Grade 10 Andhra Pradesh Mathematics 2024

Q1. What is the HCF of 37 and 49.

Solution:

$37 = 37 \times 1$

$49 = 7 \times 7 \times 1$

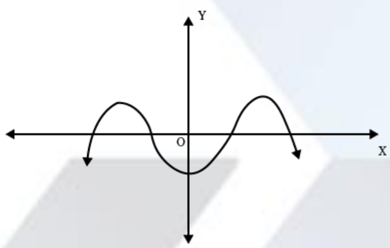
Therefore, HCF= 1

Q2. N= $\{1, 2, 3, \dots.\}$. This set is a/an _______ set. (Finite/Infinite)

Solution:

Infinite

Q3. Find the number of zeroes of the polynomial $p(x)$, whose graph is given.



Solution:

There will be four zeroes as it touches the x-axis at four different points.

Q4. Which of the following equations is not a linear equation?

(A) $5 + 4x = y + 3$

(B) $x + 2y = y - x$

(C) $3 - x = y^2 + 4$

(D) $x + y = 0$

Solution: C

$3 - x = y^2 + 4$ is a quadratic equation, not linear.

Q5. Statement – I: $2x^2 - x = 5$ is a quadratic equation.

Statement – II: General form of the quadratic equation is $ax^2 + bx + c = 0$ where $a \neq 0$.

Now choose the correct answer.

(A) Both statements are true

(B) Statement I is true and Statement II is false

(C) Statement I is false and Statement II is true
(D) Both statements are true
Solution: A
Both statements are true.

Q6. Match the following:

i. tanθ	p. $\frac{1}{\cos\theta}$
ii. secθ	q. $\frac{1}{\sin\theta}$
iii cosecθ	r. $\frac{\sin\theta}{\cos\theta}$

Choose the correct answer.
(A) i → r, ii → p, iii → q
(B) i → r, ii → q, iii → p
(C) i → p, ii → q, iii → r
(D) i → q, ii → r, iii → p
Solution: A

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\text{cosec}\theta = \frac{\sin\theta}{\cos\theta}$$

Q7. Distance between the two points (2, 0) and (6, 0) is _____ units
Solution:

Distance= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6-2)^2 + (0-0)^2}$
$= \sqrt{4^2}$
$= 4$ units

Q8. The n^{th} term is $a_n = ar^{n-1}$.Here 'r' represents ________.
Solution:
Here, 'r' is the common ratio between two consecutive terms.

Q9. Two _________ are always similar.
(A) Line segments
(B) Triangles
(C) Circles
(D) Squares

Solution: C

All circles are similar.

Q10. Number of tangents drawn from the external points to the circle is _______.

Solution:

Two

Q11. What is the length of the edge of the cube whose volume is 64 cm^3 ?

(A) 4 cm

(B) 16 cm

(C) 5 cm

(D) 6 cm

Solution: 4 cm

Volume $= a^3 = 64 \text{ cm}^3$

$a = 4$ cm

Q12. Which of the following cannot be the probability of an event?

(A) 0.3

(B) -1.5

(C) 15%

(D) $\frac{2}{7}$

Solution: B

-1.5, as probability, can never be negative.

SECTION - II

Q13. Check whether 3 and -2 are the zeroes of the polynomial $p(x) = x^2 - x - 6$.

Solution:

$p(x) = x^2 - x - 6$

$p(3) = 3^2 - 3 - 6 = 9 - 9 = 0$

$p(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$

As both 3 and -2 satisfies the given polynomial.

So, they are the zeroes of the given polynomial.

Q14. 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹ 46. Represent this information in the form of pair of linear equations in variables x and y.

Solution:

Let the cost of the pencil be Rs. x

The cost of the pens is Rs. y

5 pencils and 7 pens together cost Rs. 50,
So, we get
$5x + 7y = 50$
7 pencils and 5 pens together cost Rs. 46
$7x + 5y = 46$

Q15. Check whether $(x-2)^2 + 1 = 2x - 3$ is a quadratic equation.

Solution:

$(x-2)^2 + 1 = 2x - 3$
$x^2 - 4x + 4 + 1 = 2x - 3$
$x^2 - 6x + 8 = 0$
It is a quadratic equation.

Q16. Find the centroid of the triangle whose vertices are $(3, -2), (-2, 8)$ and $(0, 4)$.

