# Grade 10 Andhra Pradesh Mathematics 2014

**Part – A**

**SECTION - I**

**GROUP - A**

**(Statements and Sets, Functions, Polynomials)**

Q1. Prove $\sim (p \Rightarrow q) \equiv p \wedge (\sim q)$.

**Solution:**

$\sim (p \Rightarrow q) \equiv p \wedge (\sim q)$

By the definition of implication, we can rewrite:

$p \Rightarrow q \equiv \sim p \vee q$

Negating both sides:

$\sim (p \Rightarrow q) = \sim (\sim p \vee q)$

Using De Morgan's theorem:

$\sim (\sim p \vee q) = \sim\sim p \wedge \sim q$

Since $\sim\sim p = p$, we get:

$p \wedge \sim q$

$\sim (p \Rightarrow q) \equiv p \wedge (\sim q)$

Q2. If $A$ and $B$ are any two sets, show that $\mathrm{A}' - \mathrm{B}' = \mathrm{B} - \mathrm{A}$.

**Solution:**

Let $\mathrm{U} =$ the Universal Set. By definition:

$\mathrm{A}' = \mathrm{U} - \mathrm{A}$

$\mathrm{B}' = \mathrm{U} - \mathrm{B}$

$\mathrm{A}' - \mathrm{B}' = (\mathrm{U} - \mathrm{A}) - (\mathrm{U} - \mathrm{B})$

$\mathrm{A}' - \mathrm{B}' = \mathrm{U} - \mathrm{A} - \mathrm{U} + \mathrm{B}$

$\mathrm{A}' - \mathrm{B}' = \mathrm{B} - \mathrm{A}$

Q3. If $f: \mathrm{R} - \{3\} \to \mathrm{R}$ is defined by $f(x) = \frac{x+3}{x-3}$, show that $f\left[\frac{3x+3}{x-1}\right] = x$ for $x \neq 1$.

**Solution:**

Given that:

$$f(x) = \frac{x+3}{x-3} \text{ where, } f: R\{3\} \to R$$

$$\mathrm{f}\left[\frac{3x+3}{x-1}\right] = \frac{\frac{3x+3}{x-1} + 3}{\frac{3x+3}{x-1} - 3}$$

$$= \frac{3x+3+3x-3}{3x+3-3x+3}$$

$= \frac{6x}{6}$

$= x$

Hence, $f\left[\frac{3x+3}{x-1}\right] = x$

Q4. Find the value of $m$ in order that $x^4 - 2x^3 + 3x^2 - mx + 5$ may be exactly divisible by $x - 3$.

**Solution:** C

Let, $p(x) = x^4 - 2x^3 + 3x^2 - mx + 5$

If $(x - 3)$ exactly divides $p(x)$, then for $x = 3, p(x) = 0$

i.e. $p(3) = 0$

$3^4 - 2 \times 3^3 + 3 \times 3^2 - 3\,m + 5 = 0$

$81 - 54 + 27 - 3\,m + 5 = 0$

$-3\,m + 59 = 0$

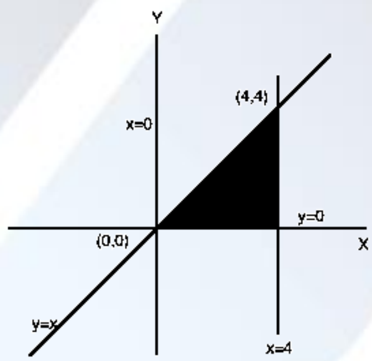
$m = \frac{59}{3}$

**GROUP - B**

**(Linear Programming, Real numbers, Progressions)**

Q5. Indicate the polygonal region represented by the system of inequations $x \geq 0,\ x \leq 4,\ x \geq y$.

**Solution:** C




Q6. If $a^x = b, b^y = c, c^z = a$, show that $xyz = 1$.

