Grade 10 Andhra Pradesh Mathematics 2022

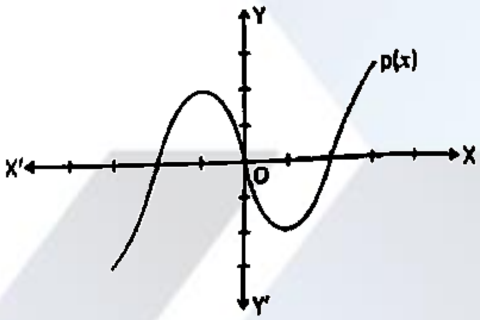
Q1. Express $\frac{7}{25}$ in decimal form.

Solution:
Decimal expansion $= 0.28$

Q2. Express the set $\mathrm{P} = \{x: x \text{ is a prime}, x < 5\}$ in roaster form.

Solution:
$\mathrm{P} = \{2, 3\}$

Q3. Find the number of zeroes of the polynomial $p(x)$, whose graph is given.



Solution:
There will be three zeroes as it touches the x-axis at three different points.

Q4. What is the common ratio of the G.P. $25, -5, 1, \frac{-1}{5}, \ldots$..?

Solution: C

$$\text{Common ratio} = -\frac{5}{25} = -\frac{1}{5}$$

Q5. Match the following :

	Equation		Solution
(a)	$x + y = 5$	(p)	$(3, 3)$
(b)	$2x - y = 9$	(q)	$(1, 4)$
(c)	$x - y = 0$	(r)	$(5, 1)$

Choose the correct answer.
(A) $a \rightarrow r,\ b \rightarrow p,\ c \rightarrow q$
(B) $a \rightarrow p,\ b \rightarrow q,\ c \rightarrow r$
(C) $a \rightarrow q,\ b \rightarrow r,\ c \rightarrow p$
(D) $a \rightarrow r,\ b \rightarrow q,\ c \rightarrow p$
Solution: C
$x + y = 1 + 4 = 5 = \text{RHS}$
$2x - y = 2(5) - 1 = 9 = \text{RHS}$
$x - y = 3 - 3 = 0 = \text{RHS}$

Q6. Assertion : $(0, 2)$ is a point on Y-axis.
Reason : Every point on Y -axis is at a distance of zero units from the Y -axis.
Now, choose the correct answer.
(A) Both Assertion and Reason are true. Reason is supporting the Assertion.
(B) Both Assertion and Reason are true, but Reason is not supporting the Assertion.
(C) Assertion is True, but Reason is False.
(D) Assertion is False, but Reason is True.

Solution: A
Every point on Y -axis is at a distance of zero units from the Y -axis.

Q7. Statement-1 : The lengths 3 cm, 4 cm, 5 cm form a right angled triangle.
Statement-II : If ' a ' is the side of an equilateral triangle, then its height is $\sqrt{3}a$.
Now, choose the correct answer.
(A) Statement-I and Statement-II both are True.
(B) Statement-I and Statement-II both are False.
(C) Statement-I is True. Statement-II is False.
(D) Statement-I is False. Statement-II is True.

Solution: C

Given, length of sides of a triangle are 3,4 and 5 respectively.
Now, $(3)^2 + (4)^2 = 9 + 16 = 25 = (5)^2$
Hence, it is a right angled triangle.

Side= a

Height= $\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}a}{2}$

Q8. The tangents drawn at the end points of a diameter are
Solution:

Parallel

Q9. Which of the following is not true ?

