V}$$

$$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

$$K = \rho \frac{dP}{\rho}$$

here  $dP = P$

$$d\rho = \frac{\rho P}{K}$$

Hence the correct answer is  $\frac{\rho P}{K}$ .

## CHEMISTRY

51. Copper crystallizes in fcc unit cell with cell edge length of  $3.60 \times 10^{-8} \text{ cm}$ . The density of copper is  $8.92 \text{ g cm}^{-3}$ . Calculate the atomic mass of copper (approx.)

- A. 63.1 u  
B. 31.55 u  
C. 60 u



D. 65 u

Key: A

Solution: Given  $a = 3.608 \times 10^{-8} \text{ cm}$

Number of atoms per fcc unit cell ( $z$ ) = 4

Atomic mass of Cu ( $M$ ) = ?

Density of Cu ( $d$ ) =  $8.929 \text{ cm}^{-3}$

$$d = \frac{zM}{a^3 N_A}$$

$$M = \frac{da^3 N_A}{z}$$

$$= \frac{8.92(3.608 \times 10^{-8})^3 (6.023 \times 10^{23})}{4}$$

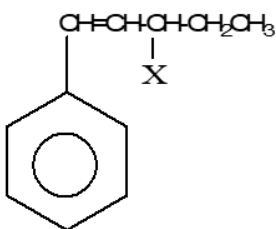
Atomic mass of Cu ( $M$ ) =  $63.1 \text{ g/mol} = 63.1 \text{ u}$

52. Which of the following practices will not come under green chemistry?
- A. If possible, making use of soap made of vegetable oils instead of using synthetic detergents.
  - B. Using  $\text{H}_2\text{O}_2$  for bleaching purpose instead of using chlorine based bleaching agents
  - C. Using bicycle for travelling small distances instead of using petrol/diesel based vehicles
  - D. Promoting the usage of tetrachloroethene as cleaning agent for dry cleaning of clothes

Key: D

Solution: Tetrachloroethylene (also known as perchloroethylene, or PCE) is an excellent solvent for organic materials. It is volatile, highly stable, and non-flammable. For these chemical properties, it is widely used in dry cleaning. It does not come under the practices of green chemistry.

53. The given compound



is an example of \_\_\_\_\_.

- A. aryl halide
- B. allylic halide
- C. vinylic halide
- D. benzylic halide

Key: B

Solution: Allylic chloride

54. Which of the following Nitrogenous base is not present in DNA ?

- A. Adenine
- B. Cytosine
- C. Thymine
- D. Uracil

Key: D

Solution: In DNA on the place thymine, uracil is present

55. For the reaction,  $X_2O_{4(l)} \rightarrow 2XO_{2(g)}$   $\Delta U = 2.1kcal, \Delta S = 20calK^{-1}$  at 300 K hence :  $\Delta G$  is:

- A. 2.7 kcal
- B. -2.7 kcal
- C. 9.3 kcal
- D. -9.3 kcal

Key: B

Solution:  $\Delta H = \Delta U + \Delta n_g RT$

$$\Delta U = 2.1kcal = 2.1 \times 10^3 cal \left[ \because 1kcal = 10^3 cal \right]$$

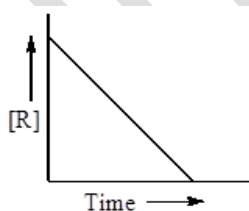
$$\Delta H = (2.1 \times 10^3) + (2 \times 2 \times 300) = 3300 cal$$

Hence;  $\Delta G = \Delta H - T\Delta S$

$$\Delta G = (3300) - (300 \times 20) = -2700$$

$$\Delta G = -2.7kcal$$

56.



If [R] is the concentration of reactant, which of the following is the correct match of given graph.

- A. first order
- B. zero-order
- C. second order
- D. third order

Key: B

Solution: Given graph represents o-order reaction

57. Which of the following is biodegradable polymer

- A. Decron
- B. Nylon – 6
- C. Nylon – 2 – Nylon – 6
- D. Nylon – 6, 6

Key: C

Solution: Nylon – 2 Nylon – 6

58. Incorrect match among the following is

- A.  $[Co(NH_3)_6]Cl_3$  - homoleptic complex – Cationic complex
- B.  $K_3[Al(C_2O_4)_3]$  - homoleptic complex – Anionic complex
- C.  $[Co(NH_3)_3Cl_3]$  - Heteroleptic complex – neutral complex
- D.  $[Pt(NH_3)_4]Cl_2$  - heteroleptic complex – cationic complex

Key: D

Solution:  $[Pt(NH_3)_4]Cl_2$  is homoleptic complex since it has only one kind of ligands.

59.  $K_H$  value for some gases at the same temperature 'T' are given,

Gas	$K_H / kbar$
Ar	40.3
CO <sub>2</sub>	1.67
HCHO	$1.83 \times 10^{-5}$
CH <sub>4</sub>	0.413

where  $K_H$  is Henry's Law constant in water. The order of their solubility in water is

A.  $Ar < CO_2 < CH_4 < HCHO$

B.  $Ar < CH_4 < CO_2 < HCHO$

C.  $HCHO < CO_2 < CH_4 < Ar$

D.  $HCHO < CH_4 < CO_2 < Ar$

Key: A

Solution: Greater the value of  $K_H$  for a gas, lesser is its solubility in water. So, order of solubility of gases in water is

$Ar < CO_2 < CH_4 < HCHO$ .

60. The bond order of  $O_2^+$  is x. The bond orders of  $O_2^-$  and  $O_2^{2+}$  are respectively

A.  $\frac{5}{3}x, \frac{5}{6}x$

B.  $\frac{3}{5}x, \frac{6}{5}x$

C.  $\frac{2}{5}x, \frac{3}{5}x$

D.  $\frac{5}{2}x, \frac{5}{3}x$

Key: B

Solution: Bond order for  $O_2^+$  is:

$$\frac{10-5}{2} = \frac{5}{2} = 2.5 = X$$

Bond order for  $O_2^-$  is:

$$\frac{10-7}{2} = \frac{3}{2} = 1.5$$

Bond order for  $O_2^{2+}$  is:

$$\frac{10-4}{2} = \frac{6}{2} = 3$$

$$2.5 \times \frac{3}{5} = 1.5 (\text{bond order of } O_2^-)$$

$$x \times \frac{3}{5} = 1.5$$

$$2.5 \times \frac{6}{5} = 3 (\text{bond order of } O_2^{2+})$$

$$x \times \frac{6}{5} = 3$$

61. Element furnishing coloured ions in the aqueous medium is

- A. Zn
- B. Hg
- C. Cu
- D. Al

Key: C

Solution:

- Generally Transition elements exhibits colours due to incompletely filled electronic configuration
- Copper exhibits blue colour in aqueous solutions ( $4s^1 3d^{10}$ )
- Zn and Hg both are exhibits no colour due to completely filled electronic configuration

62. Which has maximum number of atoms?

- A. 24 g of C(12)
- B. 56g of Fe(56)
- C. 27g of Al(27)
- D. 108g of Ag(108)

Key: A

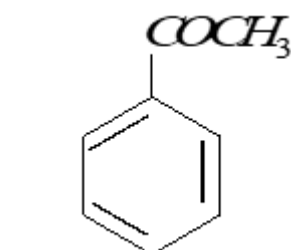
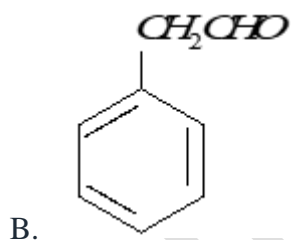
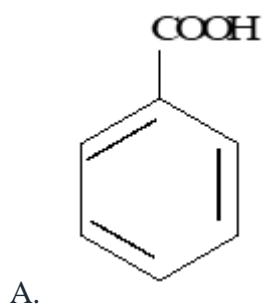
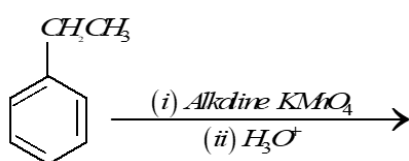
Solution: *No of C atoms* =  $\frac{24}{12} N_0 = 2 N_0$

$$\text{No of Fe atoms} = \frac{56}{56} N_0 = 1N_0$$

$$\text{No of Al atoms} = \frac{27}{27} N_0 = 1N_0$$

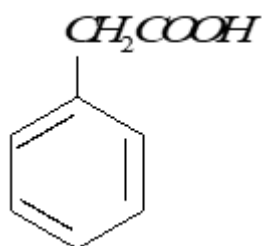
$$\text{No of Fe atoms} = \frac{108}{108} N_0 = 1N_0$$

63. The major product of the following reaction is



C.

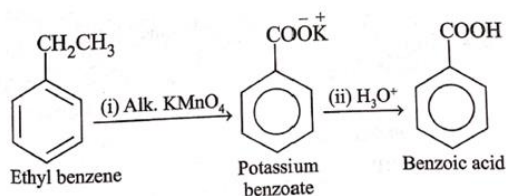




D.

