

Grade 10 Andhra Pradesh Mathematics 2023

Time Allowed: 3 Hours 15 mins Maximum Marks: 100

INSTRUCTIONS:

1. In the duration of 3 hours minutes, 15 minutes of time is allotted to read the question paper.
2. All answers shall be written in the answer booklet only.
3. Question paper consists of 4 Sections and 33 questions.
4. Internal choice is available in Section IV only.
5. Answers shall be written neatly and legibly.

Section I

Note:

1. Answer all the questions in one word or a phrase.
2. Each question carries 1 mark.

Q1. Find the LCM of 12, 15 and 21.

Solution:

2	<u>12</u>	15	21
2	<u>6</u>	15	21
3	<u>3</u>	<u>15</u>	<u>21</u>
5	1	<u>5</u>	7
7	1	1	<u>7</u>
	1	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 7$$

$$\text{LCM} = 420$$

Q2. Write the following set in roster form:

$$A = \{x: x \text{ is a natural number less than } 6\}$$

Solution:

$$\text{Roster form of Set } A = \{1, 2, 3, 4, 5\}$$

Q3. Choose the correct answer satisfying the following statements:

Statement (P) : The degree of the quadratic polynomial is 2.

Statement (Q) : Maximum no. of zeroes of a quadratic polynomial is 2.

(A) Both (P) and (Q) are true

(B) (P) is true, (Q) is false

(C) (P) is false, (Q) is true

(D) Both (P) and (Q) are false

Solution:

Statement (P) is true, Statement (Q) is true. The degree of the quadratic polynomial is 2 and has maximum 2 zeroes.

So, option (A) is correct.

Q4. Assertion : $3x + 6y = 3900, x + 2y = 1300$ represent coincident lines and have an infinite number of solutions.

Reason : If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then, these lines are coincident lines.

Choose the correct answer.

(A) Both Assertion and Reason are true, Reason is supporting the Assertion

(B) Both Assertion and Reason are true, But Reason is not supporting the Assertion

(C) Assertion is true, but the Reason is false

(D) Assertion is false, but the Reason is true

Answer: (A)

Solution:

We know that, If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then, these lines are coincident lines.

Hence Reason is true.

For the given assertion,

$$3x + 6y = 3900 \Rightarrow 3x + 6y - 3900 = 0 \dots (i)$$

$$x + 2y = 1300 \Rightarrow x + 2y - 1300 = 0 \dots (ii)$$

From (i) and (ii),

$$a_1 = 3, a_2 = 1 \Rightarrow \frac{a_1}{a_2} = \frac{3}{1}$$

$$b_1 = 6, b_2 = 2 \Rightarrow \frac{b_1}{b_2} = \frac{6}{2} = \frac{3}{1}$$

$$\text{and } c_1 = -3900, c_2 = -1300 \Rightarrow \frac{c_1}{c_2} = \frac{-3900}{-1300} = \frac{3}{1}$$

$$\text{Hence, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{3}{1}$$

Thus, Both Assertion and Reason are true, Reason is supporting the Assertion Option (A) is correct.

Q5. The number of roots of the equation $5x^2 - 6x - 2 = 0$ is

Solution:

The Quadratic equation will have 2 roots.

So, the given equation has 2 roots.

Q6. State Thales theorem.

Solution:

If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.

Q7. Find the number of tangents drawn at the end points of the diameter.

Solution:

Two tangents can be drawn at the endpoints of a diameter of a circle.

Q8. Find the volume of a cube, whose side is 4 cm.

