Grade 10 Maths Andhra Pradesh 2017

QUESTION PAPER CODE 15E(A)

SECTION – I

Q1. Find the value of $\log_2 512$.

Solution:

512 is 2^9.

$\Rightarrow \log_2 (512) = \log_2(2^9)$

By the power Rule, bring the 9 to the front of the log.

$= 9\log_2(2)$

The logarithm of a to the base a is always 1 .

$\log_2(2) = 1$

$= 9(1) = 9$

Q2. Write $A = \{1,4,9,16,25\}$ in set builder form.

Solution:

$A = \{x: x$ is the square of a natural number $\}$

$A = \{x: x = n^2$, where $n \in N\}$

Q3. Two angles are complementary, and one angle is 18° more than the other, then find the angles.

Solution:

Let one angle be x and another angle be $x + 18$.

Since the angles are complementary,

$\angle A + \angle B = 90^\circ$

$x + x + 18^\circ = 90^\circ$

$2x = 90^\circ - 18^\circ$

$2x = 72^\circ$

$x = 36^\circ$

The two angles are

$\angle A = 36^\circ$

$\angle B = 36 + 18 = 54^\circ$

Q4. Find the total surface area of a hemisphere of radius 7 cm .

Solution:

Total surface area of a hemisphere $= 3\pi r^2$

Radius $= 7$ cm

$= 3 \times \left(\frac{22}{7}\right) \times 7 \times 7$

$= 3 \times 22 \times 7$
$= 21 \times 22$
$= 462 \text{ cm}^2$

SECTION - II

Q5. Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between zeroes and coefficients.

Solution:

$P(x) = x^2 - 2x - 8$
$\Rightarrow x^2 - 4x + 2x - 8$
$\Rightarrow x(x - 4) + 2(x - 4)$
$\Rightarrow (x - 4)(x + 2) = 0$
$\Rightarrow (x - 4) = 0 \text{ or } (x + 2) = 0$
$\Rightarrow x = 4 \text{ or } x = -2$

4 and -2 are the two zeroes of the polynomial $x^2 - 2x - 8$.

The relation between the zeroes and coefficients:

Sum of zeroes $= \alpha + \beta$
$= 4 + (-2)$
$= 4 - 2$
$= \frac{2}{1}$
$= \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \alpha \times \beta$
$= 4 \times -2$
$= -\frac{8}{1}$
$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Q6. Which term of AP 21,18,15, ... is -81 ?

Solution:

Let $a_n = -81$
$d = 18 - 21 = -3$
$a = 21$

The general formula of AP is $a_n = a + (n - 1)d$
$-81 = 21 + (n - 1) - 3$
$-81 - 21 = (n - 1) - 3$
$-\frac{102}{-3} = n - 1$
$34 = n - 1$
$n = 35$

Hence -81 is 35^{th} term of an AP.

Q7. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What will be its slant height?

Solution:

Given that curved surface area of the cone is 4070 cm^2

Diameter of cone $= 70$ cm

Radius of cone $= \frac{d}{2} = \frac{70}{2} = 35$ cm

Curved surface area of cone $= \pi r l$

$\pi r l = 4070$

$\left(\frac{22}{7}\right) \times 35 \times l = 4070$

$22 \times 5 \times l = 4070$

$110 l = 4070$

$l = \frac{4070}{110}$

$l = 37$ cm

Q8. Find the discriminant of $2x^2 - 4x + 3 = 0$ and discuss the nature of its roots.

Solution:

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = 2$

$b = -4$

$c = 3$

Therefore, the discriminant is

$D = b^2 - 4ac$

$= (-4)^2 - (4 \times 2 \times 3)$

$= 16 - 24$

$= -8 < 0$

Since, $D < 0$, the equation has no real roots.

Q9. Express as algebraic expressions of the following:

(a) Five times of a number, when increased by 10 gives 20.

(b) The digits in ones and tens place of a two-digit number are x and y, then find the number.

Solution:

(a) $5x + 10 = 20$

(b) Since, a number is written as the sum of all the place value of all digits in the number,

So, a two-digit number can be written as,
$10 \times$ (tens place digit) + ones place digit,
Here, ones place digit $= x$ and the tens place digit $= y$,
Thus, the required two-digit number is,
$10 \times y + x = 10y + x$

Q10. Solve the following pair of equations by reducing them to a pair of linear equations.
$\frac{5}{x-1} + \frac{1}{y-2} = 2, \quad \frac{6}{x-1} - \frac{3}{y-2} = 1.$

