Grade 10 Andhra Pradesh Mathematics 2018

SECTION – I

Q1. Find the HCF of 60 and 100 by using Euclid's division lemma.

Solution:

Using Euclid's division lemma:

100 = (60 × 1) + 40

60 = (40 × 1) + 20

40 = (20 × 2) + 0

The remainder is 0.

The HCF of 60 and 100 is 20.

Q2. Write A = {3, 9, 27, 81} in set builder form.

Solution:

A = {3, 9, 27, 81}

Set builder form

A = {x: $x = 3n, n$ belongs to N & n < 5}

Q3. Find the value of k for which the pair of equations $2x + ky + 3 = 0$, $4x + 6y - 5 = 0$ represent parallel lines.

Solution:

On comparing the given equations,

$2x - ky + 3 = 0, 4x + 6y - 5 = 0$ with $a_1x + b_1y + c_1 = 0$ and

$a_2x + b_2y + c_2 = 0$

$a_1 = 2, b_1 = -k, c_1 = 3;$

$a_2 = 4, b_2 = 6, c_2 = -5;$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{2}{4} = \frac{-k}{6}$$

$$\frac{2 \times -6}{4} = k$$

$$k = -3 \text{ or } \frac{18}{5}$$

Q4. Find the volume of the right circular cone with radius 3 cm and height 14 cm.

Solution:

Given the radius of the cone is 3 cm and height is 14 cm.

The volume of the cone = $\left(\frac{1}{3}\right)\pi r^2 h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 3 \times 3 \times 14$

$= 22 \times 3 \times 2$

$= 132 \text{ cm}^3$

SECTION – II

Q5. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and coefficients.

Solution:

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The zeroes of the given polynomial are $\sqrt{3}$ and $-\sqrt{3}$.

Relationship between the zeroes and coefficients

Sum of zeroes = $\sqrt{3} + (-\sqrt{3}) = \sqrt{3} - \sqrt{3} = 0 = \frac{\text{coefficient of } -x}{\text{coefficient of } x^2}$

Product of zeroes = $(\sqrt{3})(-\sqrt{3}) = -(\sqrt{3})^2 = -\frac{3}{1} = \frac{\text{constant}}{\text{coefficient of } x^2}$

Q6. How many three-digit numbers are divisible by 3?

Solution:

The three-digit natural numbers start from 100 and end with 999.

The first three-digit number which is divisible by 3 is 102.

The last three-digit number which is divisible by 3 is 999.

The number of three-digit numbers divisible by 3

$= \frac{999}{3} - \left(\frac{102}{3}\right) + 1$

$= 333 - 34 + 1$

$= 300$

Q7. A solid iron rod has a cylindrical shape. Its height is 11 cm and base diameter is 7 cm. Then find the total volume of 50 rods.

Solution:

The height of the cylinder is 11 cm.

The diameter of the cylinder is 7 cm.

The volume of cylinder $= \pi r^2 h$

$= \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right) \times 11$

$= (121) \times \left(\frac{7}{2}\right)$

$= 423.5$

The volume of 50 such rods

$= 50 \times 423.5$

$= 21175 \text{ cm}^2$

Q8. Find the roots of $x + \left(\frac{6}{x}\right) = 7, x \neq 0$.

Solution:

$x + \left(\frac{6}{x}\right) = 7$

$x^2 - 7x + 6 = 0$

$x^2 - 6x - x + 6 = 0$

$x(x - 6) - 1(x - 6) = 0$

$x = 1, 6$

Q9. Length of a rectangle is 2 units greater than its breadth. If the area of the rectangle is 120 square units, then find its length.

Solution:

Breadth = x

Length = $x + 2$

Area = 120 square units

$x(x + 2) = 120$

$x^2 + 2x = 120$

$x^2 + 2x - 120 = 0$

$x^2 + 12x - 10x - 120 = 0$

$x(x + 12) - 10(x + 12) = 0$

$(x + 12)(x - 10) = 0$

$x = -12, 10$

The value of x can't be negative as it is the breadth of a rectangle.

$x = 10$

Breadth = 10 units

Length = $x + 2 = 10 + 2 = 12$ units

SECTION – III

Q10. [a] Hari went to a bank to withdraw Rs. 2000. He asked the cashier to give the cash in Rs. 50 and Rs. 100 separately. He got 25 notes in all. Can you tell how many notes, each of Rs. 50 and Rs. He received 100?

Solution:

[a] Let the number of Rs. 50 notes are 'x'.

Let the number of Rs. 100 notes are 'y'.

