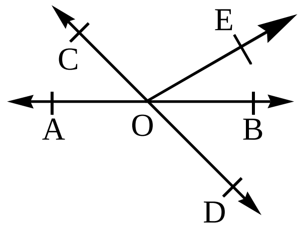
LINES AND ANGLES

EXERCISE 6.1

1. In Fig. lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



Sol. $\angle AOC = \angle BOD$ [Vertically opposite angles]

$\angle AOC = 40°$ ...(1)

[$\therefore \angle BOD = 40°$, given]

$\angle AOC + \angle BOE = 70°$ [Given]

$40° + \angle BOE = 70°$ [From (1)]

$\angle BOE = 70° - 40° = 30°$ ...(2)

$\angle AOC + \angle COB = 180°$ [Linear pair]

So, $\angle AOC + \angle COE + \angle BOE = 180°$

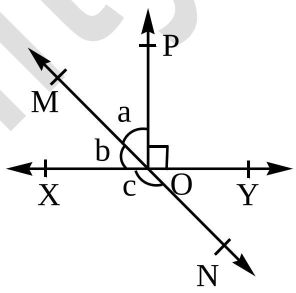
or $40° + \angle COE + 30° = 180°$

[From (1) and (2)]

So, $\angle COE = 180° - 40° - 30° = 110°$

Therefore, Reflex $\angle COE = 360° - 110° = 250°$

2. In Fig. lines XY and MN intersect at O. If $\angle POY = 90°$ and $a:b = 2:3$, find c.



Sol. Here, $a:b = 2:3$ and $a + b = \angle POX = \angle POY - 90°$

and sum of the ratios $= 2 + 3 = 5$.

$$a = \frac{2}{5} \times 90° = 2 \times 18° = 36°$$

and $$b = \frac{3}{5} \times 90° = 3 \times 18° = 54°$$

Also, MN is a line.

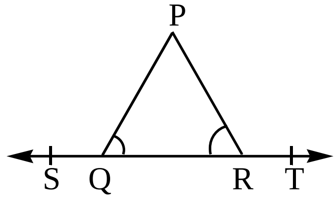
Since, ray OX stands on MN, therefore $\angle MOX + \angle XON = 180°$ [Linear pair]

or $b + c = 180°$

$\Rightarrow$ $c + 54° = 180°$ $\Rightarrow$ $c = 180° - 54° = 126°$

Hence, $c = 126°$.

3. In Fig. $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Sol.

SRT is a line.

Since QP stands on the line SRT, therefore $\angle PQS + \angle PQR = 180°$ [Linear Pair] ...(1)

Since RP stands on the line SRT, therefore $\angle PRQ + \angle PRT = 180°$ [Linear pair] ...(2)

From (1) and (2), we have

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$ [∴ Each side $= 180°$] ...(3)

Also, $\angle PQR = \angle PRQ$ [Given] ...(4)

Subtracting (4) and (3), we have

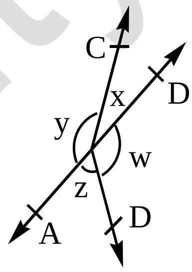
$\angle PQS = \angle PRT$

4. In Fig., if $x + y = w + z$, then prove that AOB is a line.

Sol. Since the sum of all the angles round a point is equal to $360°$, therefore

$(\angle BOC + \angle COA) + (\angle BOD + \angle AOD) = 360°$

$\Rightarrow \quad (x + y) + (w + z) = 360°$



But $x + y = w + z$ [Given]

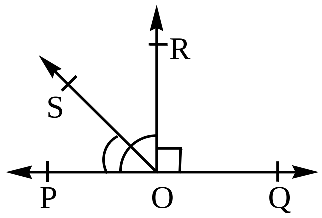
$(x + y) + (x + y) = 360°$

$2(x + y) = 360°$

$(x + y) = \frac{360°}{2} = 180°$

Thus, $\angle BOC$ and $\angle COA$ as well as $\angle BOD$ and $\angle AOD$ form linear pairs. Consequently, OA and OB are two opposite rays. Therefore, AOB is a straight line.

