Grade 10 Karnataka Math 2014

QUESTION PAPER CODE 81-E

General Instructions to the Candidates:

1. The Question-cum-Answer Booklet consists of objective and subjective types of questions having 58 questions.
2. Space has been provided against each objective type of question. You have to choose the correct choice and write the complete answer along with its alphabet in the space provided.
3. For subjective type questions, enough space for each question has been provided. You have to answer the questions in space.
4. Follow the instructions given against both the objective and subjective types of questions.
5. Candidate should not write the answer with pencil. Answers written in pencil will not be evaluated. (Except Graphs, Diagrams & Maps)
6. In case of Multiple Choice, fill in the blanks and Matching questions, scratching / rewriting / marking is not permitted, thereby rendering to disqualification for evaluation.
7. Candidates have an extra 15 minutes for reading the question paper.
8. Space for Rough Work has been printed and provided at the bottom of each page.

I. Four alternatives are given for each of the following questions / incomplete statements. Only one of them is correct or most appropriate. Choose the correct alternative and write the complete answer along with its alphabet in the space provided against each question.

Q1. If matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$ then the matrix AB' is

(A) $\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 4 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 & 3 \end{bmatrix}$

(D) $[10]$.

Solution:

Correct Answer: (D)

Given matrices:

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

First, find B^T :

$B' = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Now, compute AB' :

$AB' = (1 \times 3) + (2 \times 2) + (3 \times 1) = 3 + 4 + 3 = 10$

$AB' = [10]$.

Q2. The value of ${}^5P_1$ is

(A) 5

(B) 6

(C) 0

(D) 5!

Solution:

Correct answer: (A)

The formula for Permutation is:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Substituting $n = 5$ and $r = 1$:

$${}^5P_1 = \frac{5!}{(5-1)!} = \frac{5!}{4!}$$

Since $5! = 5 \times 4!$, we cancel $4!$:

$${}^5P_1 = 5$$

Q3. The meaningful among the following is

(A) ${}^2P_5$

(B) ${}^5P_2$

(C) ${}^{-5}P_2$

(D) ${}^5P_{-2}$

Solution:

Correct answer: (B)

${}^5P_2$

Q4. If ${}^{11}P_r = 990$, then the value of r is

(A) 3

(B) 9

(C) 4

(D) 2

Solution:

Correct answer: (A)

The formula for permutation is:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Given ${}^{11}P_r = 990$, substituting $n = 11$:

$$\frac{11!}{(11-r)!} = 990$$

Expanding the first r terms:

$$\frac{11 \times 10 \times 9!}{(11-r)!} = 990$$

$$(11-r)! = \frac{990}{11 \times 10 \times 9!}$$

$(11-r)! = 8!$
$r = 3$

Q5. The standard deviation and coefficient of variation of the scores of a player are 1.5 and 15 respectively. Then the mean score of the player is
(A) 1.5
(B) 10
(C) 15
(D) 1000
Solution:
Correct answer: (B)
The formula for coefficient of variation (CV) is:
$$CV = \frac{\sigma}{\bar{X}} \times 100$$
Given $CV = 15$ and $\sigma = 1.5$, substituting the values:
$$15 = \frac{1.5}{\bar{X}} \times 100$$
Solving for $\bar{X}$:
$$\bar{X} = \frac{1.5 \times 100}{15} = 10$$

Q6. The L.C.M. of $2ab$ and $6ac^2$ is $6abc^2$. Then their HCF is
(A) $2a$
(B) $6ab$
(C) $6a$
(D) $2ab$
Solution:
Correct answer: (A)
The relationship between L.C.M. and H.C.F. is:
L.C.M. $\times$ H.C.F. = Product of the given terms
Given terms: $2ab$ and $6ac^2$, and L.C.M. $= 6abc^2$.
Product $= (2ab) \times (6ac^2) = 12a^2bc^2$
Using the formula:
$$\text{H.C.F.} = \frac{12a^2bc^2}{6abc^2} = 2a$$

Q7. The HCF of $(p-q)$ and $(\sqrt{p} - \sqrt{q})$ is
(A) $(\sqrt{p^3} - \sqrt{q^3})$
(B) $(\sqrt{p} - \sqrt{q})$
(C) $(\sqrt{p} + \sqrt{q})$
(D) $(p-q)$.
Solution:
Correct answer: (B)

We have the expressions $(p-q)$ and $(\sqrt{p}-\sqrt{q})$.
Since $p-q$ can be factored as $(\sqrt{p}-\sqrt{q})(\sqrt{p}+\sqrt{q})$, the common factor in both expressions is $(\sqrt{p}-\sqrt{q})$

Q8. The simplified form of $\sum_{pqr}(p+q-r)+\sum_{pqr}(p-q-r)$ is
(A) $2p+2q+2r$
(B) $-p-q-r$
(C) 0
(D) $-2p-2q-2r$
Solution:
Correct answer: (C)
We have the given expression:
$$\sum_{pqr}(p+q-r)+\sum_{pqr}(p-q-r)$$
Expanding both summations:
$$\sum_{pqr} p+\sum_{pqr} q-\sum_{pqr} r+\sum_{pqr} p-\sum_{pqr} q-\sum_{pqr} r$$
The terms $\sum_{pqr} q$ cancel out, and we are left with:
$$2\sum_{pqr} p-2\sum_{pqr} r$$
Factoring out 2:
$$2\sum_{pqr}(p-r)=2\times 0=0$$

Q9. If $ab+bc+ca=1$ then $(a+b)(c+a)=$
(A) ac
(B) $1+bc$
(C) bc
(D) $1+a^2$.
Solution:
We are given $ab+bc+ca=1$ and need to find $(a+b)(c+a)$.
Expanding the expression:
$(a+b)(c+a)=ac+a^2+bc+ab$
Using the given condition $ab+bc+ca=1$, we substitute:
$(a+b)(c+a)=a^2+1$

Q10. If $\sum_{abc} a^3-3abc=0$ then the value of $\sum_{abc} a^2=$
(A) $\sum_{abc} a$
(B) $2\sum_{abc} ab$
(C) $\sum_{abc} a^2-a$
(D) $\sum_{abc} ab$.

