26th MAY, 2024 (SUNDAY)

SUBJECT: MATHEMATICS

PART : MAHTEMATICS

SECTION-1 : 12 Marks

- This section contains **FOUR (04)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct option is chosen;
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : **−1** In all other cases.

1. Let $f(x)$ be a continuously differentiable function on the interval $(0,\infty)$ such that $f(1)=2$ and $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$ for each x >0. Then, for all x >0, f(x) is equal to

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$ (B) $\frac{9}{11x}+\frac{13}{11}x^{10}$ (C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$ (D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

Ans. **(B)**

Sol. $\because \quad \lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$\Rightarrow \quad \lim\limits_{t\to x}\frac{10t^9f(x)-x^{10}f'(t)}{9.t^8}=1$

$\Rightarrow \quad \frac{10.x^9f(x)-x^{10}f'(x)}{9.x^8}=1$

$\Rightarrow \quad x^{10}f'(x)-10.x^9f(x)=-9.x^8$

$\Rightarrow \quad f'(x)-\frac{10}{x}f(x)=-\frac{9}{x^2}$

$\text{I.F.} = e^{-10\int_n \frac{1}{x}dx} = e^{-10\ell nx} = \frac{1}{x^{10}}$

$\therefore \quad y.\frac{1}{x^{10}}=\int-\frac{9}{x^2}.\frac{1}{x^{10}}dx \Rightarrow \frac{y}{x^{10}}=\frac{9}{11}\frac{1}{x^{11}}+C$

$\Rightarrow \quad y=\frac{9}{11}.\frac{1}{x}+C.x^{10} \qquad \because y(1)=2$

$\Rightarrow \quad 2=\frac{9}{11}+C \qquad \Rightarrow C=2-\frac{9}{11}=\frac{13}{11}$

$\therefore \quad y=\frac{9}{11x}+\frac{13}{11}.x^{10}$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{5}{7}$ (D) $\frac{5}{12}$

Ans. **(C)**

Sol. Let total questions = T

x (No. of questions whose answer be knows)

(T– x) number of questions whose answer be don't know

$\therefore$ reqd $= \frac{x}{T}$

$$\therefore \frac{1}{6} = \frac{\left(\frac{T-x}{T}\right)\frac{1}{2}}{\frac{x}{T} + \left(\frac{T-x}{T}\right)\frac{1}{2}} \Rightarrow \frac{x}{T} = \frac{5}{7}$$

3. Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then $\left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos\frac{11x}{2}\right)(\sin 6x + \cos 6x)$ is equal to

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$ (C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

Ans. **(B)**

Sol. Let $E = \left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \cos\left(\frac{11x}{2}\right)(\sin 6x + \cos 6x)$

$$= \left(\sin 6x \sin\frac{11x}{2} + \cos 6x \cos\frac{11x}{2}\right) + \left(\sin 6x \cos\frac{11x}{2} - \cos 6x \sin\frac{11x}{2}\right) = \cos\left(6x - \frac{11x}{2}\right) + \sin\left(6x - \frac{11x}{2}\right)$$

$E = \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)$ ____(2)

$\because \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 = 1 + \sin x$ $\quad\because x \in \left(\frac{\pi}{2}, \pi\right)$ and $\cot x = -\frac{5}{\sqrt{11}}$

$= 1 + \frac{\sqrt{11}}{6}$ $\quad\therefore \sin x = \frac{\sqrt{11}}{6}$

$= \frac{\sqrt{11}+6}{6}$

$$\therefore \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) = \frac{\sqrt{6+\sqrt{11}}}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{12+2\sqrt{11}}}{2\sqrt{3}}$$

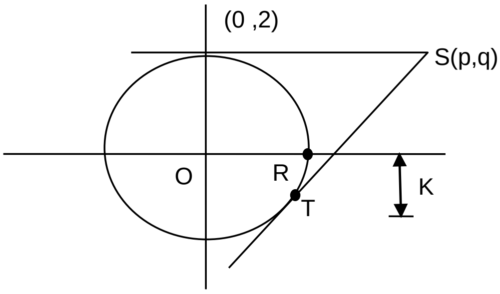
$$= \frac{\sqrt{\left(\sqrt{11}+1\right)^2}}{2\sqrt{3}} = \frac{\sqrt{11}+1}{2\sqrt{3}} \Rightarrow \quad E = \frac{\sqrt{11}+1}{2\sqrt{3}}$$

4. Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let S(p, q) be a point in the first quadrant such that $\frac{p^2}{9}+\frac{q^2}{4}>1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the centre of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct ?

