Grade 10 Math Paper 1 Andhra Pradesh 2015

SECTION- I

Q1. Find L.C.M and H.C.F of 72 and 108 by the prime factorisation method.

Solution:

H.C.F of 72 and 108

2|72, 108

2| 36, 54

3| 18, 27

3| 6.9

| 2, 3

H.C.F. of 72 and 108 $= 2 \times 2 \times 3 \times 3$

$= 36$

L.C.M. of 72 and 108 $= 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 216$

So, H.C.F. of 72 and 108 is 36 and L.C.M of 72 and 108 is 216

Q2. If $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$, then find $V - B$ and $B - V$. Are they equal?

Solution:

$\mathrm{V - B = \{e, o\}}$

$B - V = \{k\}$

No, they are not equal.

Q3. Find a quadratic polynomial, the sum and product of whose zeroes are $\frac{1}{4}$, - 1 respectively.

Solution:

Let the roots be a,b.

Then the equation of polynomial is given by

$x^2 - (a + b)x + ab$

The sum of roots $= \frac{1}{4}$

$$a + b = \frac{1}{4}$$

product of roots $= -1$

$ab = -1$

Substituting above values in (1)

$\therefore$ Equation of the required polynomial is $x^2 - \left(\frac{1}{4}\right)x - 1 = 0$ or $4x^2 - x - 4 = 0$.

Q4. Find two numbers, whose sum is 27 and product is 182.

Solution:

Let the first number be x and the second number is $27 - x$.

Therefore, their product $= x(27 - x)$

It is given that the product of these numbers is 182 .

Therefore, $x(27-x)=182$
$\Rightarrow x^2-27x+182=0$
$\Rightarrow x^2-13x-14x+182=0$
$\Rightarrow x(x-13)-14(x-13)=0$
$\Rightarrow (x-13)(x-14)=0$
Either $x-13=0$ or $x-14=0$
$\Rightarrow x=13$ or $x=14$
If first number $=13$, then the other number $=27-13=14$
If first number $=14$, then the other number $=27-14=13$
Therefore, the numbers are 13 and 14.

GROUP - B

Q5. For what value of ' k ', the pair equations $3x+4y+2=0$ and $9x+12y+k=0$ represent coincident lines.

Solution:

The coincident lines are $3x+4y+2=0$ and $9x+12y+k=0$.
Comparing the equations with the general equations
$a_1x+b_1y+c_1=0$
$a_2x+b_2y+c_2=0$
Then,
$a_1=3, b_1=4, c_1=2, a_2=9, b_2=12, c_2=k$
Since, the lines are coincident, then,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$
$$\frac{3}{9}=\frac{4}{12}=\frac{2}{k}$$
$$\frac{1}{3}=\frac{1}{3}=\frac{2}{k}$$
$$\frac{1}{3}=\frac{2}{k}$$
$k=6$

Q6. In a flower bed, there are 23 rose plants in the first row, 21 in the second row, 19 in the third row, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution:

$a_1=23, a_2=21, a_3=19$ and $a_n=5=1; S_n=?$
$a_n=a+(n-1)d$
$\Rightarrow 5=23+(n-1)(-2)$
$\Rightarrow 5=23-2n+2$
$\Rightarrow 5=25-2n$
$\Rightarrow 2n=25-5$
$$\Rightarrow n=\frac{20}{2}$$
$\Rightarrow n=10$

$S_n = \left(\frac{n}{2}\right)(2a + [n-1]d)$

$S_{10} = \left(\frac{10}{2}\right)(2 \times 23 + 9 \times (-2))$

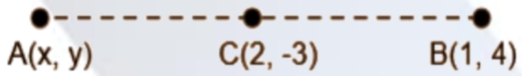
$S_{10} = 5 \times 28$

$= 140$

So, there are 140 rose plants in the flower bed.

Q7. Find the coordinates of a point A, where AB is the diameter of a circle, whose centre is $(2, -3)$ and B is $(1,4)$.

Solution:



$2 = \frac{(x+1)}{2}$

$4 - x = 1$

$x = 3$

$-3 = \frac{(y+4)}{2}$

$-6 - y = 4$

$y = -10$

The coordinates of A are $(3, -10)$.

Q8. Find the area of the triangle, whose vertices are $(-5, -1)$, $(3, -5)$, $(5, 2)$.

