VITEEE PYP 2023

Instructions:

- *This question paper contains total 125 questions divided into four parts:*

  Part I: Physics Q. No -1 to 35

  Part II: Chemistry Q. No- 36 to 70

  Part III : Mathematics Q. No- 71 to 110

  Part IV: Aptitude Test Q. No - 111 to 120

  Part V: English Q. No- 121 to 125

- *All questions are multiple choice questions with four options, only one of them is correct.*
- *For each correct response, the candidate will get 1 mark.*
- *There is no negative marking for the wrong answer.*
- *The test is of* $2\frac{1}{2}$ *hours duration.*

PART I : PHYSICS

1. Light of wavelength λ_A and λ_B falls on two identical metal plates A and B respectively. The maximum kinetic energy of photoelectrons is K_A and K_B respectively, then which one of the following relations is true ($\lambda_A = 2\lambda_B$).

A. $K_A < \frac{K_B}{2}$

B. $2K_A = K_B$

C. $K_A = 2K_B$

D. $K_A > 2K_B$

Key: A

$$KE = \frac{hc}{\lambda} - W_0 \Rightarrow K_A = \frac{hc}{\lambda_A} - W_0$$

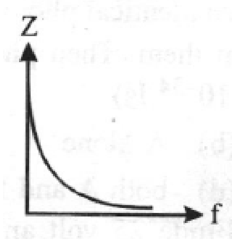
Solution: $K_A = \frac{hc}{2\lambda_B} - W_0 \ldots \text{(i)} \left(\because \lambda_A = 2\lambda_B\right)$

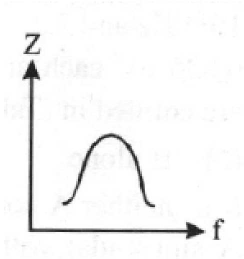
$$K_B = \frac{hc}{\lambda_B} - W_0 \ldots\ldots \text{(ii)}$$

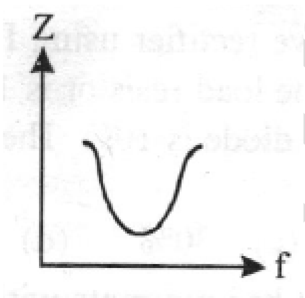
From eq. (i) and eq. (ii) we have,

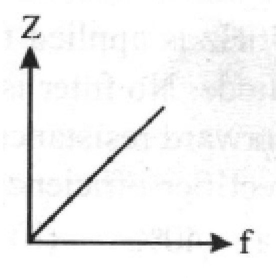
$$K_A < \frac{K_B}{2}$$

2. Which one of the following curves represents the variation of impedance (Z) with frequency f in series LCR circuit?

A. 

B. 

C. 

D. 

Key: C

Solution: Impedance at resonant frequency is minimum in series LCR circuit.

$$\text{So, } Z=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

When frequency is increased or decreased, Z increases.

3. A Carnot engine takes 3×10^6 cal. of heat from a reservoir at 627 °C, and gives it to a sink at 27 °C. The work done by the engine is.

A. 4.2×10^6 J
B. 8.4×10^6 J
C. 16.8×10^6 J
D. zero

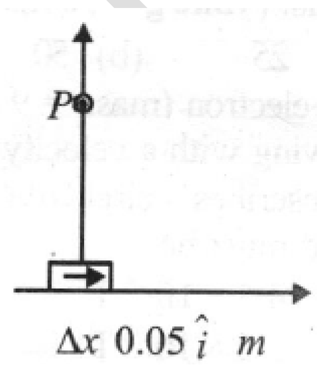
Key: B

Solution:

$$\eta=\frac{(627+273)-(273+27)}{627+273}$$
$$=\frac{900-300}{900}=\frac{600}{900}=\frac{2}{3}$$

$$\text{work}=(\eta)\times\ \text{Heat}=\frac{2}{3}\times3\times10^6\times4.2\text{J}=8.4\times10^6\,\text{J}$$

4. An element of $0.05\hat{i}m$ is placed at the origin as shown in figure which carries a large current of 10 A. distance of 1m in perpendicular direction. The value of magnetic field is



A. 4.5×10^{-8} T

B. 5.5×10^{-8} T

C. 5.0×10^{-8} T

D. 7.5×10^{-8} T

Key: C

Solution: $dB = \frac{\mu_0}{4\pi} \frac{|d|\sin\theta}{r^2}$

Here, $dI = \Delta x = 0.05\text{m}, I = 10\text{A}, r = 1\text{m} \sin\theta = \sin 90^\circ = 1,$

$$\therefore dB = 10^{-7} \times \frac{10 \times 0.05 \times 1}{(1)^2}$$

$$= 0.50 \times 10^{-7} = 5.0 \times 10^{-8}\text{T}$$

5. A sinusoidal voltage of amplitude 25 volt and frequency 50 Hz is applied to a half wave rectifier using P-n junction diode. No filter is used and the load resistor is 1000Ω. The forward resistance R_f of ideal diode is 10Ω. The percentage rectifier efficiency is

A. 40%

B. 20%

C. 30%

D. 15%

Key: A

Solution:

$$I_m = \frac{V_m}{R_f + R_L} = \frac{25}{(10+1000)} = 24.75\text{mA}$$

$$I_{dc} = \frac{I_m}{\pi} = \frac{24.75}{3.14} = 7.87\text{mA}$$

$$I_{\text{rms}} = \frac{I_m}{2} = \frac{24.75}{2} = 12.37\text{mA}$$

$$P_{dc} = I_{dc}^2 \times R_L = \left(7.87 \times 10^{-3}\right)^2 \times 10^3 = 61.9\text{mW}$$

$$P_{ac} = I_{rms}^2 \left(R_f + R_L\right) = \left(12.37 \times 10^{-3}\right)^2 \times (10+1000)$$

$$= 154.54\text{mW}$$

Rectifier efficiency

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{61.9}{154.54} \times 100 = 40.05\%$$

6. A flask contains a monoatomic and a diatomic gas in the ratio of 4:1 by mass at a temperature of 300 K. The ratio of average kinetic energy per molecule of the two gases is

A. 1:1
B. 2:1
C. 4:1
D. 1:4

Key: A

Solution: The average kinetic energy per molecule of any ideal gas is always equal to $\left(\frac{3}{2}\right)k_B T$. It depends only on the temperature and is independent of the mass and nature of the gas.

7. The potential energy of a particle (U_x) executing S.H.M. is given by

A. $U_x = \frac{k}{2}(x-a)^2$

B. $U_x = k_1 x + k_2 x^2 + k_3 x^3$

C. $U_x = Ae^{-bx}$

D. $U_x = \text{aconstant}$

Key: A

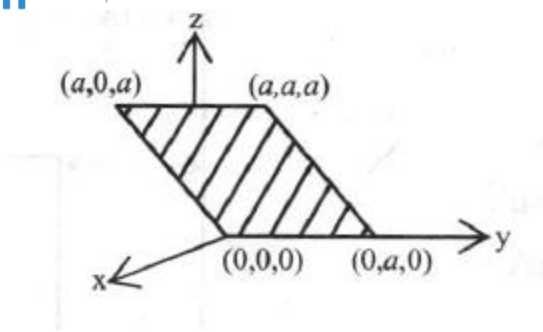
Solution: P.E. of body in S.H.M. at an instant,

$$U = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}ky^2$$

If the displacement, y=(a-x) then

$$U = \frac{1}{2}k(a-x)^2 = \frac{1}{2}k(x-a)^2$$

8. Consider an electric field $\vec{E} = E_0\hat{x}$ where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is.



A. $2E_0a_2$

B. $\sqrt{2}E_0a^2$

C. E_0a^2

D. $\frac{E_0a^2}{\sqrt{2}}$

Key: C

Solution:

9. The equation of a wave on a string of linear mass density 0.04kgm^{-1} is given by $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$ The tension in the string is

A. 4.0 N

B. 12.5 N

C. 0.5 N

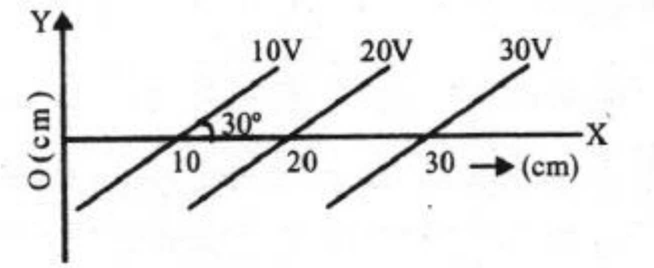
D. 6.25 N

Key: D

Solution: $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2}$

=6.25N

10. Equipotential surfaces are shown in the figure. Then the electric field strength will be



A. 100Vm^{-1} along X-axis

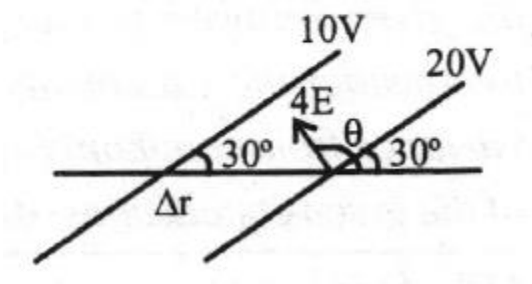
B. 100Vm^{-1} long Y-axis

C. 200Vm^{-1} at an angle 120° with X-axis

D. 50Vm^{-1} at an angle 120° with X-axis

Key: C

Solution:



Using $dV = -\vec{E}\cdot d\vec{r}$

$$\Rightarrow \Delta V = -E\Delta r\cos\theta$$

$$\Rightarrow E = \frac{-\Delta V}{\Delta r\cos\theta}$$

$$\Rightarrow E = \frac{-(20-10)}{10\times10^{-2}\cos120^{\circ}}$$

$$= \frac{-10}{10\times10^{-2}\left(-\sin30^{\circ}\right)} = 200\text{V}/\text{m}$$

Direction of E be perpendicular to the equipotential surface i.e. at 120° with X-axis.

11. Water falls from a 40 m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydroelectric energy number of 100 W lamps, that can be lit, is:(Take$g=10\text{ms}^{-2}$)

A. 25

B. 50

C. 100

D. 18

Key: B

Solution: Given

$$\frac{1}{2}\cdot\frac{\Delta U}{\Delta t}=P_{Bulb}\times N,\ \text{where}N=\text{no. of bulb}$$

$$\Rightarrow\frac{9\times10^4\times10\times40}{2\times3600}=100\times N$$

$$\left[\because\frac{\Delta U}{\Delta t}=\frac{mgh}{3600}\frac{J}{S}\right]$$

$$\Rightarrow N=\frac{36\times10^6}{72\times10^4}\Rightarrow N=\frac{1}{2}\times100\Rightarrow N=50$$

12. An electron (mass $=9\times10^{-31}$ kg charge $=1.6\times10^{-19}$ C). moving with a velocity of 10^6 m/s enters a magnetic field. If it describes a circle of radius 0.1 m, then strength of magnetic field must be.