(A) $q=2, p=3\sqrt{3}$ (B) $q=2, p=4\sqrt{3}$ (C) $q=1, p=5\sqrt{3}$ (D) $q=1, p=6\sqrt{3}$

Ans. **(A)**

Sol. $\frac{x^2}{9}+\frac{y^2}{4}=1$ ______(1)



$\because$ q = 2

$\because$ Area of $\Delta ORT = \frac{3}{2}$

$\Rightarrow \frac{1}{2}(OR)(K)=\frac{3}{2} \Rightarrow \frac{1}{2}(3)(K)=\frac{3}{2}$

$\therefore$ T(h,–1)

$\because$ T (h,–1) lies on (1) $\Rightarrow \frac{h^2}{9}+\frac{1}{4}=1 \Rightarrow h^2=\frac{3}{4}\times 9 \Rightarrow h=\frac{3\sqrt{3}}{2}$

$\therefore$ $T\left(\frac{3\sqrt{3}}{2},-1\right)$

$\because$ equation of tangent at $T\left(\frac{3\sqrt{3}}{2},-1\right)$, is

$\frac{x\left(\frac{3\sqrt{3}}{2}\right)}{9}+\frac{y(-1)}{4}=1$ $\because$ it is passes through S(p , 2)

$\therefore$ $\frac{p\left(\frac{3\sqrt{3}}{2}\right)}{9}+\frac{2(-1)}{4}=1$

$\frac{p}{2\sqrt{3}}=1+\frac{1}{2}=\frac{3}{2}$

$p=3\sqrt{3}$

$\therefore$ $p=3\sqrt{3}$, q = 2

SECTION 2 : 12 Marks

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4 ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : **−2** In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

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5. Let $S = \{a + b\sqrt{2} : a,b \in \mathbb{Z}\}$, $T_1 = \left\{\left(-1+\sqrt{2}\right)^n : n \in N\right\}$, and $T_2 = \left\{\left(1+\sqrt{2}\right)^n : n \in N\right\}$. Then which of the following statements is (are) TRUE ?

(A) $\mathbb{Z} \cup T_1 \cup T_2 \subset S$

(B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.

(C) $T_2 \cap (2024, \infty) \neq \phi$

(D) For any given $a,b \in \mathbb{Z}$, $\cos\left(\pi\left(a+b\sqrt{2}\right)\right) + i\sin\left(\pi\left(a+b\sqrt{2}\right)\right) \in \mathbb{Z}$ if any only if b=0, where $i = \sqrt{-1}$.

Ans. **(ACD)**

Sol. Binomial expansion :

$$\left(-1+\sqrt{2}\right)^n = \left({}^nC_0(-1)^n + {}^nC_2(-1)^{n-2}\left(\sqrt{2}\right)^2 + ...\right) + \sqrt{2}\left({}^nC_1(-1)^{n-1} + {}^nC_3(-1)^{n-3}\left(\sqrt{2}\right)^2 + ...\right)$$

= (integer) + $\sqrt{2}$ (integer)

$\left(-1+\sqrt{2}\right)^n$ belongs to S

Similarly $\left(1+\sqrt{2}\right)^n$ belongs to S

All integer belongs to S

$\Rightarrow \mathbb{Z} \cup T_1 \cup T_2 \subset S \Rightarrow$ (A) is True

$0 < (-1 + \sqrt{2}) < 0.414 \Rightarrow$ for sufficiently large n,

$\left(-1+\sqrt{2}\right)^n < \frac{1}{2024} \Rightarrow T_1 \cap \left(0, \frac{1}{2024}\right) \neq \phi \Rightarrow$ B is false

$1+\sqrt{2} > 2.414 \Rightarrow$ for sufficiently large n ($n \geq 9$)

$\left(1+\sqrt{2}\right)^n > 2024 \Rightarrow T_2 \cap (2024, \infty) \neq \phi \Rightarrow$ C is True

$e^{i\left(a+b\sqrt{2}\right)\pi} \in Z \Leftrightarrow \left(a+b\sqrt{2}\right)\pi = n\pi, n \in I$

$\Leftrightarrow a = n, b = 0 \Rightarrow$ D is true

6. Let R^2 denote $R \times R$. Let $S = \left\{(a,b,c) : a,b,c \in R \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x,y) \in R^2 - \{(0,0)\}\right\}$ for. Then which of the following statements is (are) TRUE ?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.