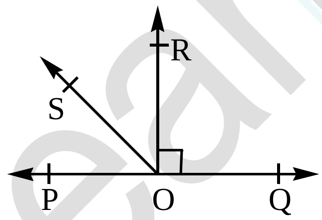
5. In fig. POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Sol. **Given.** Since, OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR.

To prove. $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Proof. $\angle QOS - \angle POS = (\angle QOR + \angle ROS) - \angle POS$



$= 90° + \angle ROS - \angle POS$

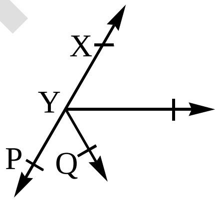
$= (90° - \angle POS) + \angle ROS$

$= (\angle ROP - \angle POS) + \angle ROS \quad [\because \angle ROP = 90°]$

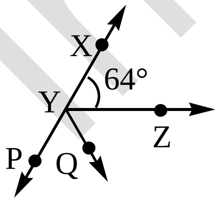
$= \angle ROS + \angle ROS$

$= 2\angle ROS$

6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given Information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.



Sol.



Since XY is produced to point P, therefore XP is a straight line.

Now ray YZ stands on XP.

$\therefore \quad \angle XYZ + \angle ZYP = 180°$ [Linear Pari]

$\Rightarrow \quad 64° + \angle ZYP = 180°$ $[\therefore \angle XYZ = 64°]$

$\Rightarrow \quad \angle ZYP = 180° - 64° = 116°$

Since ray YQ bisects $\angle ZYP$,

Therefore $\angle QYP = \angle ZYQ = \frac{116°}{2} = 58°$

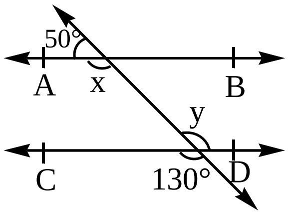
Now, $\angle XYQ = \angle XYZ + \angle ZYQ$

$\Rightarrow$ $\angle XYQ = 64° + 58° = 122°$

and reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$

EXERCISE 6.2

1. In the figure below, find the values of x and y and then show that $AB \parallel CD$.



Sol. $y = 130°$ [Vertically opposite angles]

Further, $50° + x = 180°$ [Linear pair]

$\Rightarrow$ $x = 130°$

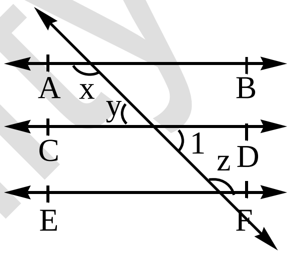
Hence, $x = y = 130°$ [From eqn. No. (1) and (2)]

Transversal intersects lines AB and CD.

Such that $x = y$. [Alternate interior angles]

Hence, $AB \parallel CD$.

2. In figure, if $AB \parallel CD, CD \parallel EF$ and $y : z = 3:7$, find x.



Sol. $AB \parallel CD$ and $CD \parallel EF$ ...(1)

[Given]

$\angle 1 = y$ [Vertically opposite angels]

$\angle 1 + \angle Z = 180$

[$CD \parallel EF$ and $\angle 1, \angle z$ are on the same side of the transversal]

$\Rightarrow$ $y + z = 180°$ ...(2)

Given: $y : z = 3:7 \Rightarrow \frac{y}{z} = \frac{3}{7} \Rightarrow y = \frac{3z}{7}$

$\Rightarrow \frac{3z}{7} + z = 180° \Rightarrow \frac{10z}{7} = 180°$

[using equation (2)]

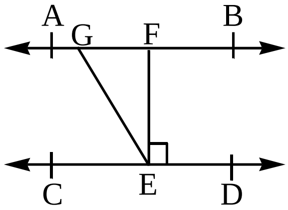
$z = 126°$

So, $y = 180° - 126° = 54°$

[Using equation (1)]

$\rightarrow \quad x+y=180° \Rightarrow x+54°=180°-54°=126°$

3. In figure, $AB \,||\, CD, EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE, \angle GEF$ and $\angle FGE$.



$AB \,||\, CD$ and GE is transversal.