Solution:

Side of cube $a = 4$ cm

Volume of cube $= a^3 = 4^3 = 64 \text{ cm}^3$

Q9. Match the following:

- | | |
|------------------|---|
| P) $\sin \theta$ | i) $\frac{1}{\sec \theta}$ |
| Q) $\cos \theta$ | ii) $\sqrt{\sec^2 \theta - 1}$ |
| R) $\tan \theta$ | iii) $\sqrt{\frac{\sec^2 \theta - 1}{\sec^2 \theta}}$ |

Choose the correct answer:

(A) $P \rightarrow$ (i), $Q \rightarrow$ (ii), $R \rightarrow$ (iii)

(B) $P \rightarrow$ (iii), $Q \rightarrow$ (i), $R \rightarrow$ (ii)

(C) $P \rightarrow$ (iii), $Q \rightarrow$ (ii), $R \rightarrow$ (i)

(D) $P \rightarrow$ (i), $Q \rightarrow$ (iii), $R \rightarrow$ (ii)

Solution:

$$\sqrt{\sec^2 \theta - 1} = \sqrt{\tan^2 \theta} = \tan \theta$$

So, $R) \rightarrow$ ii)

$$\cos \theta = \frac{1}{\sec \theta}$$

So, $Q) \rightarrow$ i)

$$\sqrt{\frac{\sec^2 \theta - 1}{\sec^2 \theta}}$$

$$= \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}}$$

$$= \frac{\tan \theta}{\sec \theta}$$

$$= \sin \theta$$

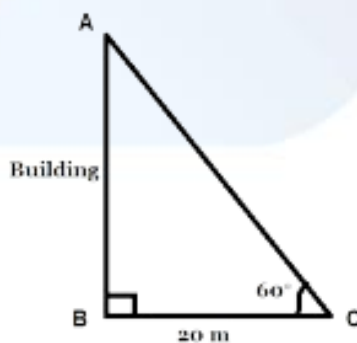
So, $P) \rightarrow$ iii)

(B) $P \rightarrow$ (iii), $Q \rightarrow$ (i), $R \rightarrow$ (ii)

- Q10. You are observing the top of your school building at an angle of elevation 60 degrees from a point which is at 20 meters distance from the foot of the building. Draw the diagram to the above situation.

Solution:

The diagram to the above situation is as follows:



- Q11. If $P(E) = 0.05$, what is the probability of not 'E'?

Solution:

$$P(\text{not } E) = 1 - P(E)$$

$$= 1 - 0.05$$

$$= 0.95$$

Q12. Find the mean of the given data.

2, 3, 7, 6, 6, 3, 8

Solution:

To find the mean of the data 2, 3, 7, 6, 6, 3, 8

We first add up all the numbers and then divide the sum by the number of data points.

$$\text{Sum of the data} = 2 + 3 + 7 + 6 + 6 + 3 + 8 = 35$$

The number of data points is 7

The mean is 35 divided by 7, which is equal to 5.

So, the mean of the data 2, 3, 7, 6, 6, 3, 8, is 5.

Section II

Note:

1. Answer all the questions.
2. Each question carries 2 marks.

Q13. If $A = \{3, 4, 5, 6\}$, $B = \{5, 6, 7, 8, 9\}$, then illustrate $A \cap B$ in the Venn diagram.

Solution:



The common portion represents $A \cap B$.

Q14. 6 pencils and 4 pens together cost Rs. 50 whereas 5 pencils and 6 pens together cost Rs. 46. Express the above statements in the form of Linear Equations.

Solution:

Let 'x' be pencils and 'y' be pens.

6 pencils and 4 pens together cost Rs. 50

$$6x + 4y = 50$$

5 pencils and 6 pens together cost Rs. 46

$$5x + 6y = 46$$

Q15. Check whether $(x - 2)^2 + 1 = 2x - 3$ is a quadratic equation or not.

Solution:

$$(x - 2)^2 + 1 = 2x - 3$$

$$x^2 + 4 - 4x + 1 = 2x - 3$$

$$x^2 - 4x + 5 = 2x - 3$$

$$x^2 - 6x + 8 = 0$$

So, given equation is a quadratic equation.

Q16. Write the formula to find n^{th} term of A.P. and explain the terms in it.

Solution:

n^{th} term of arithmetic progression is given as,

$$t_n = a + (n - 1)d$$

where a = first term of the arithmetic progression

n = number of term

d = common difference of the arithmetic progression

Q17. Find the distance between the two points $(7, 8)$ and $(-2, 3)$.