Key: A

Solution: The major product of the given reaction is benzoic acid ( $C_6H_5COOH$ ). On vigorous oxidation of alkyl benzene with acidic or alkaline  $KMnO_4$  aromatic acids are obtained. During oxidation of alkyl benzene. The aromatic nucleus remains intact and the entire chain is oxidized to  $-COOH$  group irrespective of the length of carbon chain



64. The conductivity of a solution containing 2.08 g of anhydrous barium chloride in 200 mL solution is  $6 \times 10^{-3} \text{ ohm}^{-1} \text{ cm}^{-1}$ . The molar conductivity of the solution (in  $\text{ohm}^{-1} \text{ cm}^{-1} \text{ mol}^{-1}$ ) is  $x \times 10^2$ .

The value of x is (Atomic mass of Ba = 137, Cl = 35.5)

- A. 1.2
- B. 2.4
- C. 3.6
- D. 3.0

Key: A

Solution: Mass of  $BaCl_2 = 2.08\text{g}$

Mass of Solution=200 mL

The molarity of  $BaCl_2$  is:

$$\frac{2.08 \text{ g}}{\frac{208 \text{ g/mol}}{0.2 \text{ L}}} = 0.05 \text{ mol/L}$$

Molar conductivity is calculated as:

$$\frac{k \times 1000}{M} = \frac{6 \times 10^{-3} \times 1000}{0.05} = 120 \text{ ohm cm}^{-1} \text{ mol}^{-1} = 1.2 \times 10^2 \text{ ohm cm}^{-1} \text{ mol}^{-1}$$

65. Hinsberg's reagent is

- A.  $C_6H_5SO_3H$
- B.  $C_6H_5OH$
- C.  $C_6H_5SO_2Cl$
- D.  $C_6H_5SO_2Na$

Key: C

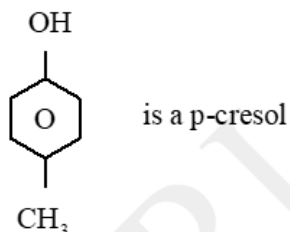
Solution: Reference NCERT (XII) Page No.401

66. 4-methyl phenol is

- A. m- cresol
- B. p-cresol
- C. m-xylene
- D. – xylene

Key: B

Solution:



67. The inter molecular forces present in inert gases are

- A. Ion – ion
- B. Ion – dipole
- C. Dipole – dipole
- D. Dispersion

Key: D

Solution: Inert gases have only london dispersion forces

68. Kraft temperature is the temperature:

- A. below which the aqueous solution of detergents starts freezing
- B. below which the formation of micelles takes place
- C. above which the aqueous solution of detergents starts boiling
- D. above which the formation of micelles takes place

Key: D

Solution: Above Kraft temperature formation of micelles takes place.

69. The atomic number of the metal whose magnetic moment is 2.84 BM in its  $M^{4+}$ .

- A. 21
- B. 24
- C. 29
- D. 22

Key: B

Solution:  $M = 24, Cr^{+4}; 3d^2$

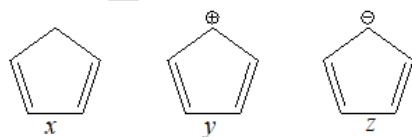
70. Density of a 2.05 M solution of acetic acid in water is 1.02g/mL. The molality of the solution is

- A.  $2.28 \text{ mol kg}^{-1}$
- B.  $0.44 \text{ mol kg}$
- C.  $1.14 \text{ mol kg}^{-1}$
- D.  $3.28 \text{ mol kg}$

Key: A

Solution:  $m = \frac{1000 \times M}{(1000 \times d) - (M \times G.M. \text{ of solute})}$

71. The order of stability of compounds x, y and z:



A.  $x > y > z$

B.  $y > z > x$

C.  $z > x > y$

D.  $x > z > y$

Key: C

Solution: Stability Order,

Aromatic > non-aromatic > Anti Aromatic

72. Which of the following statements regarding the manufacture of  $H_2SO_4$  by Contact process is not true?

A. Sulphur is burnt in air to form  $SO_2$

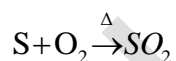
B.  $SO_2$  is catalytically oxidised to  $SO_3$

C.  $SO_3$  is dissolved in water to get 100% sulphuric acid.

D.  $H_2SO_4$  obtained by contact process is of higher purity than that obtained by other processes.

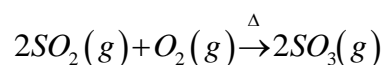
Key: C

Solution: Sulphur dioxide is produced by burning sulphur in excess of oxygen.



Since combustion reactions are irreversible, (a) is incorrect.

$SO_3$  is produced by burning  $SO_2$  in oxygen

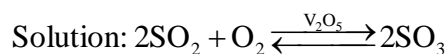


Since less number of molecule are there in the produced side than the reactant side, low temp, high pressure and a specific Catalyst like Pt. is required to produce maximum amount of  $\text{SO}_3$

73. Catalyst used in contact process during the manufacture of sulphuric acid is

- A. Pt
- B. NO
- C. CO
- D. Vanadium pentoxide

Key: D

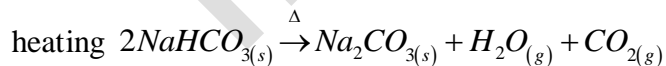


74. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

- A. Baking soda
- B. Soda ash
- C. Washing soda
- D. Caustic soda

Key: A

Solution: Baking soda i.e. sodium bicarbonate is used in fire extinguisher as it produces  $\text{CO}_2$  on



75. Which of the following sets contain only copolymers?

- A. Melamine, Bakelite, PVC

- B. Buna-N, Nylon-6, Polythene  
C. Buna-S, Nylon-6,6, Glyptal  
D. Neoprene, Styron, Polyisoprene

Key: C

Solution: if polymers are formed from two or more types of monomers it is called as co-polymers

76. When sugar 'X' is boiled with dilute  $H_2SO_4$  in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with  $HNO_3$  yields saccharic acid where as 'B' is laevorotatory. The compound 'X' is :

- A. Maltose  
B. Sucrose  
C. Lactose  
D. Starch

Key: B



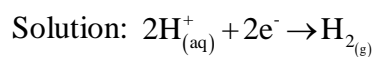
77. When an electric current is passed through acidified water, 112ml of hydrogen gas at N.T.P was collected at the cathode in 965 seconds. The current passed in ampere is

- A. 1.0  
B. 0.5

C. 0.1

D. 2.0

Key: A



At NTP, 22.4 L (or 22400 mL) of  $\text{H}_2 = 1$  mole of  $\text{H}_2$

$$112 \text{ ml of } \text{H}_2 = \frac{112}{22400} \times 1$$

$$= 0.005 \text{ moles of } \text{H}_2$$

$$\text{moles of } \text{H}_2 = \frac{i_{(\text{A})} t_{(\text{S})}}{96500 \text{ c/mole}} \times \text{mole ratio}$$

$$0.005 \text{ ml} = \frac{i_{(\text{A})} \times 965_{(\text{S})}}{96500 \text{ c/mole}} \times \frac{1 \text{ mole } \text{H}_2}{2}$$

$$i = 1 \text{ A}$$

78. In the given reaction  $\text{CH}_3 - \text{C} \equiv \text{C} - \text{CH}_3 + \text{H}_2 \xrightarrow{\text{Na/liquid.NH}_3} (\text{X})$  (X) will be

A. Butane

B. Trans - 2 - butene

C. Cis-2-butene

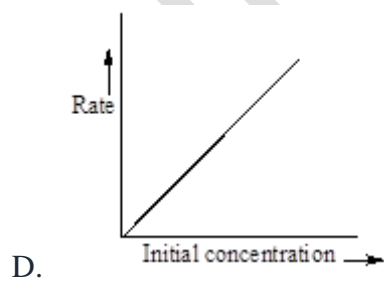
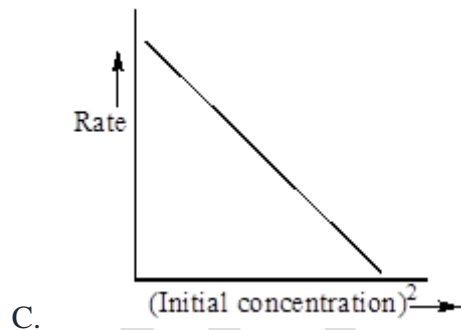
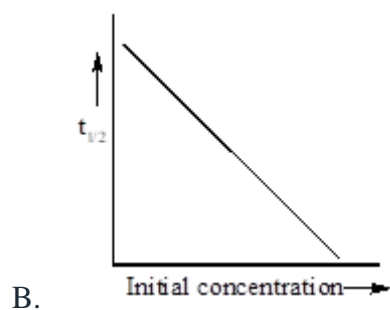
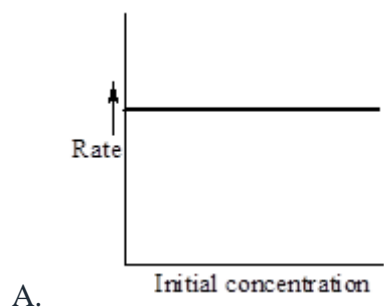
D. 1- Butene

Key: B



Solution: Na/liquid.NH<sub>3</sub> partial reduction results in the formation of trans alkenes

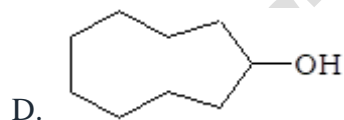
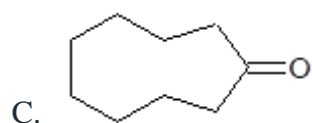
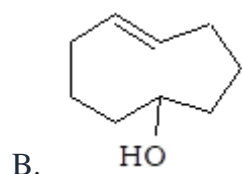
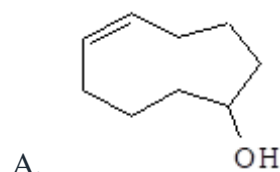
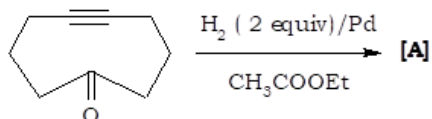
79. Which of the following represents a first order reaction?