Solution:

We are given:

$$\sum_{abc} a^3 - 3abc = 0 \Longrightarrow a^3 + b^3 + c^3 = 3abc$$

Using the identity:

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

we deduce:

$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then:

$a^2 + b^2 + c^2 = ab + bc + ca$

Thus, $\sum_{abc} a^2 = \sum_{abc} ab$.

.

Q11. The sum of $4\sqrt{2}, \sqrt{2}$ and $-\sqrt{32}$ is

(A) $-\sqrt{2}$

(B) $9\sqrt{2}$

(C) $4\sqrt{2}$

(D) $\sqrt{2}$

Solution:

We simplify and combine the terms:

$4\sqrt{2} + \sqrt{2} - \sqrt{32}$

Simplify $-\sqrt{32}$:

$-\sqrt{32} = -4\sqrt{2}$

Now combine like terms:

$4\sqrt{2} + \sqrt{2} - 4\sqrt{2} = \sqrt{2}$

Q12. If $F = \frac{mV^2}{r}$ then $V =$

(A) $\pm\sqrt{\frac{F}{m \cdot r}}$

(B) $\pm\sqrt{\frac{m}{F \cdot r}}$

(C) $\pm\sqrt{\frac{F \cdot m}{r}}$

(D) $\pm\sqrt{\frac{F \cdot r}{m}}$

Solution:

We start with:

$$F = \frac{mV^2}{r}$$

Multiply both sides by r and divide by m :

$$V^2 = \frac{F \cdot r}{m}$$

Take the square root:

$$V = \sqrt{\frac{F \cdot r}{m}}$$

Q13. The roots of the quadratic equation $x^2 - 5x - 6 = 0$ are

(A) - 3 and - 2
(B) 3 and 2
(C) 6 and - 1
(D) -6 and 1

Solution:

We solve the quadratic equation:

$x^2 - 5x - 6 = 0$

Factor the equation:

$(x - 6)(x + 1) = 0$

Solve for x :

$x = 6 \text{ or } x = -1$

Q14. The value of p for the equation $x^2 - px + 9 = 0$ to have equal roots is

(A) +6
(B) ± 6
(C) -6
(D) ± 13

Solution:

For the quadratic equation $x^2 - px + 9 = 0$ to have equal roots, the discriminant must be zero:

$D = (-p)^2 - 4(1)(9) = 0$

Simplify:

$p^2 - 36 = 0 \Rightarrow p^2 = 36$

Solve for p :

$p = \pm 6$

Q15. The sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is

(A) $\frac{c}{a}$
(B) $\frac{b}{a}$
(C) $-\frac{b}{a}$
(D) $\frac{a}{c}$

Solution:

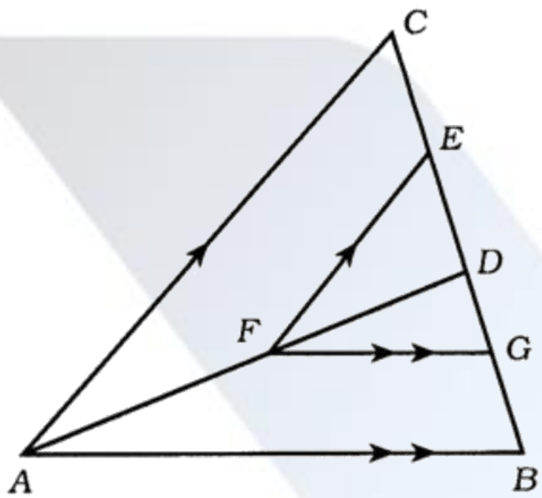
For a quadratic equation of the form:

$ax^2 + bx + c = 0$

The sum of its roots is given by the formula:

$$\text{Sum of roots } = -\frac{b}{a}$$

Q16. In the given figure, $EF \| CA$ and $FG \| AB$ then $\frac{DE}{EC} =$.



(A) $\frac{DG}{GB}$
(B) $\frac{GB}{DG}$
(C) $\frac{AF}{DF}$
(D) $\frac{AB}{AD}$

Solution:

Since EF || CA and FG || AB, we apply the Basic Proportionality Theorem (Thales' Theorem) in both triangles.

From $\triangle$ DAC, since EF| | CA :

$$\frac{DE}{EC} = \frac{DF}{FA}$$

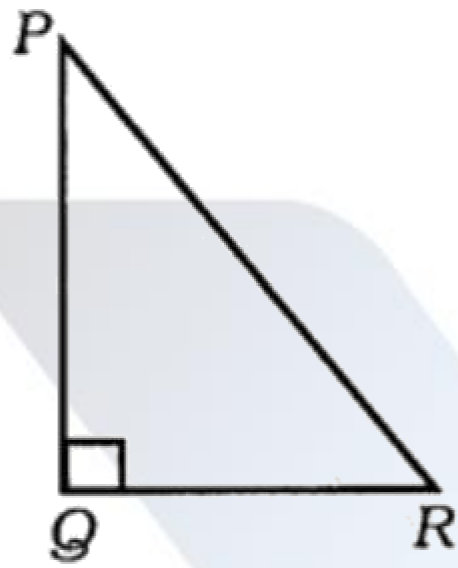
From $\triangle$ DAB, since FG||AB :

$$\frac{DF}{FA} = \frac{DG}{GB}$$

Thus,

$$\frac{DE}{EC} = \frac{DG}{GB}$$

Q17. In $\triangle PQR, \angle PQR = 90^\circ$. The correct relation with respect to $\triangle PQR$ is