(C) For any given $(a,b,c) \in S$, the system of linear equations

$ax + by = 1$

$bx + cy = -1$

has a unique solution.

(D) For any given $(a,b,c) \in S$, the system of linear equations

$(a+1)x + by = 0$

$bx + (c+1)y = 0$

has a unique solution.

Ans. **(BCD)**

Sol. $b^2 - ac < 0$ & $a > 0$ & $c > 0$

(A) $\frac{49}{4} - 12 > 0$ (B) $b^2 - \frac{1}{4} < 0 \Rightarrow |2b| < 1$ (C) $\begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$

(D) $\begin{vmatrix} a+1 & b \\ b & c+1 \end{vmatrix} \Rightarrow ac + a + c + 1 - b^2 > 0$ but, $\left.\begin{array}{l} x = \frac{0}{+\text{ive}} = 0 \\ y = \frac{0}{+\text{ive}} = 0 \end{array}\right\}$

Which gives unique solution of system of linear equations

7. Let R^3 denote the three-dimensional space. Take two points P = (1,2,3) and Q = (4,2,7). Let dist (X,Y) denote the distance between two points X and Y in R^3. Let

$S = \left\{X \in R^3 : (\text{dist}(X,P))^2 - (\text{dist}(X,Q))^2 = 50\right\}$ and

$T = \left\{Y \in R^3 : (\text{dist}(Y,Q))^2 - (\text{dist}(Y,P))^2 = 50\right\}$.

Then which of the following statements is (are) TRUE ?

(A) There is a triangle whose area is 1 and all of whose vertices are from S

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T

Ans. **(ABC)**

Sol. S : $6x_1 + 8z_1 = 105$

T : $6x_2 + 8z_2 = 5$

(A) Triangles can be formed by taking 3 points in the XZ plane, two of them lying on $6x_1 + 8z_1 = 105$

(B) Has elements of set T is also a line complete line in T.

(C) Difference between parallel planes is $\frac{100}{10} = 10$. Infinite rectangles can be formed with sides 10, 14

(D) Difference between parallel planes is $\frac{100}{10} = 10$. Squares of side 12 is not possible

SECTION-3 : 24 Marks

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on--screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4 ONLY** the correct integer value is entered;

Zero Marks : **0** In all other cases.

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8. Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{6}}$. If x, y ∈ R are such that $3x + 2y = \log_a (18)^{\frac{5}{4}}$ and $2x - y = \log_b \left(\sqrt{1080}\right)$, then 4x + 5y is equal to ________.

Ans. **(8)**

Sol. $\because a = 3\sqrt{2}$; $b = \left(5^{1/6}\sqrt{6}\right)^{-1}$

$\therefore \log_a (18)^{5/4} = \log_{3\sqrt{2}} \left(3\sqrt{2}\right)^{5/2} = 5/2$

$\Rightarrow 3x + 2y = \log_a (18)^{5/4} \Rightarrow 3x + 2y = 5/2$ ...(1)

$\because \sqrt{1080} = \sqrt{36 \times 3 \times 5 \times 2} = 6\sqrt{6}\sqrt{5} = \left(\sqrt{6}.5^{1/6}\right)^3$

$\therefore 2x - y = \log_b \sqrt{1080} = \log_{\left(\sqrt{6}.5^{1/6}\right)^{-1}} \left(\sqrt{6}.5^{1/6}\right)^3 = -3$ ....(2)

$\therefore$ solving (1) and (2)

$x = -\frac{1}{2}$, $y = 2$

So, 4x + 5y = – 2 + 10 = 8

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that f (1) = –9. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and α_4 are all the roots of the equation f(x) = 0, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ______.

Ans. **(20)**

Sol. $\because$ $f(x) = x^4 + ax^3 + bx^2 + c$

$\because$ $f(1) = -9 \Rightarrow 1+a+b+c = -9$ ….(1)

$\because$ $i\sqrt{3}$ is a root of the equation $x(4x^2+3ax+2b)=0$

$\Rightarrow$ $4(-3) + 3a(i\sqrt{3}) + 2b = 0$

$\Rightarrow$ $(2b-12) + i(3a\sqrt{3}) = 0 + i(0)$ $\because$ a, b$\in$ R

$\Rightarrow$ $2b-12 = 0$ and $3a\sqrt{3} = 0$

$\Rightarrow$ $b=6$ $a=0$

$\therefore$ from (1) $c = -16$

$\therefore$ $f(x) = 0$ becomes

$x^4 + 6x^2 - 16 = 0$

$(x^2+8)(x^2-2)=0 \Rightarrow$ Roots of $f(x) = 0$ are $\sqrt{2}$, $-\sqrt{2}$, $i(2\sqrt{2})$, $-i(2\sqrt{2})$

$\therefore$ $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 2+2+8+8 = 20$

10 Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a,b,c,d,e \in \{0,1\} \text{ and } |A| \in \{-1,1\} \right\}$, where $|A|$ denotes the determinant of A. Then the number of elements in S is ________.