Solution:
$\frac{5}{x-1} + \frac{1}{y-2} = 2$ ....(i)
$\frac{6}{x-1} - \frac{3}{y-2} = 1$ ....(ii)
Let $\frac{1}{x-1} = u, \ \frac{1}{y-2} = v$
The equations become,
$5u + v = 2$ ....(iii)
$6u - 3v = 1$ ....(iv)
From (iii), $5u + v = 2$
$v = 2 - 5u$
Putting the value of v in (4),
$6u - 3v = 1$
$6u - 3 \times (2 - 5u) = 1$
$6u + 15u = 7$
$21u = 7$
$u = \frac{1}{3}$
On putting $u = \left(\frac{1}{3}\right)$ in (3),
$5 \times \left(\frac{1}{3}\right) + v = 2$
$v = \frac{1}{3}$
$\frac{1}{x-1} = u = \frac{1}{3}$
$\Rightarrow x - 1 = 3$
$\Rightarrow x = 3 + 1 = 4$
$\frac{1}{y-2} = v = \frac{1}{3}$
$\Rightarrow y - 2 = 3$
$\Rightarrow y = 3 + 2 = 5$

OR

A well of diameter 14 m is dug 15 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 7 m to form an embankment. Find the height of the embankment.

Solution:

Inner Diameter of the well = 14 m

Inner Radius of the well $(r) = \frac{14}{2}$ m $= 7$ m

Height of the well (h) = 15 m

The volume of the earth taken out of the well $= \pi r^2 h$

$= \left(\frac{22}{7}\right) \times (7)^2 \times 15$

$= 22 \times 7 \times 15$

$= 2310 \text{ m}^3$

Width $= 7$ m

Outer radius of the embankment R $=$ inner radius + width

Outer radius (R) $= 7 + 7 = 14$ m

The embankment is in the form of cylindrical shell, so the area of embankment

Area of embankment = outer area - inner area

$= \pi \mathrm{R}^2 - \pi \mathrm{r}^2 = \pi(\mathrm{R}^2 - \mathrm{r}^2)$

$= \left(\frac{22}{7}\right)(14^2 - 7^2)$

$= \frac{22}{7}(196-49)$

$= \frac{22}{7} \times 147$

$= 22 \times 21$

$= 462 \text{ m}^2$

The volume of embankment $=$ volume of earth taken out on digging the well

Area of embankment $\times$ height of embankment $=$ volume of earth dugout

Height of embankment $= \frac{volume\ of\ earth\ dug\ out}{area\ of\ the\ embankment}$

Height of the embankment $= \frac{2310}{462}$

Height of embankment $= 5$ m

Hence, the height of the embankment so formed is 5 m.

Q11. Show that the cube of any positive integer is of the form $9m$ or $9m + 1$ or $9m + 8$, where m is an integer.

Solution:

Let a be any positive integer and $b = 3$
$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$
$\therefore r = 0,1,2$
Therefore, every number can be represented as these three forms.
There are three cases.
Case 1: When $a = 3q$,
Where m is an integer such that $m = 3q$
Case 2: When $a = 3q + 1$,
$a = (3q + 1)^3$
$a = 27q^3 + 27q^2 + 9q + 1$
$a = 9(3q^3 + 3q^2 + q) + 1$
$a = 9m + 1$ (where $m = 3q^3 + 3q^2 + q$).
Case 3: When $a = 3q + 2$,
$a = (3q + 2)^3$
$a = 27q^3 + 54q^2 + 36q + 8$
$a = 9(3q^3 + 6q^2 + 4q) + 8$
$a = 9m + 8$
Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$
Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

OR

If $A = \{3,6,9,12,15,18,21\}$, $B = \{4,8,12,16,20\}$; then check whether $A \cup B = B \cup A$ and $A - B = B - A$.

Solution:

$A \cup B = \{\mathbf{3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21}\}$
$B \cup A = \{\mathbf{3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21}\}$
B∪A = A∪B is the same.
$A - \boldsymbol{B} = \{\mathbf{3, 6, 9, 15, 18, 21}\}$
$B - A = \{\mathbf{4, 8, 16, 20}\}$
A − B is not the same as B − A.

Q12. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the 7^{th} year. Assuming that the production increases uniformly by a fixed number every year, find
i The production in the 1^{st} year.
ii Total production in the 10^{th} year.
iii Total production in the first seven years.