Key: D

Solution: Rate = k(concentration) or Rate  $\propto$  (concentration)

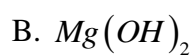
80. Compound [A] in the following reaction is.

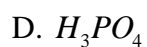


Key: C

Solution: Reactivity towards catalytic hydrogenation:  $C \equiv C > C = C > C = O$ .

81. Which of the following is a strong electrolyte?

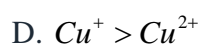
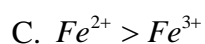
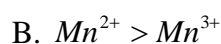
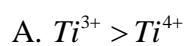




Key: C

Solution: The strong electrolyte ionizes completely in solution. Therefore,  $NH_4OH$ ,  $Mg(OH)_2$  are hydroxides which dissociate completely and  $H_3PO_4$  is strong acid, it ionizes at greater extent compared to  $BaCl_2$  which is salt of weak base and strong acid.

82. In which of the following cases, the stability of two oxidation states is correctly represented



Key: B

Solution:  $Mn^{2+} (3d^5)$  is more stable than  $Mn^{3+} (3d^4)$  due to half-filled orbitals.

83. The number of lone pair of electrons present in the valence shell of Xenon (Z=54) in  $XeOF_4$ ,  $XeF_4$ ,  $XeF_2$  and  $XeF_6$  are respectively.

A. 1,2,3,1

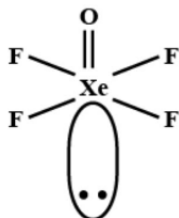
B. 2,1,2,2

C. 3,1,2,1

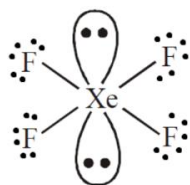
D. 1,3,2,0

Key: A

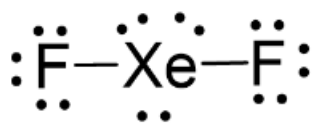
Solution: In XeOF there is only one lone pair of electrons.



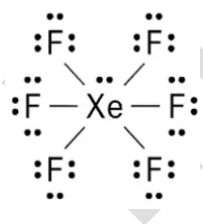
In XeF<sub>4</sub> there are two lone pair of electrons.



In XeF<sub>2</sub> there are three lone pair of electrons.



In XeF<sub>6</sub> there are three lone pair of electrons.



84. Radius of second orbit in Be<sup>3+</sup> ion will be ( r<sub>0</sub> is Bohr's radius )

A. 2 r<sub>0</sub>

B. 0.5 r<sub>0</sub>

C.  $r_0$

D.  $4 r_0$

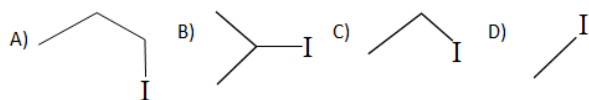
Key: C

Solution:  $r_n = \frac{n^2}{Z} r_0$  ;

$n = 2$  and  $Z = 4$  for  $Be^{3+}$

$$r_n = \frac{2^2}{4} r_0 ;$$

85.



Most reactive towards  $SN_2$  reaction is

A. C

B. B

C. D

D. A

Key: C

Solution: Rate of reactivity towards  $SN_2$  reaction  $\propto \frac{1}{\text{steric hindrance}}$

$\therefore$  reactivity of  $1^\circ RX > 2^\circ RX > 3^\circ RX$

86. Man dies in an atmosphere of carbon monoxide because it

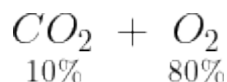
- A. combine with  $O_2$  present in the body to form  $CO_2$
- B. reduces of the organic matter of tissues
- C. combines with haemoglobin of blood, making it incapable of binding  $O_2$
- D. dries up the blood.

Key: C

Solution: Carbon monoxide is poisonous gas

Since it combines with haemoglobin of blood and forming the stable carboxy haemoglobin complex irresveresibly hence the oxygen transportation by haemoglobin decreases . Hence man dies in an atmoshere of high concentration of carbon monoxide

The anti dote used for the poisonous effect of carbon monoxide is carbogen. carbogen is a mixture of



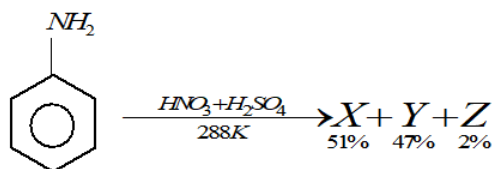
carboxy haemoglobin is 300 times more stable than Oxy haemoglobin

87. Enthalpy of reaction is always negative for

- A. Sublimation
- B. Condensation
- C. Atomization
- D. Formation of compound from it elements

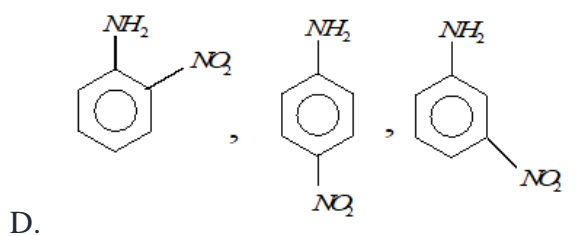
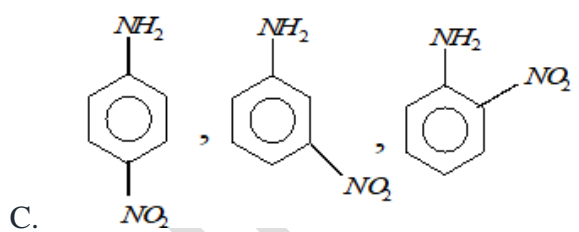
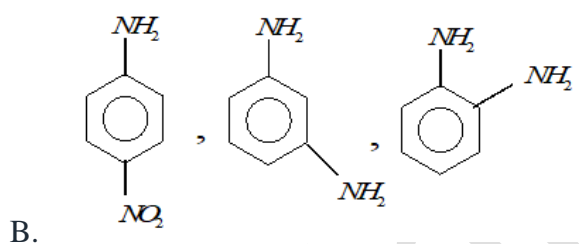
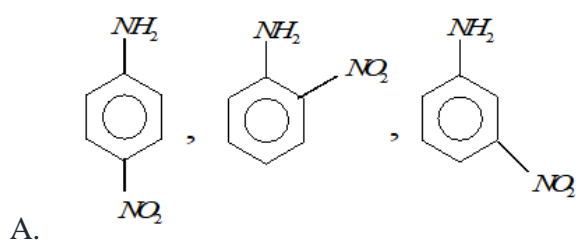
Key: B

Solution: Condensation enthalpy is negative



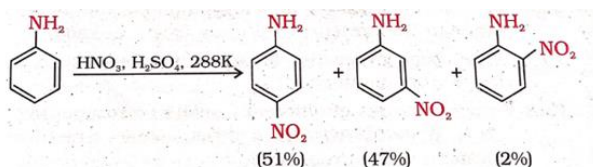
88.

X, Y, and Z are respectively,



Key: C

Solution: Nitration Direct nitration of aniline yields tarry oxidation products in addition to the nitro derivatives. Moreover, in the strongly acidic medium, aniline is protonated to form the anilinium ion which is meta directing. That is why besides the ortho and para derivatives, significant amount of meta derivative is also formed.



89. The spin only magnetic moment of  $[\text{MnBr}_4]^{2-}$  is 5.9 BM. The geometry of the complex ion is

- A. Octahedral
- B. Square planar
- C. Tetrahedral
- D. Square pyramidal

Key: C

Solution:  $\sqrt{n(n+2)} = 5.9$

$$n(n+2) = 35, n = 5$$

$n=5$  means Mn has unpaired electrons are 5. Bromine is a weak field ligand, so back pairing will not occur.

So, the geometry of the compound is tetrahedral.



90. Four gases A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively. Correct order of ease of liquefaction of these gases is their correct order is.