**Solution:** A

$a^x = b \ldots\ldots (1), b^y = c \ldots\ldots (2), c^z = a. \ldots\ldots (3)$

Now putting the value of $c$ from (2) in (3) we get,

$b^{yz} = a$

Now using the value of $b$ from (1) in the above equation we get,

$a^{xyz} = a$

or, $xyz = 1$

Q7. Solve the absolute value inequation $\left|\frac{2x-1}{3}\right| \leq 5$.

**Solution:** C

Inequality condition

$$\left|\frac{2x-1}{3}\right| \leq 5$$

$\Rightarrow |2x-1| \leq 15$

$\Rightarrow -15 \leq 2x-1 \leq +15$

$\Rightarrow -14 \leq 2x \leq +16$

$\Rightarrow -7 \leq x \leq +8$

$\therefore x=(-7,8)$ is the solution set of the given inequality.

Q8. Which term of the AP 10,8,6, ... ... is -28?

**Solution:**

$n^{th}$ term of arithmetic series is $t_n = a+(n-1)d$ where $a$ is first term and $d$ is common difference.

so $a=10$ and $d=8-10=-2$

Given $t_n = 10-(n-1)(-2) = 12-2n$

Let $r^{th}$ term be -28

So $t_r = 12-2r = -28$

$r=20$

$20^{\text{th}}$ term will be $-28$

**SECTION-II**

Q9. Write the converse and contrapositive of the following conditional If in a triangle ABC, AB $>$ AC, then $\angle$C $>$ $\angle$B.

**Solution:** A

Converse of the statement:

If in a triangle ABC, $\angle$C $>$ $\angle$B, then AB $>$ AC.

Contrapositive of the statement:

If in a triangle ABC, $\angle$C $\ngtr$ $\angle$B, then AB $\ngtr$ AC.

Q10. If a set A contains ' $m$ ' elements and B contains ' $n$ ' elements, then find the number of elements in A $\times$ B.

**Solution:** C

No of elements in set A $= m$

No of elements in set B $= n$

So, no of elements in A $\times$ B $= m \times n$

Note: A $\times$ B is not equal to B $\times$ A only in case A $=$ B they are equal.

Q11. Find the middle term in the expansion $\left(\frac{x}{a}+\frac{y}{b}\right)^6$.

**Solution:** D

Given expression,

$\left(\frac{x}{a}+\frac{y}{b}\right)^6$

No of terms will be $6+1=7$ terms

$\therefore$ middle term will be $4^{\text{th}}$ term

$t_4 = {}^6C_3\left(\frac{x}{a}\right)^3\left(\frac{y_r}{b}\right)^{6-3}$

$t_4 = 20\times\frac{x^3y^3}{a^3b^3}$ is the middle term.

Q12. Define 'Objective function.'

**Solution:**

Objective functions are necessary for solving optimization problems. A real-valued function whose value is minimised or maximised, depending on the constraints in the problem. $Z = ax + by$, where $a$ and $b$ are constraints and $x$ and $y$ are variables, is the objective function.

Q13. Simplify and obtain a numerical value $(32)^{-\frac{4}{5}}$

**Solution:**

Given: $\mathrm{E} = 32^{-\frac{4}{5}}$

The expression becomes

$$E = (2^5)^{\frac{-4}{5}} = 2^{5\times\frac{-4}{5}} = (2)^{-4} = \frac{1}{16}$$

Q14. Find the sum to infinity of the GP $5, \frac{20}{7}, \frac{80}{49}, \ldots\ldots.$

**Solution:**

Let $= 5, \frac{20}{7}, \frac{80}{49}, \ldots\ldots$

Here, $a = 5$ and $r = \frac{4}{3}$

So,

$$S = \frac{5}{1-\frac{4}{7}}\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{35}{3}$$

## SECTION – III
## GROUP - A
## (Statements and Sets, Functions, Polynomials)

Q15. Prove that $A - (B \cup C) = (A - B) \cap (A - C)$ for any three sets A, B, C.