Solution:

The distance between two points $A(7,8)$ and $B(-2,3)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2 - 7)^2 + (3 - 8)^2}$$

$$= \sqrt{(-9)^2 + (-5)^2}$$

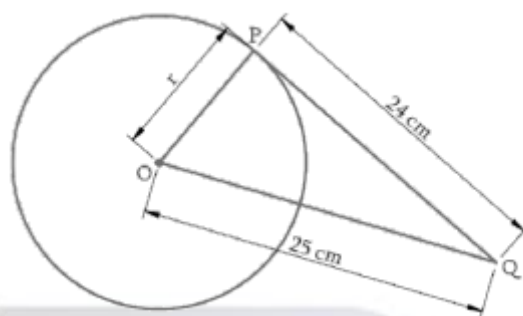
$$= \sqrt{81 + 25} \text{ units}$$

$$= \sqrt{106} \text{ units.}$$

Q18. From a point Q , the length of the tangent to a circle is 24 cm, and the distance of Q from the centre is 25 cm. Find the radius of the circle.

Solution:

Let's draw a figure as per the given question.



A tangent at any point of a circle is perpendicular to the radius at the point of contact.

Therefore, OPQ is a right-angled triangle.

By Pythagoras theorem,

$$OQ^2 = OP^2 + PQ^2$$

$$25^2 = r^2 + 24^2$$

$$r^2 = 25^2 - 24^2$$

$$r^2 = 625 - 576$$

$$r^2 = 49$$

$$r = \pm 7$$

Radius cannot be a negative value, hence, $r = 7$ cm.

Q19. If $\cos A = \frac{12}{13}$, then find $\sin A$ and $\tan A$.

Solution:

$$\cos A = \frac{12}{13}$$

We know,

$$\sin^2 A + \cos^2 A = 1$$

$$\Rightarrow \sin^2 A + \left(\frac{12}{13}\right)^2 = 1$$

[From (1)]

$$\Rightarrow \sin^2 A + \frac{144}{169} = 1$$

$$\Rightarrow \sin^2 A = 1 - \frac{144}{169}$$

$$\Rightarrow \sin^2 A = \frac{169 - 144}{169}$$

$$\Rightarrow \sin^2 A = \frac{169 - 144}{169}$$

$$\Rightarrow \sin^2 A = \frac{25}{169} \Rightarrow \sin A = \frac{5}{13}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\Rightarrow \tan A = \frac{\frac{5}{13}}{\frac{12}{13}} \Rightarrow \tan A = \frac{5}{12}$$

Q20. A die is thrown once, find the probability of getting

- i) a prime number
- ii) an odd number.

Solution:

Number of outcomes on throwing a die is $(1, 2, 3, 4, 5, 6) = 6$

Number of prime numbers on dice are 2, 3 and 5 = 3

Number of odd numbers on dice are 1, 3 and 5 = 3

$$(i) \text{ Probability of getting a prime number} = \frac{\text{Number of prime numbers}}{\text{total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

$$(ii) \text{ Probability of getting a odd number} = \frac{\text{Number of odd numbers}}{\text{total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

Section III

Note:

1. Answer all the questions.
2. Each question carries 4 marks.

Q21. Find 'x', if $2\log 5 + \frac{1}{2}\log 9 - \log 3 = \log x$.

Solution:

$$2\log 5 + \frac{1}{2}\log 9 - \log 3 = \log x$$

$$\Rightarrow \log 25 + \log 3 - \log 3 = \log x$$

$$\Rightarrow \log 25 = \log x$$

$$\Rightarrow x = 25$$

Q22. Check whether -3 and 3 are the zeroes of the polynomial $x^4 - 81$.

Solution:

$$\text{Let } p(x) = x^4 - 81$$

$$\text{If } x = 3$$

$$P(3) = (3)^4 - 81 = 0$$

$$\text{If } x = -3$$

$$P(-3) = (-3)^4 - 81 = 0$$

Therefore, 3 and -3 are the zeroes of the given polynomial.