Total notes = 25

$\Rightarrow x + y = 25$

$\Rightarrow x = 25 - y$

Total amount only in Rs. 50 notes = $50x$

Total amount only in Rs. 100 notes = $100y$

Total amount = 2000

$\Rightarrow 50x + 100y = 2000$

$\Rightarrow 50(25 - y) + 100y = 2000$

$\Rightarrow 50 \times 25 - 50y + 100y = 2000$

$\Rightarrow 1250 + 50y = 2000$

$\Rightarrow 50y = 2000 - 1250 = 750$

$\Rightarrow y = \frac{750}{50} = 15$

$x = 25 - 15 = 10$

The number of Rs. 50 notes = 10

The number of Rs. 100 notes = 15

OR

[b] How many spherical balls can be made from a solid cube of lead, whose edges measure 66 cm and each ball being 3 cm in radius?

Solution:

Radius of each ball = 3 cm

Length of the edge of cube = 66 cm

The volume of each ball is given by $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (3^3)$

$= 113.14 \text{ cm}^3$

The volume of a cube is given by $a^3 = 66^3 = 287496 \text{ cm}^3$

So, the number of spherical balls is given by

$= \frac{\text{volume of the cube}}{\text{volume of the ball}}$

$= \frac{287496}{113.14}$

$= 2541$

Hence, there are 2541 spherical balls that can be made.

Q11. [a] Show that $\sqrt{3}$ is irrational.

Solution:

[a] Assume that $\sqrt{3}$ is rational.

Then, there exist positive integers a and b such that $\sqrt{3} = \frac{a}{b}$, where a and b are coprime, their HCF is 1.

$\sqrt{3} = \frac{a}{b}$

$\Rightarrow 3 = \frac{a^2}{b^2}$

$\Rightarrow 3b^2 = a^2$

$\Rightarrow$ 3 divided a^2

$\Rightarrow$ 3 divides a ….. … (1)

$\Rightarrow a = 3c$ for some integer c

$\Rightarrow a^2 = 9c^2$

$\Rightarrow 3b^2 = 9c^2$

$\Rightarrow b^2 = 3c^2$

$\Rightarrow$ 3 divides b^2

$\Rightarrow$ 3 divides b.. …. (2)

From (1) and (2), it is observed that a and b have at least 3 as a common factor. But this contradicts the fact that a and b are co-prime. This means that our assumption is not correct.

Hence, $\sqrt{3}$ is an irrational number.

OR

[b] If A = {x: x is a natural number}

B = {x: x is an even number}

C = {x: x is an odd number}

D = {x: x is a prime number}

Then find A ∪ B, A ∩ C, B ∩ C and B ∩ D. What do you notice?

Solution:

A ∪ B = {all natural numbers}

A ∩ C = {all odd numbers}

B ∩ C = { }

$B \cap D = \{2\}$

Q12. [a] The sum of the reciprocals of Rehman's age, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find the present age.

Solution:

[a] Let the age of Rehman be x years. ———- (1)

Rehman's 3 years ago age will be x – 3. ——- (2)

Rehman's 5 years ago age will be x + 5. ——- (3)

Given, the sum of reciprocals of (1), (2), (3) will be $\frac{1}{3}$.

$$\left[\frac{1}{x-3}\right] + \left[\frac{1}{x+5}\right] = \frac{1}{3}$$

$$\frac{[x + 5 + x - 3]}{[(x - 3)(x + 5)]} = \frac{1}{3}$$

$$\frac{[2x + 2]}{[(x - 3)(x + 5)]} = \frac{1}{3}$$

$6x + 6 = x^2 + 2x - 15$

$x^2 + 2x - 15 - 6x - 6 = 0$

$x^2 - 4x - 21 = 0$

$x^2 - 7x + 3x - 21 = 0$

$x(x - 7) + 3(x - 7) = 0$

$(x - 7)(x + 3) = 0$

$x = 7$ or $x = -3$.

The value of x cannot be negative.

So, the present age of Rehman is 7 years.

OR

[b] If the sum of the first 7 terms and 15 terms of an AP are 98 and 390, respectively. Find the sum of the first 10 terms.

Solution:

Sum $= \frac{n}{2}(2a + [n - 1]d)$

$98 = \frac{7}{2}(2a + 6d)$

$28 = 2a + 6d$

$14 = a + 3d$ —— (1)

$390 = \frac{15}{2}(2a + [15 - 1]\, d)$

$52 = 2a + 14d$

$26 = a + 7d$ —— (2)

On solving the equations (1) and (2),

$a = 5$ and $d = 3$

Sum to 10 terms

$= \left[\frac{10}{2}\right] \times (2 \times 5 + (10 - 1) \times 3)$

$= 5\,(10 + 27)$

$= 37 \times 5$

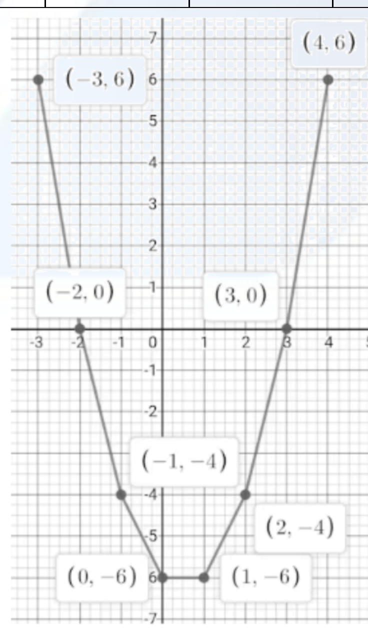
$= 185$

Q13. [a] Solve the quadratic polynomial $p(x) = x^2 - x - 6$ by graphical method.