Solution:

Let the number of sets produced in 1st year be 'a' and 'd' be the increase in the production every year.
$a+2d=600----(1)$
$a+6d=700$----- (2)
Subtracting equation (1) from (2),
$4d=100$ or $d=25$
Substituting d = 25 in equation (1),
a = 550
i Production in the first year $=a=550$
ii Production in 10^{th} year $=a+9d=550+9\times25=775$
iii Total production in first 7 years $=a+(a+d)+(a+2d)+\cdots..+(a+6d)$
$=\left(\frac{7}{2}\right)(2\times550+(7-1)25)$
$=4375$

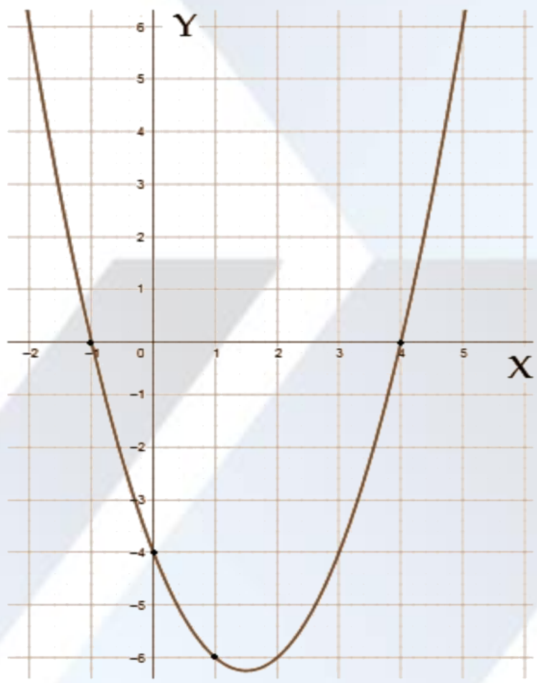
OR

There is a motorboat, whose speed in still water is 18 km/h. It takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
Solution:
Let the speed of the stream be x km/h.
Speed upstream $=(18-x)$ km/h
Speed downstream $=(18+x)$km/h
Time taken upstream: $\frac{24}{18-x}$
Time taken downstream: $\frac{24}{18+x}$
Given:
$\frac{24}{18-x}-\frac{24}{18+x}=1$
$48x=324-x^2$
$x^2+48x-324=0$
$x^2+54x-6x-324=0$
$(x+54)(x-6)=0$
$x=-54, \quad x=6$
Thus , $x=6$, because x can't be negative
speed of the stream = 6 km/h.

Q13. Solve the quadratic polynomial x^2-3x-4 by graphical method.
Solution:

If $x = 0,$
$\Rightarrow y = 0^2 - 3(0) - 4 = -4$
If $x = 1$
$\Rightarrow y = 1^2 - 3(1) - 4 = -4 = 1 - 3 - 4 = -6$
If $x = -1$
$\Rightarrow y = y = (-1)^2 - 3(-1) - 4 = 1 + 3 - 4 = 0$
If $x = 4$
$\Rightarrow y = (-1)^2 - 3(4) - 4 = 16 - 12 - 4 = 0$
The coordinate points of the given equation are

x	0	1	-1	4
y	-4	-6	0	0



OR

Half of the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution:

Perimeter $= 2(l + b)$
half of the perimeter $= 36\text{m} = (l + b)$
Let breadth $= x$, length $= 4x$
Perimeter $= 2 \times 36 = 72$ m
Perimeter $= 2(l + b)$
$72 = 2(4 + x + x)$

$36 = 4 + 2x$
$36 - 4 = 2x$
$32 = 2x$
$x = \frac{32}{2}$
$x = 16$
Breadth $= 16$
Length $= x + 4 = 16 + 4 = 20$

SECTION - IV

Q14. A rational number that equals to $(2.\overline{6})$ is

(A) $\frac{7}{3}$
(B) $\frac{8}{3}$
(C) $\frac{16}{7}$
(D) $\frac{17}{7}$

Solution: B

Let $\boldsymbol{x = 2.6\overline{6}}$
Multiply by 10: $\boldsymbol{10x = 26.6\overline{6}}$
Subtracting, $\boldsymbol{10x - x = 26.6\overline{6}}$ gives:
$\boldsymbol{9x = 24 \Rightarrow x = \frac{24}{9} = \frac{8}{3}}$
Thus, the rational number is $\boldsymbol{\frac{8}{3}}$.

Q15. The value of $\log_{25}5$ is

(A) $\frac{1}{2}$
(B) 2
(C) 5
(D) 25

Solution: A

We have $\log_{25} 5$.
Since $25 = 5^2$ we write:
$\log_{25} 5 = \log_{5^2} 5 = \frac{1}{2}\log_5 5$
Since $\log_5 5 = 1$, we get $\frac{1}{2}$.