Q23. Solve the pair of linear equations using the elimination method.

$$3x + 2y = -1 \text{ and } 2x + 3y = -9$$

Solution:

Let's begin by eliminating the y variable.

We can do this by multiplying the first equation by 3 and the second equation by 2, so that the coefficients of y in both equations become the same:

$$(3x + 2y = -1) \times 3 \Rightarrow 9x + 6y = -3$$

$$(2x + 3y = -9) \times 2 \Rightarrow 4x + 6y = -18$$

Now, we can subtract the second equation from the first equation to eliminate the y variable:

$$(9x + 6y) - (4x + 6y) = -3 - (-18)$$

Simplifying this, we get:

$$5x = 15$$

Dividing both sides by 5, we get:

$$x = 3$$

Now, we can substitute this value of x into either of the original equations to find the value of y .

Let's substitute it into the first equation:

$$3x + 2y = -1$$

$$3(3) + 2y = -1$$

$$9 + 2y = -1$$

$$2y = -10$$

$$y = -5$$

Q24. Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. Write the quadratic equation to find Rohan's present age.

Solution:

Let Rohan's age = x

Rohan's mother age = $x + 26$

After 3 years

Rohan's age = $x + 3$

$$\text{Rohan's mother age} = (x + 26) + 3 = x + 29$$

Now,

$$\text{Product of ages after 3 years} = 360$$

$$(x + 3)(x + 29) = 360$$

$$x(x + 29) + 3(x + 29) = 360$$

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 29x + 3x + 87 - 360 = 0$$

$$x^2 + 32x - 273 = 0$$

It is the form of $ax^2 + bx + c = 0$

where $a = 1, b = 32, c = -273$

Hence, it is a quadratic equation.

- Q25. Draw a tangent to a given circle with Centre 'O' from a point 'R' outside it. How many tangents can be drawn to the circle from that point?

Solution:

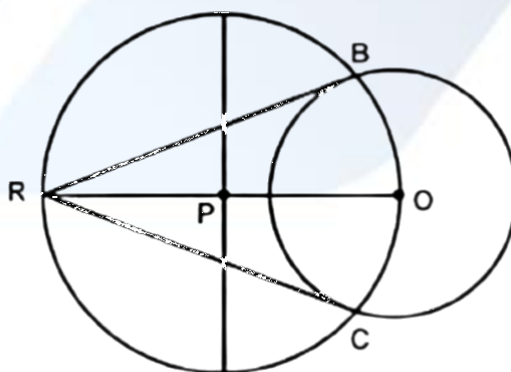
Steps of construction:

Step 1: Consider a point R from the outside the circle with centre O.

Step 2: Join points R and O, bisect the line RO. Let P be the midpoint of RO.

Step 3: Draw a circle taking P as centre and PO as a radius. This circle will intersect at two points B and C on the circle with centre O.

Step 4: Join point R with B and C. RB and RC are the required tangents through points B and C on the circle.



Two tangents can be drawn from to the circle from an external point.

Q26. An oil drum is in the shape of a cylinder having the following dimensions. Diameter is 2 m and height is 7 m. The painter charges ₹3 per m^2 to paint the drum. Find the total charges to be paid to the painter for 10 drums.

Solution:

It is given that diameter of the (oil drum) cylinder = 2 m.

$$\text{Radius of cylinder} = \frac{d}{2} = \frac{2}{2} = 1 \text{ m.}$$

$$\text{Total surface area of a cylindrical drum} = 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times 1(1 + 7)$$

$$= 2 \times \frac{22}{7} \times 8 = \frac{352}{7} \text{ m}^2 = 50.28 \text{ m}^2$$

$$\text{So, the total surface area of a drum} = 50.28 \text{ m}^2$$

$$\text{Painting charge per } 1 \text{ m}^2 = ₹3.$$

$$\text{Cost of painting of 10 drums} = 50.28 \times 3 \times 10$$

$$= ₹1508.57$$

Q27. Show that $\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$.