(A) $\sin\left(90^{\circ}-\theta\right)=\operatorname{cosec}\ \theta$

(B) $\sin^{2}\ \theta+\cos^{2}\ \theta=1$

(C) $\operatorname{cosec}\ \theta\cdot\sin\ \theta=1$

(D) $\sin\ 90^{\circ}=1$

Solution: A

$\sin(90^{\circ}-\theta)=\cos\theta$ is correct not $\sin\left(90^{\circ}-\theta\right)=\operatorname{cosec}\ \theta$

Q10. At a particular time, if the angle of elevation of the sun is 45°, then the shadow of a 5 m high tree is

(A) $5\sqrt{3}$ m

(B) 10 m

(C) 5 m

(D) $\frac{5}{\sqrt{3}}$ m

Solution: C

Let height of the tree be $\mathrm{BC}=5$ m and length of the shadow of tree be $\mathrm{AB}=x$ m.
Given that angle of elevation of sum is 45°.
i.e, $\angle\mathrm{CAB}=45^{\circ}$

$\therefore\ \tan\mathrm{A}=\frac{\mathrm{BC}}{\mathrm{AB}}$

$=\tan 45^{\circ}=\frac{5}{x}$

$=\frac{5}{x}=1(\because\ \tan 45^{\circ}=1)$

$=x=5$

Hence, the length of the shadow of 12 m high tree is 5 m.

Q11. If $P(E)=0.3$, then $P(\text{not }E)=$.

(A) 0.3

(B) $\frac{1}{3}$

(C) 0

(D) 0.7

Solution: D

$P(\text{not }E)=1-0.3=0.7$

Q12. In the classes $35-39, 40-44, 45-49, \ldots$ of a frequency distribution, the upper boundary of the class $40-44$ is

Solution:

40 is the upper limit.

SECTION - II

Q13. Evaluate: $\log_2(1+\tan^2 45^\circ)$

Solution:

$\log_2(1+\tan^2 45^\circ)=\log_2(1+1)=\log_2 2=1$

Q14. Check whether -3 and 3 are the zeroes of the polynomial x^2-9 or not.

Solution:

$(-3)^2-9=9-9=0$

And $3^2-9=9-9=0$

So, -3 and 3 are the zeroes.

Q15. "Sum of the two numbers is 82 and their difference is 38. ." Represent this information in the form of pair of linear equations with variables 'x' and 'y'.

Solution:

Let the two numbers be x and y.

Then, $x+y=82$ and $x-y=38(x>y)$.

Q16. If the slope of the line passing through the points $R(2, y)$ and $S(x, 3)$ is 2 , then find the relation between ' x ' and ' y '.

Solution:

Slope= $\frac{y_2-y_1}{x_2-x_1}$

$$2=\frac{3-y}{x-2}$$

$2x-4=3-y$

$2x+y=7$

Q17. If ABC is an isosceles triangle, right angled at C, then prove that $AB^2=2AC^2$.

Solution:

By Pythagoras theorem,

$\Rightarrow (AB)^2 = (AC)^2 + (BC)^2$

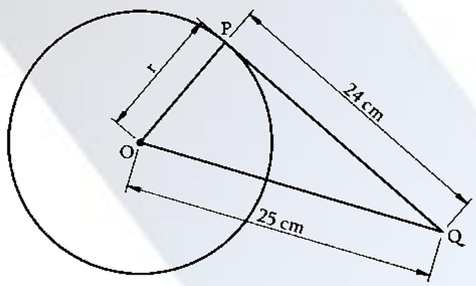
$\Rightarrow (AB)^2 = (AC)^2 + (AC)^2$

$\Rightarrow (AB)^2 = 2AC^2$

Hence, proved that $(AB)^2 = 2AC^2$.

Q18. The length of the tangent drawn from an external point R to a circle is 24 cm and the distance of R from the centre of the circle is 25 cm. Find the radius of that circle.

Solution:



A tangent at any point of a circle is perpendicular to the radius at the point of contact.

Therefore, OPQ is a right angled triangle.

By Pythagoras theorem,

$OQ^2 = OP^2 + PQ^2$

$25^2 = r^2 + 24^2$

$r^2 = 25^2 - 24^2$

$r^2 = 625 - 576$

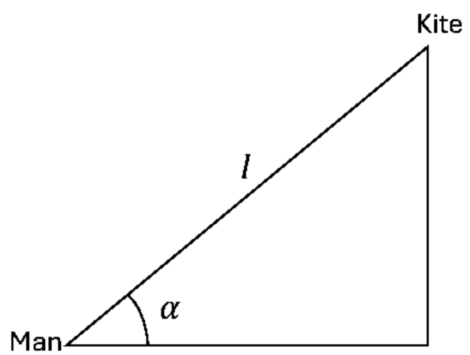
$r^2 = 49$

$r = \pm 7$

Radius cannot be a negative value, hence, $r = 7$ cm.

