MHT-CET PYP 2023

Instructions:

- *This question booklet contains 150 Multiple Choice Questions (MCQs).*
  Section-A: Physics \& Chemistry - 50 Questions each and
  Section-B: Mathematics - 50 Questions.
- *Choice and sequence for attempting questions will be as per the convenience of the candidate.*
- *Read each question carefully.*
- *Determine the one correct answer out of the four available options given for each question.*
- *Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.*
- *No mark shall be granted for marking two or more answers of same question, scratching or overwriting.*
- *Duration of paper is 3 Hours.*

SECTION-A

PHYSICS

1. The focal length f is related to the radius of curvature r of the spherical convex mirror by:

   A. $f = +\frac{1}{2}r$

   B. $f = -r$

   C. $f = -\frac{1}{2}r$

   D. $f = r$

   Key: A

   Solution: For convex mirror, focus is behind the mirror.

   So, its focal length (f) is positive.

   $\therefore f = +\frac{r}{2}$

2. When a longitudinal wave propagates through a medium, the particles of the medium execute simple harmonic oscillations about their mean positions. These oscillations of a particle are characterised by an invariant

A. Kinetic energy

B. Potential energy

C. Sum of kinetic energy and potential energy

D. Difference between kinetic energy and potential energy

Key: C

Solution: The sum of their KE and PE is a constant.

3. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to : [Use $g = 10\,ms^{-2}$]

A. $1:1$

B. $\sqrt{2}:\sqrt{3}$

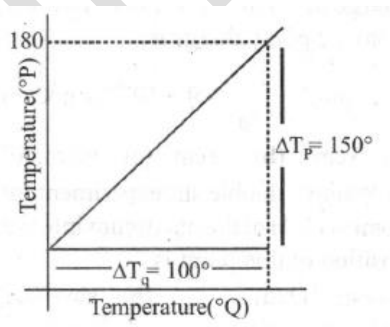
C. $\sqrt{3}:\sqrt{2}$

D. $2:3$

Key: B

Solution:

4. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by :



A. $\frac{t_Q}{150} = \frac{t_P - 180}{100}$

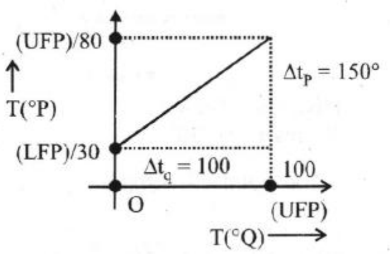
B. $\frac{t_Q}{100}=\frac{t_P-30}{150}$

C. $\frac{t_P}{180}=\frac{t_Q-40}{100}$

D. $\frac{t_P}{100}=\frac{t_Q-180}{150}$

Key: B

Solution:



As $\frac{\text{Reading} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant}$

$$\frac{t_P-30}{180-30}=\frac{t_Q-0}{100-0} \Rightarrow \frac{t_P-30}{150}=\frac{t_Q}{100}$$

5. In the uranium radioactive series, the initial nucleus is ${}_{92}U^{238}$ and that the final nucleus is ${}_{82}Pb^{206}$. When uranium nucleus decays to lead, the number of α particles and β particles emitted are

A. $8\alpha, 6\beta$

B. $6\alpha, 7\beta$

C. $6\alpha, 8\beta$

D. $4\alpha, 3\beta$

Key: A

Solution: Let no. of α-particles emitted be x and no. of β particles emitted be y.

Diff. in mass no. $4x=238-206=32 \Rightarrow x=8$

Diff. in atomic no. $2x-1y=92-82=10$

$16-y=10, y=6$

6. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is

A. 1:1

B. 1:2

C. 2:1

D. depends on the moment of inertia of the two molecules

Key: A

Solution: Each degree of rotation of diatomic molecule has energy $\frac{RT}{2}$ per mole.

7. There are two organ pipes of the same length and the same material but of different radii. When they are emitting fundamental notes.

A. broader pipe gives note of smaller frequency

B. both the pipes give notes of the same frequency

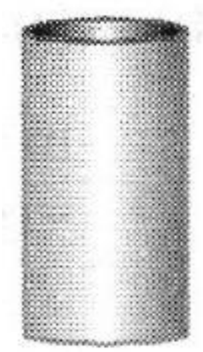
C. narrower pipe gives note of smaller frequency

D. either of them gives note of smaller or larger frequency depending on the wavelength of the wave.

Key: A

Solution: $f=\frac{v}{\lambda}=\frac{v}{(\ell+e)}$

$$f=\frac{v}{4(L+e)}(\mathrm{e}=0.3\mathrm{r})$$



Broader pipe has more value of e therefore, it gives note of smaller frequency.

8. An electron moving along the x-axis has a position given by $x = 20te^{-t}$ m, where t is in second. How far is the electron from the origin when it momentarily stop?

A. $20m$

B. $20em$

C. $\frac{20}{e}$ m

D. zero

Key: C

Solution: $x = 20te^{-t}$

$$\therefore v = \frac{dx}{dt} = 20\left(t\frac{de^{-t}}{dt} + e^{-t} \times 1\right)$$

$$\text{or } 0 = 20\left[te^{-t} \times (-1) + e^{-t}\right]$$

$$\therefore t = 1$$

$$\text{Thus } x = 20 \times 1 \times e^{-1} = \frac{20}{e}\text{ m}$$

9. Electric potential at a point 'P' due to a point charge of 5×10^{-9}C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{+9}\ Nm^2C^{-2}$)

A. 3 cm

B. 9 cm

C. 90 cm

D. 0.9 cm

Key: C

Solution: Electric potential at a point P due to a point charge, $\left(\because K = 9 \times 10^9\right)$

$$V_P = \frac{KQ}{r}$$

$$\Rightarrow 50 = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{r}$$

$$\Rightarrow 50=\frac{9\times10^{9}\times5\times10^{-9}}{r}$$

$$\Rightarrow r=\frac{45}{50}=\frac{9}{10}=0.9\,\text{m}=90\,cm$$

10. In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this point is

A. $\sin^{-1}(\lambda/d)$

B. $\sin^{-1}(\lambda/2d)$

C. $\sin^{-1}(\lambda/3d)$

D. $\sin^{-1}(\lambda/4d)$

Key: C

Solution: $\frac{I_0}{4}=I_0\cos^2(\phi/2)\Rightarrow\phi=2\pi/3$

$\Rightarrow\Delta x\times(2\pi/\lambda)=2\pi/3\Rightarrow\Delta x=\lambda/3$

$\sin\theta=\Delta x/d\Rightarrow\sin\theta=\lambda/3d\Rightarrow\theta=\sin^{-1}(\lambda/3d)$

11. A projectile is projected at 30° from horizontal with initial velocity $40\,ms^{-1}$. The velocity of the projectile at $t=2\,s$ from the start will be:

(Given $g=10\,m/s^2$)

A. $20\sqrt{3}\,ms^{-1}$

B. $40\sqrt{3}\,ms^{-1}$

C. $20\,ms^{-1}$

D. Zero

Key: A

Solution: Given,

Initial velocity of projectile, $u=40\,m/s$

Angle, $\theta = 30°$

Time of flight

$$T = \frac{2usin\theta}{g} = \frac{2\times 40\times 1}{10\times 2} = 45\left(\because g = 10\,m/s^2\right)$$

It means projectile is at maximum height at t= 2 s. At maximum height vertical component of velocity is zero.

Velocity at $t = 2s = V_x = ucos\theta = 40\cos 30°$

$= 20\sqrt{3}\,ms^{-1}$.

12. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

A. simple harmonic with period $2\pi/\omega$
B. simple harmonic with period π/ω
C. periodic but not simple harmonic
D. non-periodic

Key: B

Solution: As given that, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Velocity of the particle

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 3(-2\omega)\left[-\sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 6\omega\sin\left(\frac{\pi}{4} - 2\omega t\right)$$

So, acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega\sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= (6\omega)\times(-2\omega)\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$= -4\omega^2\left[3\cos\left(\frac{\pi}{4}-2\omega t\right)\right]$$

$a = -4\omega^2 y$ In simple harmonic motion acceleration (or force) is directly proportional to the negative of displacement of particle

$\Rightarrow$ as acceleration, $a \propto -y$

Hence, due to negative sign motion is simple harmonic motion (SHM.)
A simple harmonic motion is always periodic. So motion is periodic simple harmonic.
From the given equation,

$$y = 3\cos\left(\frac{\pi}{4}-2\omega t\right)$$

Compare it by standard equation

$$y = a\cos(\omega t+\phi)$$

So, $\omega' = 2\omega$

$$\frac{2\pi}{T'} = 2\omega \Rightarrow T'' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

Hence, the motion is SHM with period $\frac{\pi}{\omega}$.

13. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

A. 0.96 V
B. 1.25 V
C. 0.24 V
D. 1.5 V

Key: B

Solution: Given wavelength of neon lemp $\lambda_1 = 670.5\,nm$

Stopping potential $V_1 = 0.48V$.

From the Einstein's photoelectric equation

$$kE_{\max} = \frac{hc}{\lambda_1} + \phi \Rightarrow eV_1 = \frac{hc}{\lambda_1} + \phi$$

$$\Rightarrow e(0.48) = \frac{1240}{670.5} + \phi$$

For second case wavelength $\lambda_2 = 474.6\,nm$ stlopping potential $= V_2$.

$$e(V_2) = \frac{1240}{474.6} + \phi$$

Subtrating equation (i) from (ii) we get

$$e(V_2 - 0.48) = 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right)eV$$

$$\Rightarrow V_2 = 0.48 + 1240\left(\frac{670.5 - 474.6}{474.6 \times 670.5}\right) \text{ volts}$$

$$\Rightarrow V_2 = 0.48 + 0.76 = 1.25V$$

14. A stone of mass m tied to a spring of length l is rotating along a circular path with constant speed v. The torque on the stone is

A. $mv^2 l$
B. Zero
C. mlv
D. $\frac{mv^2}{l}$

Key: B

Solution: Torque $= \tau = \vec{r} \times \vec{F} = rFsin\theta = rFsin0° = 0$

15. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

A. $4Mg / k$
B. $2Mg / k$
C. Mg / k
D. $Mg / 2k$

Key: B

Solution: $\frac{1}{2}kx^2 = Mgx$

or $x = \frac{2Mg}{k}$

16. If E,L,M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are:-

A. $\left[M^0 L^1 T^0\right]$
B. $\left[M^{-1} L^{-1} T^2\right]$
C. $\left[M^1 L^1 T^{-2}\right]$
D. $\left[M^0 L^0 T^0\right]$

Key: D

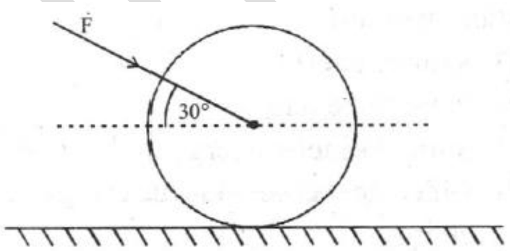
Solution: $[E] = ML^2T^{-2}$

$[L] = ML^2T^{-1}$

$$P = \frac{EL^2}{M^5G^2}$$

$$\Rightarrow [P] = \frac{\left(ML^2T^{-2}\right)\left(M^2L^4T^{-2}\right)}{M^5\left(M^{-2}L^6T^{-4}\right)} = M^0L^0T^0$$

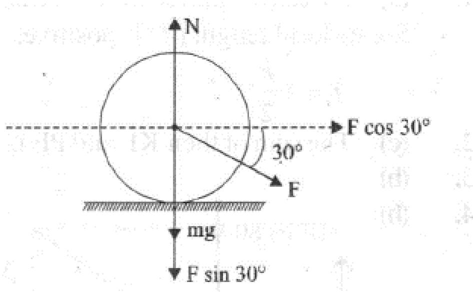
17. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200\,N$ at an angle of $30°$ with horizontal. The normal reaction on the roller is (Given $g = 10\,ms^{-2}$)



A. $800\sqrt{2}\,N$
B. $600\,N$
C. $800\,N$
D. $200\sqrt{3}\,N$

Key: C

Solution:



From FBD of roller

$$N = mg + F\sin 30^\circ$$

$$= 700 + 200 \times \frac{1}{2} = 800\,N$$

18. A convex lens is in contact with concave lens. The magnitude of the ratio of their powers is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths (in cm)?