Solution:

$$\frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

Hence Proved

Q28. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household.

Family size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
No. of families	7	8	2	2	1

Find the mode of the data.

Solution:

Here, modal class = 3 – 5

$\Rightarrow l = 3, f_0 = 7, f_1 = 8, f_2 = 2$ and $h = 2$

$$= 3 + \frac{8 - 7}{2 \times 8 - 7 - 2} \times 2$$

$$= 3 + \frac{1}{7} \times 2$$

$$= 3 + 0.286$$

$$\therefore \text{Mode} = 3.286$$

Section IV

Note:

1. Answer all the questions.
2. Each question carries 8 marks.
3. Each question has internal choice

Q29. (a) Prove that $6 + \sqrt{2}$ is irrational.

Solution:

(a) Let us assume that $6 + \sqrt{2}$ is a rational number.

So, it can be written in the form $\frac{a}{b}$

$$6 + \sqrt{2} = \frac{a}{b}$$

Here a and b are coprime numbers and $b \neq 0$

$$6 + \sqrt{2} = \frac{a}{b}$$

By solving the equation we get,

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a - 6b}{b}$$

This shows $\frac{a - 6b}{b}$ is a rational number.

But we know that $\sqrt{2}$ is an irrational number, it contradicts our assumption.

Our assumption $6 + \sqrt{2}$ is a rational number that is incorrect.

Therefore, $6 + \sqrt{2}$ is an irrational number

Hence, it is proved that $6 + \sqrt{2}$ is an irrational number.

OR

(b) Show that $a_1, a_2, a_3, \dots, a_n$ form an AP where a_n is defined as below.

i) $a_n = 3 + 4n$

ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution:

(b) A sequence that has a common difference between any two of its consecutive terms is an arithmetic progression.

Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n - 1)d]$ or $S_n = \frac{n}{2}[a + l]$, and

the n th term of an AP is $a_n = a + (n - 1)d$

Here, a is the first term, d is the common difference and n is the number of terms and l is the last term.

(i) $a_n = 3 + 4n$

Given, n^{th} term, $a_n = 3 + 4n$

$$a_1 = 3 + 4 \times 1 = 7$$

$$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$$

$$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$$

$$a_4 = 3 + 4 \times 4 = 3 + 16 = 19$$

It can be observed that

$$a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4$$

$$a_4 - a_3 = 19 - 15 = 4$$

So, the difference of a_n and a_{n-1} is constant.

Therefore, this is an AP with a common difference as 4 and first term as 7.

The sum of n terms of AP is given by the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\text{Sum of 15 terms, } S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1)4]$$

$$= \frac{15}{2}[14 + 14 \times 4]$$

$$= \frac{15}{2} \times 70$$

$$= 15 \times 35$$

$$S_{15} = 525$$

(ii) $a_n = 9 - 5n$

Given, n^{th} term is $a_n = 9 - 5n$

$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$

$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$

$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$

$$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$$

It can be observed that

$$a_2 - a_1 = (-1) - 4 = -5$$

$$a_3 - a_2 = (-6) - (-1) = -5$$

$$a_4 - a_3 = (-11) - (-6) = -5$$

So, the difference of a_n and a_{n-1} is constant.

Therefore, this is an A.P. with common difference - 5 and first term as 4.

The sum of n terms of AP is given by the formula

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{15} = \frac{15}{2} [2 \times 4 + (15 - 1)(-5)]$$

$$= \frac{15}{2} [8 + 14(-5)]$$

$$= \frac{15}{2} [8 - 70]$$

$$= \frac{15}{2} \times (-62)$$

$$S_{15} = -465$$

- Q30. (a). Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7cm.

Solution:

(a) Given: The edge of cube = 7 cm

⇒ This will be the altitude of the cone.

Now

The radius of circular cone will be $\frac{7}{2} = 3.5$ cm.

As, we know that

The volume of cone is given by formula:

$$V = \frac{1}{3} \pi R^2 H$$

where,

R is the radius and H the altitude of the cone.