A. 4.5×10^{-5} T

B. 1.4×10^{-5} T

C. 5.5×10^{-5} T

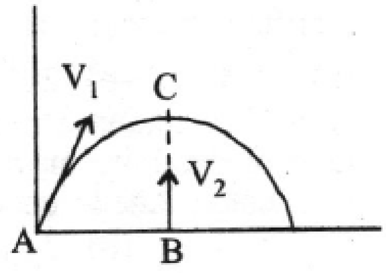
D. 2.6×10^{-5} T

Key: C

Solution: $Bqv=\frac{mv^2}{r}\text{or}B=\frac{mv}{rq}=\frac{(9\times10^{-31})\times10^6}{0.1\times(1.6\times10^{-19})}$

$=5.5\times10^{-5}\text{T}$

13. If V_1 is velocity of a body projected from the point A and V_2 is the velocity of a body projected from point B which is vertically below the highest point C. if both the bodies collide, then



A. $V_1 = \frac{1}{2} V_2$

B. $V_2 = \frac{1}{2} V_1$

C. $v_1 = v_2$

D. $V_1 = 3V_2$

Key: B

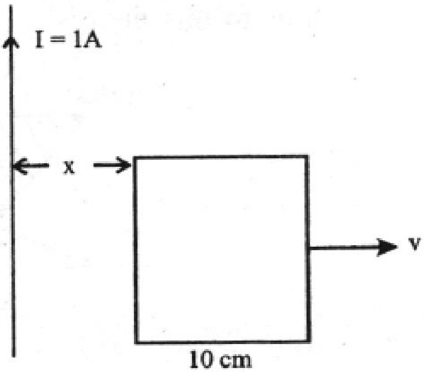
Solution: Two bodies will collide at the highest point if both cover the same vertical height in the same time.

$$\text{So} \frac{V_1^2 \sin^2 30^\circ}{2g} = \frac{V_2^2}{2g} \Rightarrow \frac{V_2}{V_1} = \sin 30^\circ = \frac{1}{2}$$

$$\therefore \quad V_2 = \frac{1}{2} V_1$$

14. A square frame of side 10 cm and a long straight wire carrying current 1A are in the plate of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 ms^{-1}(see figure).

The e.m.f induced at the time the left arm of the frame is at x=10 cm from the wire is



A. $2\mu V$

B. $1\mu V$

C. $0.75\mu V$

D. $0.5\mu V$

Key: B

Solution: In the given question,

Current flowing through the wire, I=1A

Speed of the frame, v = 10 ms^{-1}

Side of square loop, l=10 cm

Distance of square frame from current carrying wires x=10 cm.

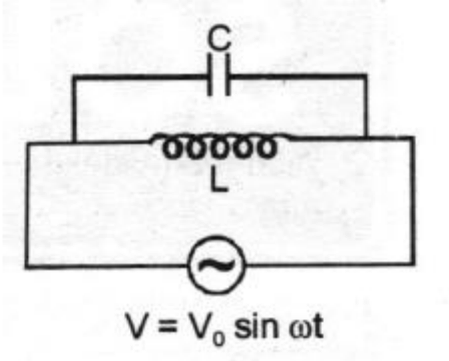
We have to find, e.m.f induced e = ?

According to Biot-Savart's law

$$B = \frac{\mu_0}{4\pi}\frac{Idsin\theta}{x^2} = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{1 \times 10^{-1}}{\left(10^{-1}\right)^2} = 10^{-6}$$

$$\text{Induced e.m.f.e} = Blv = 10^{-6} \times 10^{-1} \times 10 = 1\mu V$$

15. For the circuit shown in the fig. the current through the inductor is 0.9 A while the current through the condenser is 0.4 A. Then



A. current drawn from source I =1.13A

B. $\omega = 1/(1.5LC)$

C. $I = 0.5A$

D. $I = 0.6A$

Key: C

Solution: The current drawn by inductor and capacitor will be in opposite phase. Hence net current drawn from generator.

$= I_L - I_C = 0.9 - 0.4 = 0.5amp$

16. The ozone layer in the atmosphere absorbs

A. only the radiowaves

B. only the visible light

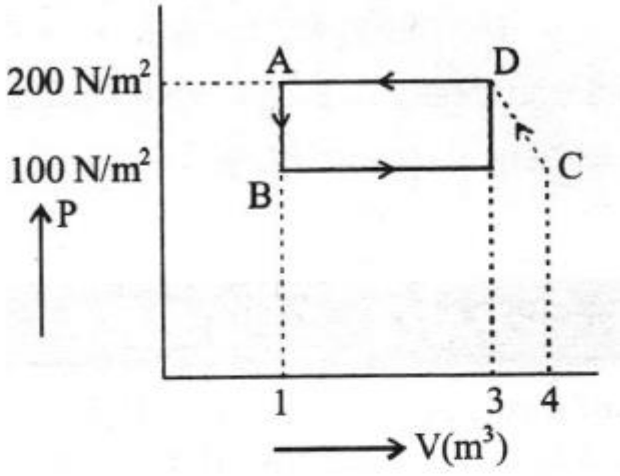
C. only the γ-rays

D. X-rays and ultraviolet rays

Key: D

Solution:

17. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (Use γ=1.4):



A. -500 J

B. 200 J

C. -400 J

D. 400 J

Key: A

Solution: Work done during adiabatic process CD,

$$W_{C'D} = \frac{P_C V_C - P_D V_D}{\gamma - 1}$$

$$= \frac{100 \times 4 - 200 \times 3}{1.4 - 1} = \frac{400 - 600}{\frac{2}{5}} J = -500J$$

18. In YDSE, how many maximas can be obtained on a screen including central maxima in both sides of the central fringe if λ=3000Å,d=5000Å

A. 2

B. 5

C. 3

D. 1

Key: C

Solution: $\Delta x_{max} = d = 5000Å$. Given $\lambda = 3000Å$

As $\lambda < d < 2\lambda \quad \therefore n = 3.$

19. A and B are two metals with threshold frequencies 1.8×10^{14} Hz and 2.2×10^{14} Hz. Two identical photons of energy 0.825eV each are incident on them. Then photoelectrons are emitted in (Take $h=6.6\times10^{-34}$ Js)

A. B alone
B. A alone
C. neither A nor B
D. both A and B

Key: B

Solution:

20. A sinusoidal voltage of amplitude 25 volt and frequency 50 Hz is applied to a half wave rectifier using P-n junction diode. No filter is used, and the load resistor is 1000Ω. The forward resistance R_f of ideal diode is 10Ω. The percentage rectifier efficiency is.

A. 40%
B. 20%
C. 30%
D. 15%

Key: A

Solution: $I_m = \dfrac{V_m}{R_f + R_L} = \dfrac{25}{(10+1000)} = 24.75\text{mA}$

$$I_{dc} = \frac{I_m}{\pi} = \frac{24.75}{3.14} = 7.87\text{mA}$$

$$I_{rms} = \frac{I_m}{2} = \frac{24.75}{2} = 12.37\text{mA}$$

$$P_{dc} = I_{dc}{}^2 \times R_L = \left(7.87\times10^{-3}\right)^2 \times 10^3 = 61.9\text{mW}$$

$$P_{ac} = I_{rms}{}^2 (R_f + R_L) = \left(12.37\times10^{-3}\right)^2 \times (10+1000)$$

=154.54 mw Rectifier efficiency

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{61.9}{154.54} \times 100 = 40.05\%$$

21. The force between two short bar magnets with magnetic moments M_1 and M_2 whose centres are r metres apart is 8 N when their axes are in same line. if the separation is increased to 2r, the force between them is reduced to

A. 4 N
B. 2 N
C. 1 N
D. 0.5 N

Key: D

Solution: As $F \propto \frac{1}{r^4}$ and r becomes twice, therefore,

$$F \text{ becomes } \frac{1}{2^4} = \frac{1}{16} \text{ times}$$

$$\therefore \frac{1}{16} \times 8 = 0.5\text{N}.$$

22. In a Rutherford scattering experiment when a projectile of charge Z_1 and mass M_1 approaches a target nucleus of charge Z_2 and mass M_2, the distance of closest approach is r_0. The energy of the projectile is

A. directly proportional to $Z_1 Z_2$
B. inversely proportional to Z_1
C. directly proportional to mass M_1
D. directly proportional to $M_1 \times M_2$

Key: A

Solution: The kinetic energy of the projectile is given by

$$\frac{1}{2}mv^2 = \frac{Ze(2e)}{4\pi\varepsilon_0 r_0} = \frac{Z_1 Z_2}{4\pi\varepsilon_0 r_0}$$

Thus, energy of the projectile is directly proportional to $Z_1 Z_2$

23. What will be the maximum speed of a car on a road turn of radius 30 m if the coefficient of friction between the tyres and the road is 0.4 (Take $g = 9.8 m/s^2$)

A. 10.84 m/s

B. 9.84 m/s

C. 8.84 m/s

D. 6.84 m/s

Key: A

Solution: $6.84\ m/s$

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.4 \times 30 \times 9.8} = 10.84 m/s$$

24. A person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120∘with the direction of flow of water. The speed of water in the stream is

A. 1 m/s

B. 0.5 m/s

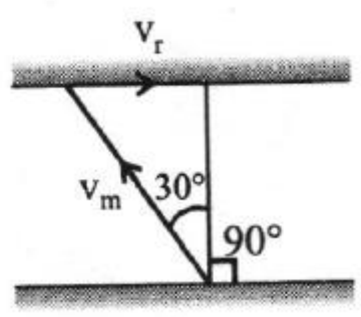
C. 0.25 m/s

D. 0.433 m/s

Key: C

Solution: $\sin 30^{\circ} = \frac{v_r}{v_m} = \frac{1}{2}$

$$\Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25 m/s$$



25. A car moves at a speed of $20 ms^{-1}$on a banked track and describes an arc of a circle of radius $40\sqrt{3}m$. The angle of banking is (g =10 ms^{-2})

A. 25^0

B. 60^0

C. 45^0

D. 30^0

Key: D

Solution: banking istan θ

$$=\frac{v^2}{rg}=\frac{20^2}{40\sqrt{3}\times 10}=\frac{1}{\sqrt{3}}$$

26. A force $\vec{F}=\alpha\hat{i}+3\hat{j}+6\hat{k}$ is acting at a point $\vec{r}=2\hat{i}-6\hat{j}-12\hat{k}$ The value of α for which angular momentum about origin is conserved is

A. 2

B. zero

C. 1

D. -1

Key: D

Solution: From Newton's second law for rotational motion,

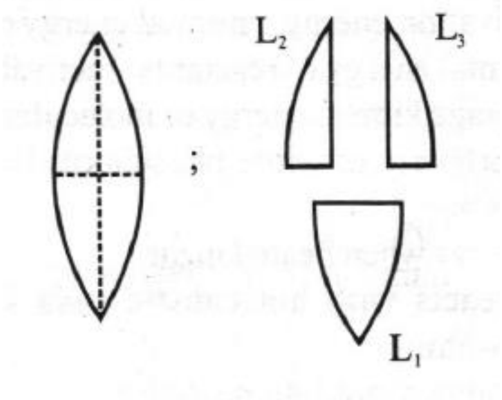
$$\vec{\tau}=\frac{d\vec{L}}{dt}, \text{ if } \vec{L}=\text{constant then } \vec{\tau}=0$$

$$\text{So, } \vec{\tau}=\vec{r}\times\vec{F}=0$$

$$\left(2\hat{i}-6\hat{j}-12\hat{k}\right)\times\left(\alpha\hat{i}+3\hat{j}+6\hat{k}\right)=0$$

$$\text{Solving we get } \alpha=-1$$

27 A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figure). Choose the incorrect option for the reported pieces.



A. Power of $L_1 = \frac{P}{2}$

B. Power of $L_2 = \frac{P}{2}$

C. Power of $L_3 = \frac{p}{2}$

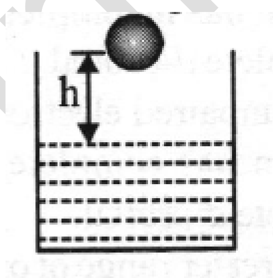
D. Power of $L_1 = P$

Key: A

Solution: If a lens is cut into two half along principal axis, its focal length remain unchanged only the intensity of image get reduced.

So, power of L_1 will remain unaffected as $P = \frac{1}{f}$

28. A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η the value of h is given by



A. $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$

B. $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

C. $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

D. $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

Key: C

Solution:

29. The pressure inside a tyre is 4 times that of atmosphere. If the tyre bursts suddenly at temperature 300K, what will be the new temperature?