A.  $C > D > B > A$

B.  $D > C > B > A$

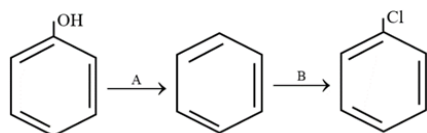
C.  $D > C > A > B$

D.  $C > B > D > A$

Key: B

Solution: Ease of liquification of gas  $\propto$  Critical temperature

91. Identify the reagents A and B used in the following reactions.



A.  $Zn, Cl_2 / FeCl_3$

B.  $Sn / HCl, Cl_2 / h\nu$

C.  $Cl_2 / Fe, FeCl_3$

D.  $Cl_2 / h\nu, Zn$

Key: A

Solution: A =  $Zn$ ; B =  $Cl_2 / FeCl_3$

92. d-block elements can act as catalysts due to their ability to

A. Exhibit variable oxidation states.

B. Coloured ion formation

C. Paramagnetic nature

D. Alloy formation

Key: A

Solution:

- Transition metals have partially filled d- orbitals so they can easily withdraw the electrons from the reagents or give electrons to them depending on the nature of the reaction.
- They also have a tendency to show large no. of oxidation states and the ability to form complexes which makes them a good catalyst.

93. Y gm of a non-volatile solute of molar mass M is dissolved in 250 g of benzene. If  $K_b$  is molal elevation constant, the value of  $\Delta T_b$  is given by:

A.  $\frac{4M}{K_b Y}$

B.  $\frac{4K_b Y}{M}$

C.  $K_b \frac{Y}{4M}$

D.  $K_b \frac{Y}{M}$

Key: B

Solution:  $\Delta T_b = K_b \times \frac{W_{solute}}{GMW_{solute}} \times \frac{1000}{W_{solvent(g)}}$

$$= \frac{K_b \times Y \times 1000}{M \times 250} = \frac{4K_b Y}{M}$$

94. The bond length of C – O ( $\text{CH}_3 - \text{OCH}_3$ ) of ether

- A. 111 A°
- B. 1.41 A°
- C. 2.8 A°
- D. 4.8 A°

Key: B

Solution: C – O bond length 1.41 A°

95. In corundum, oxide ion's have hcp arrangement, what percentage of octahedral voids are occupied by Al?

- A. 33%
- B. 50%
- C. 66%
- D. 75%

Key: C

Solution: Corundum is  $\text{Al}_2\text{O}_3$  for  $3\text{O}^{2-}$  ions octahedral sites are 3. Out of three sites only 2 are

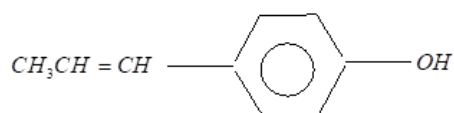
occupied by  $\text{Al}^{+3}$  atoms.

sites occupied by

$$\text{Al}^{+3} \text{ ions} = 2/3 \times 100 = 66\%$$

96. The reaction of

67 |

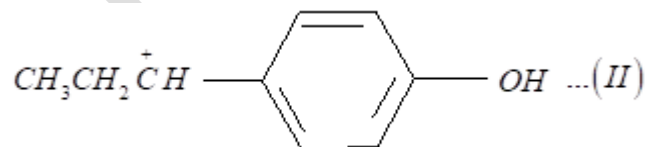
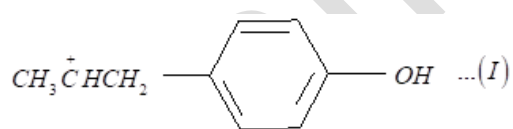


with HBr gives

- A.  $\text{CH}_3\text{CH}_2\text{CHBr}-\text{C}_6\text{H}_4-\text{Br}$
- B.  $\text{CH}_3\text{CH}_2\text{CHBr}-\text{C}_6\text{H}_4-\text{OH}$
- C.  $\text{CH}_3\text{CHBrCH}_2-\text{C}_6\text{H}_4-\text{Br}$
- D.  $\text{CH}_3\text{CH}_2\text{CHBr}-\text{C}_6\text{H}_4-\text{Br}$

Key: B

Solution: Of The two possible carbocations, benzylic type (II) is more stable than the I.



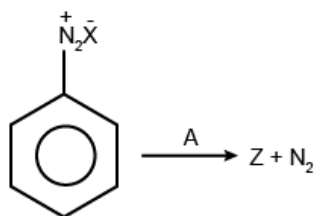
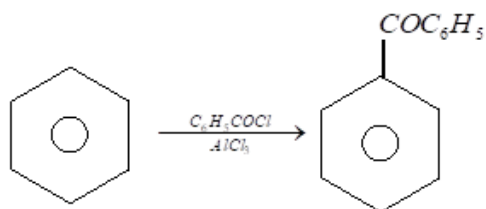
hence bromination occurs on 3rd C atom

97. Benzophenone can be obtained by \_\_\_\_\_.

- A. Benzoyl chloride + Benzene +  $AlCl_3$
- B. Benzoyl chloride + Diphenyl amine
- C. Benzoyl chloride + Methyl magnesium chloride
- D. Benzene + Carbon monoxide +  $ZnCl_2$

Key: A

Solution:



98.

Identify correct match?

- A. A = KI ; Z = Iodobenzene
- B. A = KCl ; Z = Chlorobenzene
- C. A = KBr ; Z = Bromobenzene
- D. A = KF ; Z = Fluorobenzene

Key: A

Solution: Replacement of diazonium group by iodine does not require presence of cuprous halide

99. The rate constant of a first order reaction at  $27^\circ\text{C}$  is  $10^{-3} \text{ min}^{-1}$ . The 'temperature coefficient' of this reaction is 2. What is the rate constant (in  $\text{min}^{-1}$ ) at  $17^\circ\text{C}$  for this reaction?

A.  $10^{-3}$

B.  $5 \times 10^{-4}$

C.  $2 \times 10^{-3}$

D.  $10^{-2}$

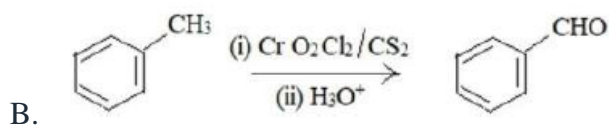
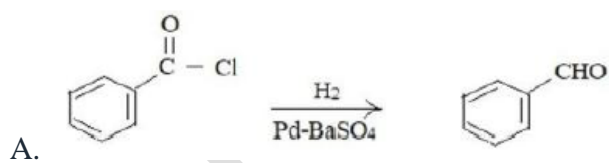
Key: B

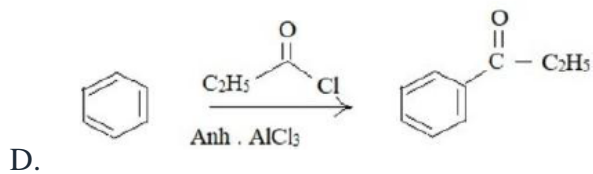
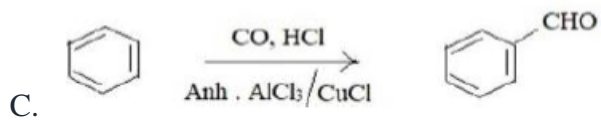
Solution:  $K = 10^{-3} \rightarrow \frac{16^3}{2} = K$

at  $27^\circ\text{C}$       at  $17^\circ\text{C}$

$\therefore K = 5 \times 10^{-4} \text{ min}^{-1}$

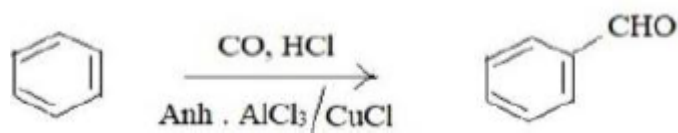
100. Which of the following represents Gatterman-Koch reaction?





Key: C

Solution: In the Gatterman – Koch reaction, benzene or its derivative is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous aluminum chloride or cuprous chloride, resulting in the formation of benzaldehyde or substituted benzaldehyde.