Also, we are asked for the largest cone, its volume must be equal or less than the volume of the cube.

$$\begin{aligned}
 V &= \frac{1}{3}\pi R^2 H \\
 &= \frac{1}{3}\pi(3.5)^2 \times 7 \\
 &= \frac{1}{3}\pi \times 12.25 \times 7 \\
 &= 89.83 \text{ cm}^3
 \end{aligned}$$

Required volume is 89.83 cm^3

OR

(b) If $A = \{1, 2, 3, 4, 5\}$; $B = \{3, 4, 5, 6, 7\}$; $C = \{1, 3, 5, 7\}$; $D = \{2, 4, 6, 8\}$

Find:

i) $A \cup B$ ii) $B \cup C$ iii) $A \cup D$ iv) $B - D$ v) $A \cap B$ vi) $B \cap D$ vii) $C \cap D$ viii) $A - D$

Solution:

(b) Given $A = \{1, 2, 3, 4, 5\}$; $B = \{3, 4, 5, 6, 7\}$; $C = \{1, 3, 5, 7\}$; $D = \{2, 4, 6, 8\}$

i) $A \cup B$: The union of sets A and B is the set containing all elements that are in A or B or both. Therefore, $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$.

ii) $B \cup C$: The union of sets B and C is the set containing all elements that are in B or C or both. Therefore, $B \cup C = \{1, 3, 4, 5, 6, 7\}$.

iii) $A \cup D$: The union of sets A and D is the set containing all elements that are in A or D or both. Therefore, $A \cup D = \{1, 2, 3, 4, 5, 6, 8\}$.

iv) $B - D$: The set difference of sets B and D is the set containing all elements that are in B but not in D. Therefore, $B - D = \{3, 5, 7\}$.

v) $A \cap B$: The intersection of sets A and B is the set containing all elements that are in both A and B. Therefore, $A \cap B = \{3, 4, 5\}$.

vi) $B \cap D$: The intersection of sets B and D is the set containing all elements that are in both B and D. Therefore, $B \cap D = \{4, 6\}$.

vii) $C \cap D$: The intersection of sets C and D is the set containing all elements that are in both C and D. Therefore, $C \cap D = \{\}$ (empty set).

viii) $A - D$: The set difference of sets A and D is the set containing all elements that are in A but not in D. Therefore, $A - D = \{1, 3, 5\}$.

Q31. (a). The distribution below gives the weights of 30 students in class. Find the median weight of the students.

Weight (in kg)	Number of students
40 – 45	2
45 – 50	3
50 – 55	8
55 – 60	6
60 – 65	3
65 – 70	2

Solution:

(a)

Weight (in kg)	Frequency	Cumulative frequency
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30

We have $N = 30$ and $\frac{N}{2} = 15$.

Median class = 55 – 60, so $l = 55$, $f = 6$, $cf = 13$ and $h = 60 - 55 = 5$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\Rightarrow \text{Median} = 55 + \frac{15 - 13}{6} \times 5$$

$$\Rightarrow \text{Median} = 55 + 1.67 = 56.67 \text{ kg}$$

OR

(b). Find the value of ' b ' for which the points A(1, 2), B(-1, b), C(-3, -4) are collinear.

Solution:

(b) Given A(1, 2), B(-1, b), C(-3, -4)

Area of $\Delta = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[1(b - (-4)) + (-1)(-4 - 2) + (-3)(2 - b)] = 0$$