Solution:

We know that the coordinates of the centroid of a triangle whose angular points are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are

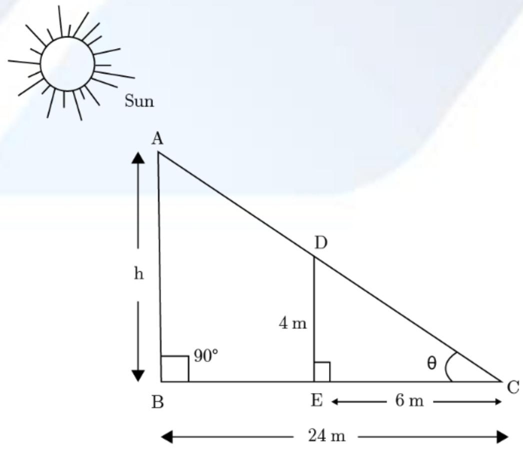
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

So, the coordinates of the centroid of a triangle whose vertices are $(0, 6), (8, 12)$ and $(8, 0)$ are

$$\left(\frac{3 + (-2) + 0}{3}, \frac{-2 + 8 + 4}{3}\right) \text{ or, } \left(\frac{1}{3}, \frac{10}{3}\right)$$

Q17. A flag pole 4 m tall casts 6 m shadow. At the same time, a nearby building casts a shadow of 24 m. How tall is the building?

Solution:



AB is the building of height h DE is a flag pole of height 4 m

In △ DEC,

$\tan\theta = \frac{4}{6} = \frac{2}{3}$

In △ ABC,

$\tan\ \theta = \frac{h}{24}$

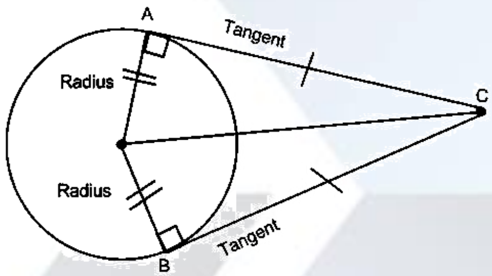
$\Rightarrow \frac{h}{24} = \frac{2}{3}$

$\Rightarrow h = 16$ m

So, the height of the pole is 16 m

Q18. Calculate the length of the tangent drawn from a point 15 cm away from the centre of a circle of radius 9 cm.

Solution:



Point from center $= 15$ cm

radius is $= 9$ cm

Applying Pythagoras theorem from figure,

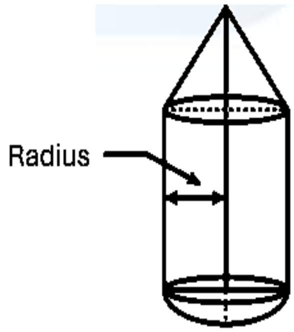
Length of tangent from a point 15 cm from center of circle is

$= \sqrt{9^2 + 15^2}$

$= 17.49$ cm

Q19. A solid toy is in the form of right circular cylinder with hemispherical shape at one end and a cone at the other end. Draw a rough diagram of this solid toy.

Solution:



Q20. Express sin 81° + tan 81° in terms of trigonometric ratios of angles between 0° and 45°.

Solution:

We know that,

$\sin A = \cos(90° - A)$

$\tan A = \cot(90° - A)$

$\sec A = \operatorname{cosec}(90° - A)$

Using these, we get

$\sin 81° = \sin(90° - 9°) = \cos 9°$

$\tan 81° = \tan(90° - 9) = \cot 9°$

$\therefore \sin 80° + \tan 81° = \cos 9° + \cot 9°$

SECTION - III

Q21. If $x^2 + y^2 = 25xy$, then prove that $2\log(x + y) = 3\log 3 + \log x + \log y$.

Solution:

$x^2 + y^2 = 25xy \ldots.(1)$

$\text{L. H. } S = 2\log(x + y)$

$= \log(x + y)^2$

$= \log(x^2 + y^2 + 2xy)$

$= \log(25xy + 2xy)$ [from (1)]

$= \log(27xy)$

$= \log(3^3 xy)$

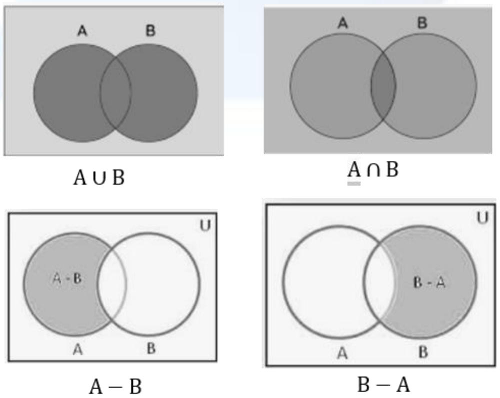
$= \log 3^3 + \log x + \log y$

$= 3\log 3 + \log x + \log y$

= R. H.S

Q22. Draw the Venn diagrams of A ∪ B, A ∩ B, A − B and B − A (Here A, B are non-empty sets).