## MATHEMATICS

101. For  $x \in \mathbb{R}$  if  $f(x) = \sqrt{\log_{10} \frac{3-x}{x}}$ , then the domain of  $f$  is

A.  $(0, \frac{3}{2}]$

B.  $(\frac{3}{2}, 3)$

C.  $[0, 1]$

D. (0,1]

Key: B

Solution: We have,  $f(x) = \sqrt{\log_{10} \frac{3-x}{x}}$

Clearly,  $f(x)$  is defined, if

$$\log_{10} \frac{3-x}{x} \geq 0 \text{ and } \frac{3-x}{x} > 0$$

$$\text{P } \frac{3-x}{x} \geq 10^0 \text{ and } \frac{x-3}{x} < 0$$

$$\text{P } \frac{3-x}{x} \geq 1 \text{ and } \frac{x-3}{x} < 0$$

$$\text{P } \frac{3-x}{x} \geq 1 \text{ and } \frac{x-3}{x} < 0$$

$$\text{P } 0 < x \leq \frac{3}{2} \text{ and } x < 3$$

102. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 1, 2 and 6, then the other two observations can be

A. 2 and 9

B. 3 and 8

C. 4 and 7

D. 5 and 6

Key: C



Solution: Let the two unknown items be x and y, then  $Mean = 4 \Rightarrow \frac{1 + 2 + 6 + x + y}{5} = 4$

$$X + y = 11$$

And variance = 5.2

$$\frac{1^2 + 2^2 + 6^2 + x^2 + y^2}{5} - (mean)^2 = 5.2$$

$$41 + x^2 + y^2 = 5 \times 5.2 + (4)^2$$

$$41 + x^2 + y^2 = 106$$

$$x^2 + y^2 = 65 \dots\dots\dots(ii)$$

Solving Equations (i) and (ii) for x and y, we get  $x = 4, y = 7$  or  $x = 7, y = 4$

103. The number of ways of arranging 8 Eamcet Question papers so that best and worst never come together is
- A. 30240
  - B. 21600
  - C. 5040
  - D. 4320

Key: A

Solution: Except the best and worst papers, the remaining 6 papers can be arranged in 6! ways. There are 7 places in between, before and after these 6 papers. In these 7 places best and worst papers can be arranged in  ${}^7P_2$  ways

Total no of ways of arranging the 8 papers =  $6!(7P_2)$

104. If  $\frac{d}{dx} \frac{ax^4 + x^2 + 1}{1 + x + x^2} = ax + b$ , then (a, b) =

- A. (-1, 2)
- B. (-2, 1)
- C. (2, -1)
- D. (1, 2)

Key: C

Solution:  $x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$

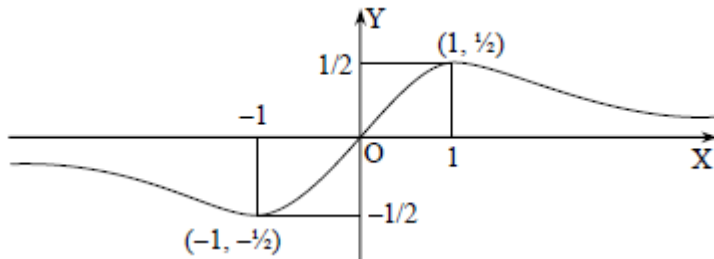
$$\frac{d}{dx} \frac{ax^4 + x^2 + 1}{1 + x + x^2} = \frac{d}{dx} (1 - x + x^2) = 2x - 1$$

105. The function  $f : \mathbb{R} \rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$  defined as  $f(x) = \frac{x}{1 + x^2}$ , is:

- A. invertible
- B. injective but not surjective
- C. surjective but not injective
- D. neither injective nor surjective

Key: C

Solution: For,  $f(x) = \frac{x}{1+x^2}$  the curve has graph as shown



Any line parallel to x-axis cuts the graph more than one point, hence the function is many-to-one.

Let  $y = \frac{x}{1+x^2}$

$x^2y - x + y = 0$

$D^3 - 0D^2 - 4y^3 = 0$

$y^2 - \frac{1}{4} = 0$

$y = \pm \frac{1}{2}$   
Range of  $f = \text{codomain}$  So  $f$  is onto

So given function is onto but not one-one for  $f : \mathbb{R} \rightarrow \left[ -\frac{1}{2}, \frac{1}{2} \right]$

106. If  $g(x) = x^2 + x - 1$  and  $(g \circ f)(x) = 4x^2 - 10x + 5$  then  $f\left(\frac{5}{4}\right)$  is

A.  $\frac{3}{2}$

B.  $-\frac{1}{2}$

C.  $\frac{1}{2}$

D.  $-\frac{3}{2}$

Key: B

Solution:  $Q g(x) = x^2 + x - 1$

$$(g \circ f)(x) = g(f(x)) = f(x)^2 + f(x) - 1$$

$$(g \circ f)\left(\frac{5}{4}\right) = g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10\left(\frac{5}{4}\right) + 5 = \frac{-5}{4}$$

$$P \quad \frac{f\left(\frac{5}{4}\right)^2}{\left(\frac{5}{4}\right)^2} + \frac{f\left(\frac{5}{4}\right)}{\frac{5}{4}} - 1 = \frac{-5}{4}$$

$$P \quad \frac{f\left(\frac{5}{4}\right)^2}{\left(\frac{5}{4}\right)^2} + \frac{f\left(\frac{5}{4}\right)}{\frac{5}{4}} - \frac{1}{4} = 0$$

$$P \quad \frac{f\left(\frac{5}{4}\right)^2}{\left(\frac{5}{4}\right)^2} + \frac{1}{2} = 0 \quad P \quad f\left(\frac{5}{4}\right) = \frac{-1}{2}$$

107.  $\tan \cos^{-1} \frac{2}{\sqrt{5}} + 2 \tan^{-1} \frac{1}{5} + 2 \tan^{-1} \frac{1}{8} = \underline{\hspace{2cm}}$

A. 1

B.  $\frac{1}{4}$

C.  $\frac{5}{4}$

D. 2

Key: D

Solution:  $\tan^{-1} \frac{1}{2} + 2 \tan^{-1} \frac{3}{4}$

$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{3}{4}$

$$= \frac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1 \cdot 3}{2 \cdot 4}} = \frac{\frac{5}{4}}{\frac{5}{8}} = 2$$

108. Number of roots of equation  $\sin^{-1}(1-x) + 2 \sin^{-1} x = \frac{\pi}{2}$ , is

A. 1

B. 2

C. 3

D. 4

Key: B

$$\text{Solution: } \sin^{-1}(1-x) + 2\sin^{-1}x = \frac{\pi}{2}$$

$$P \quad \sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1}x$$

$$P \quad 1-x = \sin\left(\frac{\pi}{2} - 2\sin^{-1}x\right)$$

$$P \quad 1-x = \sin\frac{\pi}{2}\cos(2\sin^{-1}x) - \cos\frac{\pi}{2}\sin(2\sin^{-1}x)$$

$$P \quad 1-x = \cos(2\sin^{-1}x)^2$$

$$P \quad 1-x = \cos(\cos^{-1}(1-2x^2))$$

$$P \quad 2x^2 - x = 0$$

$$P \quad x = 0 \text{ or } x = \frac{1}{2}$$

109. The values of x if the matrix  $A = \begin{pmatrix} 2x & y & z \\ x & -y & z \\ x & -y & z \end{pmatrix}$  satisfies the equation  $A^T \cdot A = I$

$$A. \pm \frac{1}{\sqrt{6}}$$

B.  $\pm \frac{1}{\sqrt{2}}$

C.  $\pm \frac{1}{\sqrt{3}}$

D.  $\pm 1$

Key: B

Solution:  $A = \begin{pmatrix} 2y & z \\ x & -z \\ x & -y & z \end{pmatrix}$

$A^T = \begin{pmatrix} x & x \\ 2y & y & -z \\ -z & z & z \end{pmatrix}$

Now,  $A^T \cdot A = I$

$$\begin{pmatrix} 2x & x \\ 2y & y & -z \\ -z & z & z \end{pmatrix} \begin{pmatrix} 2y & z \\ x & -z \\ x & -y & z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have

$$2x^2 = 1 \text{ or } x = \pm \frac{1}{\sqrt{2}}$$

$$6y^2 = 1 \text{ or } y = \pm \frac{1}{\sqrt{6}}$$

$$3z^2 = 1 \text{ or } z = \pm \frac{1}{\sqrt{3}}$$

110. If A, B, C are the cofactors of 2, 3, -5 in the matrix  $\begin{vmatrix} 1 & 0 & 5 \\ 1 & 2 & -2 \\ 4 & -5 & 3 \end{vmatrix}$  then the ascending order

of A, B, C is

A. A, B, C

B. B, C, A

C. A, C, B

D. B, A, C

Key: B

Solution: cofactor of 2 =  $\begin{vmatrix} -1 & 5 \\ -4 & 3 \end{vmatrix} = -3 + 20 = 17$

A = 17 cofactor of 3 =  $\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} = -2$

B = -2



$$\text{cofactor of } -5 = - \begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix} = - (2 - 5) = 3$$

Ascending order is B, C, A

111. If  $A = \begin{vmatrix} 2x & 3 \\ 4 & 1 \end{vmatrix}$  and  $|A^3| = 216$ , then find the value of x.

A. 11

B. 16

C. 5

D. 9

Key: D

Solution: Given,  $A = \begin{vmatrix} 2x & 3 \\ 4 & 1 \end{vmatrix}$  and  $|A^3| = 216$

$$|A| = \begin{vmatrix} 2x & 3 \\ 4 & 1 \end{vmatrix}$$

$$|A| = 2x - 12$$

As we know,  $|A^3| = |A|^3$

Given,  $|A^3| = 216$

$$P | A |^3 = 216$$

$$\text{Now, } P (2x - 12)^3 = 216$$

$$P (2x - 12) = 6$$

$$P 2x = 18$$

$$P x = 9$$

Therefore,  $x=9$

112.  $x_1 + 2x_2 + 3x_3 = a$ ,  $2x_1 + 3x_2 + x_3 = b$ ,  $3x_1 + x_2 + 2x_3 = c$  this system of equations has

A. Infinite solution

B. No solution

C. Unique solution

D. None of these

Key: C

Solution: We have,  $x_1 + 2x_2 + 3x_3 = a$

$$2ax_1 + 3x_2 + x_3 = c$$

$$3bx_1 + x_2 + 2x_3 = c$$

Let  $a = b = c = 1$ .

$$\text{Then } D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1(5) - 2(1) + 3(-7) = -18 \neq 0$$

$$D_x = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -3$$

Similarly  $D_y = D_z = -3$ . Now,  $x = \frac{D_x}{D} = \frac{1}{6}$ ,  $y = z = \frac{1}{6}$

Hence  $D \neq 0$ ,  $x = y = z$ , i.e., unique solution.