A. $300(4)^{7/2}$

B. $300(4)^{2/7}$

C. $300(2)^{7/2}$

D. $300(4)^{-2/7}$

Key: D

Solution: Under adiabatic change

$$\frac{T_2}{T_1}=\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} \text{ or } T_2=T_1\left(P_1/P_2\right)^{\frac{1-\gamma}{\gamma}}$$

$$\therefore T_2=300(4/1)^{\frac{1-(7/5)}{(7/5)}};$$

$\gamma=1.4=7/5$ for air

or $T_2=300(4)^{-2/7}$

30. A parallel plate air capacitor of capacitance C is connected to a cell of emf V and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect?

A. The energy stored in the capacitor decreases K times.

B. The chance in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K}-1\right)$.

C. The charge on the capacitor is not conserved.

D. The potential difference between the plates decreases K times.

Key: C

Solution: Capacitance of the capacitor, $C=\frac{Q}{V}$ After inserting the dielectric, new capacitance C^'=K.C

$$V'=\frac{V}{K};u_i=\frac{1}{2}CV^2=\frac{Q^2}{2C}\quad(\because Q=CV)$$

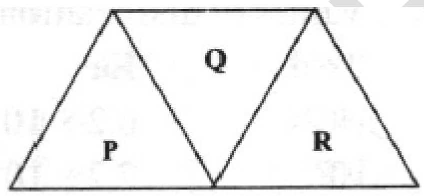
$$u_f=\frac{Q^2}{2f}=\frac{Q^2}{2kc}=\frac{C^2V^2}{2KC}=\left(\frac{u_i}{k}\right)$$

$$\Delta u=u_f-u_i=\frac{1}{2}CV^2\left\{\frac{1}{k}-1\right\}$$

As the capacitor is isolated, so change will remain conserved p.d. between two plates of the capacitor.

$$L=\frac{Q}{KC}=\frac{V}{K}$$

31. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will now suffer.



A. greater deviation

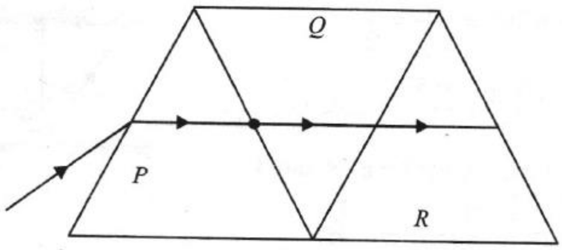
B. no deviation

C. same deviation as before

D. total internal reflection

Key: C

Solution:



There will be no refraction from P to Q and then from Q to R (all being identical). Hence the ray will now have the same deviation.

32. If m is magnetic moment and B is the magnetic field, then the torque is given by

A. $\vec{m}\cdot\vec{B}$

B. $\frac{|\vec{m}|}{|\vec{B}|}$

C. $\vec{m}\times\vec{B}$

D. $|\vec{m}|\cdot|\vec{B}|$

Key: C

Solution:

33. An α-particle of 10MeV collides head-on with a copper nucleus (Z=29) and is deflected back. Then, the minimum distance of approach between the centres of the two is:

A. 8.4×10^{-15} cm

B. 8.4×10^{-15} m

C. 4.2×10^{-15} m

D. 4.2×10^{-15} cm

Key: B

Solution:

34. A planet in a distant Solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth's surface is $11 kms^{-1}$, the escape velocity from the surface of the planet would be

A. $1.1 kms^{-1}$

B. $11 kms^{-1}$

C. $110 kms^{-1}$

D. $0.11 kms^{-1}$

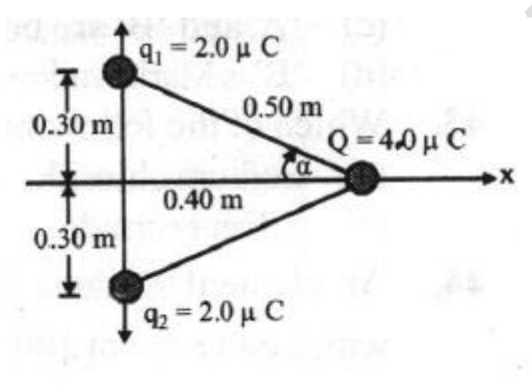
Key: C

Solution: $\frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$$= \sqrt{\frac{10M_e}{M_e} \times \frac{R_e}{R_e / 10}} = 10$$

$$\therefore (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110 km/s$$

35. In fig., two equal positive point charges $q_1 = q_2 = 2.0$ mC interact with a third point charge Q = 4.0μC. The magnitude, as well as direction, of the net force on Q is.



A. 0.23N in the +x-direction

B. 0.46N in the +x-direction

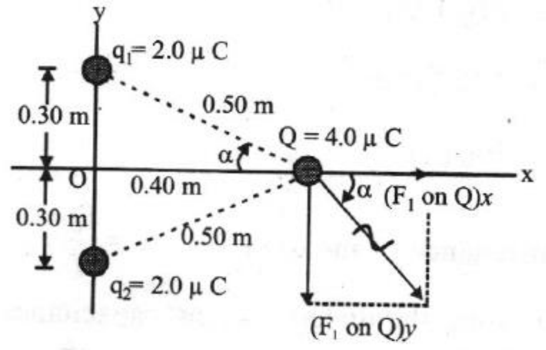
C. 0.23 N in the -x-direction

D. 0.46N in the -x-direction

Key: B

Solution: $F_{net} = 2|F_{31}|\cos$

$$\alpha = 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{2 \times 4 \times 10^{-12}}{(0.5)^2} \times \frac{4}{5} = 0.46N$$



PART II : CHEMISTRY

36. Which of the following sets of quantum numbers is correct for an electron in 4f orbital ?

A. $n = 4, l = 3, m = +1, s = +1/2$

B. $n = 4, l = 4, m = -4, s = -1/2$

C. $n = 4, l = 3, m = +4, s = +1/2$

D. $n = 3, l = 2, m = -2, s = +1/2$

Key: A

Solution: The possible quantum numbers for 4f electron are

$$n = 4, \ell = 3, m = -3, -2 -1, 0, 1, 2, 3 \text{and } s = \pm\frac{1}{2}$$

Of various possiblities only option (A.is possible.

37. Arrange the following in increasing order of ionic radii?

$C^{4-}, N^{3-}, F^{-}, O^{2-}$

A. $C^{4-} < N^{3-} < O^{2-} < F^{-}$

B. $N^{3-} < C^{4-} < O^{2-} < F^{-}$

C. $F^{-} < O^{2-} < N^{3-} < C^{4-}$

D. $O^{2-} < F^{-} < N^{3-} < C^{4-}$

Key: C

Solution: All the given species are isoelectronic. In case of isoelectronic species, ionic radii increases with increase in negative charge on anions

38. The bond dissociation energies of X_2, Y_2 and XY are in the ratio of 1 : 0.5 : 1 ΔH for the formation of XY is $-200 kJ\ mol^{-1}$. The bond dissociation energy of X_2 will be

A. $200 kJmol^{-1}$

B. $100 kJmol^{-1}$

C. $400 kJmol^{-1}$

D. $800 kJmol^{-1}$

Key: D

Solution: Let B.E o fX_2, Y_2 and XY are $xkJmol^{-1}$, $0.5xkJmol^{-1}$ and $xkJmol^{-1}$ respectively.

$$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY; \Delta H = -200 kJmol^{-1}$$

$$\Delta H = -200 = \Sigma(\text{B.E})_{\text{Reactants}} - \Sigma(\text{B.E})_{\text{Product}}$$

$$= \left[\frac{1}{2}\times(x) + \frac{1}{2}\times(0.5x)\right] - \left[1\times(x)\right]$$

On solving, $x = 800 kJmol^{-1}$

39. Values of dissociation constant, Ka are given as follows:

Acid	Ka
HCN	6.2×10^{-10}
HF	7.2×10^{-4}
HNO_2	4.0×10^{-4}

Correct order of increasing base strength of the base CN^-, F^- and NO_2^- will be :

A. $F^- < CN^- < NO_2^-$

B. $NO_2^- < CN^- < F^-$

C. $F^- < NO_2^- < CN^-$

D. $NO_2^- < F^- < CN^-$

Key: C

Solution: Higher the value of ka lower will be the value of pka i.e., higher will be the acidic nature.

Further since, CN^-, F^- and NO_2^- are conjugate base of the acids HCN, HF and HNO_2 respectively hence, the correct order of base strength will be $F^- < NO_2^- < CN^-$

($\therefore$ stronger the acid weaker will be its conjugate base)

40. The product/s formed when diborane is hydrolysed is/are

A. B_2O_3 and H_3BO_3

B. B_2O_3 only

C. H_3BO_3 and H_2

D. H_3BO_3 only

Key: C

Solution: When diborane is hydrolysed, one can get both orthoboric acid and H_2

$$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3 + 6H_2$$

41. The compounds $CH_3CH = CHCH_3$ and $CH_3CH_2CH = CH_2$

A. are tautomers

B. are position isomers

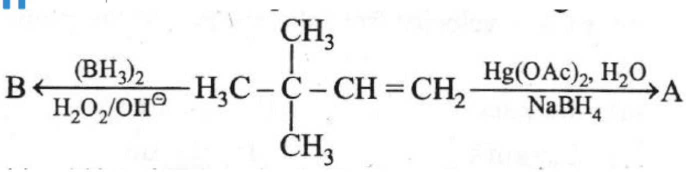
C. contain same number of $sp^3 - sp^3, sp^3 - sp^2$ and $sp^2 - sp^2$ carbon-carbon bonds

D. exist together in dynamic equilibrium

Key: B

Solution: The two isomers differ in the position of the double bond are called position isomers, so given compounds are position isomers.

42. Choose the correct option for the following reactions.



A. 'A' and ' B ' are both Markovnikov addition products.

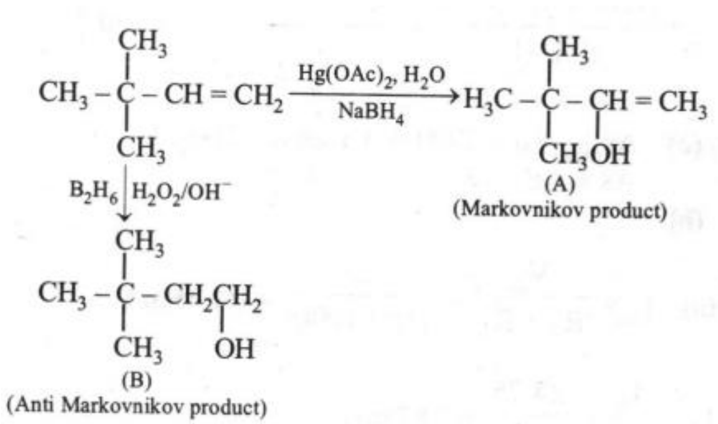
B. 'A' is Markovnikov product and ' B ' is anti-Markovnikov product.

C. 'A' and ' B ' are both anti-Markovnikov products.

D. 'B' is Markovnikovand ' A ' is anti-Markovnikov product.

Key: B

Solution:



43. Which of the following has Frenkel defects?

A. Sodium chloride

B. Graphite

C. Silver bromide

D. Damond

Key: C

Solution: AgBr exhibit Frenkel defect.

44. An element X has a body centred cubic (bcC. structure with a cell edge of200 pm. The density of the element is $5 g cm^{-3}$. The number of atoms present in 3OO g of the element X is ------------

Given: Avogadro Constant, $N_A = 6.0 \times 10^{23} mol^{-1}$.

A. $5N_A$

B. $6N_A$

C. $15N_A$

D. $25N_A$

Key: D

Solution: $d = \frac{Z \times M}{a^3 \times N_A}$; $Z = 2$ for bcc

$$5g/cm^3 = \frac{2 \times M}{\left(200 \times 10^{-10} cm\right)^3 \times 6.0 \times 10^{23}} \Rightarrow M = 12g$$.

