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How do we Prove the Existence of Irrational Numbers?

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In the previous segment, we saw if a prime number 'p' divides a square number 'a²', then will it even divide 'a'? In this segment let us prove the existence of Irrational Numbers.

How do we Prove the Existence of Irrational Numbers?

Consider a right-angled triangle ABC.

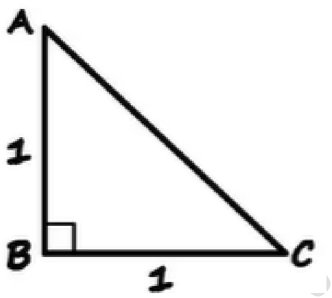

Fig 1

No.	Statement	Reason
1	△ABC is a right angled triangle	Given
2	$\therefore AC^2 = AB^2 + BA^2$ $\therefore AC^2 = 1^2 + 1^2$ $\therefore AC^2 = 2$ $\therefore AC = \sqrt{2}$	Pythagoras theorem
Let $\sqrt{2}$ be a rational number. That is, $\sqrt{2} = \frac{p}{q}$, where p and q have no common factors.		

<table>
<tr><td>3</td><td>So, $\sqrt{2} = \frac{p}{q}$
$\therefore 2 = \frac{p^2}{q^2}$
$\therefore p^2 = 2q^2$
$\therefore p^2$ is an even number.
$\therefore p$ is an even number.</td><td>Definition of rational numbers</td></tr>
<tr><td colspan="3">$\therefore p$ will be a multiple of 2. So, let $p = 2a$.</td></tr>
<tr><td>4</td><td>$\therefore (2a)^2 = 2q^2$
$\therefore 4a^2 = 2q^2$
$\therefore q^2 = 2a^2$
$\therefore q$ is an even number.</td><td>Substituting in statement 3</td></tr>
<tr><td>5</td><td>Thus, p and q are even numbers.
$\therefore$ 2 is a common factor of p and q.</td><td>From statements 3 and 4</td></tr>
<tr><td colspan="3">But we assumed that p and q have no common factors. So this is a contradiction to our hypothesis.
$\therefore \sqrt{2}$ is not a rational number. Hence, it is an irrational number.</td></tr>
</table>

Summary

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Did you know?

The Golden Ratio is an irrational number. Two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities.

What's next?

In the next segment of Class 10 Maths, we will look at **irrational numbers.**