So, $\angle FGE = \angle GED$ [Alternate angles]

$\Rightarrow \quad AGE = 126°$.

As we have, $\angle GED = \angle GEF + \angle FED$

$\Rightarrow 126° = \angle GEF + 90°$ $[EF \perp CD]$

$\Rightarrow \quad \angle GEF = 126° - 90° = 36°$.

Again, $AB \,||\, CD$ and GE is transversal.

$\therefore \quad \angle FEG + \angle GED = 180°$

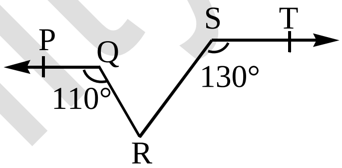
[sum of interior angles on the same side of transversal is $180°$]

$\Rightarrow \quad \angle FGE + 126° = 180°$

$\Rightarrow \quad \angle FGE = 180° - 126° = 54°$.

4. In figure, if $PQ \,||\, ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.

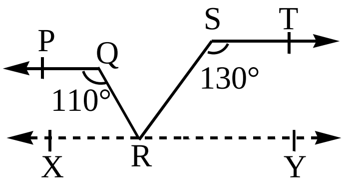
[Hint: Draw the parallel to ST through point R.]



Sol. Construction: Through R draw a line XRY parallel to PQ.

Proof: $PQ \,||\, XRY$ and QR is transversal.

$\therefore \quad \angle PQR = \angle QRY = 110° \quad ...(1)$ [Alternative angles]



Also, $PQ \,||\, ST$ [Given]

and $PQ \,||\, RY$ [Construction]

So, $ST \,||\, XRY$ and SR is transversal.

Hence $\angle TSR + \angle SRY = 180°$ [Sum of interior angles on the same side of transversal]

$\Rightarrow \quad 130° + \angle SRY = 180°$

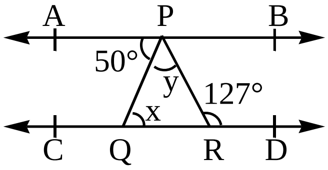
$\Rightarrow \quad \angle SRY = 180° - 130° = 50°$

Also, $\angle QRY = \angle QRS + \angle SRY$

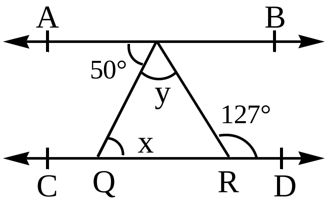
$\Rightarrow \quad 110° = \angle QRS + 50°$

$\Rightarrow \quad \angle QRS = 110° - 50° = 60°.$

5. In Fig, if $AB \parallel CD, \angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Sol.



Here, $AB \parallel CD$ and transversal PQ intersects them at P and Q respectively.

$\angle PQR = \angle APQ$ [Alternate angles]

$\Rightarrow \quad x = 50°$ [$\therefore \angle APQ = 50°$ (given)]

Also, $AB \parallel CD$ and transversal PR intersects them at P and R respectively.

$\angle APR = \angle PRD$

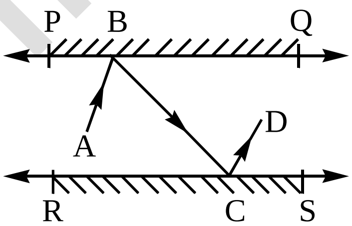
$\Rightarrow \quad \angle APQ + \angle QPR = 127°$

$\Rightarrow \quad 50° + y = 127°$

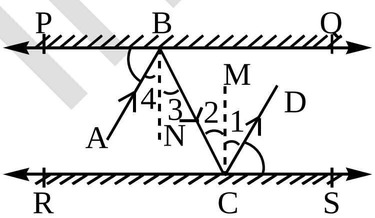
$\Rightarrow \quad y = 127° - 50° = 77°$

Hence, $x = 50°$ and $y = 77°$.

6. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Sol.



Given. Mirror $PQ \parallel$ Mirrors RS.

An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path C ad strikes

the mirror RS at C and again reflects back along CD.

To prove. $AB \parallel CD$

Construction. Draw normal BN on PQ and CM on RS.