$12g$ of element contain $= N_A$ atoms

$300g$ of element contains $= N_A \times \frac{300}{12} = 25N_A$

45. On passing current through two cells, connected in series, containing Solution:ution of $AgNO_3$, and $CuSO_4$, 0.18 g of Ag is deposited. The amount of the Cu deposited is:

A. 0.529 g

B. 10.623 g

C. 0.0529 g

D. 1.2708 g

Key: D

Solution: Using Faraday's second law ofelectrolysis,

$$\frac{\text{Weight of } Cu \text{ deposited}}{\text{Weight of } Ag \text{ deposited}} = \frac{\text{Equ. wt. of } Cu}{\text{Equ. wt. of } Ag}$$

$$\Rightarrow \frac{w_{Cu}}{0.18} = \frac{63.5}{2} \times \frac{1}{108}$$

$$\Rightarrow w_{Cu} = \frac{63.5 \times 18}{2 \times 108 \times 100} = 0.0529g.$$

46. The limiting molar conductivities of HCl, CH_3COONa and NaCl are respectively 425,90 and 125 $mhocm^2mol^{-1}$ at $25^\circ C$. The molar conductivity of $0.1M CH_3COOH$ Solution:utions is

$7.8\, mhocm^2 mol^{-1}$ at the same temperature. The degree of dissociation of 0.1M acetic acid Solution:ution at the same temperature is

A. 0.10

B. 0.02

C. 0.15

D. 0.03

Key: B

Solution: Λ° for $CH_3COOH = \lambda^\circ_{CH_3COO^-} + \lambda^\circ_{H^+}$

$$= \left(\lambda^\circ_{CH_3COO^-} + \lambda^\circ_{Na^+}\right) + \left(\lambda^\circ_{H^+} + \lambda^\circ_{Cl^-}\right) - \left(\lambda^\circ_{Na^+} + \lambda^\circ_{Cl^-}\right)$$

$$= 90 + 425 - 125 = 390 mhocm^2 mol^{-1}.$$

$$\text{Degree of dissociation}\,(\alpha) = \frac{\Lambda^c_m}{\Lambda^\circ_m} = \frac{7.8}{390} = 0.02$$

47. The rate law for a reaction between the substances A and B is given by $Rate = k[A]^n[B]^m$ On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as

A. $(m+n)$

B. $(n-m)$

C. $2^{(n-m)}$

D. $\frac{1}{2^{(m+n)}}$

Key: C

Solution: $Rate_1 = k[A]^n[B]^m ; Rate_2 = k[2A]^n[1/2B]^m$

$$\therefore \frac{Rate_2}{Rate_1} = \frac{k[2A]^n[1/2B]^m}{k[A]^n[B]^m} = [2]^n[1/2]^m = 2^n \cdot 2^{-m} = 2^{n-m}$$

48. In a reaction, the threshold energy is equal to

A. activation energy + normal energy of reactants

B. activation energy-normal energy of reactants

C. normal energy of reactants - activation energy

D. average kinetic energy of molecules of reactants

Key: A

Solution: Threshold Energy = Enogr of activation + Internal energy

49. Which property of white phosphorus is common to red phosphorous?

A. It burns when heated in air.

B. It reacts with hot caustic soda Solution:ution to give phosphine.

C. It shows chemiluminescence.

D. It is Solution:uble in carbon disulphide.

Key: A

Solution: Both white phosphorus and red phosphorus burns when heated in air

50. XeO_4 molecule is tetrahedral having :

A. Two $p\pi - d\pi$ bonds

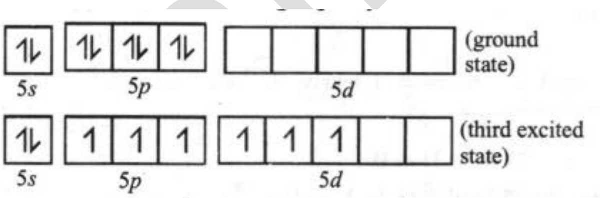
B. One $p\pi - d\pi$ bonds

C. Four $p\pi - d\pi$ bonds

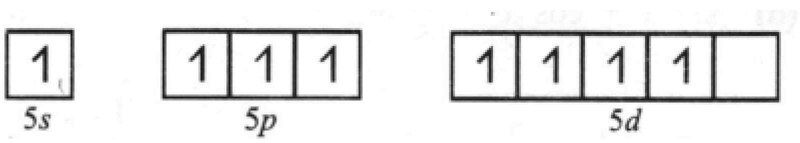
D. Three $p\pi - d\pi$ bonds

Key: C

Solution: Xenon undergo, sp3 hybridization



In the fourth excited state, xenon atom has 8 unpaired electrons



One s and three p orbital undergo sp^3 hybridization. Four sp^3 hybrid orbitals form four σ bonds with oxygen atoms. They are $\sigma sp^3 - p$.Four$p\pi - d\pi$ bonds are also formed with oxygen atoms by the urpaired electrons.

51. Cuprous ion is colourless while cupric ion is coloured because

A. both have half filled p-and d-orbitals

B. cuprous ion has incomplete d-orbital and cupric ion has a complete d-orbital

C. both have unpaired electrons in the d-orbitals

D. cuprous ion has complete d-orbital and cupric ion has an incomplete d-orbital.

Key: D

Solution: In $Cu^{+}[Ar]3d^{10}$ there is no unpaired electron, $Cu^{2+}[Ar]3d^{9}$ contains one unpaired electron hence coloured.

52. The reason for greater range of oxidation states in actinoids is attributed to :

A. actinoid contraction

B. 5f,6d and 7s levels having comparable energies

C. 4f and 5d levels being close in energies

D. the redioactive nature of actinoids

Key: B

Solution: Minimum or comparable energy gap between 5f,6d and 7s subshell makes electron excitation easier, hence there is a greatsr range ofoxidation states in actinoids.

53. The geometry and magnetic behaviour of the complex $[Ni(CO)_4]$ are

A. Square planar geometry and diamagnetic

B. Tetrahedral geometry and diamagnetic

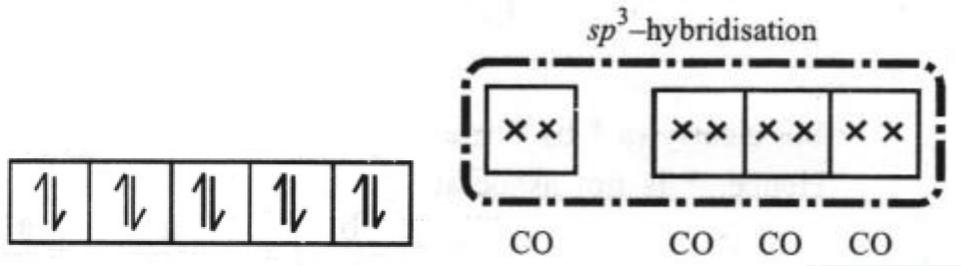
C. Tetrahedral geometry and paramagnetic

D. Square planar geometry and paramagnetic

Key: B

Solution: $Ni(28):[Ar]3d^{8}4s^{2}$

CO is a strong field ligand, so unpaired electrons get paired. Hence, configuration would be:



For, four 'CO' ligands h/ridimtiur would be sp^3 and thus complex would be diamagnetic and of tetrahedral geometry.

54. Indicate the complex ion which shows geometrical isomerism.

A. $\left[Cr\left(H_2O\right)_4 Cl_2\right]^+$

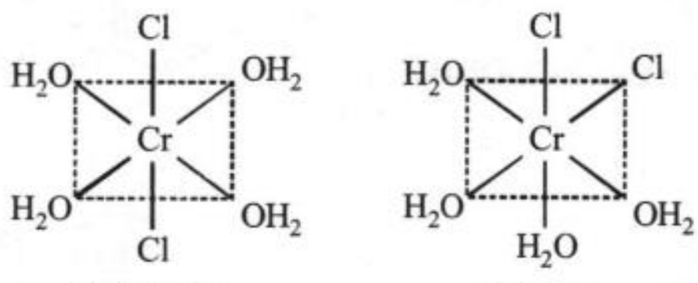
B. $\left[Pt\left(NH_3\right)_3 Cl\right]^{3-}$

C. $\left[Co\left(NH_3\right)_6\right]^{3-}$

D. $[Co(CN)_5(NC)]^{3-}$

Key: A

Solution: $\left[Cr\left(H_2O\right)_4 Cl_2\right]^+$ shows geometrical isomerism. The possible geometrical isomers are.



trans-isomer *cis*-isomer

55. Reaction of $C_6H_5CH_2Br$ with aqueous sodium hydroxide follows ....

A. S_N1 mechanism

B. S_N2 mechanism

C. Any of the above two depending upon the temperature of reaction

D. Saytzeff rule

Key: A

Solution: In $C_6H_5CH_2Br$ carbocation is $C_6H_5\overset{\oplus}{C}H_2$ which is stable due to resonance.

56. What is the correct order of reactivity of alcohols in the following reaction?

$$R-OH+HCl\xrightarrow{ZnCl_2}R-Cl+H_2O$$

A. $1^\circ>2^\circ>3^\circ$

B. $1^\circ<2^\circ<3^\circ$

C. $3^\circ>2^\circ>1^\circ$

D. $3^\circ>1^\circ>2^\circ$

Key: C

Solution: $HCl+An\cdot ZnCl_2$ s known as Lucas reagent. It isused to determine degree of an alcohol.

The reaction follow nucleophilic substitution reaction in which ------OH group is replaced by ------Cl. In this reaction carbocation is formed as intermediate. Higher the stability of intermediate carbocation higher will be the reactivity of reactant molecule. Since 3° carbocation is more stable than 2° carbocation as well as 1° carbocation, so the order of reactivity of alcohols is $3^\circ>2^\circ>1^\circ$.

57. Which of the following cannot be made by using Williamson's synthesis?

A. Methoxybenzene

B. Benzyl p-nitrophenyl ether

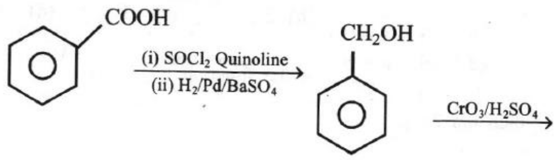
C. Methyl tertiary butyl ether

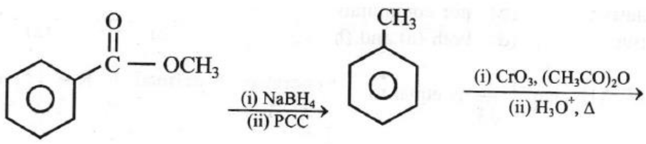
D. Di-tert-butyl ether

Key: D

Solution: The two components should be $(CH_3)_3CONa+(CH_3)_3CBr$. However tert-alkyl halides tend to undergo elimination reaction rather than substitution leading to the formation of an alkene, $Me_2C=CH_2$

58. Which of the following reactions will yield benzaldehyde as a product?





A. (B. and (C.

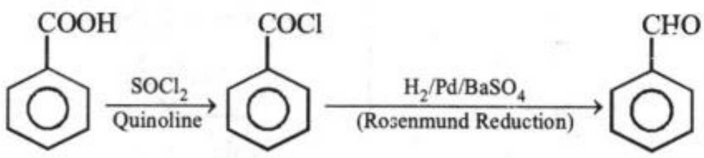
B. (C. and (D.

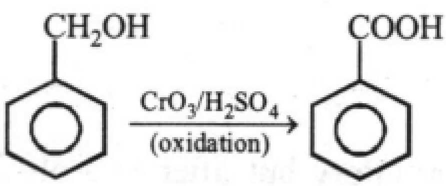
C. (A.and (D.

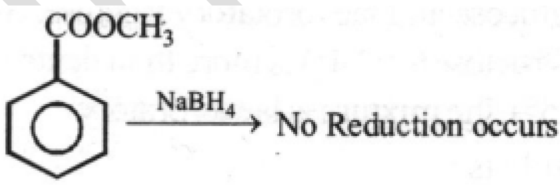
D. (A.and (C.