Ans. **(16)**

Sol. $\because$ $|A| = (d-e) + c(b-a)$ $\because$ a,b,c,d,e $\in\{0,1\}$

d	e	c	b	a
1	0	0	1	1
		0	1	0
		0	0	1
		0	0	0
1	0	1	1	1
		1	0	0
1	1	1	1	0
0	0	1	1	0

$\Rightarrow |A| = 1$

0	1	0	1	1
		0	1	0
		0	0	1
		0	0	0
0	1	1	1	1
		1	0	0
1	1	1	0	1
0	0	1	0	1

$\Rightarrow |A| = -1$

$\therefore$ Total 16 elements will be there is S

11 A group of 9 students, $s_1, s_2, \ldots, s_9$, is to be divided to form three teams X, Y, and Z of sizes 2,3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is _______.

Ans. **(665)**

Sol.

X	Y	Z
(2)	(3)	(4)

Case : 1 s_2 is in team X

$$= {}^7C_1 \times \frac{7!}{3!4!} = 245$$

Case : 2 s_2 is not in team X

$= {}^7C_2 \times {}^6C_3 = 420$

$\therefore$ Total ways = 245 + 420 = 665

12 Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in R - \{0\}$ and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right).\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane

$3x + 3y - z + \ell = 0$,

then the value of ℓ is ________.

Ans. **(5)**

Sol. $\because \overrightarrow{OP} = \left(\frac{\alpha-1}{\alpha}\right)\hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{OQ} = \hat{i} + \left(\frac{\beta-1}{\beta}\right)\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{\hat{k}}{2}$

$\therefore (\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + \ell = 0$

$\Rightarrow 3\alpha + 3\beta - 2 + \ell = 0$ ....(1)

$\because \left(\overrightarrow{OP} \times \overrightarrow{OQ}\right).\overrightarrow{OR} = 0$

$$\Rightarrow \begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{bmatrix} -\frac{1}{\alpha} & 0 & 1 \\ \frac{1}{\beta} & -\frac{1}{\beta} & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = 0 \qquad \Rightarrow -\frac{1}{\alpha}\left(-\frac{1}{2\beta} - \frac{1}{2}\right) + 1\left(\frac{1}{2\beta}\right) = 0$$

$$\frac{1}{2\alpha\beta} + \frac{1}{2\alpha} + \frac{1}{2\beta} = 0$$

$1 + \beta + \alpha = 0 \Rightarrow \alpha + \beta = -1$

$\therefore$ from (1), we get

$3(\alpha + \beta) - 2 + \ell = 0 \Rightarrow \ell = 5$

13 Let X be a random variable, and let P(X = x) denote the probability that X takes the value x. Suppose that points (x, P (X = x)), x = 0,1,2,3,4, lie on a fixed straight line in the xy-plane, and P(X = x) = 0 for all $x \in R - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ______.

Ans. **(42)**

Sol. Let $P(x=0)=a$, $P(x=1)=b$, $P(x=2)=c$, $P(x=3)=d$, $P(x=4)=e$

X	P(X)	X.P(X)	X^2.P(X)
0	a	0	0
1	b	b	b
2	c	2c	4c
3	d	3d	9d
4	e	4e	16e

Let fixed straight line be $y = mx + \lambda$ .... (1)

$\because (0,a),(1,b),(2,c),(3,d),(4,e)$ all lies on (1)

$\Rightarrow \frac{b-a}{1}=\frac{c-b}{1}=\frac{d-c}{1}=\frac{e-d}{1}$ = m = slope of line

$\Rightarrow a,b,c,d,e$ are in A.P

$\because \sum p(x)=1$

$\Rightarrow a+b+c+d+e=1 \Rightarrow (a+e)+(b+d)+c=0$

$2c+2c+c=1 \quad \Rightarrow c=1/5$

$\because$ a, b, c, d, e are AP and c = 1/5

$\frac{1}{5}-2\beta, \frac{1}{5}-\beta, \frac{1}{5}, \frac{1}{5}+\beta, \frac{1}{5}+2\beta$