Solution:



Q23. Solve the pair of linear equations $3x + 2y = 11$ and $2x + 3y = 4$.

Solution:

Given that:

$3x + 2y = 11 \ldots \ldots (1)$

On multiplying by 3, we get

$9x + 6y = 33 \ldots \ldots (2)$

$2x + 3y = 4, \ldots \ldots (3)$

On multiplying by 2, we get

$4x + 6y = 8 \ldots \ldots (4)$

On subtracting equation (4) from equation (2), we get

$5x = 25$

$x = 5$

Substituting in equation (1), we get

$15 + 2y = 11$

$y = -2$

Q24. Find the roots of the quadratic equation $2x^2 + x - 3 = 0$.

Solution:

$2x^2 + x - 3 = 0$

$2x^2 + 3x - 2x - 3 = 0$

$x(2x + 3) - 1(2x + 3) = 0$

$(2x + 3)(x - 1) = 0$

$x = -\frac{3}{2}, 1$

Q25. Find the volume and surface area of a sphere of radius 2.1 cm. (Take $\pi = \frac{22}{7}$).

Solution:

Radius of sphere, $r = 2.1$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.8 \text{ cm}^3$

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 2.1 \times 2.1 = 55.44 \text{ cm}^2$

Q26. Simplify $(1 - \cos\theta)(1 + \cos\theta)(1 + \cot^2\theta)$.

Solution:

$(1 - \cos\theta)(1 + \cos\theta)(1 + \cot^2\theta)$

$= [(1 - \cos\theta)(1 + \cos\theta)(1 + \cot^2\theta)]$

$= (1 - \cos^2\theta)(1 + \cot^2\theta)$

$= \sin^2\theta \times \operatorname{cosec}^2\theta = \sin^2\theta \times \dfrac{1}{\sin^2\theta} = 1$

Q27. A die is thrown once. Find the probability of getting

i) a prime number

ii) an odd number.

Solution:

Number of outcomes on throwing a die is $(1,2,3,4,5,6) = 6$

Number of prime numbers on dice are $1,3$ and $5 = 3$

i) Probability of getting a prime number $= \dfrac{\text{Number of prime number}}{\text{total number of outcomes}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$

ii) Total number of odd numbers are $1,3$ and $5 = 3$

Probability of getting a odd number $= \dfrac{\text{Number of odd number}}{\text{total number of outcomes}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$

Q28. Write the formula to find the median of a grouped data and explain the terms involved in it.

Solution:

Median for Grouped Data

$$\text{Median} = 1 + \left[\frac{\frac{n}{2} - c}{f}\right] \times h$$

Where,

- | = lower limit of median class
- n = total number of observations
- c = cumulative frequency of the preceding class
- f = frequency of median class
- h = class size (upper limit - lower limit)

SECTION - IV

Q29. a) Prove that $\sqrt{7}$ is irrational.

Solution:

Let us assume that $\sqrt{7}$ is rational. Then, there exist co-prime positive integers a and b such that

$\sqrt{7} = \dfrac{a}{b}$

$\Rightarrow a = b\sqrt{7}$

Squaring on both sides, we get

$a^2 = 7\, b^2$

Therefore, a^2 is divisible by 7 and hence, a is also divisible by 7

so, we can write $a = 7p$, for some integer p.

Substituting for a, we get

$49p^2 = 7b^2 \Rightarrow b^2 = 7p^2$.

This means, b^2 is also divisible by 7 and so, b is also divisible by 7.

Therefore, a and b have at least one common factor, i.e., 7.

But, this contradicts the fact that a and b are co-prime.

Thus, our supposition is wrong.

Hence, $\sqrt{7}$ is irrational.