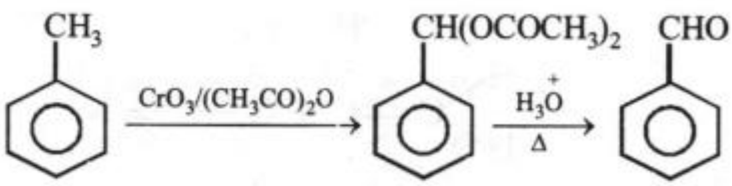
Key: C

Solution:









59. In Clemmensen reduction, carbonyl compounds is treated with ....

A. zinc amalgam +HCl
B. sodium amalgam +HCl
C. zinc amalgan + nitric acid
D. sodiumamalgam +HNO_3

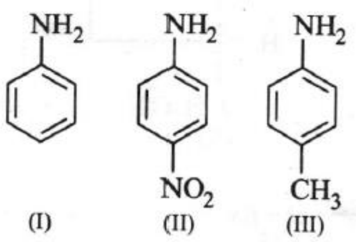
Key: A

Solution: Clemmensen reduction is used to convert carbonyl group as follows

$$>C=O \xrightarrow{Zn(Hg)+HCl} >CH_2$$

Zinc amalgam and HCI act as reagent in this reaction.

60. The correct increasing order of basic strength for the following compounds is

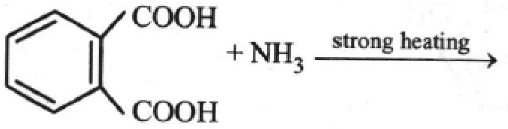


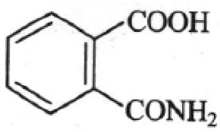
A. II < III < I
B. III < I < II
C. III < II < I
D. II < I < III

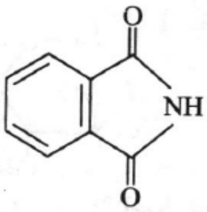
Key: D

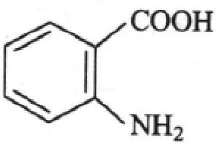
Solution: III > I > II: Higher the electron density towards ring, higher will be its basic strength. Electron donating group increases the basic strength while electron withdrawing group decreases the basic strength.

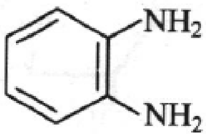
61. The major product of the following reaction is:



A. 

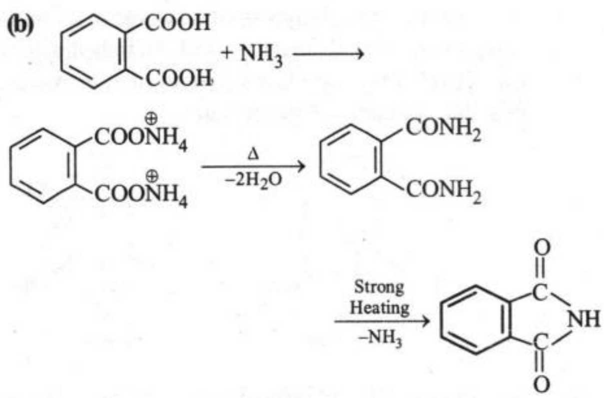
B. 

C. 

D. 

Key: B

Solution:



62. Blister copper is

A. Impure Cu

B. Cu alloy

C. Pure Cu

D. Cu having 1% impurity

Key: D

Solution: Blister-copper contains 1 - 2 % impurities. It is obtained after bessemerisation of crude copper.

63. P_A and P_B are the vapour pressure of pure liquid components, A and B, respectively of an ideal binary Solution: If XA represents the mole fraction of component A, the total pressure of will be.

A. $P_A + X_A(P_B - P_A)$

B. $P_A + X_A(P_A - P_B)$

C. $P_B + X_A(P_B - P_A)$

D. $P_B + X_A(P_A - P_B)$

Key: D

Solution: $P = P_A X_A + P_B X_B = P_A X_A + P_B(1 - X_A)$

$\Rightarrow P_A X_A + P_B - P_B X_A$

$\Rightarrow P_B + X_A(P_A - P_B)$

64. Which of the following complex shows sp^3d^2 hybridization?

A. $\left[Cr(NO_2)_6\right]^{3-}$

B. $[Fe(CN)_6]^4$

C. $[CoF_6]^{3-}$

D. $[Ni(CO)_4]$

Key: C

Solution: Among these ligands, 'F' is a weak field ligand, makes only high spin complexes which has sp^3d^2 hybridization.

65. 2- Pentene contains

A. 15σ- and oneπ-bond

B. 14σ-and oneπ- bond

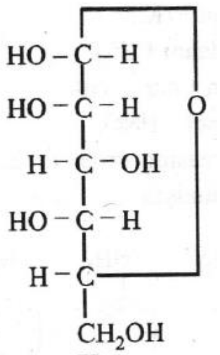
C. 15σ- and twoπ- bonds

D. 14σ- and twoπ- bonds

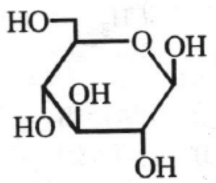
Key: B

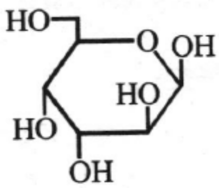
Solution: No. o fσ bonds $=14$, No. of π bond $=1$

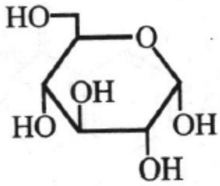
66. For the below given cyclic hemiacetal (X), the correct pyranose structure is :

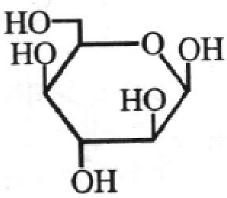


(X)

A. 

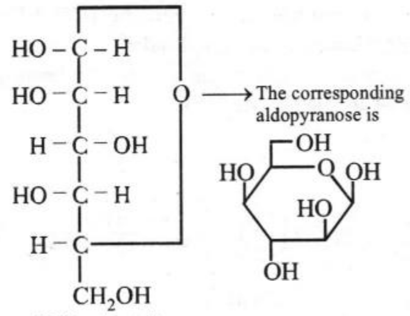
B. 

C. 

D. 

Key: D

Solution:



X (Hemiacetal)

67. Sucrose which is dextrorotatory in nature after hydrolysis gives glucose and fructose, among which

(i) Glucose is laevorotatory and fructose is dextrorotatory.

(ii) Glucose is dextrorotatory and fructose is laevorotatory

(iii) The mixture is laevorotatory.

(iv) Both are dextrorotatory.

A. (i) and (iii)

B. (ii) and (iii)

C. (iii) and (iv)

D. (iii) only

Key: B

Solution: Sucrose is dextrorotatory but after hydrolysis gives dextrorotatory glucose and laevorotatory fructose. Since the laevorotation of fructose $\left(-92.4^{\circ}\right)$ is more than dextrorotation of glucose $\left(+52.5^{\circ}\right)$ the mixture is laevororatory.

68. Allyl cyanide molecule contains

A. 9 sigma bonds, 4 pi bonds and no lone pair

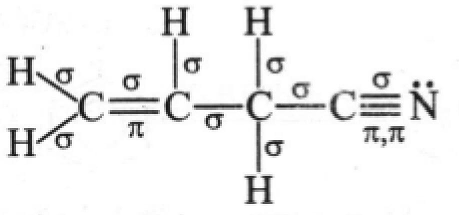
B. 9 sigma bonds, 3 pi bonds and one lone pair

C. 8 sigma bonds, 5 pi bonds and one lone pair

D. 8 sigma bonds, 3 pi bonds and two lone pairs

Key: B

Solution: Allyl cyanide is :



It contains 9 sigma bonds, 3 pi bonds and I lone pair of electrons

69. Which of the following pairs of compounds is isoelectronic and isostructural ?

A. TeI_2, XeF_2

B. IBr_2^-, XeF_2

C. IF_3, XeF_2

D. $BeCl_2, XeF_2$

Key: B

Solution: IBr_2^-, XeF_2

Total number ofvalence electrons are equal in both the spercies and both the species exhibit linear shape

70. In which case change in entropy is negative?

A. Evaporation of water

B. Expansion of a gas at constant temperature

C. Sublimation of Solution:id to gas

D. $2H(g) \rightarrow H_2(g)$

Key: D

Solution: In $2H(g) \rightarrow H_2(g)$, no. of species decreases, therefore entropy decreases.

PART III : MATHEMATICS

71. The argument of the complex number $\left(\frac{i}{2}-\frac{2}{i}\right)$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{2}$

Key: D

Solution: Since $\left(\frac{i}{2}-\frac{2}{i}\right)=\frac{i}{2}-\frac{2i}{i^2}=\frac{i}{2}+2i=\frac{5}{2}i$

So, argument is $\tan^{-1}\left(\frac{b}{a}\right)=\tan^{-1}\left(\frac{\frac{5}{2}}{0}\right)=\frac{\pi}{2}$

72. The lines $p\left(p^2+1\right)x-y+q=0$ and $\left(p^2+1\right)^2x+\left(p^2+1\right)y+2q=0$ are perpendicular to a common line for

A. exactly one value of P

B. exactly two values of P

C. more than two values of P

D. no value of P

Key: A

Solution: If the lines $p\left(p^2+1\right)x-y+q=0$

and $\left(p^2+1\right)^2x+\left(p^2+1\right)y+2q=0$

are perpendicular to a common line then these lines must be parallel to each other,

$$\therefore m_1=m_2\Rightarrow-\frac{p\left(p^2+1\right)}{-1}=-\frac{\left(p^2+1\right)^2}{p^2+1}$$

$$\Rightarrow\left(p^2+1\right)\left(p+1\right)=0\Rightarrow p=-1$$

73. The probability that a card drawn from a pack of 52 cards will be a diamond or king is

A. $\frac{1}{52}$

B. $\frac{2}{13}$

C. $\frac{4}{13}$

D. $\frac{1}{13}$

Key: C

Solution:

74. If n (A) = 4 and n (B) = 7 then the difference between maximum and minimum value of $n(A \cup B)$ is

A. 1

B. 2

C. 3

D. 4

Key: D

Solution: $n(A \cup B)$ is minimum when $A \subseteq B$ in this case, $(A \cup B) = B$ and hence minimum value of $n(A \cup B) = n(B) = 7$

$n(A \cup B)$ is maximum when A and B are disjoint.

Maximum value of $n(A \cup B) = 4 + 7 = 11$ So 11 - 7= 4

75. The domain of the function $f(x) = \frac{1}{\sqrt{9 - x^2}}$ is

A. $-3 \le x \le 3$

B. $-3 < x < 3$

C. $-9 \le x \le 9$

D. $-9 < x < 9$

Key: B

Solution: $f(x) = \frac{1}{\sqrt{9 - x^2}}$ Clearly, $9 - x^2 > 0 \Rightarrow x^2 - 9 < 0$

$\Rightarrow (x+3)(x-3) < 0$

Thus, domain of $f(x)$ is $x \in (-3,3)$.

76. If $\sin x + \cos x = \frac{1}{5}$ then $\tan 2x$ is

A. $\frac{25}{17}$

B. $\frac{7}{25}$

C. $\frac{25}{7}$

D. $\frac{24}{7}$

Key: D

Solution: $\sin x + \cos x = \frac{1}{5}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{1}{25}$

$\sin 2x = \frac{-24}{25} \Rightarrow \cos 2x = \frac{-7}{25} \Rightarrow \tan 2x = \frac{24}{7}$

77. For binary operation defined on $R - \{1\}$ such that $a * b = \frac{a}{b+1}$ is

A. not associative

B. not commutative

C. commutative

D. both (A) and (B)

Key: D

Solution: Commutative $a * b = \frac{a}{b+1}$ and $b * a = \frac{b}{a+1}$

$a * b \neq b * a \Rightarrow *$ is not commutative.