Q16. If ' 4 ' is one of the zeroes of $p(x) = x^{\mathbf{2}} + kx - 8$, then the value of k is

(A) 1
(B) -1

(C) 2
(D) -2
Solution: C
Given that 4 is a zero of $p(x) = x^2 + kx - 8$, we substitute $x = 4$ into the equation:
$4^2 + k(4) - 8 = 0$
$16 + 4k - 8 = 0$
$4k + 8 = 0 \Rightarrow 4k = -8 \Rightarrow k - 2$
Thus, the value of $k = -2$.

Q17. If the pair of equations $2x + 3y + k = 0,\ 6x + 9y + 3 = 0$ having infinite solutions, the value of k is
(A) 2
(B) 3
(C) 0
(D) 1
Solution: D
For a pair of linear equations to have infinitely many solutions, their coefficients must be proportional:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Given equations:
$\mathbf{2x + 3y + k = 0}$
$\mathbf{6x + 9y + 3 = 0}$
Comparing coefficients:
$\frac{2}{6} = \frac{3}{9} = \frac{k}{3}$
$\frac{1}{3} = \frac{1}{3} = \frac{k}{3}$
So, $\mathbf{k = 3}$.

Q18. If the roots of $x^2 + 6x + 5 = 0$ are a and b, then $a + b$ is
(A) 5
(B) -6
(C) 6
(D) -1
Solution: B
For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is given by:
$a + b = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$
Given equation:
$x^2 + 6x + 5 = 0$

Here, $a = 1$ and $b = 6$, so:

$a + b = -\frac{6}{1}$

Thus, $a + b = -6$.

Q19. Which term of GP $3, 3\sqrt{3}, 9$......... equals to 243?

(A) 6

(B) 7

(C) 8

(D) 9

Solution: D

Given the GP: $3, 3\sqrt{3}, 9, \ldots,$

The first term $a = 3$ and common ratio $r = 3$.

The general formula for the nth term is $T_n = 3 \times (3)^{n-1}$.

We set $T_n = 243$, solve for n, and find that $n = 9$.

Thus, the 9th term equals 243.

Q20. If $n(A) = 12$ and $n(A \cap B) = 5$, then $n(A - B) =$

(A) 4

(B) 7

(C) 17

(D) 0

Solution: B

We are given:

$n(A) = 12$ and $n(A \cap B) = 5$.

We want to find $n(A - B)$, which represents the number of elements in AA but not in B.

Using the formula:

$n(A - B) = n(A) - n(A \cap B)$

Substitute the given values:

$n(A - B) = 12 - 5 = 7$

Thus, $n(A - B) = 7$.

Q21. If $x, x + 2, x + 6$ are three consecutive terms in GP, find the value of x.

(A) 3

(B) 4

(C) 2

(D) 1

Solution: C

Given that $x, x + 2$, and $x + 6$ are three consecutive terms in a geometric progression (GP), the ratio of consecutive terms must be constant:

$\frac{x+2}{x} = \frac{x+6}{x+2}$

$(x + 2)^2 = x(x + 6)$

$x^2 + 4x + 4 = x^2 + 6x$

$4 = 2x \Rightarrow x = 2$

Thus, the value of x is 2.

Q22. A quadratic equation, whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3} =$

(A) $x^2 - x - 4 = 0$

(B) $x^2 - 4x + 1 = 0$

(C) $x^2 + 4x + 3 = 0$

(D) $x^2 + x - 3 = 0$

Solution: B

For a quadratic equation with roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$, the sum and product of the roots can be used to find the equation.

Sum of the roots:

$(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

Product of the roots:

$(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Using the quadratic equation formula $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$, the equation is:

$x^2 - 4x + 1 = 0$

Thus, the quadratic equation is $x^2 - 4x + 1 = 0$.

Q23. If $\mathrm{a_n} = \frac{\mathrm{n(n+3)}}{\mathrm{n+2}}$, then find $\mathrm{a_{17}}$.

(A) $\frac{340}{20}$

(B) $\frac{341}{19}$

(C) $\frac{340}{19}$

(D) $\frac{341}{20}$

Solution: C

Given $a_n = \frac{n(n+3)}{n+2}(n + 2)$, to find a_{17}, substitute $n = 17$

$a_{17} = \frac{17(17+3)}{17+2} = \frac{17\times20}{19} = \frac{340}{19}$.

Q24. The curved surface area of a sphere will be whose radius is 10 cm.

(A) 239π

(B) 400π

(C) 221π

(D) 129π

Solution: B

The formula for the curved surface area of a sphere is:

$A = 4\pi r^2$

Given the radius $r = 10$ cm, we substitute:

$A = 4\pi(10)^2 = 4\pi \times 100 = 400\pi\ cm^2$.

Q25. The volume of a cube will be if the total surface area is 216 cm^2.