Proof. $\angle 1+\angle 2=\angle 3+\angle 4 (BN \perp PQ)$

$\therefore$ $\angle 2=\angle 3$

(Angle of incidents $=$Angle of reflection)

$\Rightarrow$ $\angle 1+\angle 2=\angle 2+\angle 4 \Rightarrow \angle 1=\angle 4$ ...(1)

Similarly, $\angle 5=\angle 8$. ...(2)

But $\angle 4=\angle 5$ (Alternate angles) ...(3)

From $(1),(2)$ and (3), we have

$\angle 1=\angle 8$ ...(4)

Now, $\angle PBC=\angle BCS$ (Alternate angels) ...(5)

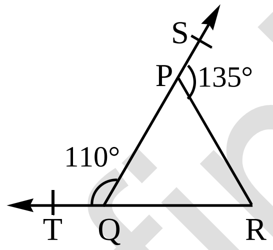
Subtracting (4) from (5), we have

$$\angle PBC-\angle 1=\angle BCS-\angle 8 \Rightarrow \angle ABC=\angle BCD$$

EXERCISE 6.3

1. In figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR=135°$ and $\angle PQT=110°$, find $\angle PRQ$.

Sol.



In $\Delta PQR, \angle PQT=\angle QPR+\angle PRQ$ [Exterior angle property]

$\Rightarrow$ $110°=\angle QPR+\angle PRQ$ ...(1)

Also, $\angle SPR+\angle QPR=180°$ [Linear pair]

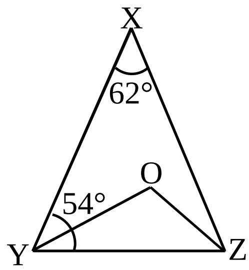
$\Rightarrow$ $135°+\angle QPR=180° \Rightarrow \angle QPR=180°-135°=45°$.

Substituting the value of $\angle QPR$ in (1), we get

$$110°=45°+\angle PRQ \Rightarrow \angle PRQ=110°-45°=65°$$

2. In figure, $X=62°$, $\angle XYZ=54°$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$

Sol.



Consider ΔXYZ $\angle YXZ + \angle XYZ + \angle XZY = 180°$ [Angle-sum property]

$\Rightarrow$ $62° + 54° + \angle XZY = 180°$ $[\because \angle YXZ = 62°, \angle XYZ = 54°]$

$\Rightarrow$ $\angle XZY = 180° - 62° - 54° = 64°$

Since YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$, therefore

$$\angle OYZ = \frac{1}{2} \times \angle XYZ = \frac{1}{2} \times 54° = 27°$$

and, $$\angle OZY = \frac{1}{2} \times \angle XZY = \frac{1}{2} \times 64° = 32°$$

Now, in ΔOYZ, we have:

$\angle YOZ + \angle OYZ + \angle OZY = 180°$

[Angle-sum property]

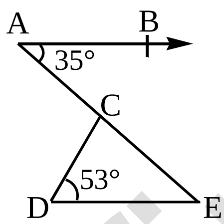
$\angle YOZ + 27° + 32° = 180°$

$\Rightarrow$ $\angle YOZ = 180° - 27° - 32° = 121°$

Hence, $\angle OZY = 32°$ and $\angle YOZ = 121°$.

3. In figure, if AB $||$ DE, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.

Sol.



Since $AB || DE$ and transversal AE intersects them at A and E respectively, therefore

$\angle DEA = \angle BAE$

[Alternate angles]

$\Rightarrow$ $\angle DEC = 35°$

$[\because \angle DEA = \angle DEC \text{ and } \angle BAE = 35°]$

In ΔDEC, we have:

$\angle DCE + \angle DEC + \angle CDE = 180°$ [Angle-sum property]

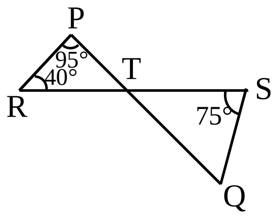
$\Rightarrow$ $\angle DCE + 35° + 53° = 180°$

$\Rightarrow$ $\angle DCE = 180° - 35° - 53° = 92°$

Hence, $\angle DCE = 92°$.