Associative:

$$\text{Now, } (a*b)*c = \left(\frac{a}{b+1}\right)*c = \frac{\left(\frac{a}{b+1}\right)}{c+1} = \frac{a}{(b+1)(c+1)}$$

$$\text{and } a*(b*c) = a*\left(\frac{b}{c+1}\right) = \frac{a}{\left(\frac{b}{c+1}\right)+1} = \frac{a(c+1)}{b+c+1}$$

So, clearly $\left(a^{*}b\right)^{*} c \neq a*(b*c)$

Hance, * is not associative

78. $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(1)+\tan^{-1}\frac{1}{\sqrt{3}}$ is equal to

A. π

B. $\frac{\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{3\pi}{4}$

Key: A

Solution: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(1)+\tan^{-1}\frac{1}{\sqrt{3}}$

$$=\frac{\pi}{3}+\frac{\pi}{2}+\frac{\pi}{6}=\frac{6\pi}{6}=\pi$$

79. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}x & 1\\ y & -1\end{bmatrix}$ and $(A+B)^2 = A^2+B^2$, then $x+y=$

A. 2

B. 3

C. 4

D. 5

Key: D

Solution: $(A+B)^2 = A^2 + B^2 \Rightarrow AB + BA = O$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix} + \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-y & 2 \\ 2x-y & 3 \end{bmatrix} + \begin{bmatrix} x+2 & -x-1 \\ y-2 & -y+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 2x - y + 2 = 0$ ....(i)

$-x+1=0$ ....(ii)

$2x-2=0$ .... (iii)

$-y+4=0$ ....(iv)

from (ii), $x=1$ and from (iv), $y=4$

Now, $x+y=1+4=5$

80. The value of $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ is

A. 0

B. abc

C. $4a^2b^2c^2$

D. None of these

Key: C

Solution: Let $\Delta = \begin{bmatrix} -a^2 & ab & ac \\ ab & --b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}$

Taking a, b, c common from R_1, R_2, and R_3, respectively, we get

$$\Delta = abc\begin{bmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{bmatrix} = a^2b^2c^2\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

[taking a, b, c common from C_1, C_2, C_3 respectively]

$$= a^2b^2c^2\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$(\text{applying} C_2 \to C_2 + C_1, C_3 \to C_3 + C_1)$

$$= a^2b^2c^2 \cdot (-1)\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = a^2b^2c^2(-1)(0-4)$$

$$\Rightarrow \Delta = 4a^2b^2c^2$$

81. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \alpha \end{bmatrix}$, then Adj. A is equal to :

A. $\begin{bmatrix} \delta & -\gamma \\ -\beta & \alpha \end{bmatrix}$

B. $\begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix}$

C. $\begin{bmatrix} -\delta & \beta \\ \gamma & -\alpha \end{bmatrix}$

D. $\begin{bmatrix} -\delta & -\beta \\ \gamma & \alpha \end{bmatrix}$

Key: B

Solution: Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$C_{11} = \delta, C_{12} = -\gamma, C_{21} = -\beta, C_{22} = \alpha$$

$$\therefore adjA = \begin{bmatrix} \delta & -\gamma \\ -\beta & \alpha \end{bmatrix}' = \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix}.$$

82. If $\sec\left(\frac{x-y}{x+y}\right) = a$, then $\frac{dy}{dx}$ is

A. $-\frac{y}{x}$

B. $\frac{x}{y}$

C. $-\frac{x}{y}$

D. $\frac{y}{x}$

Key: D

Solution: Given $\sec\left(\frac{x-y}{x+y}\right) = a \Rightarrow \frac{x-y}{x+y} = \sec^{-1} a$

Differentiating both sides, w.r.t. x, we get

$$\frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2} = 0$$

$$\Rightarrow x+y-(x+y)\frac{dy}{dx}-(x-y)-(x-y)\frac{dy}{dx} = 0$$

$$\Rightarrow 2y = \frac{dy}{dx}(x+y+x-y) \Rightarrow 2y = 2x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

83. The number of non zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ is

A. 2

B. 3

C. 4

D. 5

Key: D

Solution: Given expression

$$=2[1+{}^9C_2(3\sqrt{2}x)^2+{}^9C_4(3\sqrt{2}x)^4$$

$$=2[1+{}^9C_2(3\sqrt{2}x)^2+{}^9C_4(3\sqrt{2}x)^4+{}^9C_6(3\sqrt{2}x)^6+{}^9C_8\left(3\sqrt{2}x)^8\right]$$

$\therefore$ the number of non-zero terms is 5

84. If $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b, then the value of n is

A. 1

B. 2

C. 3

D. 4

Key: A

Solution: A. M. between a and $b=\dfrac{a+b}{2}$

$$\therefore \frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{a+b}{2}$$

$$2a^n+2b^n=a^n+ab^{n-1}+a^{n-1}b+b^n$$

$$\Rightarrow a^n a^{n-1}b-ab^{n-1}+b^n=0$$

$$\Rightarrow a^{n-1}(a-b)-b^{n-1}(a-b)=0$$

$$\Rightarrow (a-b)\left(a^n-b^{n-1}\right)+b^n=0 \quad [\because a\neq 0]$$

$$\Rightarrow a^{n-1}-b^{n-1}=0\Rightarrow a^{n-1}=b^{n-1}$$

$$\left(\frac{a}{b}\right)^{n-1}=1=\left(\frac{a}{b}\right)^0\Rightarrow n-1=0\Rightarrow n=1$$

85. The sum of the series $\dfrac{1}{1+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{4}}+\ldots$ upto 15 terms is

A. 1

B. 2

C. 3

D. 4

Key: C

Solution: Given series is

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots.$$

$$n^{\text{th}}\text{term}=\frac{1}{\sqrt{n}+\sqrt{n+1}} \quad \therefore 15^{\text{th}}\text{ term}=\frac{1}{\sqrt{15}+\sqrt{16}}$$

Thus, given series upto 15 terms is

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots+\frac{1}{\sqrt{15}+\sqrt{16}}$$

This can be re-written as

$$\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+\ldots\ldots+\frac{\sqrt{15}-\sqrt{16}}{-1}$$

(By rationalzation)

$$=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}+\ldots\ldots-\sqrt{14}+\sqrt{15}-\sqrt{15}+\sqrt{16}$$

$$=-1+\sqrt{16}=-1+4=3$$

Hence, the required sum:= 3

86. The equation of the circle with center (0, 2) and radius 2 is $x^2+y^2-my=0$ The value of m is

A. 1

B. 2

C. 4

D. 3

Key: C

Solution: Here $\text{h}=0,\text{k}=2$ and $\text{r}=2.$ Therefore, the required equation of the circle is

$(x-0)^2+(y-2)^2=(2)^2$

or $x^2+y^2-4y+4=4$ or $x^2+y^2-4y=0$

87. $\int x^x(1+\log x)dx$ is equal to

A. x^x+C

B. $x^{2x}+C$

C. $x^x\log x+C$

D. $1/2(1+\log x)^2+C$

Key: A

Solution: $I=\int x^x(1+\log x)dx$

Put $x^x=t$, then $x^x(1+\log x)dx=dt$

$\therefore\ I=\int dt \Rightarrow I=t+C \Rightarrow I=x^x+C.$

88. $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=$

A. $\frac{\pi}{\sqrt{2}}$

B. $\pi\sqrt{2}$

C. $\frac{\pi}{2}$

D. $\frac{\sqrt{2}}{\pi}$

Key: B

Solution: Let $I=\int_0^{\pi/2}\sqrt{\tan x}+\sqrt{\cot x}dx$

$$=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}dx$$

$$=\int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{1-(\sin x - \cos x)^2}} dx$$

Put, sin x - cos x:= t

$$\Rightarrow (\cos x + \sin x) dx = dt$$

$$= \int_{-1}^{1} \frac{\sqrt{2}}{\sqrt{1-t^2}} dt = \sqrt{2} \sin^{-1} \mathrm{t} |_{-1}^{1}$$

$$= \sqrt{2}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \pi\sqrt{2}$$

89. The area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$is

A. 12π

B. 3π

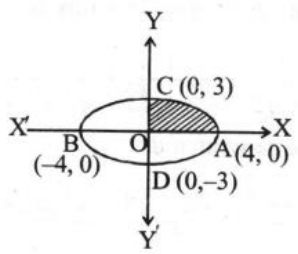
C. 24π

D. π

Key: A

Solution: The equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

The given ellipse is symmetrical to both axes as it contains only even powers of y and x



$$\text{Now, } \frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$\Rightarrow y^2 = \frac{9}{16}\left(16 - x^2\right)$$

$\Rightarrow \quad y = \pm\frac{3}{4}\left(\sqrt{16-x^2}\right)$

Now, area bounded by the ellipse = 4 (area of ellipse in first quadrant)

$= 4(\text{area OAC})$

$= 4\int_0^4 y dx = \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$

$[\because \text{y} \geq 0 \text{in first quadrant}]$

Put $x = 4\sin\theta$ so that $dx = 4\cos\theta\, d\theta,$

Now when $x = 0, \theta = 0$ and when $x = 4, \theta = \frac{\pi}{2}$

Req. area

$= \frac{4\times 3}{4}\int_0^{\frac{\pi}{2}}\sqrt{16-16\sin^2}\theta\cdot 4\cos\theta d\theta$

$= 3\int_0^{\frac{\pi}{2}} 4\sqrt{1-\sin^2\theta}\cdot 4\cos\theta d\theta$

$= 48\int_0^{\frac{\pi}{2}}\cos^2\theta \text{d}\theta = 48\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2\theta}{2}\right)\text{d}\theta$

$= 24\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)d\theta = 24\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 24\left[\left(\frac{\pi}{2}-0\right)+\frac{1}{2}(0-0)\right] = 12\pi \text{sq. units.}$

90. If vertex of a parabola is (2, -1) and the equation of its directrix is $4x-3y=21$ then the length of its latus rectum is

A. 2

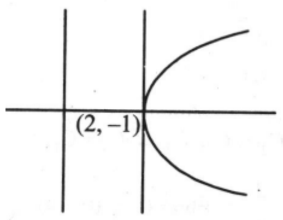
B. 8

C. 12

D. 16

Key: B

Solution:



We are given that the direction of the parabola is

$4x - 3y = 21$

and vertex ofthe parabola is (2, -1)

Now $a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2$

$\therefore$ latus rectum of the parabola $= 4\text{a} = 8$

91. Eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if it passes through point (9,5) and (12,4) is

A. $\sqrt{3/4}$

B. $\sqrt{4/5}$

C. $\sqrt{5/6}$

D. $\sqrt{6/7}$

Key: B

Solution: $\text{e} = \sqrt{1 - \frac{1}{7}} = \sqrt{\frac{6}{7}}$

92. In $\triangle ABC$ the mid-point of the sides AB,BC and CA are respectively $(l,0,0),(0,m,0)$ and $(0,0,n)$

Then, $\frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$ is equal to

A. 8

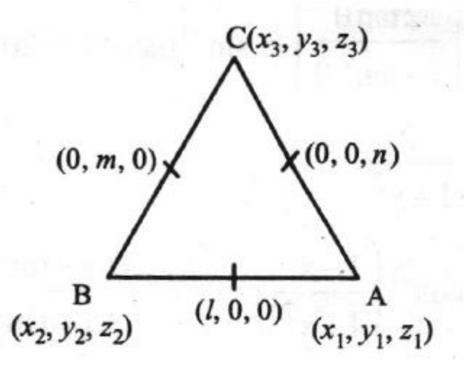
B. 16

C. 9

D. 25

Key: A

Solution: From the figure



$x_1 + x_2 = 2l, y_1 + y_2 = 0, z_1 + z_2 = 0$

$x_2 + x_3 = 0, y_2 + y_3 = 2m, z_1 + z_3 = 0$

$x_1 + x_3 = 0, y_1 + y_3 = 0, z_1 + z_3 = 2n$

On Solving, we get $x_1 = l, x_2 = l, x_3 = -1$

$y_1 = -m, y_2 = m, y_3 = m \text{and} z_1 = n, z_2 = -n, z_3 = n$

$\therefore$ Coordinates are $A(l,-m,n), B(l,m,-n)$ and $C(-l,m,n)$

$$\therefore \frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$$

$$= \frac{(4m^2 + 4n^2) + (4l^2 + 4n^2) + (4l^2 + 4m^2)}{l^2 + m^2 + n^2} = 8$$

93. If $f(x) = \frac{x + |x|}{x}$, then the value of $\lim_{x \to 0} f(x)$ is

A. 0

B. 2

C. does not exist.

D. None of these

Key: A

Solution: $\text{LHL} = \lim_{h\to 0} \frac{-h+|h|}{-h} = \lim_{h\to 0}(0) = 0$

$RHL = \lim_{\text{h}\to 0} \frac{\text{h}+|\text{h}|}{\text{h}} = 2$

$LHL \neq RHL \Rightarrow$ limit does not exist

94. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is

A. $(\sim p) \wedge q$

B. $(\text{p}) \wedge (\sim \text{q})$

C. $(\sim p) \vee (\sim q)$

D. $(\sim p) \wedge (\sim q)$

Key: D

Solution: Given expression is $p \Leftrightarrow (q \Rightarrow p)$

$\sim(\text{p} \leftrightarrow (\text{q} \to \text{p}))$

$\sim(\text{p} \leftrightarrow \text{q}) = (\text{p} \wedge \sim \text{q}) \vee (\text{q} \wedge \sim \text{p})$