A. $-15, 10$

B. $-10, 15$

C. $75, 50$

D. $-75, 50$

Key: A

Solution: $\frac{|P_1|}{|P_2|} = \frac{2}{3} \Rightarrow \frac{f_2}{f_1} = \frac{2}{3}$ ...(i)

Focal length of their combination

$$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} \Rightarrow \frac{1}{30} = \frac{1}{f_1} - \frac{1 \times 3}{2f_1} \text{ from (i)}$$

$$\Rightarrow \frac{1}{30} = \frac{1}{f_1}\left[1 - \frac{3}{2}\right] = \frac{1}{f_1} \times \left(-\frac{1}{2}\right)$$

$\therefore f_1 = -15\,cm$ and

$\therefore f_2 = \frac{2}{3} \times f_1 = \frac{2}{3} \times 15 = 10\,cm$

19. A capillary is dipped in water vessel kept in a lift going up with acceleration 2 g. Then :

A. The water will rise in the tube to the height observed under normal condition.
B. Water will rise to the maximum available height of the tube
C. Water will rise to the height one third of the height of normal condition
D. Water will rise to the height double the height of normal condition

Key: C

Solution: We know that $h = \frac{2T\cos\theta}{rgg}$

In a moving lift acceleration becomes $= (g+a) = (g+2g) = 3g$

$\therefore h' = \frac{2T\cos\theta}{r\rho(3g)} = \frac{h}{3}$

20. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

A. Cx_1^2
B. $\frac{1}{2}Cx_1^2$
C. Cx_1
D. Zero

Key: B

Solution: $\omega = \int F dx = \int_0^{x_1} Cx dx = \frac{1}{2}Cx_1^2$

21. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2 The apparent depth of this vessel when viewed from above will be-

A. $\frac{dn_1 n_2}{(n_1 + n_2)}$
B. $\frac{d(n_1 + n_2)}{2n_1 n_2}$

C. $\dfrac{dn_1n_2}{2(n_1+n_2)}$

D. $\dfrac{2d(n_1+n_2)}{n_1n_2}$

Key: B

Solution: $d_{app} = \dfrac{d_1}{n_1} + \dfrac{d_2}{n_2} = \dfrac{d}{2}\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]$

$$d_{app} = \frac{d}{2}\left[\frac{n_1+n_2}{n_1n_2}\right]$$

22. One metallic sphere A is given positive charge whereas another identical metallic sphere B of exactly same mass as of A is given equal amount of negative charge. Then

A. mass of A and mass of B still remain equal
B. mass of A increases
C. mass of B decreases
D. mass of B increases

Key: C

Solution: Sphere A is given as a positive charge so it will loss some electrons hence its mass will decrease. Sphere B is given as a negative charge so it will gain some electron hence its mass will increase.

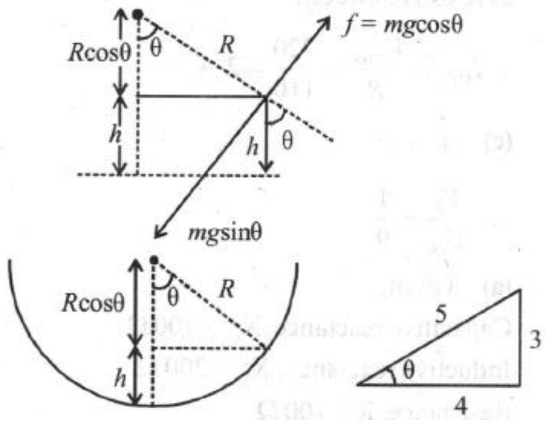
23. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75 , then h is : $\left(g = 10\,ms^{-2}\right)$

A. 0.20 m
B. 0.45 m
C. 0.60 m
D. 0.80 m

Key: A

Solution: For balancing, $mg\ \sin\theta = f = \mu mg\ \cos\theta$

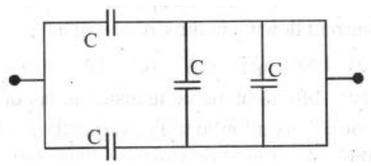
$$\Rightarrow \tan\theta = \mu = \frac{3}{4} = 0.75$$



$$h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$$\therefore h = \frac{R}{5} = 0.2\,\text{m}[\because \text{ radius, } R = 1\,\text{m}]$$

24. The equivalent capacitance of the combination shown is



A. $\frac{C}{2}$

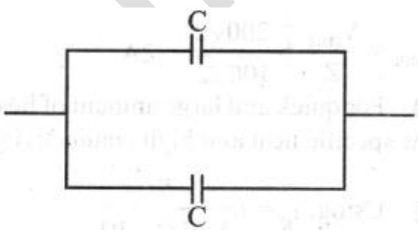
B. 4C

C. 2C

D. $\frac{3}{5}C$

Key: C

Solution: The circuit can be reduced to



The equivalent capacitance of the combination is

$$C_{eq} = C + C = 2C$$

25. Eight drops of mercury of equal radius and possessing equal charge combine to form a big drop. The capacitance of bigger drop as compared to each small drop is

A. 16 times

B. 8 times

C. 4 times

D. 2 times

Key: D

Solution: $8\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3 \Rightarrow \mathrm{R}=2\mathrm{r}$

Now $\mathrm{C}=4\pi\varepsilon_0\mathrm{r}, \mathrm{C}'=4\pi\varepsilon_0\mathrm{R}=2\times(4\pi\varepsilon_0\mathrm{r})=2\mathrm{C}$

26. A ball of mass M strikes another ball of mass m at rest. If they separate in mutually perpendicular directions, then the coefficient of impact (e) is :

A. $\frac{M}{m}$

B. $\frac{m}{M}$

C. $\frac{m}{2M}$

D. Zero

Key: A

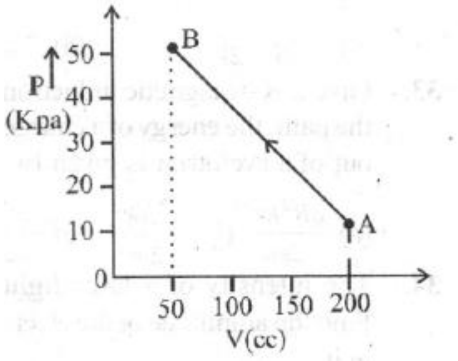
Solution: $M v_1 \sin\theta = m v_2 \cos\theta$ ...(i)

$M u = M v_1 \cos\theta + m v_2 \sin\theta$ ...(ii)

Using $e=-\left[\frac{v_1\cos\theta+v_2\sin\theta}{u-0}\right]$ ...(iii)

On solving, we get, $e=\frac{M}{m}$

27. The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



A. 6J

B. Zero

C. –4.5J

D. 4.5J

Key: D

Solution: $|\Delta W| =$ Area under P-V diagram

$$= \frac{1}{2} \times (50 + 10) \times 10^3 \times 150 \times 10^{-6}$$

$$= 30 \times 150 \times 10^{-3} = 4500 \times 10^{-3}$$

$= 4.5\text{J}$ As volume is decreasing. So $\Delta W = -4.5\text{J}$

By Ist law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow 0 = \Delta U - 4.5$$

$$\Rightarrow \Delta U = +4.5\text{J}$$

28. Ionization potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1eV. The spectral lines emitted by hydrogen atoms according to Bohr's theory will be

A. one

B. two

C. three

D. four

Key: C

Solution: $-13.6+12.1=\frac{-13.6}{n^2}$

or $n=3$

The spectral lines corresponding to $n=3$ will be

$$\frac{n(n-1)}{2}=\frac{3(3-1)}{2}=3$$

29. If two circular rings A and B are of same mass but of radii r and 2r respectively, then the moment of inertia about an axis passing through C.G. and perpendicular to its plane, of A is

A. same as that of B

B. twice that of B

C. four times that of B

D. 1/4 that of B

Key: D

Solution: Ratio of M.I is

$$\frac{M_A r^2}{M_B (2r)^2}=\frac{I_A}{I_B}=\frac{1}{4}[\because M_A=M_B]$$

or $I_A=\frac{I_B}{4}$

30. A planet having mass $9M_e$ and radius $4R_e$, where M_e and R_e are mass and radius of earth respectively, has escape velocity in km/s given by:

(Given escape velocity on earth $V_e=11.2\times10^3\,m/s$)

A. 67.2

B. 16.8

C. 33.6

D. 11.2

Key: B

Solution: Escape velocity $V=\sqrt{\dfrac{2GM}{R}}$

$$\therefore V_p=\sqrt{\frac{2GM_p}{R_p}} \text{ and } V_E=\sqrt{\frac{2GM_E}{R_E}}$$

$$\therefore \frac{V_P}{V_E}=\frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}}=\sqrt{\frac{R_E}{R_P}\times\frac{M_P}{M_E}}$$

$$\Rightarrow V_P=\sqrt{\frac{1}{4}\times 9}\times V_E=\frac{3}{2}V_E$$

$$\therefore V_P=\frac{3}{2}\times 11.2\,km/s=16.8\,km/s$$

31. In a Wheatstone's bridge, three resistances P,Q and R connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be

A. $\dfrac{P}{Q}=\dfrac{2R}{S_1+S_2}$

B. $\dfrac{P}{Q}=\dfrac{R\left(S_1+S_2\right)}{S_1S_2}$

C. $\dfrac{P}{Q}=\dfrac{R\left(S_1+S_2\right)}{2S_1S_2}$

D. $\dfrac{P}{Q}=\dfrac{R}{S_1+S_2}$

Key: B

Solution: $\dfrac{P}{Q}=\dfrac{R}{S}$ where $S=\dfrac{S_1S_2}{S_1+S_2}$

32. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{J}+k$ and $-3\hat{i}-2\hat{J}+\hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector :

A. $\hat{i}-2\hat{J}+\hat{k}$

B. $-3\hat{i}-2\hat{J}+\hat{k}$

C. $-2\hat{i}+2\hat{k}$

D. $-2\hat{i}-\hat{J}+2\hat{k}$

Key: A

Solution: Position of COM of a mass - system is given as,

$$\vec{r}_{com}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}=\frac{1\left(\hat{i}+2\hat{j}+\hat{k}\right)+3(-3\hat{i}-2\hat{j}+\hat{k}}{1+3}$$

$$=-2\hat{i}-\hat{j}+\hat{k}$$

$$|\vec{r}_{cm}|=\left|-2\hat{i}-\hat{j}+\hat{k}\right|=\sqrt{(2)^2+(1)^2+(1)^2}=\sqrt{6}$$

Only option (a) magnitude is

$$\sqrt{1^2+(-2)^2+1^2}=\sqrt{6}$$

So option (a) is correct.

33. Given, B= magnetic induction and R= radius of the path, the energy of a charged particle coming out of a cyclotron is given by

A. $\dfrac{qB^2R^2}{2m}$

B. $\dfrac{q^2BR^2}{2m}$

C. $\dfrac{q^2BR}{2m}$

D. $\frac{q^2B^2R^2}{2m}$

Key: D

Solution: We have, $R=\frac{mv}{qB}$, or $mv=RqB$

Thus kinetic energy, $K=\frac{1}{2}mv^2=\frac{P^2}{2m}=\frac{(RqB)^2}{2m}$

34. The intensity of a laser light is $53.10\,W/m^2$. Find the amplitude of the electric field oscillation in it.

A. $200\sqrt{2}\,N/C$

B. $400\,N/C$

C. $200\,N/C$

D. $100\,N/C$

Key: C

Solution: We have, $I=\frac{1}{2}\varepsilon_0E_0^2c$

or $53.10=\frac{1}{2}\times8.86\times10^{-12}\times E_0^2\times3\times10^8$

$\therefore E_0=200\,N/C.$

35. The potential energy of a satellite, having mass m and rotating at a height of $6.4\times10^6\ m$ from the earth surface, is

A. $-mgR_e$

B. $-0.67mgR_e$

C. $-0.5mgR_e$

D. $-0.33mgR_e$

Key: C

Solution: Mass of the satellite =m and height of satellite from earth $(h) = 6.4\times10^{6}$ m.