Q19. A person is flying a kite at an angle of elevation α and the length of the thread from his hand to kite is ' l '. Draw a rough diagram for the above situation.

Solution:



Q20. Median of the observations $\frac{x}{5}, x, \frac{x}{4}, \frac{x}{2}, \frac{x}{3}$ is 7. Find the value of x.

Solution:

Number of terms, n = 5.

Given median = 8

$\Rightarrow \left(\frac{n+1}{2}\right)^{x}$ term $= 8$

$\Rightarrow \left(\frac{5^{+1}}{2}\right)^{x}$ term $= 8$

$\Rightarrow 3^{x}$ term $= 8$

$\Rightarrow \frac{x}{4} = 8$

$\Rightarrow x = 32$

SECTION - III

Q21. Show that the following sets are equal :

(i) $A = \{x : x$ is a letter in the word 'FOLLOW' $\}$

(ii) $B = \{x : x$ is a letter in the word 'FLOW' $\}$

(iii) $C = \{x : x$ is a letter in the word 'WOLF' $\}$

Solution:

$A = \{F, L, O, W\}$

$B = \{F, L, O, W\}$

$C = \{F, L, O, W\}$

All elements are same in A, B and C. So, they are equal.

Q22. Which term of the A.P.: 3,8,13,18, ... is 78?

Solution:

First term: $a = 3$

Second term: $a + d = 8$

Common difference: $d = 8 - 3 = 5$

$a_n = a + (n - 1)d$

$a_n = 78, n = ?$

$3 + (n - 1)5 = 78$

$5(n - 1) = 78 - 3$

$n - 1 = 15$

$n = 16$

78 is the $16th$ term of the given AP

Q23. If (1,2), $(4, y)$, $(x, 6)$ and (3,5) are the vertices of a parallelogram taken in order, find ' x ' and ' y '.

Solution:

Let $\mathrm{A}(1,2), \mathrm{B}(4, y), \mathrm{C}(x, 6)$, and $\mathrm{D}(3,5)$ be the vertices of a parallelogram ABCD.

Since the diagonals of a parallelogram bisect each other. The intersection point O of diagonal AC and BD also divides these diagonals in the ratio 1:1.

Therefore, O is the mid-point of AC and BD.
According to the mid point formula,

$$\mathrm{O}(x, y)=\left[\frac{\left(x_1+x_2\right)}{2}, \frac{\left(y_1+y_2\right)}{2}\right]$$

If O is the mid-point of AC, then the coordinates of O are

$$\left[\frac{1+x}{2}, \frac{2+6}{2}\right]$$

$$\Rightarrow\left[\frac{x+1}{2}, 4\right] \ldots .(1)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left[\frac{4+3}{2}, \frac{5+y}{2}\right]$$

$$\Rightarrow\left[\frac{7}{2}, \frac{5+y}{2}\right] \ldots .(2)$$

Since both the coordinates are of the same point O, so,

$\frac{x+1}{2}=\frac{7}{2}$ and $4=\frac{5+y}{2}$ [From equation(1) and (2)]
$\Rightarrow x+1=7$ and $5+y=8$ (By cross multiplying & transposing)
$\Rightarrow x=6$ and $y=3$

Q24. A ladder 25 m long reaches a window of building 24 m above the ground. Determine the distance of the foot of the ladder from the building.