(A) $PR^2 = PQ^2 - QR^2$
(B) $PQ^2 = QR^2 - PR^2$
(C) $PR^2 = PQ^2 + QR^2$
(D) $QR^2 = PQ^2 - PR^2$

Solution:

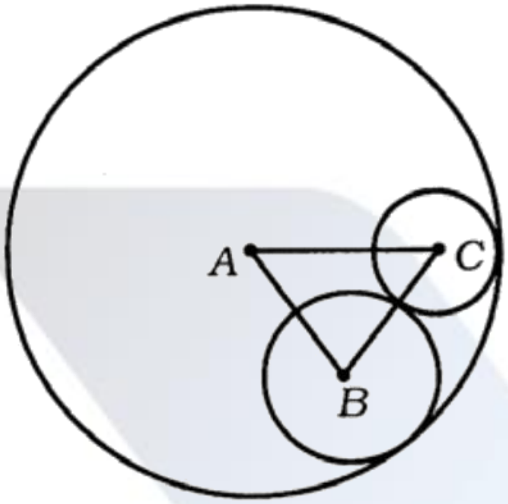
In right-angle triangle PQR
$PR^2 = PQ^2 + QR^2$

Q18. The distance between the centres of two circles of radii 3.4 cm and 1.8 cm is 3.7 cm. Then the circles are
(A) externally touching circles
(B) internally touching circles
(C) intersecting circles
(D) concentric circles

Solution:

Given radii 3.4 cm and 1.8 cm , and center distance 3.7 cm , we check:
Sum of radii $= 3.4 + 1.8 = 5.2$ cm
Difference of radii $= 3.4 - 1.8 = 1.6$ cm
Since 1.6 cm < 3.7 cm < 5.2 cm, the circles intersect at two points.

Q19. Three circles with centres A, B and C touch each other as shown in figure. If the radii of these circles are 8 cm, 3 cm and 2 cm respectively, then the perimeter of $\triangle ABC$ is.



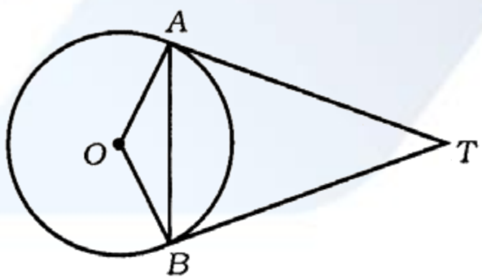
(A) 26 cm
(B) 16 cm
(C) 18 cm
(D) 14 cm

Solution:

To find the perimeter of $\triangle ABC$, we use the distances between the centers of the touching circles. The distance between centers is the sum of their radii:
$AB = 8 + 3 = 11$ cm,
$BC = 3 + 2 = 5$ cm,
$AC = 8 + 2 = 10$ cm.
The perimeter is:

$$AB + BC + AC = 11 + 5 + 10 = 26 \text{ cm}$$

Q20. In the figure O is the centre of the circle. AT and BT are the tangents at points A and B respectively. If $\angle OAB = 30^\circ$, then the measure of $\angle ATB$ is



(A) 30°
(B) 15°
(C) 60°
(D) 90°

Solution:

Given that $\angle OAB = 30^\circ$, since OA and OB are radii, $\angle OBA = 30^\circ$ as well.

The angle at the center $\angle AOB = 120^\circ$ because the sum of angles in triangle OAB is 180°.
Since AT and BT are tangents, $\angle OAT = \angle OBT = 90^\circ$.
In quadrilateral $OATB$, the sum of angles is 360°, so:
$\angle ATB = 360^\circ - 90^\circ - 90^\circ - 120^\circ = 60^\circ$.

II. Fill in the blanks with suitable answers : $10 \times 1 = 10$

Q21. If A and B are disjoint sets then $n(A \cap B) =$

Solution:

If A and B are disjoint sets, it means they have no elements in common. Therefore, their intersection $A \cap B$ is the empty set.
The number of elements in the empty set is:
$n(A \cap B) = 0$

Q22. The formula to find the n^{th} term of Harmonic progression is

Solution:

In a Harmonic Progression (HP), the reciprocals form an Arithmetic Progression (AP).
If the n^{th} term of the corresponding AP is $A_n = A + (n-1)d$, then the n^{th} term of HP is:

$$H_n = \frac{1}{A + (n-1)d}$$

Q23. P is a matrix of order 2×3. Then the order of the transposed matrix of P is

Solution:

Given that P is a 2×3 matrix, its transpose (P') is obtained by swapping rows and columns. Thus, the order of $\mathrm{P}^{\top}$ becomes 3×2.

Q24. The HCF of prime expressions is

Solution:

The HCF (Highest Common Factor) of prime expressions is the greatest factor common to all given expressions

Q25. The Σ notation form of $x^2 + y^2 + z^2$ is .

Solution:

The given expression is:
$x^2 + y^2 + z^2$
Using Σ (summation) notation, we represent it as:

$$\sum_{xyz}^{3} x^2$$

Q26. The standard form of an quadratic equation is . .

Solution:

An quadratic equation is a quadratic equation that includes both the linear term and the constant term along with the quadratic term.

The standard form of an quadratic equation is:

$ax^2 + bx + c = 0$

where:

a, b, c are real constants,

$a \neq 0$.

Q27. The angle between the radius and tangent to a circle at the point of contact is equal to

Solution:

According to the tangent-radius theorem, the angle between the radius of a circle and the tangent at the point of contact is always $90°$.

Q28. A straight line drawn parallel to one side of a triangle divides the other two sides

Solution:

According to the Basic Proportionality Theorem (Thales' Theorem), a straight line drawn parallel to one side of a triangle divides the other two sides proportionally

Q29. The curved surface area of a cone of radius r and slant height l is

Solution:

$\pi r l$

Q30. The Euler's formula for polyhedral solid is

Solution:

Euler's formula for a polyhedral solid states:

$V - E + F = 2$

Q31. If the universal set $U = \{1,2,3,4,5,6,7,8,9\}, P = \{2,3,5,7\}$ and $Q = \{1,3,5,7,9\}$ then prove that $(P \cup Q)' = P' \cap Q'$.

