Part – III
MATHEMATICS
Paper – I (B)
(English Version)

Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of **THREE Sections A, B and C.**

SECTION – A

(10×2=20)

I. Very short answer type questions.

(i) Attempt **ALL** questions.

(ii) **Each** question carries **TWO** marks.

1) Transform the equation $\sqrt{3}\,x + y = 4$ into normal form.

2) Find the distance between the parallel lines $5x - 3y - 4 = 0$ and $10x - 6y - 9 = 0$.

3) Find the ratio in which the xz-plane divides the line joining A $(-2, 3, 4)$ and B $(1, 2, 3)$.

4) Find the equation of the plane passing through the point $(1, 1, 1)$ and parallel to the plane $x + 2y + 3z - 7 = 0$.

5) Evaluate $\lim\limits_{x\to 3} \dfrac{x^3 - 3x^2}{x^2 - 5x + 6}$.

6) Evaluate $\lim\limits_{x\to\infty} \dfrac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}$.

7) Find the derivative of $y = \log[\sin(\log x)]$.

8) Find the derivative of the function $e^{\sin^{-1}x}$.

9) Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \pi/4$.

10) Show that the function $f(x) = \sin x$, $(x \in R)$ defined on R is neither increasing nor decreasing on $(0, \pi)$.

II. Short answer type questions.
   (i) Attempt **ANY FIVE** questions.
   (ii) **Each** question carries **FOUR** marks.

11) A(2, 3) and B(−3, 4) are two given points. Find the equation of locus of P so that the area of the triangle PAB is 8.5.

12) A(1, 2), B(2, −3) and C(−2, 3) are three points. A point P moves such that $PA^2 + PB^2 = 2PC^2$. Show that the equation to the locus of P is $7x − 7y + 4 = 0$.

13) A triangle of area 24 sq.units is formed by a straight line and the coordinate axis in the first quadrant. Find the equation of the straight line, if it passes through (3, 4).

14) Check the continuity of the following function at 2.

$$f(x) = \begin{cases} \dfrac{1}{2}(x^2 - 4), & \text{if } 0 < x < 2 \\ 0, & \text{if } x = 2 \\ 2 - 8x^{-3}, & \text{if } x > 2 \end{cases}$$

15) Find the derivative of tan2x from the first principle.

16) Show that the curves $6x^2 − 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at (½, ½).

17) Find the interval in which the function $6 − 9x − x^2$ is strictly increasing or strictly decreasing.

III. Long answer type questions.
   (i) Attempt **ANY FIVE** questions.
   (ii) **Each** question carries **SEVEN** marks.

18) Find the circumcenter of the triangle whose vertices are (−2, 3), (2, −1) and (4, 0).

19) Show that the pair of straight lines $6x^2 − 5xy − 6y^2 = 0$ and $6x^2 − 5xy − 6y^2 + x + 5y − 1 = 0$ form a square.

20) Find the value of K, if the lines joining the origin to the points of intersection of the curve $2x^2 − 2xy + 3y^2 + 2x − y − 1 = 0$ and the line $x + 2y = K$ are mutually perpendicular.

21) Find the direction cosines of two lines which are connected by the relations $l − 5m + 3n = 0$ and $7l^2 + 5m^2 − 3n^2 = 0$.

22) If $y = x^{\tan x} + (\sin x)^{\cos x}$, find $\dfrac{dy}{dx}$.

23) If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the coordinate axis in A and B, then show that the length AB is constant.

24) A wire of length l is cut into two parts which are bent respectively in the form of a square and a circle. What are lengths of the pieces of the wire respectively so that the sum of the areas is the least ?