$\sim(\text{p} \leftrightarrow (\text{q} \to \text{p}))$

$= (\text{p} \wedge \sim(\text{q} \to \text{p})) \vee ((\text{q} \to \text{p}) \wedge \sim \text{p})$

$(\text{p} \wedge \sim(\text{q} \to \text{p})) = \text{p} \wedge (\text{q} \wedge \sim \text{p})$

$= (\text{p} \wedge \sim \text{p}) \wedge \text{q} = \text{c}$

$(q \to p) \wedge \sim p = (\sim q \vee p) \wedge \sim p$

$= \sim p \wedge (\sim q \vee p)$

$= (\sim \text{p} \wedge \sim \text{q}) \vee (\sim \text{p} \wedge \text{p}) = \sim \text{p} \wedge \sim \text{q}$

$\sim\left(p \leftrightarrow (q \to p)\right) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

95. If $R = \{(x, y) : x$ is exactly 7 cm taller than y}, then R is

A. not symmetric
B. reflexive
C. symmetric but not transitive
D. an equivalence relation

Key: D

Solution: Here, R is not reflexive as x is not 7cm taller than x. R is not symmetric as if x is exactly 7cm taller than y, then y cannot be 7cm taller than x and R is not transitive as if x is exactly 7" " cm taller than y and y is exactly 7cm taller than z, then x is exactly 14 cm taller than z.

96. The particular solution of $\log \frac{dy}{dx} = 3x + 4y, y(0) = 0$ is

A. $e^{3x} + 3e^{-4y} = 4$
B. $4e^{3x} - 3^{-4y} = 3$
C. $3e^{3x} + 4e^{4y} = 7$
D. $4e^{3x} + 3e^{-4y} = 7$

Key: D

Solution: $\frac{dy}{dx} = e^{3x+4y} = e^{3x} \cdot e^{4y}$

$$\Rightarrow \quad e^{-4y} dy = e^{3x} dx \Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$$

put$x = 0$

We have $-\frac{1}{4} - \frac{1}{3} = c \Rightarrow c = -\frac{7}{12},$

$$\therefore \quad \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow 7 = 3e^{-4y} + 4e^{3x}$$

97. $\tan^{-1} x + \tan^{-1} y = c$ is the general solution of the differential equation

A. $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

B. $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

C. $\left(1+x^2\right)dy+\left(1+y^2\right)dx=0$

D. $\left(1+x^2\right)dx+\left(1+y^2\right)dy=0$

Key: C

Solution: $\tan^{-1}x+\tan^{-1}y=c$

differentiating w.r.t.**x**

$$\frac{1}{1+x^2}+\frac{1}{1+y^2}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dx}{1+x^2}+\frac{dy}{1+y^2}=0$$

$$\Rightarrow \left(1+y^2\right)dx+\left(1+x^2\right)dy=0$$

98. If $|\vec{a}|=3, |\vec{b}|=4,$ then value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$ is

A. $\frac{9}{16}$

B. $\frac{3}{4}$

C. $\frac{3}{2}$

D. $\frac{4}{3}$

Key: C

Solution: If $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, then

$$\left(\vec{a}+\lambda\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=\left|\vec{a}+\lambda\vec{b}\right|\left|\vec{a}-\lambda\vec{b}\right|\cdot\cos 90^\circ$$

$$\Rightarrow\left(\vec{a}+\lambda\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=0$$

$\Rightarrow \quad \vec{a}\cdot\vec{a}-\lambda\cdot\vec{a}\cdot\vec{b}+\lambda\cdot\vec{b}\cdot\vec{a}-\lambda^2\cdot\vec{b}\cdot\vec{b}=0$

$\Rightarrow \quad a^2-\lambda^2 b^2=0 \Rightarrow \lambda^2=\frac{a^2}{b^2} \Rightarrow \lambda^2=\frac{3^2}{4^2}$

$\Rightarrow \lambda=\frac{3}{4}.$

99. The area of the parallelogram whose diagonals are $\frac{3}{2}\hat{i}+\frac{1}{2}\hat{j}-k$ and $2\hat{i}-6\hat{j}+8\hat{k}$ is

A. $5\sqrt{3}$

B. $5\sqrt{2}$

C. $25\sqrt{3}$

D. $25\sqrt{2}$

Key: C

Solution: Given diagonals of parallelogram are $d_1=\frac{3}{2}\hat{i}+\frac{1}{2}\hat{j}-\hat{k}$ and $d_2=2\hat{i}-6\hat{j}+8\hat{k}$

$\therefore$ Required area $A=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{3}{2} & \frac{1}{2} & -1\\ 2 & -6 & 8\end{vmatrix}$

$=\frac{1}{2}\left[(4-6)\hat{i}-(12+2)\hat{j}+(-9-1)\hat{k}\right]$

$=-\hat{i}-7\hat{j}-5\hat{k}$

$\therefore A=\sqrt{49+25}=\sqrt{75}=5\sqrt{3}$ sq. unit

100. Bag P contains 6 red and 4 blue balls and bag Q contains 5 red and 6 blue balls. A ball is transferred from bag P to bag Q and then a ball is drawn from bag Q. What is the probability that the ball drawn is blue?

A. $\frac{7}{15}$

B. $\frac{8}{15}$

C. $\frac{4}{19}$

D. $\frac{8}{19}$

Key: C

Solution: Let E_1, E_2 and A be the events defined as follows:

E_1 = red ball is transferred from bag P to bag Q

E_2 = blue ball is transferred from bag P to bag Q

A= the ball drawn from bag Q is blue

As the bag P contains 6 red and 4 blue balls,

$$P(E_1) = \frac{6}{10} = \frac{3}{5} \text{ and } P(E_2) = \frac{4}{10} = \frac{2}{5}$$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6 red and 6 blue balls, So, $P(A|E_1) = \frac{6}{12} = \frac{1}{2}$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q, then bag Q will contain 5 red and 7 blue balls, So, $P(A|E_2) = \frac{7}{12}$

By using law of total probability, we get

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$$

$$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{7}{12} = \frac{8}{15}$$

101. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X=1)$ is

A. $\frac{1}{4}$

B. $\frac{1}{32}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

Key: B

Solution: $\left.\begin{matrix} np=4 \\ npq=2 \end{matrix}\right\} \Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$$P(X=1)={}^{8}C_{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{7}=8\cdot\frac{1}{2^{8}}=\frac{1}{2^{5}}=\frac{1}{32}$$

102. The value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)=$

A. $\frac{6}{17}$

B. $\frac{7}{16}$

C. $\frac{16}{7}$

D. None of these

Key: D

Solution: $\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]=\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}=\frac{17}{6}$$

103. If the function $f(x)=\begin{cases} 1, & x\le 2 \\ ax+b & 2<x<4 \\ 7, & x\ge 4 \end{cases}$ is continuous at x=2 and 4, then the values of a and b are

A. $a=3, b=-5$

B. $a=-5, b=3$

C. $a=-3, b=5$

D. $a=5, b=-3$

Key: A

Solution: Since f(x) is continuous at x=2

$$\therefore f(2)=\lim_{x\to 2^+} f(x)=1=\lim_{x\to 2^+}(ax+b)$$

$$\therefore 1=2a+b.......(i)$$

Again f(x) is continuous at x=4,

$$\therefore f(4)=\lim_{x\to 4^-} f(x)=7=\lim_{x\to 4^-}(ax+b)$$

$$\therefore 7=4a+b........(ii)$$

Solving (i) and (ii), we get a=3, b=-5

104. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ is equal to :

A. 1

B. -1

C. 2

D. None of these

Key: A

Solution: Let $s=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $t=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

We have to find out $\frac{ds}{dt}$

Putting $x=\tan\theta$, we get

$$s=\sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}x$$

$\therefore \frac{ds}{dx} = \frac{2}{1+x^2}$ and $t = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$

$\therefore \quad \frac{dt}{dx} = \frac{2}{1+x^2}$

$\therefore \quad \frac{ds}{dt} = \frac{ds/dx}{dt/dx} = \frac{2}{1+x^2} \times \frac{1+x^2}{2} = 1$

105. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

A. 5
B. 7
C. 1
D. 3

Key: D

Solution: Given equation is$x^7 - 7x - 2 = 0$

$x^7 - 7x = 2$

$f(x) = x^7 - 7x \text{and} y = 2$

Differentiate w.r.t. x.

$f'(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$

A function f(x) is strictly decreasing at [-1,1] and increasing at some points of x<-1 and x>1.

y=2 intersects at 3 points.

f(x)=2 has 3 real distinct Solution:ution.

106. The minimum value of the function $y = x^4 - 2x^2 + 1$ in the interval $\left[\frac{1}{2}, 2\right]$ is

A. 0
B. 2
C. 8
D. 9

Key: A

Solution: $\frac{dy}{dx}=\frac{d}{dx}\left(x^4-2x^2+1\right)=4x^3-4x=4x\left(x^2-1\right)$

For max. or min, $\frac{dy}{dx}=0\Rightarrow 4x\left(x^2-1\right)=0$

either $x=0$ or $x=\pm 1$

x = 0 and x = −1 does not belong to $\left[\frac{1}{2},2\right]$

$\frac{d^2y}{dx^2}=12x^2-4$

$\Rightarrow\left(\frac{d^2y}{dx^2}\right)_{x=1}=12(1)^2-4=8>0$

$\therefore$ there is minimum value of function at x=1

$\therefore$ minimum value is

$y(1)=1^4-2(1)^2+1=1-2+1=0$

107. $\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=$

A. $\tan x+\cot x+c$

B. $cosecx+scex+c$

C. $\tan x+\sec x+c$

D. $\tan x+cosecx+c$

Key: A

Solution: Consider $\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$

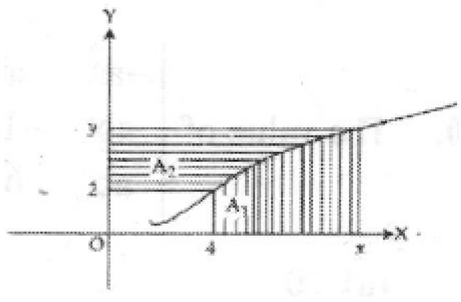
$=\int\frac{\sin^2 xdx}{\sin^2 x\cos^2 x}-\int\frac{\cos^2 xdx}{\sin^2 x\cos^2 x}$

$=\int\sec^2 xdx-\int cosec^2 xdx$

Let $I=\int\left(\sec^2 x-cosec^2 x\right)dx$

$= \tan x - (-\cot x) + c = \tan x + \cot x + c.$

108. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2 Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does NOT pass through the point.