We know that gravitational potential energy of the satellite at height

$$h = -\frac{GM_e m}{R_e + h} = -\frac{gR_e^2 m}{2R_e}$$

$$= -\frac{gR_e m}{2} = -0.5mgR_e$$

$$\left(where, GM_e = gR_e^2 \text{ and } h = R_e\right)$$

36. An electron with energy 0.1keV moves at right angle to the earth's magnetic field of $1\times10^{-4} Wbm^{-2}$. The frequency of revolution of the electron will be

$\left(\text{Take mass of electron} = 9.0\times10^{-31}\, kg\right)$

A. $1.6\times10^{5}\, Hz$

B. $5.6\times10^{5}\, Hz$

C. $2.8\times10^{6}\, Hz$

D. $1.8\times10^{6}\, Hz$

Key: C

Solution: $f = \frac{1}{T} = \frac{eB}{2\pi m}\left[\because T = \frac{2\pi m}{qB}\right]$

$$= \frac{1.6\times10^{-19}\times10^{-4}}{2\pi\times9\times10^{-31}} = 2.8\times10^{6}\, Hz$$

37. For an RLC circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance the quality factor, Q is given by

A. $\frac{\omega_0}{L}$

B. $\frac{\omega_0 L}{R}$

C. $\frac{R}{\omega_0 C}$

D. $\frac{CR}{\omega_0}$

Key: B

Solution: Quality Factor $(Q) = \frac{1}{R}\sqrt{\frac{L}{C}}$

38. The ratio of magnetic field and magnetic moment at the centre of a current carrying circular loop is x. When both the current and radius is doubled then the ratio will be

A. $\frac{x}{8}$

B. $\frac{x}{4}$

C. $\frac{x}{2}$

D. $2x$

Key: A

Solution: $B = \frac{\mu_0 I}{2R}$ so, $B \propto \frac{1}{R^3}$

$\therefore \frac{B}{M} = \frac{\mu_0}{2\pi R^3} \therefore B_2 = \frac{B_1}{8}$

39. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z axis. The direction of induced current as seen from the z axis will be

A. clockwise of the +vez axis.

B. anticlockwise of the +vez axis.

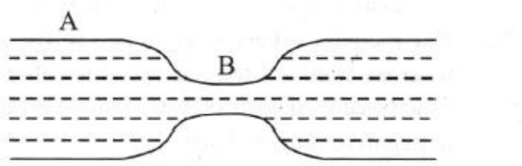
C. zero emf.

D. along the magnetic field.

Key: C

Solution: As there is no change in flux in the cylinder and so induced emf in it will be zero.

40. Water flows in a horizontal tube (see figure). The pressure of water changes by $700\,Nm^{-2}$ between A and B where the area of cross section are $40\,cm^2$ and $20\,cm^2$, respectively. Find the rate of flow of water through the tube. (density of water : $1000\ kgm^{-3}$)



A. $3020\,cm^3/s$

B. $2720\,cm^3/s$

C. $2420\,cm^3/s$

D. $1810\,cm^3/s$

Key: B

Solution: According to question, area of cross section at $A, a_A = 40\,cm^2$ and $B, a_B = 20\,cm^2$

Let velocity of liquid flow at $A, = V_A$ and at $B, = V_B$

Using equation of continuity $a_A V_A = a_B V_B 40V_A = 20V_B \Rightarrow 2V_A = V_B$

Now, using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$\Rightarrow \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right) \Rightarrow \Delta P = 500 \times \frac{3V_B^2}{4}$$

$$\Rightarrow V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}}\ m/s$$

$$= 1.37 \times 10^2\ cm/s$$

Volume flow rate $Q = a_B \times v_B$

$$= 20 \times 100 \times V_B = 2732\, cm^3 / s \approx 2720\, cm^3 / s$$

41. The magnetic flux linked with a vector area $\vec{A}$ in a uniform magnetic field $\bar{B}$ is

A. $\vec{B} \times \vec{A}$

B. AB

C. $\vec{B} \cdot \vec{A}$

D. $\frac{B}{A}$

Key: C

Solution:

42. An LCR circuit contains resistance of 110Ω and a supply of 220 V at 300rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45°. with the applied voltage. The rms current flowing in the circuit will be

A. 1.5 A

B. 1 A

C. 2 A

D. 2.5 A

Key: C

Solution: LCR circuit is in resonance circuit behaves as resistive circuit.

$$\therefore I_{rms} = \frac{V_{rms}}{R} = \frac{220}{110} = 2\,A$$

43. Two spheres of the same material, but of radii R and 3R are allowed to fall vertically downwards through a liquid of density σ. The ratio of their terminal velocities is

A. 1:3

B. 1:6

C. 1:9

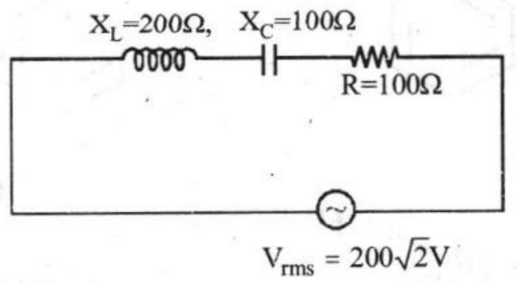
D. 1:1

Key: C

Solution: $V \propto r^2$

$$\therefore \frac{V_R}{V_{3R}} = \frac{1}{9}$$

44. In the given circuit, rms value of current $\left(I_{rms}\right)$ through the resistor R is :



A. $2\,A$

B. $\frac{1}{2}A$

C. $20\,A$

D. $2\sqrt{2}\,A$

Key: A

Solution: Given,

Capacitive reactance, $X_C = 100\Omega$

Inductive reactance, $X_L = 200\Omega$

Resistance, $R = 100\Omega$

Impedance, $Z = \sqrt{\left(R^2 + \left(X_L - X_C\right)^2\right)}$

$$Z = \sqrt{100^2 + (200 - 100)^2} = 100\sqrt{2}\Omega$$

RMS value of current,

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2\,A$$

45. For cooking the food, which of the following type of utensil is most suitable?

A. High specific heat and low conductivity

B. High specific heat and high conductivity

C. Low specific heat and low conductivity

D. Low specific heat and high conductivity

Key: D

Solution: For quick and large amount of heat needed low specific heat and high conductivity.

46. A moving coil galvanometer of resistance 20Ω gives a full scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale deflection. If shunt wire of 0.005Ω only is available then what resistance should be connected in series with the galvanometer coil

A. 99.955Ω
B. 79.995Ω
C. 59.295Ω
D. 19.955Ω

Key: B

Solution: Using, $i_g = i\dfrac{S}{S+(G+R)}$

or $10^{-3} = 20\left(\dfrac{0.005}{0.005+20+R}\right)$

or $R = 79.995\Omega.$

47. In the wave equation $y = 0.5\sin\dfrac{2\pi}{\lambda}(400 t - x)\,m$ the velocity of the wave will be :

A. $200\,m/s$

B. $200\sqrt{2}\,m/s$

C. $400\,m/s$

D. $400\sqrt{2}\,m/s$

Key: C

Solution: Compairing the given

$y = 0.5\sin\left(\dfrac{2\pi}{\lambda}400t - \dfrac{2\pi}{\lambda}x\right)$

with standard equation

$\omega = \dfrac{2\pi}{\lambda}400$ and $k = \dfrac{2\pi}{\lambda}$

$\therefore$ velocity of the wave $v=\frac{\omega}{k}=\frac{\frac{2\pi}{\lambda}400}{\frac{2\pi}{\lambda}}$

$\therefore v=400\,m/s$

48. An intersteller spacecraft far away from the influence of any star or planet is moving at high speed under the influence of fusion rockets (due to thrust exerted by fusion rockets, the spacecrafts is accelearting). Suddenly the engine malfunctions and stops. The spacecraft will.

A. immediately stops, throwing all of the occupants to the front

B. begins slowing down and eventually comes to rest

C. keep accelerating for a while, and then begins to slow down

D. keeps moving forever with constant speed

Key: D

Solution: Due to malfunctioning of engine, the process of rocket fusion stops hence net force experienced by the spacecraft becomes zero. Afterwards the spacecraft continues to move with a constant speed.

49. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t)=y_0\sin^2\omega t$ where ' y ' is measured from the lower end of unstretched spring. Then ω is

A. $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

B. $\sqrt{\frac{g}{y_0}}$

C. $\sqrt{\frac{g}{2y_0}}$

D. $\sqrt{\frac{2g}{y_0}}$

Key: C

Solution: $y=y_0\sin^2\omega t$

$\Rightarrow y=\frac{y_0}{2}(1-\cos 2\omega t)$

$$\left(\because \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}\right)$$

$$\Rightarrow y - \frac{y_0}{2} = \frac{-y_0}{2}\cos 2\omega t$$

So from this equation we can say mean position is shifted by $\frac{y_0}{2}$

distance and frequency of this SHM is 2ω.

So, at equilibrium $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$$

50. The bulk modulus of a liquid is $3\times10^{10}\,Nm^{-2}$. The pressure required to reduce the volume of liquid by 2% is :

A. $3\times10^{8}\,Nm^{-2}$

B. $9\times10^{8}\,Nm^{-2}$

C. $6\times10^{8}\,Nm^{-2}$

D. $12\times10^{8}\,Nm^{-2}$

Key: C

Solution: Bulk modulus, $B = -\frac{\Delta P}{\frac{\Delta V}{V}}$

$$\Rightarrow \Delta P = -B\frac{\Delta V}{V}$$

$$|\Delta P| = +3\times10^{10}\times0.02 = 6\times10^{8}\,Nm^{-2}$$

CHEMISTRY

51. The cubic unit cell of a metal $\left(\text{molar mass} = 63.55\,g\,mol^{-1}\right)$ has an edge length of 362pm. Its density is $8.92\,g\,cm^{-3}$.

The type of unit cell is

A. primitive
B. face centered
C. body centered
D. end centered

Key: B

Solution: $\rho = \dfrac{ZM}{N_A V}$

$$Z = \frac{\rho N_A V}{M} = \frac{8.92 \times 6.02 \times 10^{23} \times (362)^3 \times 10^{-30}}{63.55} = 4$$

$\therefore$ It has *fcc* unit cell.

52. Green chemistry means such reactions which :

A. produce colour during reactions

B. reduce the use and production of hazardous chemicals

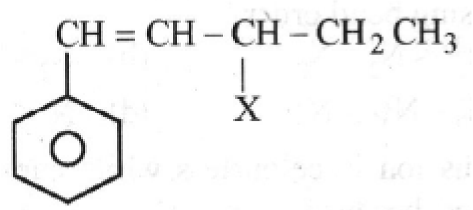
C. are related to the depletion of ozone layer

D. study the reactions in plants

Key: B

Solution: Green chemistry may be defined as the programme of developing new chemical products and chemical processes or making improvements in the already existing compounds and processes so as to make less harmful to human health and environment. This means the same as to reduce the use and production of hazardous chemicals.

53. The given compound



is an example of ______.

A. Benzylic halide
B. Aryl halide
C. Allylic halide
D. Vinylic halide

Key: C

Solution: Allylic halides are the compound where halogen atom attached to sp^3 hybridised carbon atom next to double bond.

54. DNA and RNA contain four bases each. Which of the following bases is not present in RNA?
A. Adenine
B. Uracil
C. Thymine
D. Cytosine

Key: C

Solution: RNA does not contain thymine.