Solution:

[a] $y = x^2 - x - 6$

x	−3	−2	−1	0	1	2	3
y	6	0	−4	−6	−6	−4	0



The given graph cuts x-axis at 3 and -2.

Therefore, 3 and -2 are the required zeroes.

.OR

[b] The perimeter of a rectangular plot is 32 m. If the length is increased by 2 m and the breadth is decreased by 1 m, then the area of the plot remains the same. Find the length and breadth of the plot.

Solution:

Let the dimensions of the plot be: length = 'l' m and breadth = 'b' m

Perimeter = 32 m (given)

$2(l + b) = 32$

$l + b = 16$ — (1)

If the length is increased by 2 m and the breadth is decreased by 1 m, the area of the plot remains the same.

$(l + 2)(b - 1) = lb$

$lb - l + 2b - 2 = lb$

$-l + 2b = 2$ — (2)

Add equations (1) and (2),

$3b = 18$

$b = \frac{18}{3}$

$b = 6$

Put $b = 6$ in equation (1),

$l + b = 16$

$l + 6 = 16$

$l = 10$

Therefore, the length of the rectangle plot $= l = 10$ m and breadth $= b = 6$ m.

SECTION – IV

Q14. Which of the following is a terminating decimal?

(A) $\frac{10}{81}$ (B) $\frac{41}{75}$ (C) $\frac{8}{125}$ (D) $\frac{3}{14}$

Solution: C

A terminating decimal is a decimal that ends. It's a decimal with a finite number of digits. Here only $\frac{8}{125} = 0.064$ is terminating.

Q15. The value of $\log_2 32$ is ____

(A) 2 (B) 32 (C) $\frac{1}{5}$ (D) 5

Solution: D

$\log_2 32 = \log_2 2^5 = 5$

Q16. If 3 is one of the zeroes of $p(x) = x^2 + kx - 9$, the the value of k is

(A) 0 (B) 1 (C) 2 (D) 3

Solution: A

$p(3) = 3^2 + k(3) - 9 = 0$ (Since 3 is a zero of the given polynomial.)

$9 + 3k - 9 = 0$

$k = 0$

Q17. The pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, are consistent, then

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(D) A and C

Solution: D

Condition for unique solution = $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ and for infinitely many solutions = $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. So, A and C are the correct conditions to have consistent solutions.

Q18. If α, β, γ are the zeroes of the cubic polynomial $ax3 + bx2 + cx + d$ and $(a \neq 0)$, then $\alpha\beta\gamma$ is

(A) $\frac{d}{a}$ (B) $-\frac{d}{a}$ (C) $-\frac{b}{a}$ (D) $\frac{c}{a}$

Solution: B

Since $\alpha\beta\gamma = -\frac{d}{a}$ (Product of zeroes of cubic polynomials.)

Q19. Which term of an AP 18, 15, 12 ... equals to 0?

(A) 4 (B) 5 (C) 6 (D) 7

Solution: D

$a = 18, d = 15 - 18 = -3, a_n = 0$

$a_n = a + (n-1)d$

$0 = 18 + (n-1)(-3)$

$\frac{-18}{-3} = n - 1$

$n = 6 + 1 = 7$

Hence, & 7th term will be 0.

Q20. If $A \subset B, n(A) = 4, n(B) = 6$, then $n(A \cup B) =$ ___

(A) 10 (B) 6 (C) 4 (D) 2

Solution: B

Since $A \subset B$, therefore $n(A \cup B) = n(B) = 6$

Q21. If $k, 2k + 1, 2k + 3$ are three consecutive terms in AP, then the value of k is

(A) 1 (B) 0 (C) 2 (D) 3

Solution: A

Since $k, 2k + 1, 2k + 3$ are three consecutive terms in AP.