A. $(6, 21)$

B. $(8, 9)$

C. $(10, -4)$

D. $(12, -15)$

Key: C

Solution: Given condition is $\mathrm{A}_1 = 2\mathrm{A}_2$

Given graph is a rectangle then the required area Given graph is a rectangle then the required area

$A_1 + A_2 = xy - 8$

Now, Put $A_1 = 2A_2$ in the above eq.

$$\Rightarrow \frac{3}{2}A_1 = xy - 8 \Rightarrow A_1 = \frac{2}{3}xy - \frac{16}{3}$$

Now, take $\int_4^x f(x)\,dx = \frac{2}{3}xy - \frac{16}{3}$

$$\Rightarrow f(x) = \frac{2}{3}\left(x\frac{dy}{dx} + y\right) \Rightarrow \frac{2}{3} \times \frac{dy}{dx} = \frac{y}{3}$$

Take integral both sides.

$$\Rightarrow 2\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow 2\ln y = \ln x + \ln c$$

$$\Rightarrow y^2 = cx$$

From given graph f(4)=2

$$\Rightarrow c = 1$$

from(i)

so $y^2 = x$

slope of normal =-6

$$y = -6(x) - \frac{1}{2}(-6) - \frac{1}{4}(-6)^3$$

$$\Rightarrow y = -6x + 3 + 54 \Rightarrow y + 6x = 57$$

Put coordinate (10,-4) in above equation

$$\Rightarrow -4 + 60 = 56 \neq 57$$

109. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is

A. $\frac{1}{2}$

B. 2

C. 1

D. $\frac{3}{2}$

Key: A

Solution: The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$ and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here, $\vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$

Here, $\vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\text{Shortest distance} = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 - \vec{b}_2)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \frac{\left|(-\hat{i} + 2\hat{j} - \hat{k})\cdot(-12i + 18\hat{j} - 36\hat{k})\right|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{|+12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

110. The angle between two lines $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$

A. $\cos^{-1}\left(\frac{1}{9}\right)$

B. $\cos^{-1}\left(\frac{4}{9}\right)$

C. $\cos^{-1}\left(\frac{2}{9}\right)$

D. $\cos^{-1}\left(\frac{3}{9}\right)$

Key: B

Solution: The angle θ between the two lines

$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3}$ and $\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3}$ is given by:

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Now in the given equation: $a_1 = 2, a_2 = 2, a_3 = -1$

$b_1 = 1, b_2 = 2, b_3 = 2$

$$\therefore \cos\theta = \frac{2\times 2 + 2\times 2 + (-2)\times 1}{\sqrt{4+4+1}\sqrt{4+4+1}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right).$$

PART IV : APTITUDE TEST

DIRECTIONS (Qs. 111to 113): The following table gives the annual production (in thousands) of 5 products of a famous toy company. Study the table and then answer the questions that follow:

Year	**Ludo**	**Scrabble**	**Chess**	**Monopoly**	**Carrom**
1992	200	150	78	90	65
1993	150	180	100	105	70
1994	180	175	92	110	85
1995	195	160	120	125	75
1996	220	185	130	135	80

111. What is the approximate percentage increase in the production of Monopoly form 1993 to 1995?

A. 10

B. 20

C. 30

D. 25

Key: B

Solution: Percentage increase in the production of monopoly

$$=\frac{(125-105)}{105}\times 100=\frac{20}{105}\times 100$$

$$=19.05\% \approx 20\%$$

112. For which toy category there has been a continuous increase in the production over the years?

A. Ludo

B. Chess

C. Monopoly

D. Carrom

Key: C

Solution: Production of monopoly has shown continuous increase over the years.

113. What is the percentage drop in the production of Ludo from 1992 to 1994 ?

A. 30

B. 50

C. 20

D. 10

Key: D

Solution: $\%\text{drop} = \frac{200-180}{200} \times 100 = 10\%$

DIRECTION (Qs. 114-117): In question, a series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.

114. 285, 253, 221, 189,?

A. 150

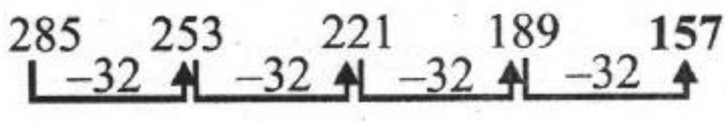
B. 182

C. 157

D. 156

Key: C

Solution:



115. In a certain code language PRESENTATION is written as ENESTAITPRON. How would INTELLIGENCE be written in that code language?

A. TETGLLTNENCE

B. LLKKTGTEEBTB

C. LLENLLTNTETG

D. LLTEIGENINCE

Key: D

Solution: $\frac{PR}{1}\frac{ES}{2}\frac{EN}{3}\frac{TA}{4}\frac{TI}{5}\frac{ON}{6} \rightarrow \frac{EN}{3}\frac{ES}{2}\frac{TA}{4}\frac{IT}{5}\frac{PR}{1}\frac{ON}{6}$

Similarly, $\frac{IN}{1}\frac{TE}{2}\frac{LL}{3}\frac{IG}{5}\frac{EN}{6}\frac{CE}{} \rightarrow \frac{LL}{3}\frac{TE}{2}\frac{IG}{4}\frac{EN}{5}\frac{IN}{1}\frac{CE}{6}$

116. Ram moves from a point X to 20 metres towards North. Then he moves 40 metres towards West. Then he moves 20 metres North. Then he moves 40 metres towards East and then 10 metres towards right and he reaches to a point Y. Find the distance and direction of Y from X ?

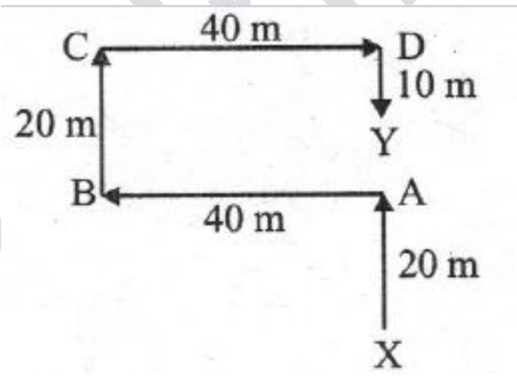
A. 30 metres, North

B. 30 metres, South

C. 40 metres, North

D. 40 metres, South

Key: A



Solution:

Required distance

$= XY = AX + AY$

$= 20 + 10$

$= 30$m, North

117. If the 5th date of a month is Tuesday, what date will be 3 days after the 3rd Friday in the month?

A. 17

B. 22

C. 19

D. 18

Key: D

Solution: 5th date of a month is Tuesday Friday will be on =5+3
=8th of a month
1st Friday is on 1st of a month
2nd Friday is on 8th of a month
3rd Friday will be on 15th of a month
3 days after 15th =15+3=18

DIRECTION (Q. 118): In the following question, two statements are given followed by four conclusions I, II, III and IV. You have to consider the statements to be true even if they seem to be at variance from commonly known facts. You have to decide which of the given conclusions, if any follow from the given statements.

118. **Statements:**
I. Some cats are dogs.
II. No dog is a toy.
Conclusions:
I. Some dogs are cats.
II. Some toys are cats.
III. Some cats are not toys.
IV. All toys are cats.

A. Only Conclusions I and either II or III.

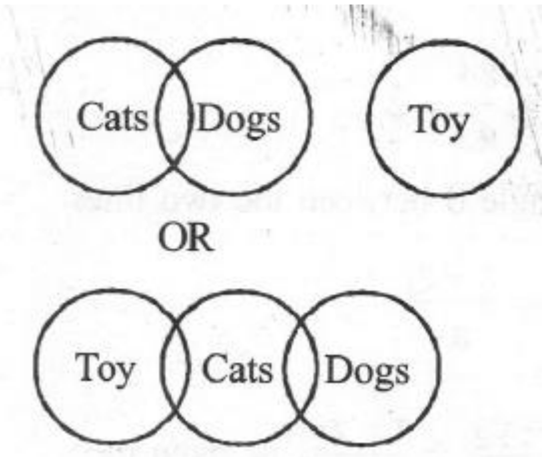
B. Only Conclusions II and III follow

C. Only Conclusions I and II follow

D. Only Conclusion I follow

Key: A

Solution:



Conclusion II: Complementary pair

Conclusion I : True

III : Complementary Pair

IV : False

So, only conclusion I and either II or III.

DIRECTION (Qs. 119-120): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question

Give answer (c) if the data in Statement I alone or in Statement II alone are sufficient to answer the question.

Give answer (d) if the data in both the Statements I and II together are not sufficient to answer the question.

Give answer (e) if the data in both the Statements I and II together are necessary to answer the question.

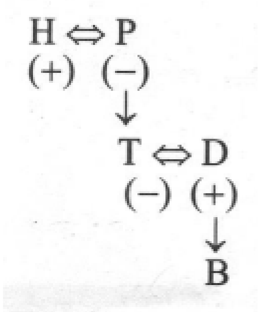
119. How is H related to B ?

I. H is married to P. P is the mother of T. T is married to D. D is the father of B.

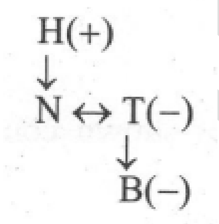
II. B is the daughter of T. T Is the sister of N. H is the father of N.

Key: C

Solution: From statement I



Hence, His the grandfather of B From statement II



Hence H is the grandfather of B

So data in statement I alone or in statement II alone

are sufficient to answer the question.

120. Among five persons D, E, F, G and H each of whom having different height, who is the second tallest?

I. D is taller than only G and E. F is not the tallest.

II. H is taller than F.G is taller than E but shorter than D.

Key: A

Solution: From statement I

$$\boxed{H} > \boxed{F} > \boxed{D} > \boxed{G}, \boxed{E}$$

Clearly, H is the tallest and F is the second tallest.

From statement II

$$\boxed{H} > \boxed{F}$$
$$\boxed{D} > \boxed{G} > \boxed{E}$$

No answer.

So data in statement I alone are sufficient to answer the question.

PART V : ENGLISH

PASSAGE
(Question Nos. 121-125)

If an opinion contrary to your own makes you angry, that is a sign that you are subconsciously aware of having no good reason for thinking, as you do. If someone maintains that two and two are five, or that Iceland is on the Equator, you feel pity rather than anger, unless you know so little of arithmetic or geography that his opinion shakes your own contrary conviction.

121. If someone else's opinion makes us angry, it means that

A. we are subconsciously aware of having no good reason for becoming angry

B. there may be good reasons for his opinion, but we are not consciously aware of them

C. our own opinion is not based on good reason, and we know this subconsciously

D. we are not consciously aware of any reason for our own opinion

Key ; C

Solution: The very first line of the passage reveals that we can become angry on someone's opinion contrary to ours only when our own opinion is not based on good reason and we are aware of this subconsciously.

122. "Your own contrary conviction" refers to

A. the fact that you feel pity rather than anger

B. the opinion that two and two are four and that Iceland is a long way from the Equator

C. the opinion that two and two are five and that Iceland is on the Equator

D. the fact that you know so little about arithmetic or geography

Key: A

Solution: 'Your own contrary conviction' refers to the fact that you feel pity rather than anger.

123. Conviction means

A. persuasion

B. disbelief

C. strong belief

D. ignorance

Key: C

Solution: Conviction means a firmly held belief or opinion.

124. The writer says if someone maintains that two and two are five you feel pity because you

A. have sympathy

B. don't agree with him

C. want to help the person

D. feel sorry for his ignorance

Key: D

Solution: If someone maintains that two and two are five, you feel pity because you feel sorry for his ignorance of the subject i.e. Arithmetic.

125. The second sentence in the passage

A. builds up the argument of the first sentence by restating it from the opposite point of view

B. makes the main point which has only been introduced by the first sentence

C. simply adds, a further point to the argument already stated in the first sentence.

D. illustrates the point made in the first sentence

Key: D

Solution: The second sentence in the passage elaborates the hidden i.e. the main point in the first sentence.