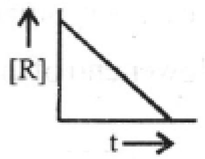
55. For the reaction $2NO_2(g) \rightleftharpoons N_2O_4(g)$ when $\Delta S = -176.0\,J\,K^{-1}$ and $\Delta H = -57.8\,kJ\,mol^{-1}$ the magnitude of ΔG at 298 K for the reaction is $kJmol^{-1}$. (Nearest integer)
A. 2
B. -5
C. 8
D. 10

Key: B

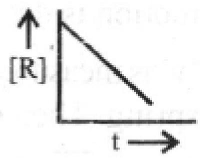
Solution: $\Delta G = \Delta H - T\Delta S$

$\Delta G = -57.8 - 298 \times (-176 \times 10^{-3}) = -5\,kJ\,mol^{-1}$

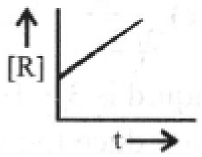
56. The plot that represents the zero order reaction is:



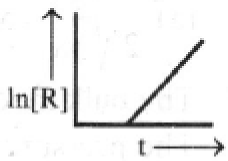
A.



B.



C.



D.

Key: B

Solution: For zero order reaction,

rate, $r = k[R]^{\circ} \Rightarrow$ rate $= \frac{-d[R]}{dt} = k \times 1$

$\Rightarrow d[R] = k\,dt \Rightarrow [R] = -kt + R_0$

where R_0 is the concentration of reactant at time t=0.

Thus, [R] decreases with time t.

57. A biodegradable polyamide can be made from :

A. Glycine and isoprene
B. Hexamethylene diamine and adipic acid
C. Glycine and aminocaproic acid
D. Styrene and caproic acid

Key: C

Solution: Nylon 2-Nylon 6 (Polyamide copolymer) is biodegradable polymer.

Its monomer units are: Glycine + Aminocaproic acid

$$\left[\underset{\text{Glycine}}{H_2N-CH_2-COOH} + \underset{\text{Aminocaproic acid}}{NH_2(CH_2)_5-COOH}\right]$$

58. Homoleptic complex from the following complexes is

A. Potassium trioxalatoaluminate (III)
B. Diamminechloridonitrito-N-platinum (II)
C. Pentaamminecarbonatocobalt (III) chloride
D. Triamminetriaquachromium (III) chloride

Key: A

Solution: Complexes in which a metal is bound to only one kind of donor groups are called as homoleptic complexes.

Potassium trioxalatoaluminate (III). It is a homoleptic complex.

59. Value of Henry's constant K_H

A. increases with increase in temperature
B. decreases with increase in temperature
C. remains constant
D. first increases then decreases

Key: A

Solution: The value of Henry's constant K_H increases with increase in temperature.

60. According to MO theory which of the following lists ranks the nitrogen species in terms of increasing bond order?

A. $N_2^{2-} < N_2^{-} < N_2$

B. $N_2 < N_2^{2-} < N_2$

C. $N_2^- < N_2^{2-} < N_2$

D. $N_2 < N_2 < N_2^{2-}$

Key: A

Solution: Molecular orbital configuration of

$N_2^{2-} = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2$

$\begin{cases} \pi 2p_x^2 \\ \pi 2p_y^2 \end{cases} \sigma 2p_z^2 \begin{cases} \pi^* 2p_x^1 \\ \pi^* 2p_y^1 \end{cases}$

Bond order $= \dfrac{10-6}{2} = 2$

$N_2^- = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2$

$\begin{cases} \pi 2p_x^2 \\ \pi 2p_y^2 \end{cases} \sigma 2p_z^2 \begin{cases} \pi^* 2p_x^1 \\ \pi^* 2p_y^0 \end{cases}$

Bond order $= \dfrac{10-5}{2} = 2.5$

$N_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \begin{cases} \pi 2 \cdot p_x^2 \\ \pi 2p_y^2 \end{cases}, \sigma 2p_z^2$

Bond order $= \dfrac{10-4}{2} = 3$

$\therefore$ The correct order is $= N_2^{2-} < N_2^- < N_2$

61. Cuprous ion is colourless while cupric ion is coloured because

A. both have half filled p-and d-orbitals

B. cuprous ion has incomplete d-orbital and cupric ion has a complete d-orbital

C. both have unpaired electrons in the d-orbitals

D. cuprous ion has complete d-orbital and cupric ion has an incomplete d-orbital.

Key: D

Solution: In $Cu^+[Ar]3d^{1C}$ here is no unpaired electron, $Cu^{2+}[Ar]3d^9$ contains one unpaired electron hence coloured

62. In which case is number of molecules of water maximum?

A. 18 mL of water

B. 0.18 g of water

C. 10^{-3} mol of water

D. 0.00224 L of water vapours at 1 atm and 273 K

Key: A

Solution: (a) Mass of water $= 18 \times 1 = 18\,g$

Molecules of water = mole $\times N_A = \dfrac{18}{18} N_A = N_A$

(b) Molecules of water $=$ mole $\times N_A$

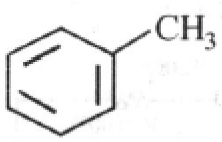
$$= \frac{0.18}{18} N_A = 10^{-2} N_A$$

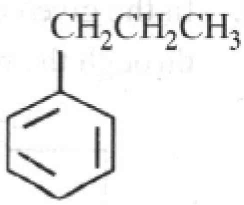
(c) Molecules of water $=$ mole $\times N_A = 10^{-3} N_A$

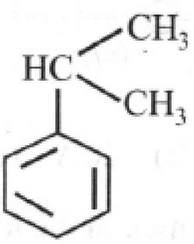
(d) Moles of water $= \frac{0.00224}{22.4} = 10^{-4}$

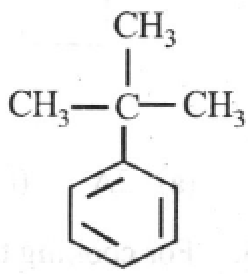
Molecules of water $=$ mole $\times N_A = 10^{-4} N_A$

63. Which of the following can not be oxidised to give carboxylic acid?


A.


B.


C.


D.

Key: D

Solution: Primary and secondary alkyl groups are oxidised to give carboxylic acid, while tertiary alkyl group remains unaffected.

64. The resistance of a conductivity cell with cell constant $1.14\,cm^{-1}$, containing 0.001MKCl at 298 K is 1500Ω. The molar conductivity of 0.001 MKCl solution at 298K in $Scm^2\,mol^{-1}$ is ______. (Integer answer)

A. 86
B. 860
C. 920
D. 760

Key: D

Solution: $\kappa = \frac{1}{R} \times l/A = \left(\left(\frac{1}{1500}\right) \times 1.14\right) Scm^{-1}$

$$\Rightarrow \Lambda_m = \frac{1000 \times \kappa}{C} = 1000 \times \frac{\left(\frac{1.14}{1500}\right)}{0.001} S\,cm^2\,mol^{-1}$$

$$= 760\,S\,cm^2\,mol^{-1}$$

65. Hinsberg's method to separate amines is based on the use of
   A. benzene sulphonyl chloride
   B. benzene sulphonic acid
   C. ethyl oxalate
   D. acetyl chloride

   Key: A

   Solution: Hinsberg's method is based on the use of benzene sulphonyl chloride.

66. IUPAC name of m-cresol is ____.
   A. 3-methylphenol
   B. 3-chlorophenol
   C. 3-methoxyphenol
   D. benzene-1,3-diol

   Key: A

   Solution: Functional group gets the priority.

67. Iodine molecules are held in the crystals lattice by ______.
   A. London forces
   B. dipole-dipole interactions
   C. covalent bonds
   D. coulombic forces

   Key: A

   Solution: Iodine molecules belongs to a class of non - polar molecular solids in which constituents molecule are held together by London or dispersion forces.

68. The formation of micelles takes place only above
   A. inversion temperature
   B. Boyle temperature
   C. critical temperature
   D. Kraft temperature

   Key: D

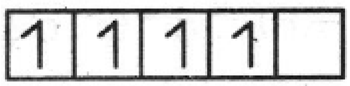
   Solution: The formation of micelles takes place only above a particular temperature called kraft temperature (T_K).

69. The calculated spin only magnetic moment of Cr^{2+} ion is

A. 4.90BM
B. 5.92BM
C. 2.84BM
D. 3.87BM

Key: A

Solution: Electronic configuration of $Cr^{2+} - [Ar]3d^4$



$n = 4$

$$\mu_n = \sqrt{n(n+2)}$$

$$\therefore \mu_n = \sqrt{4(4+2)} = \sqrt{24}BM = 4.9BM$$

70. Molarity of H_2SO_4 is 18M. Its density is 1.8 g/ml. Hence molality is

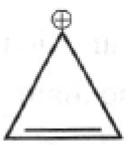
A. 36
B. 200
C. 500
D. 18

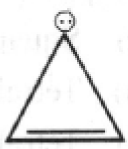
Key: C

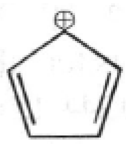
Solution: $\text{Molality}(m) = \dfrac{\text{Molarity}}{\text{Density} - \dfrac{\text{Molarity} \times \text{ Molecular mass}}{1000}}$

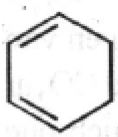
$$= \frac{18}{1.8 - \dfrac{18 \times 98}{1000}} = 500$$

71. Which of the following is most stable?

A. 

B. 

C. 

D. 

Key: D

Solution: Planar molecules having (4n+2)πe- are aromatic in nature.

Aromatic ion

Antiaromatic ion

Antiaromatic ion

Non-aromatic

72. Which of the following is the key step in the mauufacture of sulphuric acid?
   A. Burning of sulphur or sulphide ores in air to generate SO_2
   B. Conversion of SO_2 to SO_3 by the reaction with oxygen in presence of catalyst.
   C. Absorption of SO_3 in H_2SO_4 to give oleum.
   D. Both (b) and (c)

Key: B

Solution: The key step in the manufacture of H_2SO_4 is catalytic oxidation of SO_2 with O_2 to give SO_3 in presence of V_2O_5.

73. V_2O_5 catalyst is used for the manufacture of
   A. HNO_3
   B. *Polyethylene*
   C. H_2SO_4
   D. NH_3

Key: D

Solution: V_2O_5 catalyses the oxidation of SO_2 in the manufacture of H_2SO_4.

74. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?
   A. Baking Soda
   B. Soda ash
   C. Washing Soda
   D. Caustic Soda

Key: A

Solution: Baking soda or $NaHCO_3$ is used in the fire extinguishers.

75. Which of the following is not a copolymer?
   A. Buna-S
   B. Baketite
   C. Neoprene

D. Dacron

Key: C

Solution: Neoprene is a homopolymer of chloroprene.

76. Sucrose in water is dextro-rotatory, $[\alpha]_D = +66.4°$. When boiled with dilute HCl, the solution becomes leavo-rotatory, $[\alpha]_D = -20°$. In this process the sucrose molecule breaks into

A. L-glucose +D-fructose
B. L-glucose +L-fructose
C. D-glucose +D-fructose
D. D-glucose +L-fructose

Key: C

Solution: The hydrolysis of sucrose by boiling with mineral acid or by enzyme invertase or sucrase produces a mixture of equal molecules of D(+) glucose and D(-) Fructose.

$$\underset{\substack{\text{Sucrose} \\ [\alpha_D]=+66.5°}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{HCl} \underbrace{\underset{\substack{\text{D-Glucose} \\ [\alpha_D]=+52.5°}}{C_6H_{12}O_6} + \underset{\substack{\text{D-Fructose} \\ [\alpha_D]=-92°}}{C_6H_{12}O_6}}_{\text{Invert sugar, } [\alpha_D]=-20°}$$

77. When electric current is passed through acidified water, 112 mL of hydrogen gas at STP collected at the cathode in 965 seconds. The current passed in amperes is

A. 1.0
B. 0.5
C. 0.1
D. 2.0

Key: A

Solution: $2H^+ + 2e^- \rightarrow H_2$

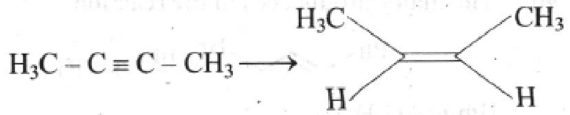
$$E_H\left(Eq{\cdot}wt\right) = \frac{2}{2} = 1g = \frac{22400}{2} = 11200\,mL\left(STP\right)$$

$$\text{Total charge passed} = \frac{96500 \times 112}{11200} = 965 \text{ coulomb}$$

$Q = It = 965$ and $t = 965s$.

$$I = \frac{965}{965} = 1amp$$

78. The most suitable reagent for the following conversion, is:



cis-2-butene

(a) Na/liquid NH_3 (b) H_2, Pd/C, quinoline

(c) Zn/HCl (d) $Hg^{2+}/H^+, H_2O$

Key: B

Solution: Alkynes can be reduced to cis-alkenes with the use of Lindlar's catalyst.