113. Let  $f(x) = \begin{cases} \frac{\log(1+5x) - \log(1+ax)}{x}, & x \neq 0 \\ 10, & x = 0 \end{cases}$  be continuous at  $x = 0$ . Then  $a = \underline{\hspace{2cm}}$

A. 10

B. -10

C. 5

D. -5

Key: D

Solution:  $\lim_{x \rightarrow 0} f(x) = f(0)$   
 $5 - a = 10 \Rightarrow a = -5$

114. If  $x = \cos^3 \theta, y = \sin^3 \theta$  then  $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} =$

A.  $\tan^2 \theta$

B.  $\sec^2 \theta$

C.  $\sec \theta$

D.  $|\sec \theta|$

Key: D

Solution:  $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\sin^2 \theta \cdot \cos \theta}{-3\cos^2 \theta \cdot \sin \theta} = -\tan \theta$

115. A stone is dropped into a quiet pond and waves move in circles outward from the place where it strikes, at a speed of 30 cm per second. At that instant when the radius of the wave ring is 50 mt, the rate of increase in the area of the wave ring is

A.  $0.75\pi$  sq.cm / sec

B. 30 sq.cm/sec

C.  $30\pi$ sq.m / sec

D.  $0.4\pi$ sq.cm / sec

Key: C

Solution:  $A = \pi r^2$   $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(50) \frac{dr}{dt}$   
 $= 30\pi \frac{dr}{dt}$

116.  $f(x) = \frac{2 - |x^2 + 5x + 6|}{a^2 + 1}$   $x = -2$ . Then the range of a so that f(x) has a maxima at x = -2

A.  $|a|^3 > 1$

B.  $|a| < 1$

C.  $a > 1$

D.  $a < 1$

Key: A

Solution:  $a^2 + 1 > 2$   $|a|^3 > 1$

117.  $\int \frac{x^2}{(x^2 + 1)(x^2 + 9)} dx = \frac{1}{p} \tan^{-1} \frac{x}{3} + \frac{1}{r} \tan^{-1} \frac{x}{9} + C$ , where C is constant of integration, then p +

q + r =

A. 12

B. 22

C. 24

D. 26

Key: A

Solution:  $x^2 = t$

$$\frac{t}{(t+1)(t+9)} = \frac{A}{t+1} + \frac{B}{t+9}$$

$$A = \frac{-1}{8}, B = \frac{9}{8}$$

$$\int I = \frac{1}{8} \int \frac{9}{x^2+9} - \frac{1}{x^2+1} dx = \frac{1}{8} \left[ 3 \tan^{-1} \frac{x}{3} - \tan^{-1} \frac{x}{1} \right] + C$$

$$p = 8, q = 3, r = 1$$

118.  $\int_{e^{-1}}^{e^2} \left| \frac{\ln x}{x} \right| dx =$

A.  $\frac{2}{5}$

B. 2

C. 5

D.  $\frac{5}{2}$

Key: D

$$\text{Solution: } \int_{\frac{1}{e}}^1 \frac{\log x}{x} dx + \int_1^{e^2} \frac{\log x}{x} dx$$

$$= - \frac{(\log x)^2}{2} \Big|_{\frac{1}{e}}^1 + \frac{(\log x)^2}{2} \Big|_1^{e^2}$$

119. The area bounded by the curves  $y = e^x$ ,  $y = 2x - x^2$  and the lines  $x = 0, x = 2$  is  $K - \frac{7}{3}$  where  $K =$

A.  $e^2$

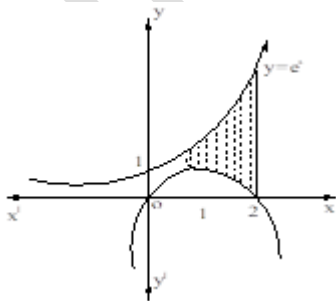
B.  $e$

C.  $\frac{e^2}{2}$

D.  $-e^2$

Key: A

Solution:



$$A = \int_0^2 (e^x - 2x + x^2) dx$$

$$= e^x - x^2 + \frac{x^3}{3} \Big|_0^2 = e^2 - 4 + \frac{8}{3} - 1$$

$$= e^2 - \frac{7}{3}$$

120. Solution of the differential equation  $\frac{dy}{dx} = \frac{x + y + 7}{2x + 2y + 3}$  is \_\_\_\_\_.

A.  $6(x + y) + 11 \log(3x + 3y + 10) = 9x + c$

B.  $6(x + y) - 11 \log(3x + 3y + 10) = 9x + c$

C.  $(x + y) - \log(x + y + \frac{10}{3}) = x + c$

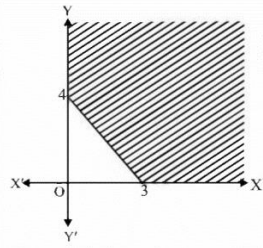
D.  $3(x + y) - \log(x + y + 10) = 9x + c$

Key: B

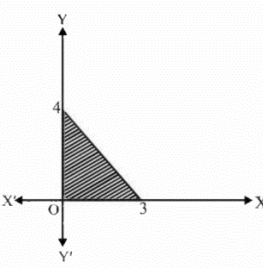
Solution:  $x + y = t$

121. Graph of the constraints  $\frac{x}{3} + \frac{y}{4} \leq 1, x \geq 0, y \geq 0$  is

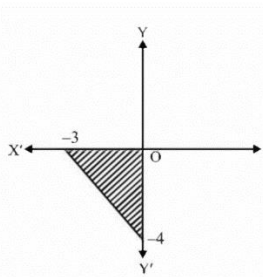




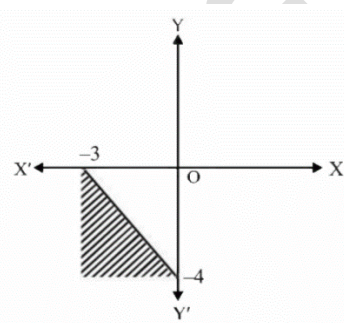
A.



B.



C.



D.

Key: B

Solution: Take a test point  $O(0,0)$ .

Equation of the constraint is  $\frac{x}{3} + \frac{y}{4} \leq 1$

$\text{P } 4x + 3y \leq 12$

Since  $4(0) + 3(0) \leq 12$ , the feasible region lies below

the line  $4x + 3y = 12$

Since,  $x \geq 0, y \geq 0$

the feasible region lies in the first quadrant.

122. If  $5 \tan \theta = 1$ , then  $\frac{5 \sin \theta - \cos \theta}{7 \cos \theta + 3 \sin \theta} =$

A. 0

B. 1

C. 2

D. 3

Key: A

Solution:  $\tan \theta = \frac{1}{5} \Rightarrow \frac{5 \tan \theta - 1}{7 + 3 \tan \theta} = \frac{5 \left(\frac{1}{5}\right) - 1}{7 + 3 \left(\frac{1}{5}\right)} = 0$

123. The distance between the parallel lines  $4y - 2x + 1 = 0, x - 2y + 1 = 0$  is

A.  $\frac{3}{2\sqrt{5}}$

B.  $\frac{2}{\sqrt{13}}$

C.  $\frac{11}{2}$

D.  $\frac{4}{\sqrt{5}}$

Key: A

Solution:  $2x - 4y - 1 = 0$   
 $2x - 4y + 2 = 0$

Formulae: Distance between parallel lines =  $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

Distance between parallel lines =  $\frac{|1 + 2|}{\sqrt{4 + 16}} = \frac{3}{2\sqrt{5}}$

124. The number of common tangents to the circles  $x^2 + y^2 - 2x - 6y + 9 = 0$   
and  $x^2 + y^2 + 6x - 2y + 1 = 0$  is

A. 1

B. 2

C. 3

D. 4

Key: D

$$\text{Solution: } C_1 C_2 = \sqrt{(1+3)^2 + (3-1)^2}$$

$$= \sqrt{20} = 2\sqrt{5} > r_1 + r_2$$

Circles are not touching each other so,

Number of common tangents are 4.