(A) 216

(B) 196

(C) 212

(D) 144

Solution: A

The total surface area of a cube is given by:

$A = 6a^2$

where a is the side length. Given the total surface area is 216 cm^2:

$6a^2 = 216 \Rightarrow a^2 = 36 \Rightarrow a = 6\, cm$

The volume of the cube is:

$V = a^3 = 6^3 = 216\, cm^3$

Thus, the volume of the cube is 216 cm^3.

Q26. A famous book was written by ancient mathematician Aryabhata is

(A) Arya tharkam

(B) Aryabhattiyam

(C) Siddhanta Shiromani

(D) Karana Kuthuhalam

Solution: B

The famous book written by the ancient mathematician Aryabhata is "Aryabhatiyam". It covers topics in mathematics, astronomy, and trigonometry.

Q27. The degree of the polynomial $\sqrt{2}x^2 - 3x + 1 =$

(A) $\sqrt{2}$

(B) 3

(C) 1

(D) 2

Solution: D

The degree of a polynomial is the highest power of x in the polynomial.

For the polynomial $\sqrt{2}x^2 - 3x + 1$, the highest power of x is x^2.

Thus, the degree of the polynomial is 2.

Q28. Which of the following equations has the solution of (2, -3) ?

(A) $2x - 3y = 10$

(B) $2x + 3y = 13$

(C) $2x - 3y = 13$

(D) $2x + 3y = -13$

Solution: C

Substitute $x = 2$ and $y = -3$ into each equation:

(A) $2(2) - 3(-3) = 4 + 9 = 13$ (False)

(B) $2(2) + 3(-3) = 4 - 9 = -5$ (False)

(C) $2(2) - 3(-3) = 4 + 9 = 13$ (True)

(D) $2(2) + 3(-3) = 4 - 9 = -5$ (False)

Thus, the correct answers are (C).

Q29. If $\mathrm{A} = \{x: x$ is the letter of the word HEADMASTER, then its roster form is

(A) $\mathrm{A} = \{\mathrm{H, E, A, D, M, A, S, T, E, R}\}$

(B) $\mathrm{A} = \{\mathrm{H, E, A, D, M, S, T, R}\}$

(C) $\mathrm{A} = \{\mathrm{H, E, A, D, M, S, T, E, R}\}$

(D) $\mathrm{A} = \{\mathrm{H, E, A, D, M, A, S, T, R}\}$

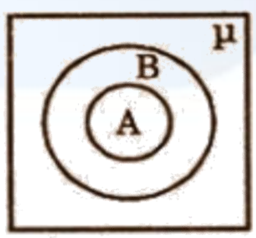
Solution: B

The word "HEADMASTER" has the distinct letters H, E, A, D, M, S, T, R.

Thus, the roster form of the set A is:

$\mathrm{A} = \{\mathrm{H, E, A, D, M, S, T, R}\}$

Q30. The following Venn diagram indicates



(A) $\mathrm{A} \subset \mathrm{B}$

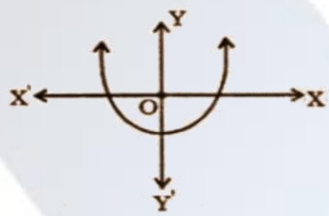
(B) $B \subset A$

(C) A, B are disjoint sets

(D) $\mu \subset B$

Solution: B

Q31.



The above diagram shows

(A) $\mathbf{b}^2 - 4ac > 0$

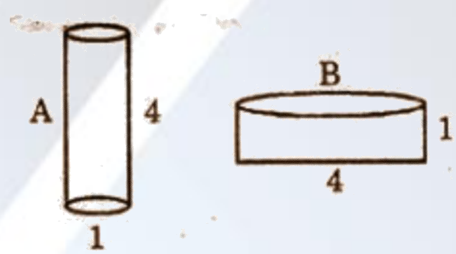
(B) $\mathbf{b}^2 - 4\mathbf{ac} = \mathbf{0}$

(C) $b^2 - 4ac < 0$

(D) None of the above

Solution: A

Q32. Which of the following vessels can be filled with more water (A, B are in cylindrical shape)?



(A) A

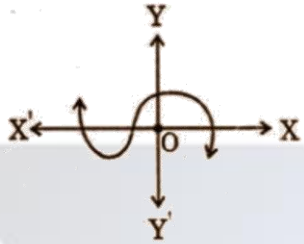
(B) B

(C) Both

(D) cannot be determined

Solution: B

Q33.



The number of zeroes can be identified by the adjacent figure.

(A) 0

(B) 1

(C) 2

(D) 3

Solution: D