Solution:

Given:

Universal set: $U = \{1,2,3,4,5,6,7,8,9\}$

Sets: $P = \{2,3,5,7\}, Q = \{1,3,5,7,9\}$

Compute $P \cup Q$ and its complement

$P \cup Q = \{1,2,3,5,7,9\},\ (P \cup Q)' = U - (P \cup Q) = \{4,6,8\}$

Compute P' and Q' and their intersection

$P' = U - P = \{1,4,6,8,9\},\ Q' = U - Q = \{2,4,6,8\}$

$P' \cap Q' = \{4,6,8\}$

Since $(P \cup Q)' = P' \cap Q'$, the given statement is proved.

Q32. A florist has certain number of garlands. 110 of them have champak flowers, 50 have jasmine flowers and 30 garlands have both the flowers. Find the total number of garlands with him.

Solution:

Let:

A be the set of garlands with champak flowers → 110

B be the set of garlands with jasmine flowers → 50

$A \cap B$ (garlands with both flowers) → 30

Using the principle of inclusion-exclusion:

Total garlands $= |A| + |B| - |A \cap B|$

$= 110 + 50 - 30 = 130$

Final Answer: The florist has 130 garlands.

Q33. In a geometric progression if $S_\infty = \frac{2}{3}$ and $a = 1$ then find the geometric progression

Solution:

In an infinite geometric progression (G.P.), the sum is given by:

$$S_\infty = \frac{a}{1-r}$$

Given:

$$S_\infty = \frac{2}{3}$$

$$a = 1$$

Substituting the values:

$$\frac{1}{1-r} = \frac{2}{3}$$

Solving for r :

$$1 - r = \frac{3}{2} \Rightarrow r = \frac{1}{3}$$

Thus, the geometric progression is:

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$$

Q34. In a geometric progression the 3rd term is 8 times the 6th term, and 4th term is 4 times the 6th term. Find the common ratio of the geometric progression.

Solution:

In a geometric progression, the nth term is given by:

$$a_n = ar^{n-1}$$

Given:

3rd term $= 8 \times 6$ th term

$$ar^2 = 8 \times ar^5$$

4 th term $= 4 \times 6$ th term

$$ar^3 = 4 \cdot ar^5$$

Dividing the first equation by the second:

$$\frac{ar^2}{ar^3} = \frac{8 \times ar^5}{4 \times ar^5}$$
$$\frac{1}{r} = \frac{8}{4} = 2 \Rightarrow r = \frac{1}{2}$$

Q35. Find the value of x if $12, \frac{1}{x-1}, 20$ are in Harmonic progression.

Solution:

In Harmonic Progression (H.P.), the reciprocals form an Arithmetic Progression (A.P.).

Given H.P.:

$$12, \frac{1}{x-1}, 20$$

Taking reciprocals, they form an A.P.

$$\frac{1}{12}, \frac{1}{x-1}, \frac{1}{20}$$

In an A.P the middle term is the arithmetic mean of the first and third terms:

$$\frac{1}{x-1} = \frac{\frac{1}{12} + \frac{1}{20}}{2}$$

Solving:

$$\frac{1}{x-1} = \frac{\frac{5}{60} + \frac{3}{60}}{2} = \frac{8}{120} = \frac{2}{30} = \frac{1}{15}$$

Thus,

$$x - 1 = 15 \Rightarrow x = 16$$

Q36. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}$ then find $A + 2B$

Solution:

Given matrices:

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}$$

First, multiply B by 2 :

$$2B = \begin{bmatrix} 2(1) & 2(2) \\ 2(7) & 2(8) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 14 & 16 \end{bmatrix}$$

Now, add A + 2B:

$$A + 2B = \begin{bmatrix} 3+2 & 4+4 \\ 5+14 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 \\ 19 & 22 \end{bmatrix}$$

Q37. If $a + b + c = abc$ then prove that $1 + a^2 = (1 - ab)(1 - ac)$.

Solution:

Expanding the RHS:

$$(1 - ab)(1 - ac) = 1 - ab - ac + a^2bc$$

Using the given equation $a + b + c = abc$, rewrite bc as:

$$bc = \frac{a+b+c}{a}$$

Substituting in the equation:

$$1 - ab - ac + a^2 \times \frac{a+b+c}{a}$$
$$= 1 - ab - ac + a^2 + a^2b + a^2c - a^2$$
$$= 1 + a^2 - ab - ac$$

Since $a + b + c = abc$, it simplifies to:

$$1 + a^2 = (1 - ab)(1 - ac)$$

Q38. Find the product of $\sqrt[3]{3}$ and $\sqrt[4]{2}$

Solution:

The given expression is:

$$\sqrt[3]{3} \times \sqrt[4]{2} = 3^{\frac{1}{3}} \times 2^{\frac{1}{4}}$$

Since the bases are different, the expression remains as it is.

Q39. Rationalise the denominator and simplify :

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

Solution:

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$
$$\Rightarrow \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2}$$
$$\Rightarrow \frac{5+3+2\sqrt{15}}{5-3}$$
$$\Rightarrow \frac{8+2\sqrt{15}}{2}$$
$$\Rightarrow 4 + \sqrt{15}$$

Q40. What is a pure quadratic equation? Give one example.

Solution:

A pure quadratic equation is a quadratic equation of the form $ax^2 + c = 0$, where the linear term bx is missing (i.e., $b = 0$).

Example:

$$3x^2 - 12 = 0$$

Q41. If m and n are the roots of the quadratic equation $x^2 - 3x + 1 = 0$, then find the value of $\frac{m}{n} + \frac{n}{m}$.