Solution:

Suppose that AB is the ladder, B is the window and CB is the building. Then, triangle ABC is a right triangle with right-angle at C.
By Pythagoras theorem, we have
$\therefore \mathrm{AB}^2=\mathrm{AC}^2+\mathrm{BC}^2$
$\Rightarrow 25^2=\mathrm{AC}^2+20^2$

$\Rightarrow AC^2 = 625 - 400 = 225$
$\Rightarrow AC = \sqrt{225}\ m = 15\ m$
Hence, the foot of the ladder is at a distance 15 m from the building.

Q25. If $\tan\ \theta = \frac{5}{12}$, then find $\sec\ \theta$ and $\operatorname{cosec}\ \theta$.

Solution:

$\tan\theta = \frac{5}{12}$

$\Rightarrow \tan^2\theta = \frac{25}{144}$

$\Rightarrow \sec^2\theta - 1 = \frac{25}{144} [\because 1 + \tan^2\ \theta = \sec^2\ \theta]$

$\Rightarrow \sec^2\theta = \frac{25}{144} + 1 = \frac{25 + 144}{144}$

$\Rightarrow \sec^2\theta = \frac{\frac{169}{144}}{169} = \frac{13}{12}$

$\Rightarrow \sec\theta = \sqrt{\frac{169}{144}} = \frac{13}{12}$

$\cos\theta = \frac{1}{\sec\theta} = \frac{12}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \frac{5}{12} = \frac{\sin\theta}{\frac{12}{13}}$

$\Rightarrow \sin\theta = \frac{5}{13}$

Q26. A tower stands vertically on the ground. From a point which is 15 meters away from the foot of the tower, the angle of elevation of the top of the tower is 45°. What is the height of the tower?

Solution:

Let, the height of the tower be H meter.
So, AB = H metre
Distance of the point
from the foot of the tower = 15 m
Hence, $CB = 15$ m
Angle of elevation = 60°
$\angle ACB = 60°$
Since tower is vertical to ground,
So, $\angle ABC = 90°$

Now,

$\tan \mathrm{C} = \dfrac{\text{Side opposite to angle C}}{\text{Side ad jacent to angle } C}$

$\tan \mathrm{C} = \dfrac{\mathrm{AB}}{\mathrm{CB}}$

$\tan 60^{\circ} = \dfrac{\mathrm{AB}}{\mathrm{CB}}$

$\sqrt{3} = \dfrac{H}{13}$

$15\sqrt{3} = H$

$H = 15\sqrt{3}$

Hence, the height of the tower $= H = 15\sqrt{3}$ m

Q27. A die is thrown once. Find the probability of getting (i) a Prime number (ii) a number lying between 1 and 5.

Solution:

Number of outcomes on throwing a die is $(1, 2, 3, 4, 5, 6) = 6$

Number of prime numbers on dice are $2, 3$ and $5 = 3$

i) Probability of getting a prime number $= \dfrac{\text{Number of prime number}}{\text{total number of outcomes}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$

ii) Total number of odd numbers are $2, 3$, and $4 = 3$

Probability of getting a odd number $= \dfrac{\text{Number of number between 1 and 5}}{\text{total number of outcomes}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$

Q28. Write the formula to find the 'mode' of a grouped data and explain the terms involved in it.

Solution:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$$

where,

- L is the lower limit of the modal class
- h is the size of the class interval
- f_1 is the frequency of the modal class
- f_0 is the frequency of the class preceding the modal class
- f_2 is the frequency of the class succeeding the modal class

SECTION - IV

Q29. (a) Prove that $\sqrt{3}$ is irrational.

Solution:

Let us assume that $\sqrt{3}$ is rational. Then, there exist co-prime positive integers a and b such that

$\sqrt{3}=\frac{a}{b}$

$\Rightarrow a=b\sqrt{3}$

Squaring on both sides, we get

$a^2=3\,b^2$

Therefore, a^2 is divisible by 7 and hence, a is also divisible by 3

so, we can write $a=3p$, for some integer p.

Substituting for a, we get

$9p^2=3b^2\Rightarrow b^2=3p^2$.

This means, b^2 is also divisible by 3 and so, b is also divisible by 3.

Therefore, a and b have at least one common factor, i.e., 3.