Mean = 5/2

$\Rightarrow b+2c+3d+4e=\frac{5}{2} \Rightarrow \frac{1}{5}-\beta+\frac{2}{5}+\frac{3}{5}+3\beta+\frac{4}{5}+8\beta=\frac{5}{2}$

$10\beta+2=\frac{5}{2} \Rightarrow \beta=\frac{1}{20}$

$\therefore b=\frac{1}{5}-\frac{1}{20}=\frac{3}{20}$

$e=\frac{6}{20}$

$\because$ variance $(\alpha)=\sum X^2.p(X)-(mean)^2$

$\Rightarrow \alpha = b+4c+9d+16e-\frac{25}{4}$

$\Rightarrow \alpha = \frac{3}{20}+4\left(\frac{4}{20}\right)+9\left(\frac{5}{20}\right)+16\left(\frac{6}{20}\right)-\frac{25}{4}$

$\Rightarrow \alpha = \frac{3+16+45+96}{20}-\frac{25}{4} \Rightarrow \alpha = 8-\frac{25}{4}=\frac{7}{4}$

$\therefore 24\alpha=24\left(\frac{7}{4}\right)=42$

SECTION 4 : 12 Marks

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3 ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : **–1** In all other cases.

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14 Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i =1,2,3 and j = 1, 2,3.

Match each entry in **List-I** to the correct entry in **List-II.**

	List-I	**List-II**
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1) 1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2) 12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for i > j. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in R,\ M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is	(3) infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4) 6
		(5) 0

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)

(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)

(C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

(D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

Ans. **(C)**

Sol. $\alpha + \beta = -1$

$1 + \alpha + \beta = 0$

$P = 3! \times 2 = 12$

$Q = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & & \\ \beta & & \end{bmatrix} 3! \times 1 = 6$

(S) = obviously

(R) $\begin{bmatrix} 0 & -1 & -\alpha \\ 1 & 0 & -\beta \\ \alpha & \beta & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -\beta \end{bmatrix}$

D = 0, Infinite such (C)

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1.

Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let

$\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		**List-II**
(P)	α equals	(1)	(–2, 4)
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	(–2,6)
(S)	B_1 equals	(4)	5
		(5)	(2,4)

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

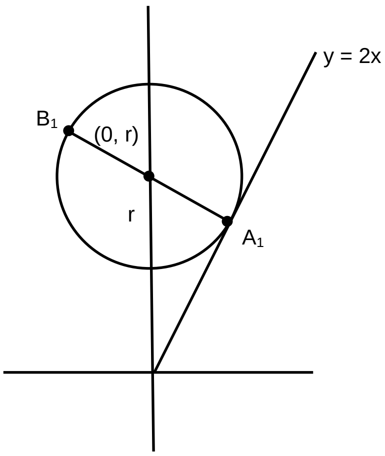
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)

(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)

(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

Ans. **(C)**

Sol.



$\alpha + r = 5 + \sqrt{5}$

$\frac{\alpha}{\sqrt{5}} = r$

$\alpha = \sqrt{5}\, r$

$\sqrt{5}\, r + r = 5 + \sqrt{5}$

$r = \frac{5+\sqrt{5}}{\sqrt{5}+1} = \sqrt{5}$

$\alpha = 5$

$\Rightarrow x^2 + (y-5)^2 = 5$

$x^2 + y^2 - 10y + 20 = 0$

(T) $2x - y = 0$

$(A_1B_1) \Rightarrow x + 2y = 10$

$5x = 10$

$x = 2, y = 4$

A_1 (2, 4)

$B_1 = (-2, 6)$

16. Let $\gamma \in$ R be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let O = (0,0,0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		**List-II**
(P)	γ equals	(1)	$-\hat{i}-\hat{j}+\hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1}.\hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

(A) (P) → (3) (Q) → (4) (R) → (1) (S) → (2)
(B) (P) → (5) (Q) → (4) (R) → (1) (S) → (2)
(C) (P) → (3) (Q) → (4) (R) → (1) (S) → (5)
(D) (P) → (3) (Q) → (1) (R) → (4) (S) → (5)

Ans. **(C)**

Sol. Point on line L_1 P($\lambda_1 - 11$, $2\lambda_1 - 21$, $3\lambda_1 - 29$)