Solution:

Area $= \left(\frac{1}{2}\right)[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \left(\frac{1}{2}\right)[-5(-5-2) + 3(2+1) + 5(-1+5)]$

$= \left(\frac{1}{2}\right)[35 + 9 + 20]$

$= \left(\frac{1}{2}\right) \times 64$

$= 32\ \text{cm}^2$

SECTION - II

Q9. Expand log15.

Solution:

$\log 15 = \log(5 \times 3)$

$= \log 5 \times \log 3$

Q10. Write roster and set builder form of "The set of all-natural numbers, which divide 42".

Solution:

(i) $x = \{1,2,3,6,7,14,21,42\}$

(ii) $x = \{x : x \in \text{N} \mid x \text{ divides } 42\}$

Q11. If $p(t)=t^{3}-1$, find the values of $p(1), p(-2)$.

Solution:

$p(1)=1^{3}-1=0$

$p(-2)=(-2)^{3}-1=-8-1=-9$

Q12. Formulate a pair of linear equations in two variables " 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46".

Solution:

Let pencil $=x$

pen $=y$

$5x+7y=50 \quad \ldots . \times 5$

$25x+35y=250 \qquad \ldots .$ (i)

$7x+5y=46 \quad \ldots . \times 7$

$49x+35y=322 \qquad \ldots .$(ii)

From (i) and (ii)

$24x=72$

$x=\frac{42}{24}$

$x=3$

Pencil $=3$

$y=\frac{(46-7x)}{5}$

$=\frac{(46-7\times 3)}{5}$

$=\frac{25}{5}$

$=5$

$y=5$

Pen $=5$

Q13. If $\mathrm{b}^{2}-4ac \geq 0$, then write the roots of a quadratic equation $ax^{2}+bx+c=0$.

Solution:

Real, equal and distinct.

Q14. Write the formula for the sum of first ' n ' positive integers.

Solution:

$$\frac{n[n+1]}{2}$$

SECTION - III

Answer any 4 questions, choosing atleast two from each of t he following groups.

GROUP - A

Q15. Prove that $\sqrt{5}$ is irrational by the method of Contradiction.

Solution:

Say, $\sqrt{5}$ is a rational number.

$\therefore$ It can be expressed in the form $\frac{p}{q}$ where p, q are coprime integers.

$\Rightarrow \sqrt{5} = \frac{p}{q}$

$\Rightarrow 5 = \frac{p^2}{q^2}$ {Squaring both the sides}

$\Rightarrow 5q^2 = p^2$

$\Rightarrow p^2$ is a multiple of 5. {Euclid's Division Lemma}

$\Rightarrow p$ is also a multiple of 5. {Fundamental Theorem of arithmetic}

$\Rightarrow p = 5\,m$

$\Rightarrow p^2 = 25\,m^2$

From equations (1) and (2), we get,

$5q^2 = 25\,m^2$

$\Rightarrow q^2 = 5\,m^2$

$\Rightarrow q^2$ is a multiple of 5. {Euclid's Division Lemma}

$\Rightarrow q$ is a multiple of 5. {Fundamental Theorem of Arithmetic}

Hence, p, q have a common factor 5.

This contradicts that they are co-primes.

Therefore, $\frac{p}{q}$ is not a rational number.

So, $\sqrt{5}$ is an irrational number.

Q16. If A = {3,6,9,12,15,18,21}; B = {4,8,12,16,20}; C = {2,4,6,8, 10,12,14,16}; D = {5,10,15,20}; find

(i) A − B

(ii) B − A

(iii) C − A

(iv) D − A

(v) B − C

(vi) B − D

(vii) C − B

(viii) D − B

Solution:

(i) A − B = {3,6,9,15,18,21}

(ii) B − A = {4,8,16,20}

(iii) C − A = {2,4,8,10,14,16}

(iv) D − A = {5,10,20}

(v) B − C = {20}

(vi) B − D = {4,8,12,16}

(vii) C − B = {2,6,10,14}

(viii) D − B = {5,10,15}

Q17. Verify that 1, −1 and −3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relationship between zeroes and the coefficient.