But, this contradicts the fact that a and b are co-prime.

Thus, our supposition is wrong.

Hence, $\sqrt{3}$ is irrational.

OR

(b) If $\sec\ \theta+\tan\ \theta=p$, then prove that $\sin\ \theta=\frac{p^2-1}{p^2+1}$.

Solution:

$\frac{p^2-1}{p^2+1}=\frac{(\sec\ \theta+\tan\ \theta)^2-1}{(\sec\ \theta+\tan\ \theta)^2+1}$

$\frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$

$=\frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(\tan^2\theta+1)+2\sec\theta\tan\theta}$

$=\frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$

$=\frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$

$=\frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)}$

$=\frac{2\tan\theta}{2\sec\theta}$

$=\tan\theta\times\cos\theta$

$$= \frac{\sin \theta}{\cos \theta} \times \cos \theta$$
$$= \sin \theta$$

Q30. (a) If $A = \{2,3,4,5,6\}, B = \{1,3,5,7\}, C = \{2,4,6,8\}, D = \{2,3,5,7\}$, then find

(i) $A \cup B$
(ii) $B \cap D$
(iii) $C \cap D$
(iv) $D - A$

Solution:

i) $\mathrm{A} \cup \mathrm{B} = \{3,5\}$
ii) $\mathrm{B} \cap \mathrm{D} = \{3,5,7\}$
iii) $\mathrm{C} \cap \mathrm{D} = \{2\}$
iv) $\mathrm{D} - \mathrm{A} = \{7\}$

OR

(b) Find x so that $x, x+2, x+6$ are consecutive terms of a Geometric Progression.

Solution:

Given $x, x+2$ and $x+6$ are in G.P. but read it as $x, x+2$ and $x+6$.

$$\therefore r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\Rightarrow \frac{x+2}{x} = \frac{x+6}{x+2}$$

$$\Rightarrow (x+2)^2 = x(x+6)$$

$$\Rightarrow x^2 + 4x + 4 = x^2 + 6x$$

$$\Rightarrow 4x - 6x = -4 = -2x = -4$$

$$\therefore x = 2$$

Q31. (a) Find the value of ' k ' for which the points $(7,-2),(5,1),(3,k)$ are collinear.

Solution:

Given coordinates are $(x_1, y_1) = (4,-1)$ and $(x_2, y_2) = (-2,-3)$

Means of trisection:- A line segment in three equal parts then ratio is 1: 2 and 2 : 1 internally.

Case (1) If $m_1 : m_2 = 1 : 2$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{1 \times (-2) + 2 \times 4}{1 + 2}, \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-2 + 8}{3}, \frac{-3 - 2}{3}\right)$$

$$(x, y) = \left(2, -\frac{5}{3}\right)$$

Case (2):-

If $m_1 : m_2 = 2 : 1$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{2 \times (-2) + 1 \times 4}{2 + 1}, \frac{2 \times (-3) + 1 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-4 + 4}{3}, \frac{-6 - 1}{3}\right)$$

$$(x, y) = \left(0, -\frac{7}{3}\right)$$

Hence, this is the answer.

OR

(b) The table below shows the daily expenditure on food of 30 households in a locality:

Daily Expenditure (in Rupees)	100 − 150	150 − 200	200 − 250	250 − 300	300 − 350
Number of households	4	5	12	6	3

Find the mean daily expenditure on food by a suitable method.

Solution:

Daily Expense	Households (f_i)	Mid Point (x_i)	$f_i x_i$
$100 - 150$	4	125	500
$150 - 200$	5	175	875
$200 - 250$	12	225	2700
$250 - 300$	6	275	1650
$300 - 350$	3	325	975
	30		6700

$$\text{Mean } = \frac{\sum f_i x_i}{\sum f_i} = \frac{6700}{30} = 223.34$$

Q32. (a) 5 pencils and 7 pens together cost ₹95. Whereas 7 pencils and 5 pens together cost ₹85. Find the cost of one pencil and that of one pen.