Point on line L_2 Q($3\lambda_2 - 16$, $2\lambda_2 - 11$, $\gamma\lambda_2 - 4$)

P = Q : $\lambda_1 - 11 = 3\lambda_2 - 16$, $2\lambda_1 - 21 = 2\lambda_2 - 11$

$\lambda_1 - 3\lambda_2 = -5$, $2\lambda_1 - 21 = 2\lambda_2 - 11$

$\lambda_1 - \lambda_2 = 5$

$-2\lambda_2 = -10$

$\lambda_2 = 5$ and $\lambda_1 = 10$

also $3\lambda_1 - 29 = \gamma\lambda_2 - 4$

$30 - 29 = 5\gamma - 4 \Rightarrow \gamma = 1$

Point of intersection P = Q = (– 1, – 1,1)

Plane contain L_1 & L_2

$$\begin{vmatrix} x+11 & y+21 & z+29 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

(x + 11) (2 – 6) – (y + 21) (1 – 9) + (z + 29) (2 – 6) = 0 (x + 11) (– 4) – (y + 21) (– 8) + (z + 29) (–4) = 0

x + 11 – 2y – 42 + z + 29 = 0

x – 2y + z – 2 = 0

(P) y = 1 (P) → (3)

(Q) $\hat{n} = \frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$ (Q) → (4)

(R) $\overrightarrow{OR_1} = -\hat{i} - \hat{j} + \hat{k}$ (R) → (1)

(S) $\overrightarrow{OR}.\hat{n} = \frac{-1+2+1}{\sqrt{6}} = \sqrt{2/3}$ (S) → (5)

17. Let f : R → R and g: R → R be functions defined by $f(x) = \begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0 \end{cases}$ and

$g(x) = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$. Let a, b, c, d ∈ R. Define the function h : R → R by

$h(x) = a\, f(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + d g(x), x \in R$.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I	**List-II**
(P) If a = 0, b =1, c = 0, and d = 0, then	(1) h is one- one.
(Q) If a = 1, b =0, c = 0, and d = 0, then	(2) h is onto.
(R) If a = 0, b =0, c = 1, and d = 0, then	(3) h is differentiable on R.
(S) If a = 0, b =0, c = 0, and d = 1, then	(4) the range of h is [0, 1].
	(5) the range of h is {0, 1}

The correct option is

(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

Ans. **(C)**

Sol. (P) If a = 0, b = 1, c = 0, d = 0

$h(x) = g(x) + g(1/2 - x) : x \in R$

$= \begin{cases} 1 \;;\; 0 \le x \le 1/2 \\ 0 \;;\; \text{otherwise} \end{cases}$

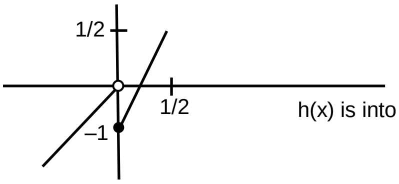
$g(1/2 - x) = \begin{cases} 1 - 2(1/2 - x) \;;\; 0 \le x \le \frac{1}{2} \\ 0 \;;\; \text{otherwise} \end{cases}$

$g(1/2 - x) = \begin{cases} 2x \;;\; 0 \le x \le 1/2 \\ 0 \;;\text{otherwise} \end{cases}$

(P) → (5)

(Q) a = 1, b = 0, c = 0, d = 0 = h(x) = f(x) = $\begin{cases} -x^2 \sin(1/x) & ; \ x < 0 \\ 0 & ; \ x = 0 \\ +x^2 \sin(1/x) & ; \ x > 0 \end{cases}$ cont. add dff. at $x \in R$ (Q) $\rightarrow$ (3)

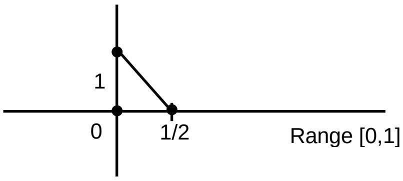
(R) a = 0, b = 0, c = 1, d = 0 h(x) = x – g(x) = $\begin{cases} x - 1 + 2x & ; \ 0 \le x \le 1/2 \\ x & ; \ \text{otherwise} \end{cases} = \begin{cases} 3x - 1 & ; \ 0 \le x \le 1/2 \\ x & ; \ \text{otherwise} \end{cases}$



(R) $\rightarrow$ (2)

(S) a = 0, b = 0, c = q, d = 1

h(x) = g(x)



(S) $\rightarrow$ (4)