Solution:

To verify roots, put root inside equations $x^3 + 3x^2 - x - 3$

Root $(1) = 1^3 + 3(1)^2 - 1 - 3 = 1 + 3 - 1 - 3 = 0$

Root $(-1) = -1^3 + 3(-1)^2 + 1 - 3 = -1 + 3 + 1 - 3 = 0$

Root $(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3 = -27 + 27 + 3 - 3 = 0$

Sum of root $= -\frac{b}{a} = -\frac{3}{1} = -3$

From roots $= (1 - 1 - 3) = -3$

Product of roots $= -\frac{d}{a} = \frac{3}{1} = 3$

From roots $= 1 \times -1 \times -3 = 3$

Product of roots $= pq + qr + rp = \frac{c}{a} = -\frac{1}{1} = -1$

From roots $= (1 \times -1) + (-1 \times -3) + (-3 \times 1) = -1 + 3 - 3 = -1$

Hence verified.

Q18. Find the root of equation $2x^2 - x - 4 = 0$ by the method of completing the square.

Solution:

$2x^2 - 1x - 4 = 0$

$a \neq 1$ so divide through by 2

$\left(\frac{2}{2}\right)x^2 - \left(\frac{1}{2}\right)x - \left(\frac{4}{2}\right) = \frac{0}{2}$

$x^2 - \left(\frac{1}{2}\right)x - 2 = 0$

Keep x terms on the left and move the constant to the right side by adding it on both sides

$x^2 - \left(\frac{1}{2}\right)x = 2$

Take half of the x term and square it

$\left\{\left(-\frac{1}{2}\right) \times \left(\frac{1}{2}\right)\right\}^2 = \frac{1}{16}$

Add the result to both sides

$x^2 - \left(\frac{1}{2}\right)x + \left(\frac{1}{16}\right) = 2 + \left(\frac{1}{16}\right)$

Rewrite the perfect square on the left

$\left[x - \left(\frac{1}{4}\right)\right]^2 = 2 + \left(\frac{1}{16}\right)$

Combine terms on the right

$\left[x - \left(\frac{1}{4}\right)\right]^2 = \frac{33}{16}$

Take the square root of both sides

$x - \left(\frac{1}{4}\right) = \pm\sqrt{\frac{33}{16}}$

Simplify the Radical term (0):

$x - \left(\frac{1}{4}\right) = \pm\frac{\sqrt{33}}{4}$

Isolate the x on the left side and solve for x (1)

$x = \left(\frac{1}{4}\right) \pm \frac{\sqrt{33}}{4}$

$$x = \left(\frac{1}{4}\right) + \frac{\sqrt{33}}{4}$$
$$x = \left(\frac{1}{4}\right) - \frac{\sqrt{33}}{4}$$

GROUP - B

Q19. Solve the equations $\frac{5}{x-1} + \frac{1}{y-2} = 2$ and $\frac{6}{x-1} - \frac{3}{y-2} = 1$.

Solution:

$\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

Let assume, $\frac{1}{x-1} = u, \quad \frac{1}{y-2} = v$

$5u + 1v = 2 \quad \ldots\ldots \times 3$

$15u + 3v = 6 \quad \ldots\ldots (i)$

$6u - 3v = 1 \quad \ldots\ldots (ii)$

Add equations (i) and (ii)

$21u = 7$

$u = \left(\frac{1}{3}\right)$

$x - 1 = 3$

$x = 4$

$v = \frac{1}{3}$

$y - 2 = 3$

$y = 5$

Q20. A fraction becomes $\frac{4}{5}$ if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction?

Solution:

Let the fraction be $\frac{p}{q}$.

The fraction will become $\frac{4}{5}$ if 1 is added to numerator and denominator

So,

$$\frac{4}{5} = \frac{p+1}{q+1}$$

$4q + 4 = 5p + 5$

$$\frac{p-5}{q-5} = \frac{1}{2}$$

$2p - 10 = q - 5$

$q = 2p - 5$

$4(2p - 5) + 4 = 5p + 5$

$8p - 20 + 4 = 5p + 5$

$3p = 21$
$p = 7$
Therefore $q = 2 \times 7 - 5$
$q = 9$
So the fraction is $\frac{7}{9}$.

Q21. The sum of the 4^{th} and 8^{th} terms of an A.P is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the A.P.

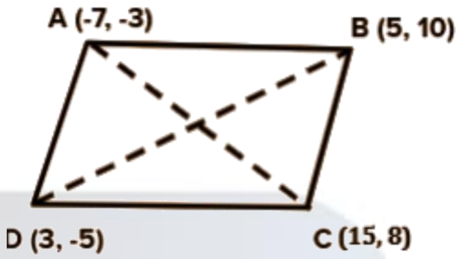
Solution:

Let the first term of an AP $= a$ and the common difference of the given AP $= d$.