$\frac{k + 2k + 3}{2} = 2k + 1$

$3k + 3 = 4k + 2$

$k = 1$

Q22. A quadratic polynomial, whose zeroes are $\sqrt{2}$ and $-\sqrt{2}$ is

(A) $x^2 - 4$ (B) $x^2 + 4$ (C) $x^2 - 2$ (D) $x^2 + 2$

Solution: C

$\alpha = \sqrt{2}, \beta = -\sqrt{2}$

Then, $\alpha + \beta = \sqrt{2} + (-\sqrt{2}) = 0$

$\alpha\beta = \sqrt{2} \times -\sqrt{2} = -2$

Sum of zeroes $= \alpha + \beta = 0$

Product of zeroes $= \alpha\beta = -2$

Then, the quadratic polynomial = $x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - (0)x + (-2) = x^2 - 2$

Q23. If $a_n = \frac{n}{n+1}$, then $a_{2017} =$

(A) $\frac{2017}{2016}$ (B) $\frac{2017}{2018}$ (C) $\frac{2017}{2019}$ (D) $\frac{2018}{2017}$

Solution: B

$a_{2017} = \frac{2017}{2017+1} = \frac{2017}{2018}$

Q24. A cylinder and cone have bases of equal radii and are of equal heights, then their volumes are in the ratio

(A) 1: 1 (B) 1: 3 (C) 3: 1 (D) 1: 9

Solution: C

Volume of the cylinder $= \pi r^2 h$

Ratio of volume of cylinder and cone= $\pi r^2 h : \frac{1}{3}\pi r^2 h$

$\Rightarrow 1 : \frac{1}{3}$

$\Rightarrow 3 : 1$

∴ Their volumes are in the ratio 3: 1.

Q25. Total surface area of a solid hemisphere of radius 7cm is ____ cm^2.

(A) 21π (B) 49π (C) 147π (D) 98π

Solution: C

Total surface area of a solid hemisphere= $3\pi r^2$

$= 3\pi \times 7 \times 7$

$= 147\pi$ cm^2

Q26. A quadratic equation $ax^2 + bx + c = 0$ has 2 distinct real roots, if

(A) $b^2 - 4ac > 0$ (B) $b^2 - 4ac < 0$ (C) $b^2 - 4ac = 0$ (D) None of these

Solution: A

When $b^2 - 4ac > 0$, then the quadratic equation will have two real and distinct roots.

Q27. The degree of the polynomial $5x^7 - 6x^5 + 7x - 4$ is

(A) 5 (B) 6 (C) 7 (D) 4

Solution: C

The degree of the polynomial $5x^7 - 6x^5 + 7x - 4$ is 7 as its highest power of variable is 7.

Q28. The n^{th} term of a progression a, ar, ar^2, is

(A) ar (B) ar^2 (C) $a + (n - 1)r$ (D) ar^{n-1}

Solution: D

The n^{th} term of a progression a, ar, ar^2, is ar^{n-1}.

Q29. Which of the following equations has the solution of $(1, -1)$?

(A) $3x - 2y = 6$ (B) $3x + 2y = 6$ (C) $3x - 2y = 5$ (D) $3x + 2y = 5$

Solution: C

Point for which LHS = RHS.

$3x - 2y = 5$: LHS $3 \times 1 - 2(-1)$

$= 5$

= RHS

Q30. If A = {x: x is a letter in the word EXAMINATION, then its roster form is

(A) A = $\{e, x, m, i, n, a, t, o, s\}$

(B) A = $\{e, x, m, i, n, a, t, o\}$

(C) A = $\{e, x, m, a, i, n, t, s\}$

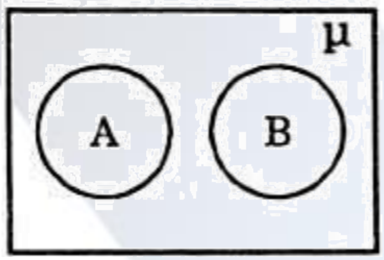
(D) A = $\{e, x, m, i, n, t, o\}$

Solution: B

Letter in the word EXAMINATION are e, x, m, i, n, a, t, o, then its roster form is

$A = \{e, x, m, i, n, a, t, o\}$.

Q31. The following Venn diagram indicates.



(A) $A \subset B$ (B) $B \subset A$ (C) A, B are disjoint sets (D) $A = B$

Solution: C

Since there is no element common to both A and B.

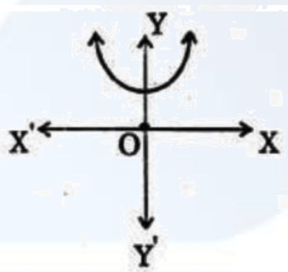
Q32. The discriminant of the quadratic equation $px^2 + qx + r = 0$ is

(A) $p^2 - 4qr$ (B) $q^2 - 4pr$ (C) $q^2 + 4pr$ (D) $r^2 - 4pq$

Solution: B

The discriminant of the quadratic equation $px^2 + qx + r = 0$ is $q^2 - 4pr$.

Q33. Number of zeroes that can be identified by the following figure.



(A) 0 (B) 1 (C) 2 (D) 3

Solution: A

The given graph is not touching the x-axis. So, it has no zero.