Solution:

Let the cost of 1 book $= x$

And the cost of 1 pen $= y$

$\Rightarrow 5x + 7y = 95$

$\Rightarrow 7x + 5y = 85$

Equation (1) $\times$ 7 : $35x + 49y = 95 \times 7$

Equation (2) $\times$ 5 : $35x + 25y = 85 \times 5$

Subtract two equations;

$\Rightarrow 24y = 240$

$\Rightarrow y = 10$

$\Rightarrow x = 5$

OR

(b) One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of black colour

(ii) a face card

(iii) a spade

(iv) a card not a heart

Solution:

The probability of an event E

$$P(E)=\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}=\frac{n(E)}{n(S)}$$

i) Probability of getting a king of black colour:

∴ Number of a king of black colour, $n(E)=2$

Also, total number of cards, $n(S)=52$

∴ Probability of getting a face card $=\frac{n(E)}{n(S)}=\frac{2}{52}=\frac{1}{26}$

Hence, the required probability is $\frac{1}{26}$.

ii) Probability of getting a face card:

∴ Number of a face card, $n(E)=12$

Also, total number of cards, $n(S)=52$

∴ Probability of getting a spade $=\frac{n(E)}{n(S)}=\frac{12}{52}=\frac{3}{13}$

Hence, the required probability is $\frac{3}{13}$.

iii) Probability of getting a spade:

We know that, there are 13 cards of spades.

∴ Number of spades, $n(E)=13$

Also, total number of cards, $n(S)=52$

∴ Probability of getting a spade $=\frac{n(E)}{n(S)}=\frac{13}{52}=\frac{1}{4}$

Hence, the required probability is $\frac{1}{4}$.

iv) Probability of getting a card not a heart:

∴ Number of a card not a heart, $n(E)=39$

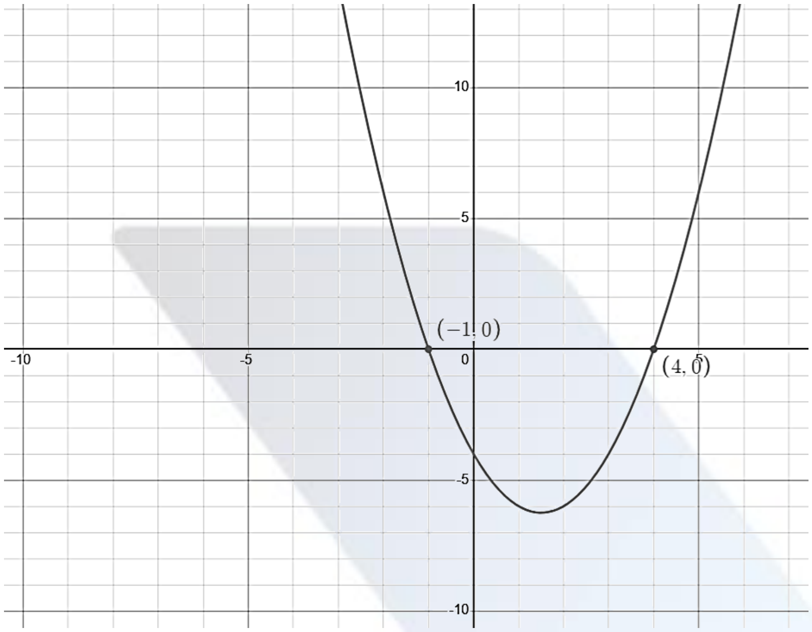
Also, total number of cards, $n(S)=52$

∴ Probability of getting the king of hearts $=\frac{n(E)}{n(S)}=\frac{39}{52}=\frac{3}{4}$

Hence, the required probability is $\frac{3}{4}$.

Q33. (a) Draw the graph of the polynomial $\mathrm{P}(x)=x^2-3x-4$ and find the zeroes.

Solution:



The above graph cuts x-axis at $x = -1$ and 4. So they are zeroes.

OR

(b) Draw a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle 60°.

Solution: