166

I

Total No. of Questions : 24

Total No. of Printed Pages : 3

Reg. No. | | | | | | | | | | |

Part – III
MATHEMATICS
Paper – I (A)
(English Version)

Time : 3 Hours | Max. Marks : 75

Note : This question paper consists of **THREE Sections** – **A**, **B** and **C**.

SECTION – A **(10×2=20)**

I. Very short answer type questions :

(i) Answer **ALL** the questions.

(ii) **Each** question carries **TWO** marks.

1) If A = {−2, −1, 0, 1, 2} and f : A → B is a surjection defined by $f(x) = x^2 + x + 1$ then find B.

2) Find the domain of the real valued function $f(x) = \log(x^2 - 4x + 3)$.

3) Define trace of matrix. Find the trace of A if $A = \begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$.

4) Find the Rank of $\begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$.

5) If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors, then find λ and μ.

6) Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.

7) If $\bar{a} = 2\bar{i} - 3\bar{j} + 5\bar{k}$, $\bar{b} = -\bar{i} + 4\bar{j} + 2\bar{k}$ then find $\bar{a} \times \bar{b}$ and unit vector perpendicular to both $\bar{a}$ and $\bar{b}$.

8) Find the period of the function $\tan(x + 4x + 9x + ... + n^2x)$ where n is any positive integer.

9) Find the maximum and minimum values of $3\sin x - 4\cos x$.

10) Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.

SECTION – B

(5×4=20)

II. Short answer type questions :

(i) Answer **ANY FIVE** questions.

(ii) **Each** question carries **FOUR** marks.

11) If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then show that $(aI + bE)^3 = a^3I + 3a^2bE$, where I is unit matrix of order 2.

12) Let ABCDEF be a regular hexagon with centre 'O', show that

$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\,\overline{AD} = 6\,\overline{AO}$.

13) $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then find $(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{c})$.

14) Find the value of $\sin^2\frac{\pi}{10} + \sin^2\frac{4\pi}{10} + \sin^2\frac{6\pi}{10} + \sin^2\frac{9\pi}{10}$.

15) Prove that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$.

16) If $\sin x + \sin y = \frac{1}{4}$ and $\cos x + \cos y = \frac{1}{3}$ then show that

(i) $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$ (ii) $\cot(x+y) = \frac{7}{24}$.

17) In ΔABC, prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$.

SECTION – C

(5×7=35)

III. Long answer type questions :

(i) Answer **ANY FIVE** questions.

(ii) **Each** question carries **SEVEN** marks.

18) If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$ then find

(i) $f + g$

(ii) $f - g$

(iii) $2f + 4g$

(iv) $f + 4$

(v) fg

(vi) f/g

(vii) $|f|$.

19) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then show that $A^{-1} = A^3$.

20) Solve the following simultaneous linear equations by using Cramer's rule

$x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

21) Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$, $\bar{a} + 2\bar{b} - 5\bar{c}$ intersect at the point $-4\bar{c}$ when $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors.

22) If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$.

23) If $A + B + C = \pi$, then prove that

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right).$$

24) In ΔABC if $a = 13$, $b = 14$, $c = 15$, show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.