79. For a first order reaction, the plot of logk against 1/ T is a straight line. The slope of the line is equal to

A. $-\frac{E_a}{R}$

B. $-\frac{2.303}{E_a R}$

C. $-\frac{E_a}{2.303}$

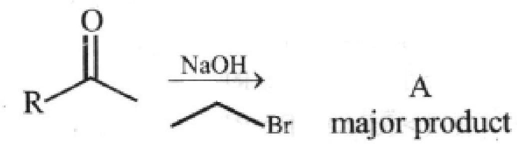
D. $\frac{-E_a}{2.303R}$

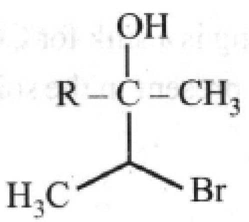
Key: D

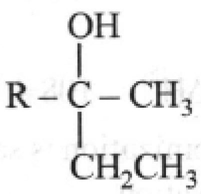
Solution: $k = Ae^{-E_a/RT}$ $\log k = \log A - \frac{E_a}{2.303R}\cdot\frac{1}{T}$

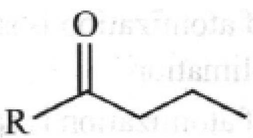
Equation of straight line slope $= \frac{-E_a}{2.303R}$.

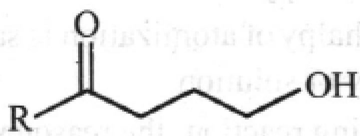
80. The structure of A in the given reaction is:



A. 

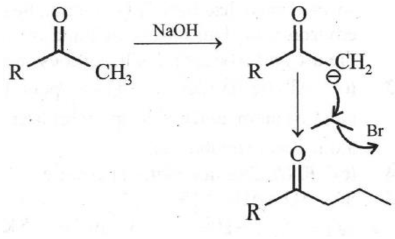
B. 

C. 

D. 

Key: C

Solution:



81. In weak electrolytes, equilibrium is established between ions and the unionized molecules. This type of equilibrium
    A. is due to complete ionization oelectrolyte.
    B. is called non ionic equilibrium.
    C. is called physical equilibrium.
    D. involves ions in aqueous solution.

    Key: D

    Solution: The equilibrium is established between ions and unionised molecules involves ions in aqueous solution.

82. In which of the compounds does 'manganese' exhibit highest oxidation number?
    A. MnO_2
    B. Mn_3O_4
    C. K_2MnO_4
    D. $MnSO_4$

    Key: C

    Solution: O.N. of Mn in K_2MnO_4 is $+6$

83. In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively
    A. 2,3, 1
    B. 1,2,3
    C. 4,1,2
    D. 3,2,1.

    Key: D

    Solution: Xe has $8e^-$ in its valence shell.

84. The radius of hydrogen atom in the ground state is $0.53Å$. The radius of Li^{2+} ion (atomic number =3) in a similar state is
    A. $0.17Å$.
    B. $0.265Å$.
    C. $0.53Å$.

D. 1.06Å.

Key: A

Solution: For hydrogen atom (n)=1 (ground state)

Radius of hydrogen atom (r)=0.53 Å.

Atomic number of Li(Z)=3.

Radius of Li^{2+} ion

$$= r \times \frac{n^2}{Z} = 0.53 \times \frac{(1)^2}{3} = 0.17 \backslash AA$$

85. The order of reactivities of the following alkyl halides for a $S_N 2$ reaction is

A. $RF > RCl > RBr > RI$
B. $RF > RBr > RCl > RI$
C. $RCl > RBr > RF > RI$
D. $RI > RBr > RCl > RF$

Key: D

Solution: Weaker the C-X bond, greater is the reactivity.

86. Which of the following is a sink for CO ?

A. Microorganism present in the soil
B. Oceans
C. Plants
D. Haemoglobin

Key: A

Solution: Microorganisms present in the soil is a sink for CO.

87. Which of the following statements is true for the given reaction?

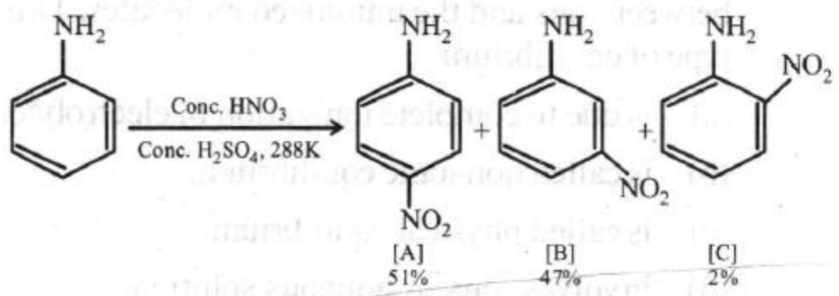
$Na(s) \rightarrow Na(g); \Delta H^{1} = 108.4 kJ\, mol^{-1}$

A. The enthalpy of atomization is same as the enthalpy of vaporisation
B. The enthalpy of atomization is same as the enthalpy of sublimation.
C. The enthalpy of atomization is same as the bond enthalpy
D. The enthalpy of atomization is same as the enthalpy of solution

Key: B

Solution: Metallic bonding breaks in this reaction.

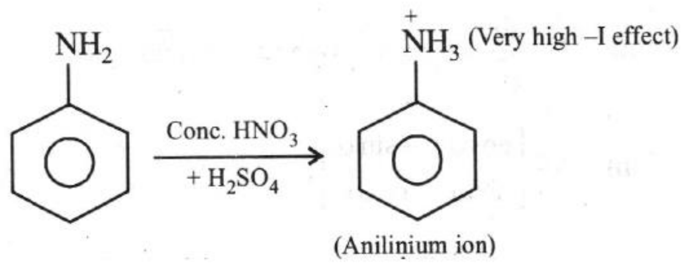
88. In the following reaction, the reason why metanitro product also formed is



A. *low temperature*
B. $-NH_2$ *group is highly meta – directive*
C. Formation of anilinium ion
D. $-NO_2$ *substitution always takes place at meta – position*

Key: C

Solution:



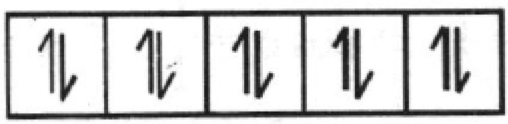
The positive anilinium nitrogen deactivates the benzene ring due to -I effect. -I effect is more on -o and -m position and less on p - position. Therefore, -m product is also formed along with o and -p products.

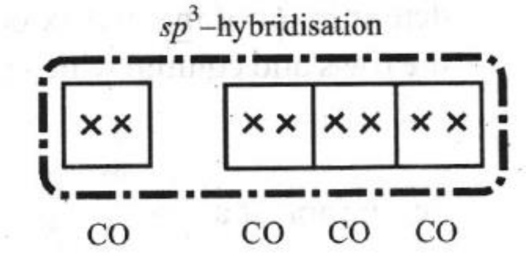
89. The geometry and magnetic behaviour of the complex $[Ni(CO)_4]$ are
A. Square planar geometry and diamagnetic
B. Tetrahedral geometry and diamagnetic
C. Tetrahedral geometry and paramagnetic
D. Square planar geometry and paramagnetic

Key: B

Solution: $Ni(28):[Ar]3d^8 4s^2$

$\because$ CO is a strong field ligand, so unpaired electrons get paired. Hence, configuration would





For, four 'CO' ligands hybridisation would be sp^3 and thus the complex would be diamagnetic and of tetrahedral geometry.

90. Given van der Waals constants for NH_3, H_2, O_2 and CO_2 are respectively 4.17, 0.244, 1.36 and 3.59 , which one of the following gases is most easily liquefied?

A. NH_3

B. H_2

C. CO_2

D. O_2

Key: A

Solution: van der Waal constant 'a', signifies intermolecular forces of attraction.

Higher is the value of 'a', easier will be the liquefaction of gas.

91. Phenol does not undergo nucleophilic substitution reaction easily due to:

A. acidic nature of phenol
B. partial double bond character of C-OH bond
C. partial double bond character of C-C bond
D. instability of phenoxide ion

Key: B

Solution: Due to partial double bond character of C-OH bond.

92. The catalytic activity of transition metals and their compounds is mainly due to

A. their magnetic behaviour
B. their unfilled d-orbitals
C. their ability to adopt variable oxidation state
D. their chemical reactivity

Key: C

Solution: The transition metals and their compounds are used as catalysts. Because of the variable oxidation states, they may form intermediate compound with one of the reactants. These intermediate provides a new path with low activation energy.

$$V_2O_5 + SO_2 \rightarrow V_2O_4 + SO_3 2V_2O_4 + O_2 \rightarrow 2V_2O_5$$

93. The rise in the boiling point of a solution containing 1.8 g of glucose in 100 g of solvent is $0.1^\circ C$. The molal elevation constant of the liquid is

A. 0.01 K/m
B. 0.1 K/m
C. 1 K/m
D. 10 K/m

Key: C

Solution: $K_b = \dfrac{0.1 \times 180 \times 100}{1.8 \times 1000} = 1 K/m$

94. The C-O-C angle in ether is about

A. 180°
B. $190^\circ 28'$
C. 110°
D. 115°

Key: C

Solution: The repulsion between two lone pair of electrons on oxygen atom decreases the bond angle whereas the -R groups at 'oxygen' atom shows repulsive interaction. As a result, there is a slight increase in the bond angle.

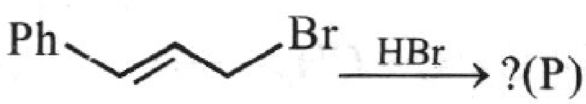
95. A metallic crystal crystallizes into a lattice containing a sequence of layers AB AB AB......Any packing of spheres leaves out voids in the lattice. What percentage of volume of this lattice is empty space?

A. 74%
B. 26%
C. 50%
D. None of these

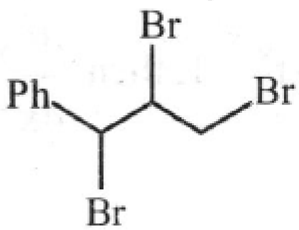
Key: B

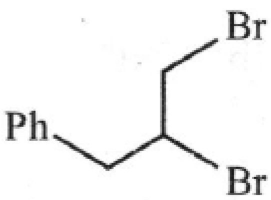
Solution: In ABAB packing (hcp), spheres occupy $74\%.26\%$ is empty.

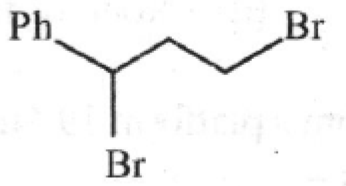
96. The major product (P) in the reaction

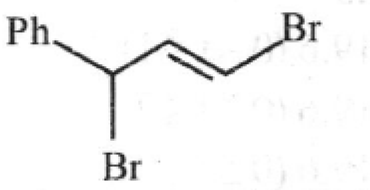


$[Ph \text{ is} - C_6H_5]$ is

A 

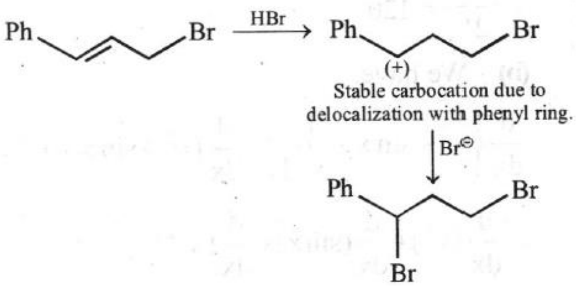
B. 

C 

D. 