25. In a single throw of a pair of dice. The probability of getting the sum a perfect square is

A.  $\frac{1}{18}$

B.  $\frac{7}{36}$

C.  $\frac{1}{6}$

D.  $\frac{2}{9}$

Key: B

Solution: Total no. of out com  $n(s) = 6^2 = 36$

Favourable outcomes =  $\{(1,3)(3,1)(2,2)(4,5)(5,4)(3,6)(6,3)\} = 7$

$$\text{Probability } P(E) = \frac{n(E)}{n(S)} = \frac{7}{36}$$

126. Let  $Z = 1 + i$  and  $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$  then the value of  $\frac{12}{\pi} \text{Arg}(z_1)$  is

A. 9

B. 5

C. 6

D. -5

Key: A

$$\text{Solution: } z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{z(1 + i\bar{z})}{z\bar{z}(1 - z) + 1}$$

$$z_1 = \frac{z + i|z|^2}{|z|^2(-i) + 1}$$

$$z_1 = \frac{1 + i + 2i}{1 - 2i} =$$

$$z_1 = \frac{(1+3i)(1+2i)}{5}$$

$$z_1 = \frac{-5+5i}{5} = -1+i$$

$$\text{Arg}(z_1) = \frac{3\pi}{4}$$

127.  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1-x+x^2}}{3^x - 1}$  is equal to

A.  $\frac{1}{\log_e^3}$

B.  $\log_e^9$

C.  $\frac{1}{\log_e^9}$

D.  $\log_e^3$

Key: C

Solution:  $\text{Lt}_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1-x+x^2}}{3^x - 1}$

Rationalize we get

$$\lim_{x \rightarrow 0} \frac{x}{3^x - 1} \cdot \frac{1}{\sqrt{1-x^2} + \sqrt{1-x+x^2}}$$

$$\frac{1}{\log_e 3} \cdot \frac{1}{2} = \frac{1}{2 \log_e 3} = \frac{1}{\log_e 9}$$

$$= \frac{1}{2 \log_e 3} = \frac{1}{\log_e 9}$$

128. Let  $R = \{(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)\}$  be relation on the set  $A = \{3,6,9,12\}$ . Then relation is

- A. Reflexive and transitive
- B. Reflexive and symmetric
- C. Equivalence
- D. symmetric and transitive

Key: A

Solution: By concept.

129. For a relation to be an equivalence relation, which of the following conditions should it satisfy?

- A. Reflexive, Symmetric, and Transitive
- B. Reflexive and Symmetric
- C. Reflexive, and Transitive

D. Only Inverse

Key: A

Solution: Let A be a relation defined on set X and Y such that

$$A = \{(x, y) \text{ where } x \in X \text{ and } y \in Y\}$$

Now, we know that:

A relation is known as an **equivalence relation** when it satisfies the condition of being **Reflexive, Symmetric, and Transitive**.

Thus, A will be an **equivalence relation** only if it is:

**i. Reflexive:**  $x \sim x$

**ii. Symmetric:** [ If  $x \sim y$ , then  $y \sim x$  ] and

**iii. Transitive:** [ If  $x \sim y$  &  $y \sim z$ , then  $x \sim z$  ].

130. The principal value of  $\sin^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is

A.  $-\frac{2\pi}{3}$

B.  $-\frac{\pi}{3}$



C.  $\frac{4\pi}{3}$

D.  $\frac{5\pi}{3}$

Key: B

Solution:  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$

131.  $\sec\left[\tan^{-1}5 + \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{3}{4}\right] =$

A.  $\frac{3}{5}$

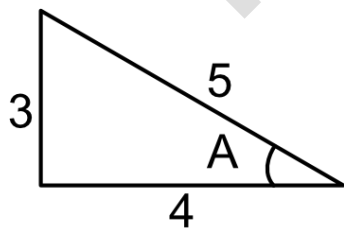
B.  $\frac{5}{3}$

C.  $\frac{4}{5}$

D.  $\sqrt{2}$

Key: B

Solution:



$$\sec\left[\tan^{-1}5 + \cot^{-1}5 - \tan^{-1}\frac{3}{4}\right]$$

$$\sec\left[\frac{\pi}{2} - \tan^{-1}\frac{3}{4}\right] = \operatorname{cosec}\left(\tan^{-1}\frac{3}{4}\right)$$

$$\tan^{-1}\frac{3}{4} = A \Rightarrow \tan A = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{5}{3}$$

132. If  $A - 2B = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$  and  $2A - 3B = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$  then  $B =$

A.  $\begin{pmatrix} -5 & 7 \\ 5 & 1 \end{pmatrix}$

B.  $\begin{pmatrix} -5 & 7 \\ -5 & -1 \end{pmatrix}$

C.  $\begin{pmatrix} -5 & 7 \\ 5 & -1 \end{pmatrix}$

D.  $\begin{pmatrix} -5 & -7 \\ -5 & -1 \end{pmatrix}$

Key: B

Solution:  $A - 2B = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ ,  $2A - 3B = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$

$$2A - 4B - (2A - 3B) = \begin{bmatrix} 2 & -4 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$$

$$-B = \begin{bmatrix} 5 & -7 \\ 5 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 7 \\ -5 & -1 \end{bmatrix}$$

133. If  $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$  then  $A^2 =$

A.  $A^2$

B.  $A$

C.  $0$

D.  $1$

Key: B

Solution:

$$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

134. If the matrix  $A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix}$  satisfies  $A(A^3 + 3I) = 2I$ , then the value of  $k$  is :

A.  $1$

B.  $\frac{1}{2}$

C.  $-1$

D.  $-\frac{1}{2}$

Key: B

Solution:  $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$$= \begin{bmatrix} 2k & -2 \\ -k & 2k+1 \end{bmatrix}$$

$$A^4 = A^2 A^2$$

$$= \begin{bmatrix} 2k & -2 \\ -k & 2k+1 \end{bmatrix} \begin{bmatrix} 2k & -2 \\ -k & 2k+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4k^2 + 2k & -4k - 4k - 2 \\ -2k^2 + 2k^2 + k & 2k + (2k+1)^2 \end{bmatrix}$$

$$\therefore A^4 + 3A = 2I$$

$$\therefore 4k^2 + 2k = 2$$

$$2k^2 + k - 1 = 0$$

$$k = -1, \frac{1}{2}$$

135.  $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$  is equal to

A.  $4abc$

B.  $abc$

C.  $a^2b^2c^2$

D.  $4a^2bc$

Key: A

Solution:

$$\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2 - R_3$$

$$\begin{vmatrix} 0 & -2c & -2b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$$

$$\begin{vmatrix} 0 & -2c & -2b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$$

$$= 2c(ab + b^2 - bc) - 2b(bc - c^2 - ac)$$

$$= 2abc + 2cb^2 - 2bc^2 - 2b^2c + 2bc^2 + 2abc$$

$$= 4abc$$

136. If  $\text{Adj} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$  then  $[a, b] =$

- A.  $[-4,1]$
- B.  $[-4,-1]$
- C.  $[4,1]$
- D.  $[4,-1]$

Key: C

Solution:  $a = (-1)^{2+1}(0-4) = 4$

$b = (-1)^{3+3}(1-0) = 1$

137. If  $f(x) = \begin{cases} 1+6x-3x^2, & x \leq 1 \\ x + \log_2(b^2+7), & x > 1 \end{cases}$  is continuous at all real  $x$ , then  $b =$

- A.  $\pm 1$
- B. 0
- C.  $\pm 5$
- D.  $\pm 2$

Key: A

Solution:

$$\text{If } f(x) = \begin{cases} 1+6x-3x^2, & x \leq 1 \\ x + \log_2(b^2+7), & x > 1 \end{cases}$$

$$= \lim_{x \rightarrow 1^-} (1+6x-3x^2)$$

$$= 4$$

$$= \lim_{x \rightarrow 1^+} (x + \log_2(b^2 + 7))$$

$$= 1 + \log_2(b^2 + 7)$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$4 = 1 + \log_2(b^2 + 7)$$

$$b = \pm 1$$

138. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$  then  $\frac{dy}{dx} =$

A.  $\sqrt{\frac{1-x^2}{1-y^2}}$

B.  $\sqrt{(1-x^2)(1-y^2)}$

C.  $\sqrt{\frac{1-y^2}{1-x^2}}$

D.  $\frac{1}{\sqrt{(1-x^2)(1-y^2)}}$

Key: C

Solution: Put  $x = \sin A$ ,  $y = \sin B$

$$\cos A + \cos B = a(\sin A - \sin B)$$

$$2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) = a \left(2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\right)$$

$$\cot\left(\frac{A-B}{2}\right) = a$$

$$A - B = 2 \cot^{-1} a$$

it implies that  $\sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$

Differentiate both sides with respect to  $x$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

139. The function,  $f(x) = (3x-7)x^{2/3}$ ,  $x \in \mathbf{R}$  is increasing for all  $x$  lying in :

A.  $\left(-\infty, \frac{14}{15}\right)$

B.  $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

C.  $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

D.  $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

Key: C



Solution:  $f(x) = (3x - 7) \cdot x^{2/3}$

$$f'(x) = 3x^{2/3} + (3x - 7) \cdot \frac{2}{3} x^{-1/3}$$

$$\therefore f(x) \text{ is increasing in } (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right) = \frac{5x - 14}{3x^{1/3}}$$

$$\therefore f'(x) > 0 \text{ then } x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