$a_n = a + (n - l)d$
$a_4 = a + (4 - 1)d$
$a_4 = a + 3d$
Similarly,
$a_8 = a + 7d$
$a_6 = a + 5d$
$a_{10} = a + 9d$
The sum of 4^{th} and 8^{th} term $= 24$
$a_4 + a_8 = 24$
$a + 3d + a + 7d = 24$
$2a + 10d = 24$
$a + 5\,d = 12.$ ....(i)
The sum of 6^{th} and $10t^{\text{h}}$ term $= 44$
$a_6 + a_{10} = 44$
$a + 5d + a + 9d = 44$
$2a + 14d = 44$
$a \; + 7d = 22$ ....(ii)
Solving (i) and (ii),
$a + 7d = 22$
$a + 5d = 12$
$2d = 10$
$d = \frac{10}{2}$
$d \; = 5$
From equation (i), we get,
$a + 5d = 12$
$a + 5(5) = 12$
$a + 25 = 12$
$a = -13$
$a_2 = a + d = -13 + 5 = -8$
$a_3 = a_2 + d = -8 + 5 = -3$

Q22. Prove that the points $(-7,-3)$, $(5,10)$, $(15,8)$ and $(3,-5)$ taken in order are the vertices of a parallelogram.

Solution:



Let say ABCD be the points of the parallelogram.

To prove: Correct order of vertices, diagonals bisect each other in the parallelogram.

Hence point O must be some from A and C.

$C = \frac{(-7+25)}{2}, \frac{(-3+8)}{2}$

$= \left[4, \left(\frac{5}{2}\right)\right]$

From B and D

$C = \frac{(-5+3)}{2}, \frac{(10-5)}{2}$

$= \left[4, \left(\frac{5}{2}\right)\right]$

Hence, both are correct.

SECTION - IV

Answer any one question from the following.

Q23. Draw the graph of $y = x^2 - x - 6$ and find zeroes. Justify the answer.

Solution:

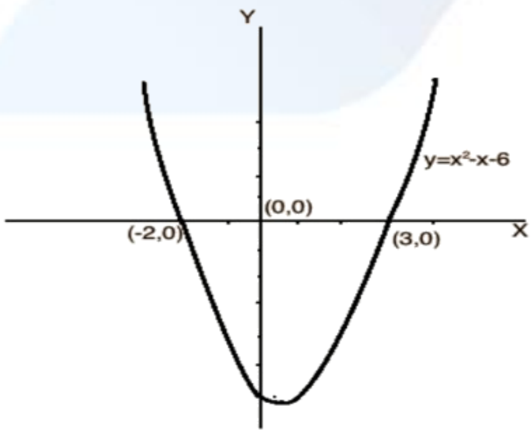
Clearly the graph of $y = x^2 - x - 6$ cut the $x-$ axis at $x = -2,3$.

$\Rightarrow x = -2, 3$ are zeros of $y = x^2 - x - 6$.

Justification:

when $x = -2; y = 4 + 2 - 6 = 0$

when $x = 3; y = 9 - 3 - 6 = 0$

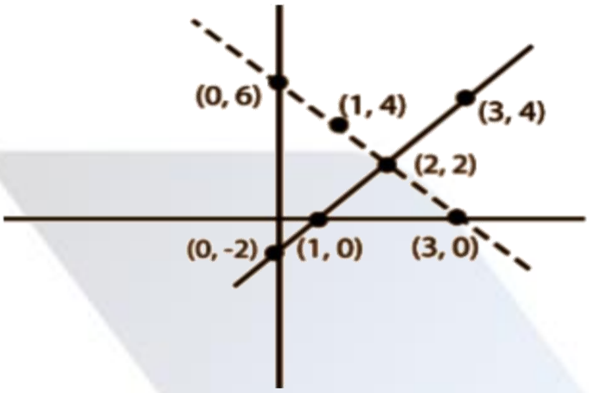


Q24. Solve the pair linear equations graphically.

$2x + y - 6 = 0$

$4x - 2y - 4 = 0$

Solution:



Hence, the graph solution is (2,2).