Key: C

Solution:

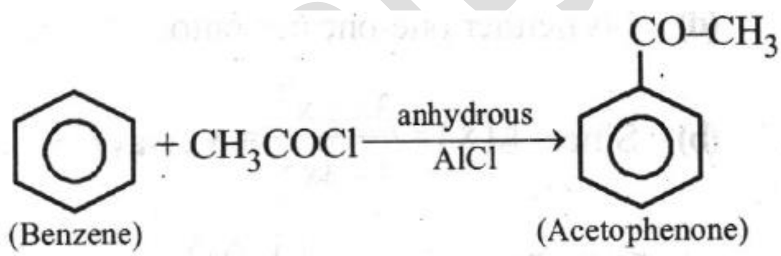


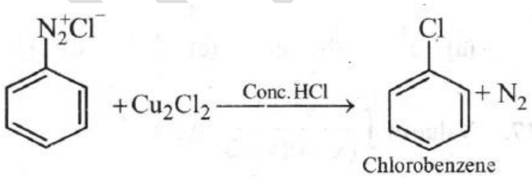
97. Benzene reacts with $CH_3COCl + AlCl_3$ to give

A. chlorobenzene
B. toluene
C. benzyl chloride
D. acetophenone

Key: D

Solution:





98. Above reaction is known as:

A. Strecker's reaction
B. Sandmeyer's reaction
C. Wohl-Ziegler reaction
D. Stephenis reaction

Key: B

Solution: The given reaction is known as Sandmeyer's reaction.

99. $t_{1/4}$ can be taken as the time taken for the concentration of a reactant to drop to $\frac{3}{4}$ of its initial value.

If the rate constant for a first order reaction is k, the $t_{1/4}$ can be written as

A. 0.75/k
B. 0.69/k
C. 0.29/k
D. 0.10/k

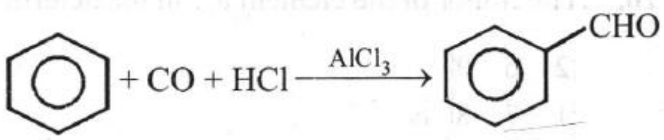
Key: C

Solution: $t_{1/4} = \frac{2.303}{k}\log\frac{1}{3/4} = \frac{2.303}{k}\log\frac{4}{3}$

$= \frac{2.303}{k}(\log 4 - \log 3) = \frac{2.303}{k}(2\log 2 - \log 3)$

$= \frac{2.303}{k}(2\times 0.301 - 0.4771) = \frac{0.29}{k}$

100. The reaction



A. Rosenmund's reaction
B. Stephen's reaction
C. Cannizzaro's reaction
D. Gatterman-Koch reaction

Key: D

Solution: This reaction proceeds in the presence of anhydrous $AlCl_3$ or $CuCl$.

SECTION-B

MATHEMATICS

1. The domain of the function $f(x) = \frac{1}{\sqrt{9-x^2}}$

A. $-3 \le x \le 3$
B. $-3 < x < 3$
C. $-9 \le x \le 9$
D. $-9 < x < 9$

Key: B

Solution: $f(x)=\frac{1}{\sqrt{9-x^2}}$ Clearly, $9-x^2>0$

$\Rightarrow x^2-9<0$

$\Rightarrow (x+3)(x-3)<0$

Thus, domain of f(x) is

$x\in(-3,3)$

2. The number which indicates variability of data or observations, is called
   A. measure of central tendency
   B. mean
   C. median
   D. measure of dispersion
   Key: D

   Solution: Variability is another factor which is required to be studied under Statistics. Like 'measure of central tendency' we want to have a single number to describe variability. This single number is called a 'measure of dispersion'.

3. Number of words from the letters of the words BHARAT in which B and H will never come together is
   A. 210
   B. 240
   C. 422
   D. 400
   Key: B

   Solution: There are 6 letters in the word BHARAT, 2 of them are identical. Hence total number of words $=6!/2!=360$ Number of words in which B and H come together

   $=\frac{5!2!}{2!}=120$

4. Derivative of $x^2+sinx+\frac{1}{x^2}$
   A. $2x+\cos x$
   B. $2x+\cos x+(-2)x^{-3}$
   C. $2x-2x^{-3}$
   D. None of these
   Key: B

   Solution: $\frac{d}{dx}\left\{x^2+sinx+\frac{1}{x^2}\right\}=\frac{d}{dx}\left(x^2+sinx+x^{-2}\right)$

$$=\frac{d}{dx}\left(x^{2}\right)+\frac{d}{dx}(\sin x)+\frac{d}{dx}\left(x^{-2}\right)$$

$$=2x+\cos x+(-2)x^{-3}$$

5. Let $f: R \rightarrow R$ be defined as $f(x)=\frac{x^{2}+1}{2}$, then

   A. f is one-one onto
   B. f is one-one but not onto
   C. f is onto but not one-one
   D. f is neither one-one nor onto

   Key: D

   Solution: f is neither one-one nor onto.

6. Given $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^{3}}{1+3x^{2}}$ then $fog(x)$ equals

   A. $-f(x)$
   B. $3f(x)$
   C. $[f(x)]^{3}$
   D. None of these

   Key: B

   Solution: Since, $g(x)=\frac{3x+x^{3}}{1+3x^{2}}=y$ (say) ..... (1)

$$\therefore f[g(x)]=f(y)=\log\left(\frac{1+y}{1-y}\right)$$

$$=\log\left\{\frac{1+\frac{3x+x^{3}}{1+3x^{2}}}{1-\frac{3x+x^{3}}{1+3x^{2}}}\right\}$$

$$\Rightarrow f(g(x))=\log\left(\frac{1+x}{1-x}\right)^{3}$$

$$=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

7. $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(1)+\tan^{-1}\frac{1}{\sqrt{3}}$ is equal to

A. π

B. $\frac{\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{3\pi}{4}$

Key: A

Solution: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(1)+\tan^{-1}\frac{1}{\sqrt{3}}$

$=\frac{\pi}{3}+\frac{\pi}{2}+\frac{\pi}{6}=\frac{6\pi}{6}=\pi$

8. If $\tan^{-1}\left(\frac{a}{x}\right)+\tan^{-1}\left(\frac{b}{x}\right)=\frac{\pi}{2}$, then x is equal to

A. $\sqrt{ab}$

B. $\sqrt{2ab}$

C. $2ab$

D. ab

Key: A

Solution: Let $\tan^{-1}\left(\frac{a}{x}\right)+\tan^{-1}\left(\frac{b}{x}\right)=\frac{\pi}{2}$

$\tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right)=\frac{\pi}{2}\Rightarrow\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}=\tan\frac{\pi}{2}$

$\Rightarrow 1-\frac{ab}{x^2}=0\Rightarrow x^2=ab\Rightarrow x=\sqrt{ab}$

9. If $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A+A'=I$ then the value of α is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{3\pi}{2}$

Key: B

Solution: $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$,

$$A'=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A+A'=\begin{bmatrix} \cos\alpha+\cos\alpha & -\sin\alpha+\sin\alpha \\ \sin\alpha-\sin\alpha & \cos\alpha+\cos\alpha \end{bmatrix}$$

$$=\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix}=I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ (given)}$$

$$\Rightarrow 2\cos\alpha=1, \Rightarrow \cos\alpha=\frac{1}{2} \therefore \alpha=\frac{\pi}{3}.$$

10. The minor of the element a_{11} in the determinant $\begin{vmatrix} 2 & 6 & 9 \\ 1 & 7 & 8 \\ 1 & 4 & 5 \end{vmatrix}$ is

A. 0
B. 3
C. 5
D. 7
Key: B

Solution: The element $a_{11}=2.$ Its minor is given by determinant of the matrix obtained by deleting the rows and column which contain element $a_{11}=2$

i.e., minor of $a_{11}=\begin{vmatrix} 7 & 8 \\ 4 & 5 \end{vmatrix}=35-32=3$

11. If $(x+9)=0$ is a factor $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0,$ then the other factor is:

A. $(x-2)(x-7)$
B. $(x-2)(x-a)$
C. $(x+9)(x-a)$
D. $(x+2)(x+a)$
Key: A

Solution: Let $A=\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$

$\Rightarrow x\left(x^2-12\right)-3(2x-14)+7(12-7x)=0$

$\Rightarrow x^3-12x-6x+42+84-49x=0$

$\Rightarrow x^3-67x+126=0$

If $(x+9)$ is a factor of the given equation then

$(x+9)\left(x^2-9x+14\right)=0 \Rightarrow x^2-9x+14=0$

Thus $(x-7)(x-2)=0$ is the other factor.

12. Consider the system of linear equations

$x_1+2x_2+x_3=3$

$2x_1+3x_2+x_3=3, 3x_1+5x_2+2x_3=1$

The system has

A. exactly 3 solutions

B. a unique solution

C. no solution

D. infinite solutions

Key: C

Solution: $D=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix}=0; D_1=\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix}\neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

13. $f(x)=\dfrac{1}{1+\tan x}$

A. *is a continuous, real-valued function for all* $x\in(-\infty,\infty)$

B. *is discontinuous only at* $x=\dfrac{3\pi}{4}$

C. *has only finitely many discontinuities on* $(-\infty,\infty)$

D. *has infinitely many discontinuities on* $(-\infty,\infty)$

Key: D

Solution: tan x is not continuous at $x=\frac{\pi}{2},3\frac{\pi}{2},5\frac{\pi}{2}$ etc.

So, tan x has infinitely many discontinuities on $(-\infty,\infty)$

$\Rightarrow f(x)=\frac{1}{1+tanx}$ has infinitely many discontinuities on $(-\infty,\infty)$

14. Let $y=t^{10}+1$ and $x=t^8+1$ then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. None of these

Key: C

Solution: $y=t^{10}+1, x=t^8+1$

$$\frac{dy}{dt}=10t^9, \frac{dx}{dt}=8t^7$$

$$\Rightarrow \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{10t^9}{8t^7}=\frac{5}{4}t^2$$

$$\Rightarrow \frac{d^2y}{dx}=\frac{5}{4}(2t)\frac{dt}{dx}$$

$$=\frac{5}{4}\times 2t\times\frac{1}{8t^7}=\frac{5}{16t^6}$$

15. A ball is dropped from a platform 19.6 m high. Its position function is –

A. $x=-4.9t^2+19.6.(0\le t\le 1)$

B. $x=-4.9t^2+19.6(0\le t\le 2)$

C. $x=-9.8t^2+19.6(0\le t\le 2)$

D. $x=-4.9t^2-19.6(0\le t\le 2)$

Key: B

Solution: The initial conditions are $x(0)=19.6$ and $v(0)=0$

So, $v=\frac{dx}{dt}=-9.8t+v(0)=-9.8t$

$\therefore x=-4.9t^2+x(0)=-4.9t^2+19.6$

Now, the domain of the function is restricted since the ball hits the ground after a certain time. To find this time we set $x=0$ and solve for $t; 0=-4.9t^2+19.6 \Rightarrow t=2$

16. The maximum value of $\frac{lnx}{x}$ in $(2,\infty)$

A. 1
B. e
C. 2/e
D. 1/e
Key: D

Solution: Let $y=\frac{lnx}{x}$

$$\Rightarrow \frac{dy}{dx}=\frac{x\cdot\frac{1}{x}-lnx\cdot 1}{x^2}=\frac{1-logx}{x^2}$$

For maxima, put $\frac{dy}{dx}=0$

$$\Rightarrow \frac{1-lnx}{x^2}=0 \Rightarrow x=e$$

Now, $\frac{d^2y}{dx^2}=\frac{x^2\left(-\frac{1}{x}\right)-(1-lnx)2x}{\left(x^2\right)^2}$

At $x=e$ we have $\frac{d^2y}{dx^2}<0$

$\therefore$ The maximum value at $x=e$ is $y=\frac{1}{e}$

17. Value of $\int \frac{x^2+1}{(x-1)(x-2)}dx$ is

A. $x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$

B. $x+\log\left[\frac{(x-1)^2}{(x-2)^5}\right]+C$

C. $x-\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$

D. None of these
Key: A

Solution: Here since the highest powers of x in numerator and denominator are equal and coefficients of x^2 are also equal, therefore

$$\int \frac{x^2+1}{(x-1)(x-2)} \equiv 1+\frac{A}{x-1}+\frac{B}{x-2}$$

On solving we get $A=-2, B=5$

thus $\int \frac{x^2+1}{(x-1)(x-2)} \equiv 1-\frac{2}{x-1}+\frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non-repeated linear factor in the denominator.