Solution:

Given quadratic equation:
$x^2 - 3x + 1 = 0$
From sum and product of roots:
$m + n = 3,\ mn = 1$
Using the identity:
$$\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn}$$
$m^2 + n^2 = (m+n)^2 - 2mn = 9 - 2 = 7$
Thus,
$$\frac{m}{n} + \frac{n}{m} = \frac{7}{1} = 7$$

Q42. Construct Cayley's table on $A = \{2,4,6,8\}$ under $\oplus$ mod10.

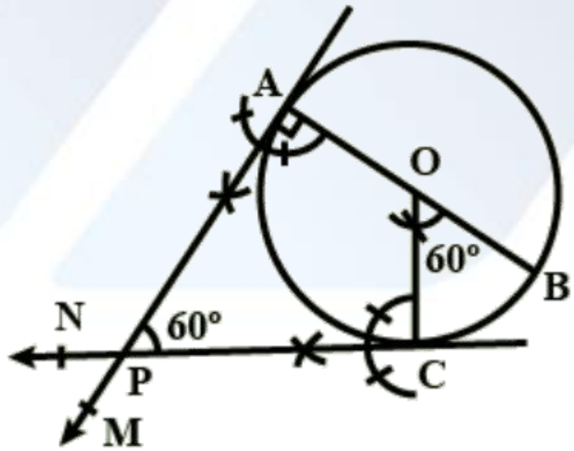
Solution:

To construct Cayley's table for $A = \{2,4,6,8\}$ under $\oplus$ mod10, we use:
$a \oplus b = (a + b)$ mod10

$\oplus$ mod10	2	4	6	8
2	4	6	8	0
4	6	8	0	2
6	8	0	2	4
8	0	2	4	6

Q43. In a circle of radius 3 cm draw two radii such that the angle between them is 60°. Construct tangents at the ends of the radii:

Solution:



Draw a Circle: Use a compass to draw a circle with radius 3 cm and center O.
Draw Two Radii: Mark a point A on the circle and draw OA. Using a protractor, mark a 60° angle at O and draw OB.

Construct Tangents: At points A and B, draw lines perpendicular to OA and OB, as tangents are always perpendicular to the radius at the point of contact.

Q44. A mansion has 12 cylindrical pillars, each having the circumference 50 cm and height 3.5 m . Find the cost of painting the lateral surface of all pillars at Rs. 150 per sq. m.

Solution:

Given the circumference $C = 50$ cm, we use the formula for circumference:

$C = 2\pi r \Longrightarrow r = \frac{C}{2\pi}$

Substitute $C = 50$ cm :

$$r = \frac{50}{2\pi} = \frac{25}{\pi} \text{ cm}$$

The lateral surface area A of a cylinder is given by:

$A = 2\pi r h$

where $h = 3.5$ m $= 350$ cm.

Substitute $r = \frac{25}{\pi}$ cm and $h = 350$ cm :

$A = 2\pi\left(\frac{25}{\pi}\right) \times 350 = 2 \times 25 \times 350 = 17500 \text{ cm}^2$

$A = \frac{17500}{10000} = 1.75 \text{ m}^2$

Multiply the lateral surface area of one pillar by 12 :

Total area $= 12 \times 1.75 = 21 \text{ m}^2$

The cost of painting is Rs. 150 per square meter. Multiply the total area by the cost:

Cost $= 21 \times 150 = 3150$Rs

Q45. 21 spheres of equal radii are melted to form a cylinder of radius 14 cm and height 49 cm. Find the radius of sphere.

Solution:

Volume of 21 spheres = Volume of the cylinder

$21 \times \frac{4}{3}\pi r^3 = \pi \times 14^2 \times 49$

$21 \times \frac{4}{3} r^3 = 14^2 \times 49$

$r^3 = \frac{14^2 \times 49 \times 3}{21 \times 4}$

$r^3 = \frac{14 \times 14 \times 49 \times 3}{84}$

$r^3 = \frac{14 \times 49 \times 3}{6}$

$r^3 = \frac{14 \times 49}{2}$

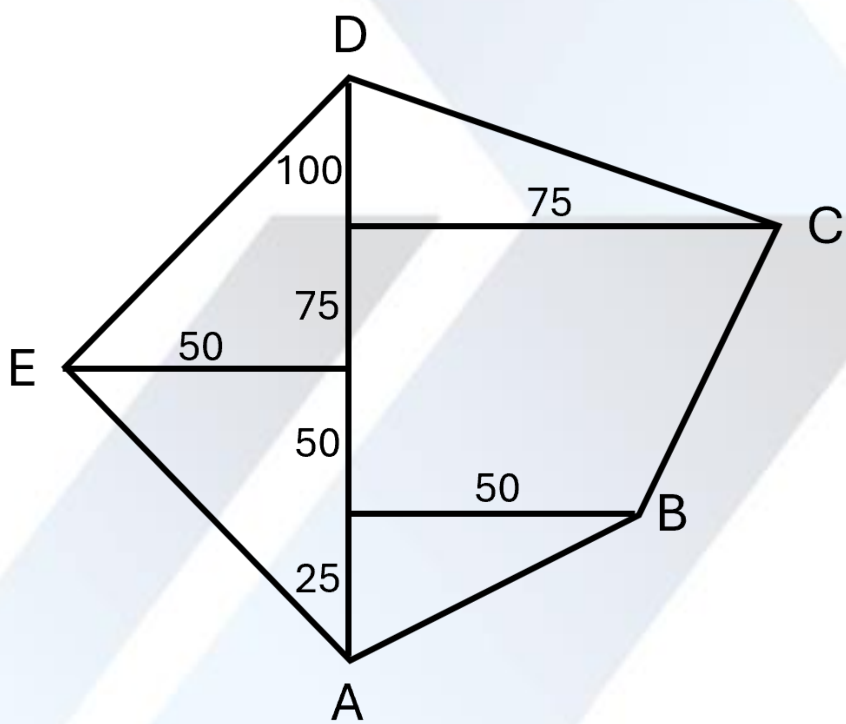
$r^3 = 14 \times 24.5 = 343$

$r = \sqrt[3]{343} = 7$ cm

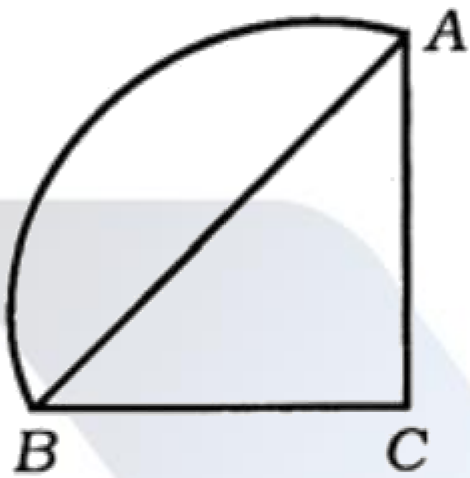
Q46. Draw the plan of the field for the following data. [Scale: 25 m = 1 cm]

	To D (in metres)	
	100	
	75	75 to C
50 to E	50	
	25	50 to B
	From A	

Solution:



Q47. In the given network, write the number of regions and number of arcs.



Solution:

In the given network:

Number of regions: The figure is divided into three regions: one inside the quarter-circle, one inside the right triangle, and one in the overlapping area.

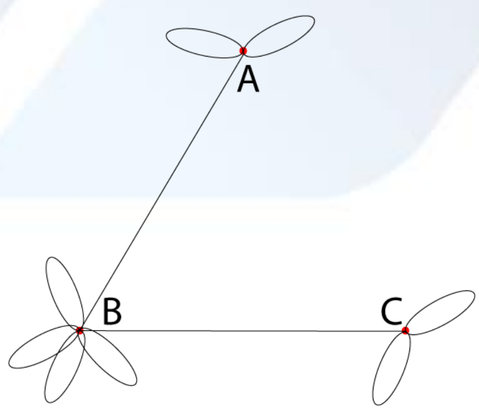
Number of arcs: The diagram contains one curved arc (quarter-circle from B to A) and two straight-line segments (BA and BC). Thus, the total number of arcs is one.

Q48. Draw the graph for the given matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Solution:

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array}$$



Q49. A basket contains 3 white and 5 red flowers. 4 flowers are removed from the basket at random.

a) In how many ways can 4 flowers be removed ?

b) Out of 4 flowers, how many of them may contain 2 white flowers ?

Solution:

Total flowers in the basket = 3 white +5 red = 8

(a) Number of ways to remove 4 flowers:

Since we are selecting 4 flowers from 8 , we use the combination formula:

$$^{8}C_{4}=\frac{8!}{4!\,(8-4)!}=\frac{8!}{4!\,4!}=\frac{8\times7\times6\times5}{4\times3\times2\times1}=70$$

So, 70 ways to remove 4 flowers.

(b) Ways to select 2 white flowers out of 4:

We select 2 white flowers from 3 and 2 red flowers from 5 :

$$^{3}C_{2}\times\ ^{5}C_{2}=\frac{3!}{2!\,1!}\times\frac{5!}{2!\,3!}=\frac{3\times2}{2\times1}\times\frac{5\times4}{2\times1}=3\times10=30$$

So, 30 ways to choose 4 flowers with exactly 2 white flowers.

Q50. Find the standard deviation for the following data

Scores (x)	32	37	42	47	52
Frequency (f)	2	5	6	5	2

Solution:

Scores (x)	Frequency (f)	fx	$x-$ Mean (d)	d^2	
32	2	64	-10	$f(d^2)$	
37	5	185	-5	100	200
42	6	252	0	25	125
47	5	235	5	0	
52	2	104	10	25	
Total	20	840		100	125
					200

Calculate the Mean

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{840}{20} = 42$$

Calculate Variance ($\boldsymbol{\sigma}^2$)

$$\sigma^2 = \frac{\sum fd^2}{\sum f} = \frac{650}{20} = 32.5$$

Find Standard Deviation

$$\sigma = \sqrt{32.5} \approx 5.7$$

Q51. Find the L.C.M. of $a^3 - 3a^2 - 10a + 24$ and $a^3 - 2a^2 - 9a + 18$ by division method

Solution:

Given polynomials:
$f(a) = a^3 - 3a^2 - 10a + 24$
$g(a) = a^3 - 2a^2 - 9a + 18$
Perform polynomial division: Divide $f(a)$ by $g(a)$
Quotient: 1, Remainder: $-a^2 - a + 6$
Divide $g(a)$ by $-a^2 - a + 6$, remainder becomes 0 .
So, GCD $= a^2 + a - 6 = (a+3)(a-2)$.
Use the LCM formula:
$$\text{LCM } (f, g) = \frac{f(a) \times g(a)}{GCD(f, g)}$$
LCM $= (a+3)(a-2)(a-4)(a+2)$

Q52. The perimeter of a right angled triangle is 30 cm and its hypotenuse is 13 cm. Find the length of other two sides of the triangle

Solution:

Let the sides of the right-angled triangle be a, b, and hypotenuse $c = 13$ cm.
The perimeter is given as:
$a + b + 13 = 30$
$a + b = 17$ (Equation 1)
Using Pythagoras theorem:
$a^2 + b^2 = 13^2$
$a^2 + b^2 = 169$ (Equation 2)
Now, solving equations (1) and (2):
Since a and b are the roots of the quadratic equation:
$x^2 - 17x + 169 = 0$
Solving:
$$x = \frac{17 \pm 7}{2} \Rightarrow x = 10,7$$
Thus, the two sides are 7 cm and 10 cm .