**Solution:**

L.H.S $= A - (B \cup C)$

$= A \cap (B \cup C)' \because (A - B) = A \cap B'$

$= A \cap (B' \cap C') \because (B \cup C)' = (B' \cap C')$

$= (A \cap B') \cap (A \cap C')$

$= (A - B) \cap (A - C)$

$=$ R.H.S

Q16. Let $f, g, h$ are functions defined by $f(x) = x - 1, g(x) = x^2 - 2$ and $h(x) = x^3 - 3$, show that $(f \circ g) \circ h = f \circ (g \circ h). x^6$

**Solution:**

$f \circ g = f[g(x)]$

$= x^2 - 2 - 1$

$= x^2 - 3$

$(f \circ g) \circ h = f \circ g[h(x)]$

$= f \circ g(x^3 - 3)$

$= [(x^3 - 3)^2 - 3]$

$= x^6 - 6x^3 + 9 - 3$

$= x^6 - 6x^3 + 6$ ------------ (1)

$g \circ h = g[h(x)]$

$= g(x^3 - 3)$

$= (x^3 - 3)^2 - 2$

$= x^6 - 6x^3 + 7$

$f \circ (g \circ h) = f(g[h(x)])$

$= f[x^6 - 6x^3 + 7]$

$= x^6 - 6x^3 + 7 - 1$

$= x^6 - 6x^3 + 6$ ------------ (2)

From eqn. (1) and (2),

$(f \circ g) \circ h = f \circ (g \circ h)$

Q17. If a function $f: R \rightarrow R$ is defined by $f(x) = 3x + 4$, show that $f^{-1}$ the inverse function of $f$ exists and a rule that defines $f^{-1}$.

**Solution:**

Let, $y = f(x) = 3x + 4 \Rightarrow f^{-1}(y) = x$

$\Rightarrow y = 3x + 4$

So, $y - 4 = 3x$

$$\frac{y-4}{3} = x = f^{-1}(y)$$

$$\therefore f^{-1}(x) = \frac{x-4}{3}$$

Hence this is the inverse.

$f^{-1}$ exists $\forall x \in R$

Note: Function must be injective to be inversed.

Q18. Factorise $3x^4 - 10x^3 + 5x^2 + 10x - 8$.

**Solution:**

Factorisation:

$3x^4 - 10x^3 + 5x^2 + 10x - 8$

$= 3x^4 - 3x^3 - 7x^3 + 7x^2 - 2x^2 + 2x + 8x - 8$

$= 3x^3(x-1) - 7x^2(x-1) - 2x(x-1) + 8(x-1)$

$= (x-1)(3x^3 - 7x^2 - 2x + 8)$

$= (x-1)(3x^3 + 3x^2 - 10x^2 - 10x + 8x + 8)$

$= (x-1)\{3x^2(x+1) - 10x(x+1) + 8(x+1)\}$

$= (x-1)(x+1)(3x^2 - 10x + 8)$

$= (x-1)(x+1)(3x^2 - 6x - 4x + 8)$

$= (x-1)(x+1)\{3x(x-2) - 4(x-2)\}$

$= (x-1)(x+1)(x-2)(3x-4)$

## GROUP - B

## (Linear Programming, Real Numbers, Progressions)

Q19. A shopkeeper sells not more than 30 shirts of each colour. Atleast twice as many white ones are sold as green ones. If the profit on each of the white be Rs. 20 and that of green be Rs. 25; then find out how many of each kind be sold to give him a maximum profit. (Graph need not be drawn)