OR

b) ABC is a right triangle right angled at C. Let $\mathrm{BC} = a$, $\mathrm{CA} = b$, $\mathrm{AB} = c$ and let p be the length of perpendicular from C on AB. Prove that

i) $pc = ab$

ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Solution:

Let $\mathrm{CD} \perp \mathrm{AB}$. Then, $\mathrm{CD} = p$

$\therefore$ Area of $\triangle \mathrm{ABC} = \frac{1}{2}(\text{ Base } \times \text{ Height })$

$= \frac{1}{2}(\mathrm{AB} \times \mathrm{CD}) = \frac{1}{2}cp$

Also,

Area of $\triangle \mathrm{ABC} = \frac{1}{2}(\mathrm{BC} \times \mathrm{AC}) = \frac{1}{2}ab$

$\therefore \frac{1}{2}cp = \frac{1}{2}ab$

$\Rightarrow cp = ab$

(ii) Since $\triangle$ ABC is a right triangle, right angled at C.

$\therefore \mathrm{AB}^2 = \mathrm{BC}^2 + \mathrm{AC}^2$

$\Rightarrow c^2 = a^2 + b^2$

$$\Rightarrow \left(\frac{ab}{p}\right)^2 = a^2 + b^2 \left[\because cp = ab \Rightarrow c = \frac{ab}{p}\right]$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Q30. a) If $A = \{1, 3, 4, 5, 7\}, B = \{2, 4, 5, 6\}, C = \{4, 5, 8, 9\}, D = \{1, 3, 7, 8\}$, then find

i) $A \cup B$

ii) $B \cap D$

iii) $A \cap D$

iv) $C - D$

Solution:

i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

ii) $B \cap D = \{\}$

iii) $A \cap D = \{1, 3, 7\}$

iv) $C - D = \{4, 5, 9\}$

OR

b) A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let the value of the prices be

$x, x - 20, x - 40, \ldots.$

This is an arithmetic sequence with first term (a) $= x, n = 7$ and common difference (d) $= -20$.

Given,

$S_n = 700$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 700$

$\Rightarrow \frac{7}{2}[2(x) + (7-1)(-20)] = 700$

$\Rightarrow 2x - 120 = 700 \times \frac{2}{7}$

$\Rightarrow 2x = 200 + 120$

$\Rightarrow x = \frac{320}{2} = 160$

Thus, the values of the prizes are Rs. 160, Rs. 140, Rs. 120, Rs. 100, Rs. 80 , Rs, 60 and Rs. 40.

Q31. a) Find the co-ordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Solution:

Given coordinates are $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (-2, -3)$

Means of trisection:- A line segment in three equal parts then ratio is 1: 2 and 2 : 1 internally.

Case (1) If $m_1 : m_2 = 1 : 2$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{1 \times (-2) + 2 \times 4}{1 + 2}, \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-2 + 8}{3}, \frac{-3 - 2}{3}\right)$$

$$(x, y) = \left(2, -\frac{5}{3}\right)$$

Case (2):-

If $m_1 : m_2 = 2 : 1$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{2 \times (-2) + 1 \times 4}{2 + 1}, \frac{2 \times (-3) + 1 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-4 + 4}{3}, \frac{-6 - 1}{3}\right)$$

$$(x, y) = \left(0, -\frac{7}{3}\right)$$

Hence, this is the answer.

OR

b) The table below shows the daily expenditure on food of 25 households in a locality.

Daily Expenditure (in Rupees)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
No. of Households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

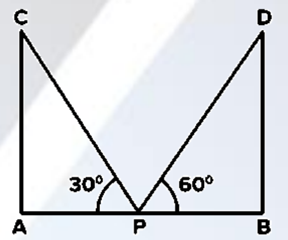
Solution:

Daily Expense	Households (f_i)	Mid Point (x_i)	$f_i\ x_i$
100 – 150	4	125	500
150 – 200	5	175	875
200 – 250	12	225	2700
250 – 300	2	275	550
300 – 350	2	325	650
	25		5275

$$\text{Mean } = \frac{\sum f_i x_i}{\sum f_i} = \frac{5275}{25} = 211$$

Q32. a) Two poles of equal heights are standing opposite to each other on either side of the road, which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles.

Solution:



Let AP $= x$ feet .

Therefore, PB $= (120 - x)$ feet

In $\triangle$ APC,

$$\tan 30^\circ = \frac{AC}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \quad \ldots\ldots (1)$$

In $\triangle$ BPD

$$\tan 60^\circ = \frac{BD}{PB}$$

$$\sqrt{3} = \frac{h}{120 - x}$$

$h = (120 - x)\sqrt{3} \ldots . (2)$

Since heights are same,

$\frac{x}{\sqrt{3}} = (120 - x)\sqrt{3}$

$x = 360 - 3x$

$4x = 360$

$x = 90$

put $x = 90$ in (1)

$h = \frac{90}{\sqrt{3}} = 30\sqrt{3}$ feet

Therefore, the height of both poles is $30\sqrt{3}$ feet.