Now, $I=\int\left(1-\frac{2}{x-1}+\frac{5}{x-2}\right)dx$

$$=x-2\log(x-1)+5\log(x-2)+C$$

$$=x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$$

18. $\int_0^{\pi/2}\left(\sqrt{tanx}+\sqrt{cotx}\right)dx=$

A. $\frac{\pi}{\sqrt{2}}$

B. $\pi\sqrt{2}$

C. $\frac{\pi}{2}$

D. $\frac{\sqrt{2}}{\pi}$

Key: B

Solution: Let $I=\int_0^{\pi/2}\sqrt{tanx}+\sqrt{cotx}dx$

$$=\int_0^{\pi/2}\frac{sinx+cosx}{\sqrt{sinxcosx}}dx$$

$$=\int_0^{\pi/2}\frac{\sqrt{2}(sinx+cosx)}{\sqrt{\sin 2x}}dx$$

Put, $\sin x-\cos x=t$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

$$= \int_{-1}^{1} \frac{\sqrt{2}}{\sqrt{1-t^2}} dt = \sqrt{2}\sin^{-1} t\Big|_{-1}^{1}$$

$$= \sqrt{2}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \pi\sqrt{2}.$$

19. The area enclosed between the graph of $y = x^3$ and the lines $x = 0, y = 1, y = 8$ is

A. $\frac{45}{4}$

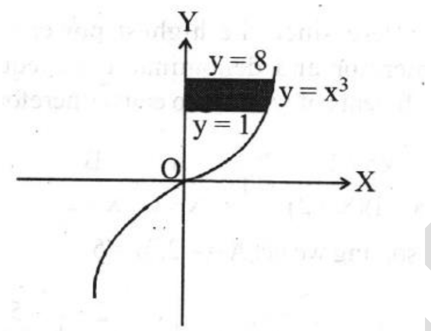
B. 14

C. 7

D. None of these

Key: A

Solution: Given curve is $y = x^3$ and $x = y^{1/3}$



Considering the areas with y-axis, we find that required area

$$= \int_1^8 y^{1/3} dy = \left[\frac{y^{4/3}}{4/3}\right]_1^8 = \frac{3}{4}\left[8^{4/3} - 1^{4/3}\right]$$

$$= \frac{3}{4} \times (16 - 1) = \frac{3}{4} \times 15 = \frac{45}{4} \text{ sq.units}$$

20. The solution of the equation $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ is

A. $(x - y^2) + c = \log(3x - 4y + 1)$

B. $x - y + c = \log(3x - 4y + 4)$

C. $(x - y + c) = \log(3x - 4y - 3)$

D. $x - y + c = \log(3x - 4y + 1)$

Key: D

Solution: Hint: Put $3x-4y=x$

$$\Rightarrow 3-4\frac{dy}{dx}=\frac{dX}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{4}\left(3-\frac{dX}{dx}\right)$$

$$\Rightarrow\frac{3}{4}-\frac{1}{4}\frac{dX}{dx}=\frac{X-2}{X-3}$$

$$\Rightarrow-\frac{1}{4}\frac{dX}{dx}=\frac{4X-8-3(X-3)}{4(X-3)}=-\frac{1}{4}\frac{dX}{dx}$$

21. The solution set of constraints $x+2y\geq 11, 3x+4y\leq 30, 2x+5y\leq 30$ and $x\geq 0, y\geq 0$, includes the point

A. (2,3)
B. (3,2)
C. (3,4)
D. (4,3)

Key: C

Solution: Obviously, solution set of constraints included the point (3,4).

22. If $5\tan\theta=4$ then $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=$

A. 0
B. 1
C. $\frac{1}{6}$
D. 6

Key: C

Solution: $5\tan\theta=4\Rightarrow\tan\theta=\frac{4}{5}$

$$\therefore \sin\theta=\frac{4}{\sqrt{41}} \text{ and } \cos\theta=\frac{5}{\sqrt{41}}$$

$$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{5\times\frac{4}{\sqrt{41}}-3\times\frac{5}{\sqrt{41}}}{5\times\frac{4}{\sqrt{41}}+2\times\frac{5}{\sqrt{41}}}=\frac{1}{6}$$

23. The distance between the parallel lines $3x-4y+7=0$ $3x-4y+5=0$ is $\frac{a}{b}$ value of $a+b$ is

A. 2
B. 5
C. 7
D. 3

Key: C

Solution: Given parallel lines are $3x-4y+7=0$ and $3x-4y+5=0$

Required distance $=\dfrac{|7-5|}{\sqrt{(3)^2+(-4)^2}}=\dfrac{2}{5}\Rightarrow a=2,$

$b=5$

24. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$ is
    A. 3
    B. 4
    C. 1
    D. 2
    Key: A

Solution: $x^2+y^2-4x-6y-12=0$ ...(i)

Centre, $C_1=(2,3)$ Radius, $r_1=5units$

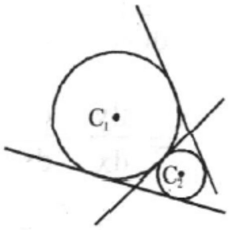
$x^2+y^2+6x+18y+26=0$ ...(ii)

$Centre, C_2=(-3,-9)$

$Radius, r_2=8units$

$C_1C_2=\sqrt{((2+3)^2+(3+9)^2)}=13units$

$r_1+r_2=5+8=13$



25. Find the probability of getting the sum as a perfect square number when two dice are thrown together.
    A. 5/12
    B. 7/18
    C. 7/36
    D. None of these
    Key: C

Solution: (c) Sample space in this case is 6×6=36

Sum of two results is a perfect square number we have following cases-

Case (i): If sum is 4 then we have following 3 cases (1,3),(2,2) and (3,1)

Case (ii): If sum is 9 then we have following 4 cases (3,6),(4,5),(5,4) and (6,3)

So total number of ways is 3+4=7

So required probability is 7/36

26. If $z = 2+i$, then $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$ is equal to

A. 2
B. 7
C. -1
D. -4
Key: D

Solution: $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$

$= 2\operatorname{Re}\left[(z-1)(\bar{z}-5)\right]$

$\left[\because z_1\bar{z}_2 + z_2\bar{z}_1 = 2\operatorname{Re}(z_1 z_2)\right]$

$= 2\operatorname{Re}\left[(1+i)(-3-i)\right] = 2(-2) = -4\left[\text{ Given } z = 2+i\right]$

27. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x+x^2}-1}{x} =$

A. $\dfrac{1}{2}$

B. $-\dfrac{1}{2}$

C. 0
D. ∞
Key: A

Solution: By rationalisation of numerator, given expression

$= \lim\limits_{x\to 0}\dfrac{\sqrt{1+x+x^2}-1}{x}.\dfrac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$

$= \lim\limits_{x\to 0}\dfrac{1+x+x^2-1}{x\left(\sqrt{1+x+x^2}+1\right)}$

$= \lim\limits_{x\to 0}\dfrac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$

$= \lim\limits_{x\to 0}\dfrac{1+x}{\sqrt{1+x+x^2}+1} = \dfrac{1}{2}z$

28. Let R be the relation in the set $\{1,2,3,4\}$ given by.

$R=\{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$.

A. R is reflexive and symmetric but not transitive
B. R is reflexive and transitive but not symmetric
C. R is symmetric and transitive but not reflexive
D. R is equivalence relation
Key: B

Solution: Here, $R=(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)$

Since, $(a,a)\in R,$ for every $a\in 1,2,3,4.$ Therefore, R is reflexive.

Now, since $(1,2)\in R$ but $(2,1)\notin R.$ Therefore, R is not symmetric.

Also, it is observed that $(a,b),(b,c)\in R \Rightarrow (a,c)\in R$ for all $a,b,c\in 1,2,3,4$

Therefore, R is transitive. Hence, R is reflexive and transitive but not symmetric.

29. Which one of the following relations on the set of real numbers R is an equivalence relation ?

A. $aR_1 b \Leftrightarrow |a|=|b|$
B. $aR_2 b \Leftrightarrow a \ge b$
C. $aR_3 b \Leftrightarrow$ a divides b
D. $aR_4 b \Leftrightarrow a < b$
Key: A

Solution: The relation R_1 is an equivalence relation $\forall a\in R, |a|=|a|$, i.e. $aR_1 a \forall a\in R$

$\therefore R_1$ is reflexive.

Again $\forall a,b\in R, |a|=|b| \Rightarrow |b|=|a|$

$\therefore aR_1 b \Rightarrow bR_1 a.$ Therefore R is symmetric.

Also, $\forall a,b,c\in R, |a|=|b|$ *and* $|b|=|c|$

$\Rightarrow |a|=|c| \therefore aR_1 b$ and $bR_1 c \Rightarrow aR_1 c$

$\Rightarrow R_1$ is transitive

R_2 and R_3 are not symmetric.

R_Δ^2 is neither reflexive nor symmetric.

30. Principal value of $cosec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is equal to

A. $-\frac{\pi}{3}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

Key: A

Solution: Let $cosec^{-1}\left(\frac{-2}{\sqrt{3}}\right)=\theta$

$$\Rightarrow cosec\theta=\frac{-2}{\sqrt{3}}=-cosec\frac{\pi}{3}=cosec\left(\frac{-\pi}{3}\right)$$

$$\Rightarrow\theta=\frac{-\pi}{3}\in\left[\frac{-\pi}{3},\frac{\pi}{2}\right]-\{0\}$$

$\therefore$ Principal value of $cosec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is $\left(\frac{-\pi}{3}\right)$

31. If $sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1,$ then the value of x is

A. -1

B. $\frac{2}{5}$

C. $\frac{1}{3}$

D. $\frac{1}{5}$

Key: D

Solution: We have, $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x=\sin^{-1}(1)$$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\frac{\pi}{2}-\sin^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\frac{\pi}{2}-\sin^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{1}{5}=\sin^{-1}x \Rightarrow x=\sin\left(\sin^{-1}\frac{1}{5}\right)$$

$$\Rightarrow x=\frac{1}{5}$$

32. If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$ then the values of k, a and b respectively are:

A. $-6,-12,-18$

B. $-6,-4,-9$

C. $-6,4,9$

D. $-6,12,18$

Key: B

Solution: $kA=\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$

$$\Rightarrow k=-6, a=-4 \text{ and } b=-9$$

33. For the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & m & -1\end{bmatrix}$, A^2 is equal to

A. 0

B. A

C. I

D. None of these

Key: C

Solution: We have $A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & m & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & m & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$

34. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then the value of k so that $A^2=8A+kI$ is

A. $k=7$

B. $k=-7$

C. $k=0$

D. None of these

Key: B

Solution: $A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

and $8A + kI = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$

Thus,

$A^2 = 8A + kI \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$

$\Rightarrow 1 = 8 + k$ and $56 + k = 49 \Rightarrow k = -7$

35. The value of $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

A. 0
B. abc
C. $4a^2b^2c^2$
D. None of these

Key: C

Solution:

Let $\Delta = \begin{bmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}$

Taking a, b, c common from R_1, R_2 and R_3 respectively, we get.