**Solution:**

Let the number of white shirts be sold be $x$

Then number of green shirts sold $= 2x$

Profit on each green shirt $=$ Rs. 25

Profit on each white shirt $=$ Rs. 20

$\therefore$ Total profit $= 2x \times 25 + x \times 20 = 70x$

Shopkeeper sells no more than 30 shirts. so $x \leq 30$ and $2x \leq 30$

so $x \leq 15$

Maximum profit $= 70 \times 15 =$ Rs. 1050

Q20. If $lmn = 1$, show that $\frac{1}{1+l+m^{-1}} + \frac{1}{1+m+n^{-1}} + \frac{1}{1+n+l^{-1}} = 1$

**Solution:**

First term: $\frac{1}{1+l+m^{-1}} = \frac{1}{1+l+\frac{1}{m}} = \frac{m}{m+lm+1}$

Second term: $\frac{1}{1+m+n^{-1}} = \frac{1}{1+m+\frac{1}{n}} = \frac{1}{1+m+lm}\left(lmn = 1 \therefore \frac{1}{n} = lm\right)$

Third term:

$$\frac{1}{1+n+l^{-1}} = \frac{1}{1+n+l^{-1}} \times \frac{lm}{lm} = \frac{lm}{lm+lm\ n+l^{-1}lm} = \frac{lm}{lm+1+m}$$

L.H.S.

$$= \frac{1}{1+l+m^{-1}} + \frac{1}{1+m+n^{-1}} + \frac{1}{1+n+l^{-1}}$$

$$= \frac{m}{m+lm+1} + \frac{1}{m+lm+1} + \frac{lm}{m+lm+1}$$

$$= \frac{m+1+lm}{m+lm+1}$$

$= 1$

$=$ R.H.S

Q21. If the sum of the first ' $n$ ' natural numbers is $S_1$ and that of their squares $\mathrm{S}_2$ and cubes $\mathrm{S}_3$, then show that $9\ \mathrm{S}_2^2 = \mathrm{S}_3(1+8\ \mathrm{S}_1)$.

**Solution:**

Sum of first n natural numbers is $\frac{n(n+1)}{2}$

Sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

Sum of cubes of first n natural numbers is $\left(\frac{n(n+1)^2}{2}\right)^2$

RHS:

$= \mathrm{S}_3(1+8\ \mathrm{S}_1)$

$$= \left(\frac{n(n+1)}{2}\right)^2 \times \left(1+8\times\left(\frac{n(n+1)}{2}\right)\right)$$

$$= \left(\frac{n(n+1)}{2}\right)^2 \times (4n^2+4n+1)$$

$$= \left(\frac{n(n+1)}{2}\right)^2 \times (2n+1)^2$$

$$= \left(\frac{n(n+1)(2n+1)}{2}\right)^2$$

$= 9 \times \left(\frac{n(n+1)(2n+1)}{6}\right)^2$

$= 9\ S_2^2$

Hence proved

Q22. Find the sum to $n$ terms $0.7 + 0.77 + 0.777 + \ .........$

**Solution:**

We know that,

$$0.1 + 0.1^2 + 0.1^3 + \cdots = \frac{0.1 \times (1 - 0.1^n)}{0.9} = \frac{(1 - 0.1^n)}{9}$$

$$= 7 \times (0.1 + 0.11 + 0.111 + \cdots)$$

$$= 7 \times \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots\right)$$

$$= \frac{7}{9} \times \left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ..\right)$$

$$= \frac{7}{9} \times \left(\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + ..\right)$$

$$= \frac{7}{9} \times \left((1 + 1 + \cdots n \text{ terms }) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots n \text{ terms}\right)\right)$$

$$= \frac{7}{9} \times \left(n - \frac{(1 - 0.1^n)}{9}\right)$$

## SECTION - IV

## (Linear Programming, Quadratic Equations and Inequations)

Q23. Using graph $y = x^2$, solve the equation $x^2 - x - 2 = 0$.

**Solution:**

The roots of the equation $x^2 - x - 2 = 0$ are the $x$-coordinates of the points of the intersection of the parabola $y^2 = x$ and the straight line $y = x + 2$.