$$[b + 4 + 4 + 2 - 6 + 3b] = 0$$

$$[4b + 4 + 6 - 6] = 0$$

$$4b + 4 = 0$$

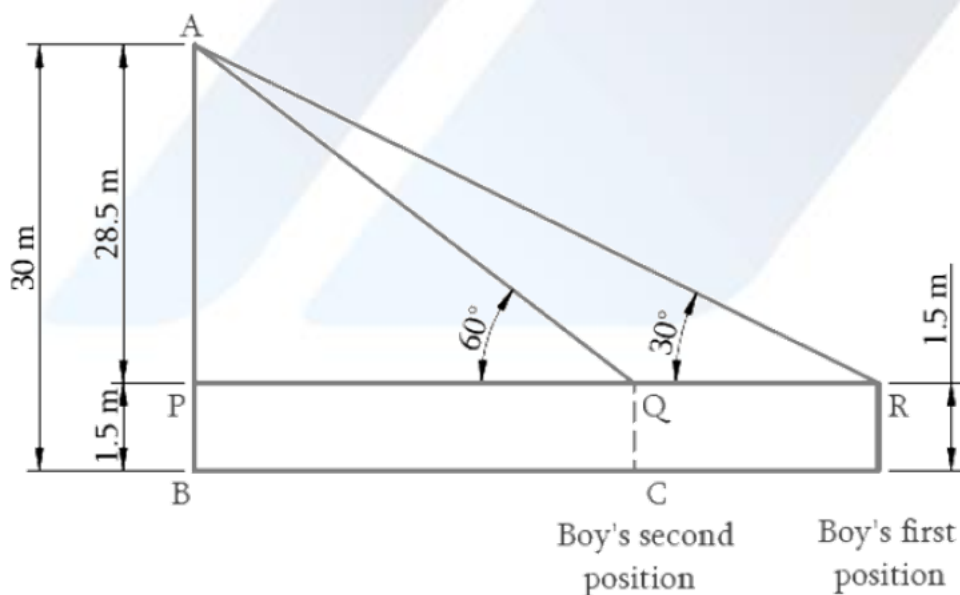
$$4b = -4$$

$$b = -\frac{4}{4}$$

$$b = -1$$

Q32. (a) A 1.5 m tall boy is looking at the top of a temple which is 30 meters in height from a point at a certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30° to 60° as he walks towards the temple. Find the distance he walked towards the temple.

Solution:



(a) Height of the temple = 30 m

Height of the boy = 1.5 m

Let initial distance between the man and the temple = d m

Let the distance walked = x m

From the figure

$$\tan 30^\circ = \frac{30 - 1.5}{d}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{d}$$

$$\Rightarrow d = 28.5 \times \sqrt{3} \text{ m} \dots \dots \dots (1)$$

$$\tan 60^\circ = \frac{28.5}{d - x}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{d - x}$$

$$\Rightarrow \sqrt{3}(d - x) = 28.5$$

$$\Rightarrow \sqrt{3}(28.5 \times \sqrt{3} - x) = 28.5 \text{ (from (1))}$$

$$\Rightarrow 28.5 \times 3 - \sqrt{3}x = 28.5$$

$$\Rightarrow \sqrt{3}x = 57$$

$$\Rightarrow x = \frac{57}{\sqrt{3}} = \frac{19 \times 3}{\sqrt{3}} = 19\sqrt{3}$$

$$\Rightarrow x = 19 \times 1.732 = 32.908 \text{ m}$$

\therefore Distance walked towards the temple = 32.908 m

OR

(b) One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

- i) a king of red colour
- ii) a face card
- iii) a jack of hearts
- iv) a spade.

Solution:

(b) We know that,

$$\text{The probability of an event } E, P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}$$

Applying the concept of probability as shown above, let's calculate all the probabilities one by one.

(i) Probability of getting a king of red colour:

We know that there are 26 red cards, 13 each of hearts and diamonds. Hence, there will be 1 king each in hearts and diamonds.

$$\therefore \text{Number of kings of red colour, } n(E) = 2$$

$$\text{Also, total number of cards, } n(S) = 52$$

$$\therefore \text{Probability of getting a king of red colour} = \frac{n(E)}{n(S)} = \frac{2}{52}$$

$$\Rightarrow \frac{1}{26}$$

Hence, the required probability is $\frac{1}{26}$.

(ii) Probability of getting a face card:

We know that each suit has 3 face cards (Jack, Queen and King). Hence, there will be 12 face cards in total.

$$\therefore \text{Number of face cards, } n(E) = 12$$

$$\text{Also, total number of cards, } n(S) = 52$$

$$\therefore \text{Probability of getting a face card} = \frac{n(E)}{n(S)} = \frac{12}{52} \Rightarrow \frac{3}{13}$$

Hence, the required probability is $\frac{3}{13}$.