$\Delta = abc\begin{bmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{bmatrix} = a^2b^2c^2\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

[taking a, b, c common from C_1, C_2, C_3 respectively]

$= a^2b^2c^2\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

(applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$)

$$=a^2b^2c^2\cdot(-1)\begin{vmatrix}0 & 2\\ 2 & 0\end{vmatrix}=a^2b^2c^2(-1)(0-4)$$

$$\Rightarrow \Delta = 4a^2b^2c^2$$

36. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & \alpha\end{bmatrix}$, then Adj. A is equal to:

A. $\begin{bmatrix}\delta & -\gamma\\ -\beta & \alpha\end{bmatrix}$

B. $\begin{bmatrix}\delta & -\beta\\ -\gamma & \alpha\end{bmatrix}$

C. $\begin{bmatrix}-\delta & \beta\\ \gamma & -\alpha\end{bmatrix}$

D. $\begin{bmatrix}-\delta & -\beta\\ \gamma & \alpha\end{bmatrix}$

Key: B

Solution: Let $A=\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}$

$$C_{11}=\delta, C_{12}=-\gamma, C_{21}=-\beta, C_{22}=\alpha$$

$$\therefore adj\,A=\begin{bmatrix}\delta & -\gamma\\ -\beta & \alpha\end{bmatrix}'=\begin{bmatrix}\delta & -\beta\\ -\gamma & \alpha\end{bmatrix}.$$

37. If $f(x)=\dfrac{\sqrt{4+x}-2}{x}$, $x\neq 0$ be continuous at x= 0, then f(0)

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. 2

D. $\frac{3}{2}$

Key: B

Solution: $f(0)=\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}$

$$=\lim_{x\to 0}\left(\frac{\sqrt{4+x}-2}{x}\times\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)$$

$$=\lim_{x\to 0}\frac{\sqrt{4+x}-4}{x\left(\sqrt{4+x}+2\right)}=\lim_{x\to 0}\frac{x}{x\left(\sqrt{4+x}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}=\frac{1}{2+2}=\frac{1}{4}.$$

38. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}=$

A. $\frac{x+1}{x}$

B. $\frac{1}{1+x}$

C. $\frac{-1}{(1+x)^2}$

D. $\frac{x}{1+x}$

Key: C

Solution: Given $x\sqrt{1+y}+y\sqrt{1+x}=0$

$\Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y)=y^2(1+x)$

$\Rightarrow x^2-y^2+x^2y-xy^2=0\Rightarrow(x-y)(x+y+xy)=0$

$\Rightarrow y=x \text{ or } y(1+x)=-x\Rightarrow y=x \text{ or } y=-\frac{x}{1+x}$

$\Rightarrow \frac{dy}{dx}=\frac{-(1+x)\cdot 1+x\cdot 1}{(1+x)^2}=\frac{-1}{(1+x)^2}$

39. The function $f(x)=xe^{x(1-x)}, x\in R$ is

A. increasing in $\left(-\frac{1}{2},1\right)$

B. decreasing in $\left(\frac{1}{2},2\right)$

C. increasing in $\left(-1,-\frac{1}{2}\right)$

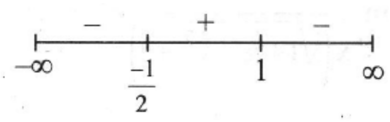
D. decreasing in $\left(-\frac{1}{2},\frac{1}{2}\right)$

Key: A

Solution: $f(x)=xe^{x(1-x)}$

$$f'(x) = -e^{x(1-x)}(2x+1)(x-1) = 0$$

$$x = 1, -\frac{1}{2}$$



$$\therefore f(x) \, is \, increasing \, in \left(-\frac{1}{2}, 1\right)$$

40. The angle of intersection of the curve $y^2 = x$ and $x^2 = y$ is

A. $\tan^{-1}\left(\frac{3}{2}\right)$

B. $\tan^{-1}\left(\frac{3}{4}\right)$

C. $\tan^{-1}\left(\frac{1}{2}\right)$

D. $\tan^{-1}\left(\frac{1}{5}\right)$

Key: B

Solution: $y^2 = x$ and $x^2 = y \Rightarrow x^4 = x$

or $x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x = 0, x = 1$

Therefore, y= 0, y= 1

i.e points of intersection are (0,0) and (1,1).

Further $y^2 = x \Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

and $x^2 = y \Rightarrow \frac{dy}{dx} = 2x$

At (0,0) the slope of the tangent to the curve $y^2 = x$ is parallel to Y-axis and the tangent to the curve $x^2 = y$ is parallel to X-axis

$$\Rightarrow \text{Angle of Intersection} = \frac{\pi}{2}$$

At (1,1) slope of the tangent to the curve $y^2 = x$

is equal to $\frac{1}{2}$ *and that of* $x^2 = y$ *is* 2.

$$tan\theta = \left|\frac{2-\frac{1}{2}}{1+1}\right| = \frac{3}{4} \Rightarrow \theta = \tan^{1}\left(\frac{3}{4}\right)$$

41. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

A. $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

B. $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

C. $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

D. $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

Key: C

Solution: $I = \int \frac{dx}{cosx + \sqrt{3}sinx}$

$$\Rightarrow I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$$

$$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]}$$

$$= \frac{1}{2}\cdot\int \frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}$$

$$\Rightarrow I = \frac{1}{2}\cdot\int cosec\left(x+\frac{\pi}{6}\right)dx$$

$$\because \int cosecxdx = \log\left|(\tan x/2)\right| + C$$

$$\therefore I = \frac{1}{2}\cdot logtan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$$

42. $\int \sin^{-1} xdx$ is equal to

A. $x\sin^{-1}x+\sqrt{1-x^2}+c$

B. $x\sin^{-1}x-\sqrt{1-x^2}+c$

C. $\cos^{-1}x+c$

D. $\frac{1}{\sqrt{1-x^2}}+c$

Key: B

Solution: Let $\sin^{-1}x=\theta \Rightarrow x=sin\theta$

$\Rightarrow dx=\cos\theta d\theta$

$\therefore I=\int\theta\,\cos\theta d\theta=\theta\int\cos\theta d\theta+\int\left(\frac{d\theta}{d\theta}\int\cos\theta d\theta\right)d\theta$

$=\theta\,sin\theta+\int\sin\theta d\theta=\theta\,sin\theta-cos\theta+c$

$=xsin^{-1}x-\sqrt{1-x^2}+c.$

43. The area bounded by the curve $y=\frac{3}{2}\sqrt{x}$, the line x =1 and x-axis is ___________ sq. units.

A. 2
B. 1
C. 6
D. None of these

Key: B

Solution: Area $=\int_0^1 ydx=\int_0^1\frac{3}{2}\sqrt{x}dx$

$=\frac{3}{2}\left[\frac{2}{3}x^{3/2}\right]_0^1=\left[x^{3/2}\right]_0^1$

$=1^{3/2}-0=1$ sq. unit

44. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^4+\left(\frac{d^2y}{dx^2}\right)^5+\frac{dy}{dx}+y=0$ is

A. 2
B. 4
C. 6
D. 8

Key: B

Solution: The degree of a differential equations is the exponent of the highest order in the differential equation. Therefore the degree of the given differential equation is 4.

45. In triangle ABC, which of the following is not true ?

A. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

B. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

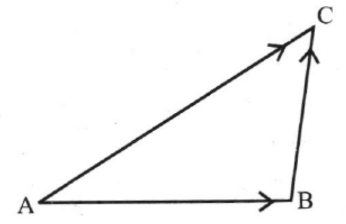
C. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$

D. $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

Key: A

Solution: By triangle law of vector addition

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$



or $\overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA}$

or $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

46. If (2,3,9),(5,2, l), (1, λ , 8)and(λ ,2.3) are coplanar, then the product of all possible values of λ is :

A. $\frac{21}{2}$

B. $\frac{59}{8}$

C. $\frac{57}{8}$

D. $\frac{95}{8}$

Key: D

Solution: Given points are

$$A(2,3,9);B(5,2,1);C(1,\lambda,8);D(\lambda,2,3)$$

$$\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]=0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix}=0$$

$$\Rightarrow\left[-6(\lambda-3)-1\right]-8\left(1-(\lambda-3)(\lambda-2)\right)+(6+(\lambda-2)=0$$

$$3(-6\lambda+17)-8(-\lambda^2+5\lambda-5)+(\lambda+4)=8$$

$$8\lambda^2-57\lambda+95=0$$

Apply the rule of product whose roots are $\alpha\beta$.

$$\alpha\beta=\frac{95}{8}$$

47. If $P(B)=\frac{3}{5}, P(A|B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$ then $P(A\cup B)'+P(A'\cup B)=$

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{2}$

D. 1

Key: D

Solution: $P(B)=\frac{3}{5}, P(A|B)=\frac{1}{2}$

and $P(A\cup B)=\frac{4}{5}$

$$P(A\cap B)=P(A|B)P(B)=\frac{1}{2}\cdot\frac{3}{5}=\frac{3}{10}$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

We know, $P(A\cap B)+P(A'\cap B)=P(B)$

[as $A\cap B$ and $A'\cap B$ are mutually exclusive events]

$$\Rightarrow\frac{3}{10}+P(A'\cap B)=\frac{3}{5}$$

$$\Rightarrow P(A'\cap B)=\frac{3}{5}-\frac{3}{10}=\frac{3}{10}$$

Now, $P(A'\cup B)=P(A')+P(B)-P(A'\cap B)$

$$=\frac{1}{2}+\frac{3}{5}-\frac{3}{10}=\frac{5+6-3}{10}=\frac{4}{5}$$

$$P\left((A\cup B)'\right)=1-P\left(A\cup B\right)=1-\frac{4}{5}=\frac{1}{5}$$

$$\therefore P\left((A\cup B)'\right)+P\left(A'\cup B\right)=\frac{1}{5}+\frac{4}{5}=1$$

48. If $|\vec{a}|=3, |\vec{b}|=4,$ then a value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$ is

A. $\frac{9}{16}$

B. $\frac{3}{4}$

C. $\frac{3}{2}$

D. $\frac{4}{3}$

Key: B

Solution: If $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, then

$$\left(\vec{a}+\lambda\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=\left|\vec{a}+\lambda\vec{b}\right|\left|\vec{a}-\lambda\vec{b}\right|\cdot\cos 90^\circ$$

$$\Rightarrow\left(\vec{a}+\lambda\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=0$$

$$\Rightarrow\vec{a}\cdot\vec{a}-\lambda\vec{a}\cdot\vec{b}+\lambda\vec{b}\cdot\vec{a}-\lambda^2\vec{b}\cdot\vec{b}=0$$

$$\Rightarrow a^2-\lambda^2b^2=0\Rightarrow\lambda^2=\frac{a^2}{b^2}\Rightarrow\lambda^2=\frac{3^2}{4^2}$$

$$\Rightarrow\lambda=\frac{3}{4}.$$

49. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is

A. $2\sqrt{29}$

B. 1

C. $\sqrt{\frac{37}{29}}$

D. $\frac{\sqrt{29}}{2}$

Key: A

Solution: Here, $\vec{a}_1=-7\hat{i}+6\hat{j}, \vec{b}_1=-6\hat{i}+7\hat{j}+\hat{k}$

$\vec{a}_2 = 7\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{b}_2 = -2\hat{i} + \hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 14\hat{i} - 4\hat{j} + 6\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 6\hat{i} + 4\hat{j} + 8\hat{k}$$

$$\text{Distance} = \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} = \frac{116}{\sqrt{116}}$$

$= \sqrt{116} = 2\sqrt{29}$

50. Let E and F be two independent events. The I probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{12}$, then a value of is $\frac{P(E)}{P(F)}$ is :

A. $\frac{4}{3}$

B. $\frac{3}{2}$

C. $\frac{1}{3}$

D. $\frac{5}{12}$

Key: A

Solution: $P(E \cap F) = P(E) \cdot P(F) = \frac{1}{12}$

$P(\bar{E} \cap \bar{F}) = P(\bar{E}) \cdot P(\bar{F}) = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \frac{1}{2}$

Let $P(E) = x$

$P(F) = y$

$$\Rightarrow 1-x-y+xy=\frac{1}{2}\Rightarrow 1-x-y=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

$$\Rightarrow x+y=\frac{7}{12}\Rightarrow x+\frac{1}{12x}=\frac{7}{12}$$

$$\left[\because x\cdot y=\frac{1}{12}\right]$$

$$\Rightarrow 12x^2-7x+1=0$$

$$\Rightarrow 12x^2-4x-3x+1=0\Rightarrow(4x-1)(3x-1)=0$$

$$\Rightarrow x=\frac{1}{3},x=\frac{1}{4}$$

and $y=\frac{1}{4},y=\frac{1}{3}$

$$\therefore\frac{x}{y}=\frac{1/3}{1/4}=\frac{4}{3}\text{ or }\frac{1/4}{1/3}=\frac{3}{4}$$