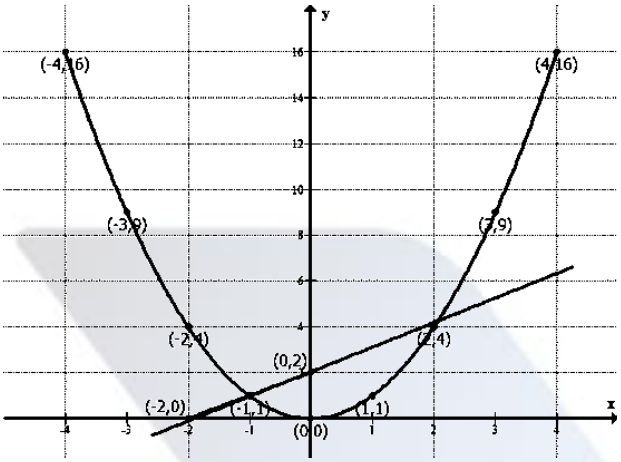
Consider $y = x^2$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 2 | 3 | 4 | -1 | -2 | -3 | -4 |
| $y$ | 0 | 4 | 9 | 16 | 1 | 4 | 9 | 16 |

Consider $\mathrm{y} = \mathrm{x} + 2$

| | | |
|---|---|---|
| $x$ | 0 | -2 |
| $y$ | 2 | 0 |

Hence, the solution set is $\{-1, 2\}$.




Q24. Maximize $f = 4x - y$, subject to the constraints

$7x + 4y \leq 28,\ 2y \leq 7,\ x \geq 0,\ y \geq 0$

**Solution:**

Converting inequations into equations we get,

$7x + 4y = 28$

$2y = 7 \Rightarrow y = \frac{7}{2} = 3 \cdot 5$

$x = 0, y = 0$

Consider, $7x + 4y = 28$

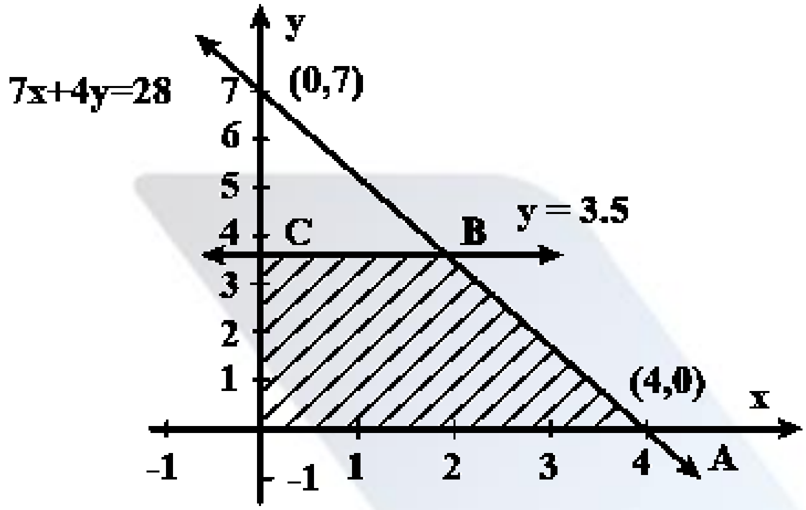
| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 7 | 0 |

The feasible region is OABC.

The values of the objective functions are given below:

| Points | Value of objective function, 4x − y |
|---|---|
| O(0,0) | $4 \times 0 - 0 = 0$ |
| A(4,0) | $4 \times 4 - 0 = 16$ |
| B(2,3 − 5) | $4 \times 2 - 3.5 = 4.5$ |
| (0,3.5) | $4 \times 0 - 3.5 = -3.5$ |

Thus, $f$ is maximum at A(4,0) i.e. $f_{max} = 16$



y
7x+4y=28
(0,7)
y = 3.5
C
B
(4,0)
x
A
7
6
5
4
3
2
1
-1
-1
1
2
3
4