Q53. $ABCD$ is a trapezium in which $AB \| CD$ and $BC \perp AB$. If $AB = 7.5$ cm, $AD = 13$ cm and $CD = 12.5$ cm, find the length of BC

Solution:

Given:

$ABCD$ is a trapezium with $AB \parallel CD$ and $BC \perp AB$.

$\mathrm{AB} = 7.5$ cm, $\mathrm{AD} = 13$ cm, $CD = 12.5$ cm.

Since $BC \perp AB$, BCD forms a right triangle with BC as one leg.

Let $BC = h$ and DA be divided into two parts:

$$DC = AB + x + x \Rightarrow x = \frac{(CD - AB)}{2} = \frac{(12.5 - 7.5)}{2} = 2.5 \text{ cm}$$

Now, applying the Pythagoras theorem in $\triangle BCD$:

$BC^2 + (2.5)^2 = (13)^2$

$BC^2 + 6.25 = 169$

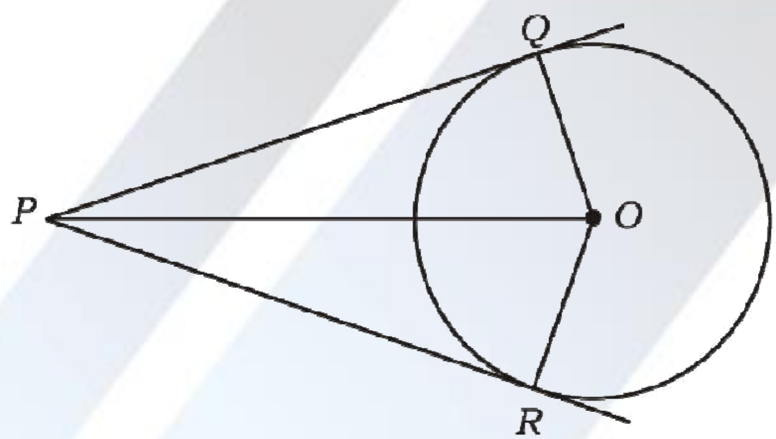
$BC^2 = 162.75$

$BC = \sqrt{162.75} = 12.75$ cm

Thus, the length of BC is 12.75 cm .

Q54. Prove that the tangents drawn from an external point to a circle are equal

Solution:



Given

O is the centre of the circle. PQ and PR are tangents drawn from external point 'P.

To prove: PQ=PR

Construction: Join OP, OQ and OR.

Proof In the figure

$\angle \mathrm{OQP} = \angle \mathrm{ORP} = 90^\circ$	$\begin{bmatrix} OQ \perp PQ \\ OR \perp PR \end{bmatrix}$
OQ=OR	[radii of same circle]
OP=OP	[common side]
$\triangle$ OQP $\cong \triangle$ ORP	
PQ=PR	[CPCT]

Q55. Three numbers are in arithmetic progression and their sum is 18 and the sum of their squares is 140. Find the numbers

Solution:

Let the three numbers in A.P. be $(a-d), a, (a+d)$.

Given, sum of numbers:

$(a-d)+a+(a+d)=18 \Rightarrow 3a=18 \Rightarrow a=6$

Given, sum of squares:

$(6-d)^2+6^2+(6+d)^2=140$

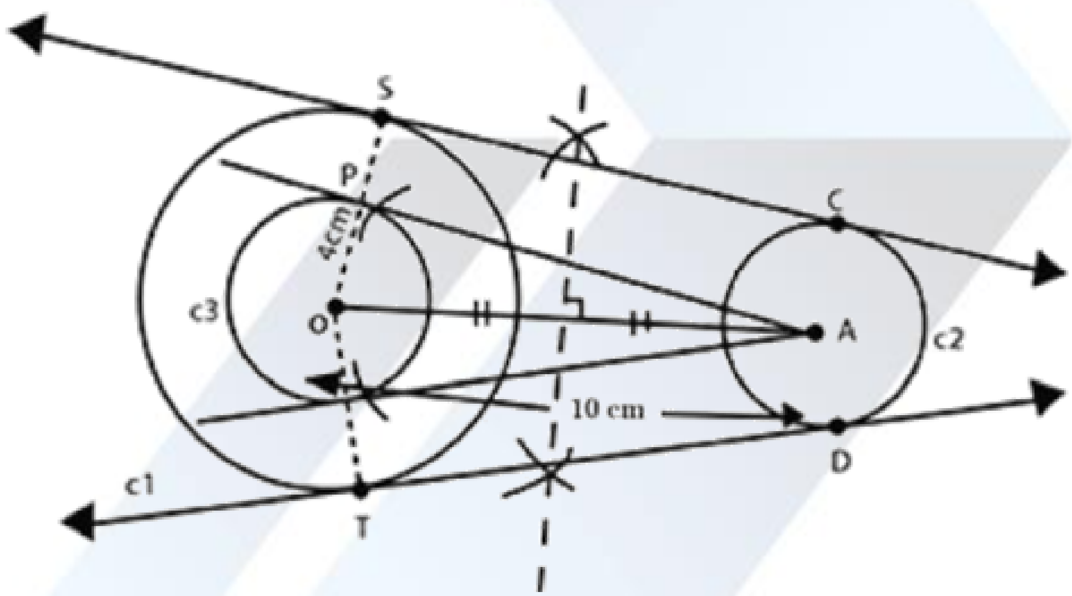
Simplifying:

$2d^2+108=140 \Rightarrow 2d^2=32 \Rightarrow d^2=16 \Rightarrow d=\pm 4$

The numbers are 2,6,10 or 10,6,2.

Q56. Construct two direct common tangents to two circles of radii 4 cm and 2 cm such that the distance between their centres is 10 cm. Measure the length of the tangents and write

Solution:



Draw two circles with radii 4 cm and 2 cm having the centre O and A respectively, so that the centres are 10 cm apart from each other.