4. In figure, if lines PQ and Rs intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.

Sol.



In ΔPRT, $\angle P + \angle R + \angle PTR = 180°$ [Sum of angles of a triangle is $180°$.]

$\Rightarrow \quad 95° + 40° + \angle PTR = 180°$

$\Rightarrow \quad \angle PTR = 180° - 135° = 45°$ ...(1)

Also, $\angle STQ = \angle PTR$ [Vertically opposite angles]

$\Rightarrow \quad \angle STQ = 45°$ ...(2)

[using eqn. (1)]

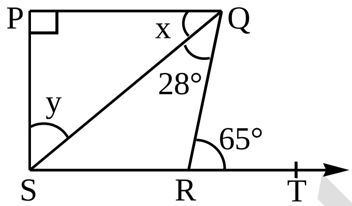
In ΔTSO, $\Delta STO + \angle S + \angle TQS = 180°$ [ASP of Δ]

$\Rightarrow \quad 45° + 75° + \angle TQS = 180°$

$\Rightarrow \quad \angle TQS = 180° - 120° = 60°$

5. In figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.

Sol.



Using exterior angle property in ΔSRQ, we have:

$$\angle QRT = \angle RQS + \angle QSR$$

$\Rightarrow \quad 65° = 28° + \angle QSR$ $[\because \angle QRT = 65°, \angle RQS = 28°]$

$\Rightarrow \quad \angle QSR = 65° - 28° = 37°$

Now, $PQ \parallel SR$ and the transversal PS intersects them at P and S respectively.

$\therefore \quad \angle PSR + \angle SPQ = 180°$ [Sum of the interior angles on the same side of the transversal is $180°$]

$\Rightarrow \quad (\angle PSQ + \angle QSR) + 90° = 180°$

$\Rightarrow \quad y + 37° + 90° = 180°$

$\Rightarrow \quad y = 180° - 90° - 37° = 53°$

In the right triangles SPQ, we have:

$$\angle PQS + \angle PSQ = 90°$$

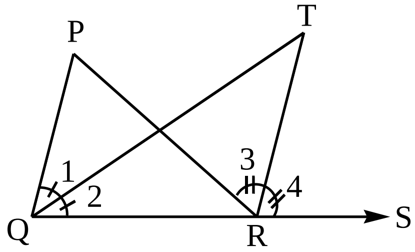
$\Rightarrow \quad x + 53° = 90°$

$\Rightarrow \quad x = 90° - 53° = 37°$

Hence, $x = 37°$ and $y = 53°$.

6. In figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at the point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.

Sol.



Let $\angle PQT = \angle 1$

$\angle TQR = \angle 2$

$\angle PRT = \angle 3$

$\angle TRS = \angle 4$

Also, not that: $2\times\angle 3 = 2\times\angle 4 = \angle PRS$ {∴ RT bisects $\angle PRS$}...(1)

and $2\times\angle 1 = 2\times\angle 2 = \angle PQR$ {∴ QT bisects $\angle PQR$}...(2)

$\angle PRS = \angle P + \angle PQR$ [By exterior angle property]...(3)

Now, from equation (1) we can put either $2\times\angle 3$ or $2\times\angle 3$ or $2\times\angle 4$ in place of $\angle PRS$ and also

$2\times\angle 1$ or $2\times\angle 2$ in place of $\angle PQR$ in equation (3).

Hence, by suitable replacement, we get

$$2\times\angle 4 = \angle P + 2\times\angle 2$$

...(4)

Now, in $\Delta TQR,\ \angle 4 = \angle 2 + \angle T$

...(5)

Value of $\angle 4 = \angle 2 + \angle T$ from equation (5) to equation (3).

$$2\times[\angle 2 + \angle T] = \angle P + 2\times\angle 2$$

$$\Rightarrow 2\times\angle 2 + \angle 2\times\angle T = \angle P + 2\times\angle 2$$

On cancelling $2\times\angle 2$ from both sides, we get

$$2\times\angle T = \angle P$$

or, $2\times\angle QTR = \angle QPR$

$$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR$$