OR

b) One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting:

i) a face card

ii) a spade

iii) the queen of diamonds

iv) the king of hearts.

Solution:

The probability of an event E

$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}$

i) Probability of getting a face card:

We know that, each suit has 3 face cards (Jack, Queen and King). Hence, there will be 12 face cards in total.

$\therefore$ Number of face cards, $n(E) = 12$

Also, total number of cards, $n(S) = 52$

$\therefore$ Probability of getting a face card $= \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

Hence, the required probability is $\frac{3}{13}$.

ii) Probability of getting a spade:

We know that, there are 13 cards of spades.

$\therefore$ Number of spades, $n(E) = 13$

Also, total number of cards, $n(S) = 52$

$\therefore$ Probability of getting a spade $= \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Hence, the required probability is $\frac{1}{4}$.

iii) Probability of getting the queen of diamonds:

We know that, there are 13 cards of diamonds. There is 1 queen of diamonds.
∴ Number of queens in diamonds, $n(E) = 1$
Also, total number of cards, $n(S) = 52$

∴ Probability of getting the queen of diamonds $= \frac{n(E)}{n(S)} = \frac{1}{52}$

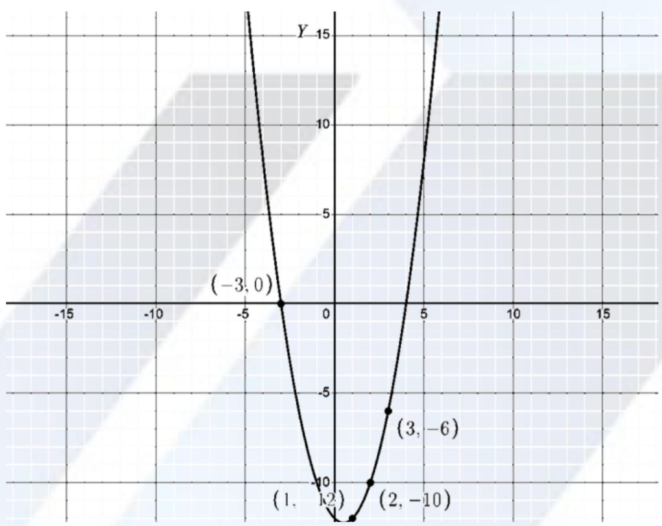
Hence, the required probability is $\frac{1}{52}$.

iv) Probability of getting the king of hearts:
We know that, there is only one king in hearts.
∴ Number of king of hearts, $n(E) = 1$
Also, total number of cards, $n(S) = 52$
∴ Probability of getting the king of hearts $= \frac{n(E)}{n(S)} = \frac{1}{52}$

Hence, the required probability is $\frac{1}{52}$.

Q33. a) Draw the graph of the polynomial $p(x) = x^2 - x - 12$ and find its zeroes.

Solution:

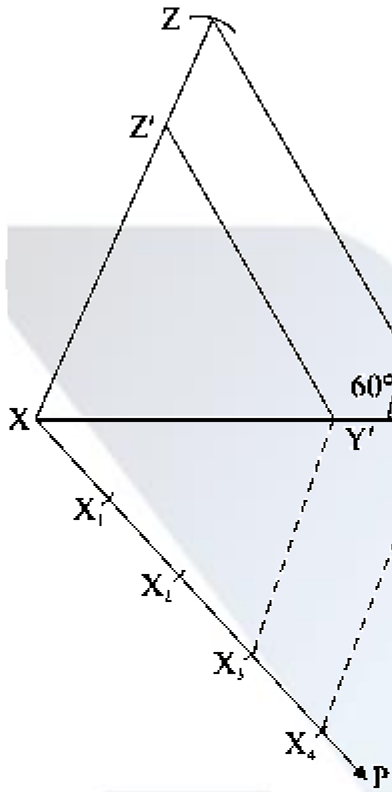


The above graph cuts x-axis at $x = -3$ and 4. So they are zeroes.

OR

b) Construct a triangle of sides 4 cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.

Solution:


Z
Z'
60°
X
Y'
Y
X1
X2
X3
X4
P