140. The angle between curves  $x^2 = 8y$  and  $y^2 = 8x$  at  $(8, 8)$  is

A. 0

B.  $\tan^{-1}(3)$

C.  $\tan^{-1}\left(\frac{3}{4}\right)$

D.  $\tan^{-1}\left(\frac{4}{3}\right)$

Key: C

Solution:  $x^2 = 8y$  —(1),  $y^2 = 8x$  —(2)

P.I =  $(8, 8)$

Diff (1) w.r.t.  $x$

$$2x = 8 \frac{dy}{dx} \text{ diff. w.r.t. 'x'}$$

$$\frac{dy}{dx} = \frac{x}{4}$$

$$m_1 = \left( \frac{dy}{dx} \right)_{(8,8)} = \frac{8}{4} = 2$$

$$y^2 = 8x$$

Diff. w.r.t. 'x'

$$2y \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{4}{y}$$

$$m_2 = \left( \frac{dy}{dx} \right)_{(8,8)} = \frac{4}{8}$$

$$= \frac{1}{2}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} \right|$$

$$= \frac{3/2}{2}$$

$$= \frac{3}{4}$$

$$\theta = \tan^{-1}(3/4)$$

141.  $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$  equals

A.  $\frac{1}{2} \log \tan \left( \frac{x}{2} + \frac{\pi}{12} \right) + c$

B.  $\frac{1}{2} \log \tan \left( \frac{x}{2} - \frac{\pi}{12} \right) + c$

C.  $\log \tan \left( \frac{x}{2} + \frac{\pi}{12} \right) + c$

D.  $\log \tan \left( \frac{x}{2} - \frac{\pi}{12} \right) + c$

Key: A

Solution:  $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$

$$= \frac{1}{2} \int \sec \left( x - \frac{\pi}{3} \right) dx$$

$$= \frac{1}{2} \log \tan \left( \frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4} \right) + c$$

$$= \frac{1}{2} \log \tan \left( \frac{x}{2} + \frac{\pi}{12} \right) + c.$$

142. If  $\int x \cos^{-1} x dx = \frac{1}{m} \left[ (2x^2 - 1) \cos^{-1} x - x \sqrt{1 - x^2} \right] + c$  then m =

A. 2

B. 3

C. 4

D. 5

Key: C

$$\text{Solution: } \frac{1}{4} \left[ (2x^2 - 1) \cos^{-1} x - x \sqrt{1 - x^2} \right] + c$$

143. The area bounded by the curve  $y = x^3$ , the x-axis and the ordinate at  $x = -2$  and  $x = 1$  is

A.  $\frac{9}{2}$

B.  $\frac{15}{2}$

C.  $\frac{15}{4}$

D.  $\frac{17}{4}$

Key: D

Solution: The function  $y = x^3$  is negative for  $x < 0$  and is positive for  $x > 0$

$$\text{Required area} = \int_{-2}^1 |x^3| dx = -\int_{-2}^0 x^3 dx + \int_0^1 x^3 dx$$

$$= \left[ \frac{-x^4}{4} \right]_{-2}^0 + \left[ \frac{x^4}{4} \right]_0^1 = -\left[ 0 - \frac{(-2)^4}{4} \right] + \left[ \frac{1}{4} - \frac{0}{4} \right]$$

$$= -\left[\frac{-16}{4}\right] + \frac{1}{4} = \frac{16}{4} + \frac{1}{4} = \frac{17}{4} \text{ sq.units}$$

144. The order and degree of the differential equation  $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$  are

A.  $\left(1, \frac{2}{3}\right)$

B. (3,1)

C. (3,3)

D. (1,2)

Key: C

Solution:  $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$

Cubing on b.s  $\left(1 + 3\frac{dy}{dx}\right)^2 = 4^3\left(\frac{d^3y}{dx^3}\right)^3$

Order = 3, degree = 3

145. P, Q, R are the midpoints of the sides  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  of the triangle ABC and O is a point

within the triangle, then  $\overline{OA} + \overline{OB} + \overline{OC} =$

A.  $2(\overline{OP} + \overline{OQ} + \overline{OR})$

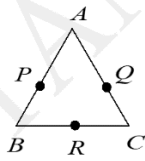
B.  $\overline{OP} + \overline{OQ} + \overline{OR}$

C.  $4(\overline{OP} + \overline{OQ} + \overline{OR})$

D.  $3(\overline{OP} + \overline{OQ} + \overline{OR})$

Key: B

Solution:



$$\overline{OP} = \frac{\overline{OA} + \overline{OB}}{2}, \overline{OQ} = \frac{\overline{OB} + \overline{OC}}{2}, \overline{OR} = \frac{\overline{OC} + \overline{OA}}{2}$$

$$\Rightarrow \overline{OP} + \overline{OQ} + \overline{OR} = \overline{OA} + \overline{OB} + \overline{OC}$$

146. If the points  $(\alpha - x, \alpha, \alpha), (\alpha, \alpha - y, \alpha), (\alpha, \alpha, \alpha - z)$  and are coplanar,  $(\alpha - 1, \alpha - 1, \alpha - 1) \alpha \in R$

then

A.  $xy + yz + zx = 1$

B.  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

C.  $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$

D.  $xyz = 1$

Key: B

Solution:  $A(\alpha - x, \alpha, \alpha), B(\alpha, \alpha - y, \alpha) C(\alpha, \alpha, \alpha - z), D(\alpha - 1, \alpha - 1, \alpha - 1)$

$$\overline{AB} = (x, -y, 0) \quad \overline{AC} = (x, 0, -z) \quad \overline{AD} = (x-1, -1, -1)$$

$$\begin{vmatrix} x & -y & 0 \\ x & 0 & -2 \\ x-1 & -1 & -1 \end{vmatrix} = 0$$

147. If A and B are two events such that  $P(A) = \frac{3}{8}$ ,  $P(B) = \frac{5}{8}$  and  $P(A \cup B) = \frac{3}{4}$  then  $P\left(\frac{B}{\overline{A}}\right) =$

A.  $\frac{1}{4}$

B.  $\frac{3}{8}$

C.  $\frac{3}{5}$

D.  $\frac{1}{5}$

Key: C

Solution:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4}$$

$$P\left(\frac{B}{\overline{A}}\right) = \frac{P(\overline{A} \cap B)}{P(\overline{A})}$$

$$= \frac{P(B) - P(A \cap B)}{1 - P(A)}$$

$$= \frac{3}{5}$$

148. Let  $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}$ ,  $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$  and a unit vector  $\vec{c}$  be coplanar. If  $\vec{c}$  is perpendicular to  $\vec{a}$ , then  $\vec{c}$  is

A.  $\frac{1}{\sqrt{2}}(-\vec{j} + \vec{k})$

B.  $\frac{1}{\sqrt{3}}(-\vec{i} - \vec{j} - \vec{k})$

C.  $\frac{1}{\sqrt{5}}(\vec{i} - 2\vec{j})$

D.  $\frac{1}{\sqrt{5}}(\vec{i} - \vec{j} - \vec{k})$

Key: A

Solution: Let  $\vec{c} = \alpha\vec{a} + \beta\vec{b}$  and  $\vec{c} \cdot \vec{a} = 0 \Rightarrow (\alpha\vec{a} + \beta\vec{b}) \cdot \vec{a} = 0$

$$\alpha|\vec{a}|^2 + \beta(\vec{a} \cdot \vec{b}) = 0$$

$$\alpha(6) + \beta(2 + 2 - 1) = 0 \Rightarrow 6\alpha + 3\beta = 0$$

$$1 \beta = -2\alpha, \therefore \vec{c} = \alpha(\vec{a} - 2\vec{b}) = \alpha(-3\hat{j} + 3\hat{k}), \text{ but } |\vec{c}| = 1 \Rightarrow \alpha = \pm \frac{1}{3\sqrt{2}}$$



$$\therefore \bar{c} = \pm \left( \frac{-\hat{j} + \hat{k}}{\sqrt{2}} \right)$$

149. The shortest distance between the lines  $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$  and  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$  is

A.  $3\sqrt{3}$

B.  $4\sqrt{3}$

C.  $2\sqrt{3}$

D.  $5\sqrt{3}$

Key: B

Solution:  $\bar{a} = (1, -8, 4), \bar{c} = (1, 2, 6),$

$\bar{b} = (2, -7, 5), \bar{d} = (2, 1, -3),$

Shortest distance =  $\frac{[\bar{a} - \bar{c} \ \bar{b} \ \bar{d}]}{|\bar{b} \times \bar{d}|}$

=  $\frac{16(12)}{16\sqrt{3}}$

=  $4\sqrt{3}$

150. Let E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2 then

A.  $P(E) = 1/3, P(F) = 1/5$

B.  $P(E) = 1/2, P(F) = 1/6$

C.  $P(E) = 1/6, P(F) = 1/2$

D.  $P(E) = 1/4, P(F) = 1/3$

Key: D

Solution:  $P(E \cap F) = P(E) \cdot P(F) = \frac{1}{12}$

$$P(\bar{E} \cap \bar{F}) = 1 - P(E \cup F) = \frac{1}{12}$$

$$\Rightarrow P(E) = \frac{1}{4}, P(F) = \frac{1}{3}$$