(iii) Probability of getting the jack of hearts:

We know that there is only one jack in hearts

$$\therefore \text{Number of jack of hearts, } n(E) = 1$$

$$\text{Also, total number of cards, } n(S) = 52$$

$$\therefore \text{Probability of getting the jack of hearts} = \frac{n(E)}{n(S)} = \frac{1}{52}$$

Hence, the required probability is $\frac{1}{52}$.

(iv) Probability of getting a spade:

We know that there are 13 cards of spades.

$$\therefore \text{Number of spades, } n(E) = 13$$

$$\text{Also, total number of cards, } n(S) = 52$$

$$\therefore \text{Probability of getting a spade} = \frac{n(E)}{n(S)} = \frac{13}{52} \Rightarrow \frac{1}{4}$$

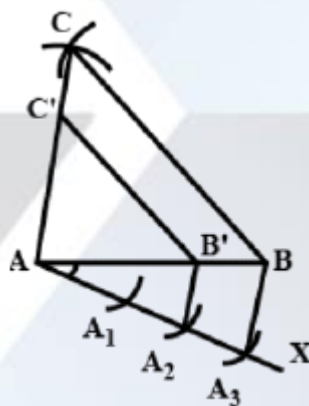
Hence, the required probability is $\frac{1}{4}$.

- Q33. (a) Construct a triangle of sides 5 cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle.

Solution:

Steps of Construction:

- 1 Draw a line segment $AB = 5$ cm
- 2 Draw an arc at 5 cm from point A.
- 3 Draw an arc at 6 cm from point B so that it intersects the previous arc.
- 4 Joint the point of intersection from A and B.
- 5 This gives the required $\triangle ABC$.
- 6 Dividing the base in 2: 3 ratio:
- 7 Draw a ray AX at an acute angle from AB .
- 8 Plot three points on AX so that $AA_1 = A_1A_2 = A_2A_3$
- 9 Join A_3 to B .
- 10 Draw a line from point A_2 so that this line is parallel to A_3B and intersects AB at point B' .
- 11 Draw a line from point B' parallel to BC so that this line intersects AC at point C' .



OR

(b) Draw a graph of $p(x) = x^2 - 3x - 4$ and hence find the zeroes of the polynomial.

Solution:

If $x = 0$,

$$\Rightarrow p(0) = 0^2 - 3(0) - 4 = -4$$

If $x = 1$

$$\Rightarrow p(1) = 1^2 - 3(1) - 4 = -4 = 1 - 3 - 4 = -6$$

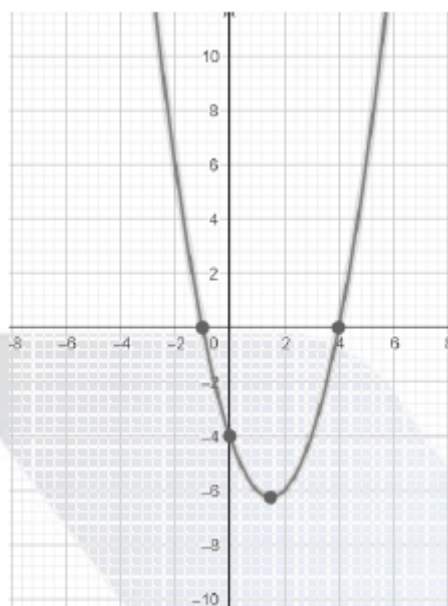
If $x = -1$

$$\Rightarrow y = y = (-1)^2 - 3(-1) - 4 = 1 + 3 - 4 = 0$$

If $x = 4$

$$\Rightarrow y = (4)^2 - 3(4) - 4 = 16 - 12 - 4 = 0$$

Plotting graphs using above coordinates, we get



From the graph, zeroes of the polynomial are $x = -1$ and $x = 4$.