Draw a third circle concentric with the first circle with centre O and radius $4-2=2$ cm

Join O - A. Draw perpendicular bisector of OA. With one end of the compass at the midpoint of OA and other end at A, draw ares on the third circle.

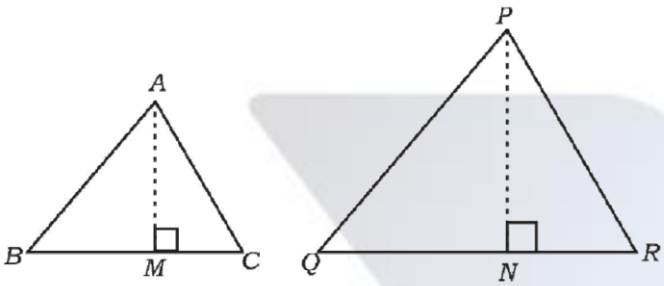
Join A to the intersection of the ares with the circle. Now draw parallel lines to these tangents at a distance of $4-2=2$ cm away from the centre 0 .

CS and DT are the required tangents.

Measure their lengths which come out to be nearly 9.8 cm .

Q57. Prove that the areas of similar triangles are proportional to the squares of their corresponding sides

Solution:



Solution:

Given

$\triangle ABC \sim \triangle PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

To prove:

$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{BC^2}{QR^2}$

Construction: Draw $AM \perp BC$ and $PN \perp QR$

Proof:

$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$

$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{BC}{QR} \times \frac{AM}{PN}$ ……….1

In $\triangle ABM$ and $\triangle PQN$

$\angle B = \angle Q$

$\angle M = \angle N = 90°$ [by construction]

by AA criterion of similarity

$\triangle ABM \sim \triangle PQN$

$\frac{AM}{PN} = \frac{AB}{PQ}$ ………..2

But $\frac{BC}{QR} = \frac{AB}{PQ}$ 3

From 2 and 3

$\frac{AM}{PN} = \frac{BC}{QR}$ ………… 4

Substituting 4 in 1

$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{BC}{QR} \times \frac{BC}{QR}$

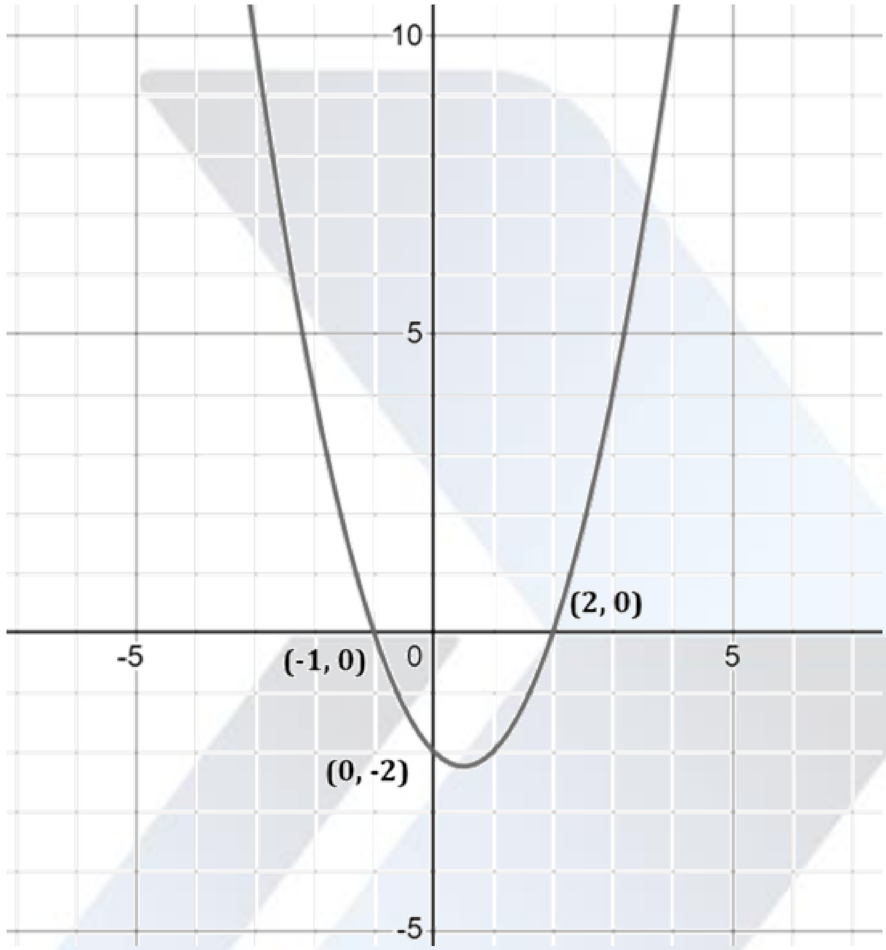
$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{BC^2}{QR^2}$

Hence proved

Q58. Solve graphically

$x^2 - x - 2 = 0$

Solution:



The given quadratic equation is $x^2 - x - 2 = 0$.

Let $y = x^2 - x - 2$, when we substitute different values of x in the equation $y = x^2 - x - 2$ then value of y changes accordingly.

When x = o then

$y = (0)^2 - 0 - 2 = 0 - 0 - 2 = -2$

The point is $(0, -2)$

When x = 2 then

$y = (2)^2 - 2 - 2 = 4 - 2 - 2 = 0$

The point is $(2,0)$

When $x = -1$ then

$y = (-1)^2 + 1 - 2 = 1 + 1 - 2 = 0$

The point is $(-1,0)$

Therefore, the coordinates are $(0, -2)$, $(2,0)$ and $(-1,0)$ and the graph of the quadratic equation $y = x^2 - x - 2$ is as shown above.

Hence, from above graph, we get that $x = -1$ or $x = 2$.