SSLC KARNATAKA-2015

Mathematics

General Instructions to the Candidate:

1. This Question Paper consists of 40 objective and subjective of questions.
2. This questions paper has been sealed by reverse jacket. You have to cut on the right side to open the paper at the time of commencement of the examination. Check whether all the pages of the question paper are intact.
3. Follow the instructions given against both the objective and subjective types of questions.
4. Figures in the right hand margin indicate maximum marks for the questions.
5. The maximum time to answer the paper is given at the top the question paper. It includes 15 for minutes for reading the question paper.

SECTION - A

Four alternatives are given for each of the following questions / incomplete statements. Only one of them is correct or most appropriate. Choose the correct alternative and write the complete answer along with its letter in the space provided against each question.

Q1. If $A = \{a, b, c\}$ and $B = \{c, d, e\}$, then A ∩ B is

(A) $\{a, b, c, d, e\}$

(B) $\{c\}$

(C) $\{a, b\}$

(D) $\{d, e\}$

Solution:

The intersection of two sets A and B, denoted as A∩B, consists of the elements that are common to both sets.

Given:

A = {*a, b, c*}

B = {*c, d, e*}

The only element common to both sets is c.

Thus,

A∩B = {c}

Q2. In a geometric progression, if $T_4 = 10$ and common ratio is 2 then T_3 is

(A) 5

(B) 20

(C) 8
(D) 12

Solution:

Given:

$T_4 = 10$

$r = 2$

We use the formula:

$T_4 = T_1 \times 2^{\{(4-1)\}}$

$10 = T_1 \times 2^3$

$T_1 = \frac{10}{8} = \frac{5}{4}$

Now, to find T_3:

$T_3 = T_1 \times r^{\{(3-1)\}}$

$T_3 = \frac{5}{4} \times 2^2$

$T_3 = \frac{5}{4} \times 4 = 5$

Thus, $T_3 = 5$.

Q3. Which of the following is an identity (unit) matrix?

(A) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

The identity matrix has 1s on the main diagonal and 0s elsewhere.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q4. The value of ${}^5P_1 - {}^3P_0$ is

(A) 4
(B) 2
(C) 3
(D) 0

Solution:

The permutation formula is:

${}^nP_r = \frac{\{n!\}}{\{(n-r)!\}}$

Now, calculating each term:

$^5P_1 = \frac{5!}{(5-1)!} \quad = \frac{5!}{4!} = \frac{5\times4!}{4!} = 5$
$^3P_0 = \frac{3!}{(3-0)!} = \frac{3!}{3!} = 1$
Now, subtracting:
$^5P_1 - {}^3P_0 = 5 - 1 = 4$

Q5. The HCF and LCM of two expressions A and B are H and L respectively. Then their relation can be written as
(A) $\mathrm{A} \times \mathrm{H} = \mathrm{B} \times \mathrm{L}$
(B) $\mathrm{A} \times \mathrm{B} = \mathrm{H} \times \mathrm{L}$
(C) $\frac{\mathrm{A}}{H} = \frac{\mathrm{B}}{\mathrm{L}}$
(D) $\frac{\mathrm{A}}{\mathrm{L}} = \frac{\mathrm{B}}{H}$
Solution:
The relationship between the HCF (H), LCM (L), and the two expressions A and B is given by the fundamental property of HCF and LCM:
$\mathrm{HCF} \times \mathrm{LCM} = \mathrm{A} \times \mathrm{B}$
$\mathrm{H} \times \mathrm{L} = \mathrm{A} \times \mathrm{B}$

Q6. The L.C.M of two expressions $a^2 - 2ab + b^2$ and $a^2 - b^2$ is $(a-b)^2\ (a+b)$. Their H.C.F is
(A) $(a+b)$
(B) $(a-b)$
(C) $(a-b)^2$
(D) (a^2-b^2)
Solution:
We have:
$A = (a-b)^2$ and B= $(a-b)(a+b)$
LCM is given as $(a-b)^2(a+b)$, and the HCF is the common factor $(a-b)$.

Q7. The value of $\sum_{a,b,c}(a-b)$ is
(A) $a+b+c$
(B) 0
(C) $a-b-c$
(D) $a-b$.
Solution:
The given summation $\sum_{a,b,c}(a-b)$ represents the sum of all cyclic permutations of $(a-b)$ over a, b, c.
Expanding, we get:

$(a-b)+(b-c)+(c-a)$
Since all terms cancel out, the result is 0.

Q8. The factors of a^3+b^3 are
(A) $(a+b)(a^2+ab+b^2)$
(B) $(a+b)(a^2+ab-b^2)$
(C) $(a+b)(a^2-ab+b^2)$
(D) $(a+b)(a^2-ab-b^2)$
Solution:
The sum of cubes formula is:
$a^3+b^3=(a+b)(a^2-ab+b^2)$
Thus, the factors of a^3+b^3 are $(a+b)$ and (a^2-ab+b^2).

Q9. The product of $\sqrt{2}$ and $\sqrt[3]{3}$ is
(A) $\sqrt[6]{6}$
(B) $\sqrt[6]{9}$
(C) $\sqrt[6]{72}$
(D) $\sqrt[6]{27}$
Solution:
The product of $\sqrt{2}$ and $\sqrt[3]{3}$ is:
$\sqrt{2}\times\sqrt[3]{3}=2^{\frac{1}{2}}\times 3^{\frac{1}{3}}=2^{\frac{3}{6}}\times 3^{\frac{2}{6}}=8^{\frac{1}{6}}\times 9^{\frac{1}{6}}=72^{\frac{1}{6}}=\sqrt[6]{72}$

Q10. Which of the following is a pure quadratic equation?
(A) $x(x-2)=0$
(B) $x+\frac{1}{x}=5$
(C) $x+\frac{1}{x}=0$
(D) $(x-2)^2=0$
Solution:
The pure quadratic equation in the form of $ax^2+bx+c=0$
So, $(x-2)^2=0 \Rightarrow x^2-4x+4=0$

Q11. Square of a number is added to three times the same number and their sum is 28 . This statement can be written in the equation form as
(A) $3a^2+3a+28=0$
(B) $a^2=3a+28$
(C) $a^2+3a=28$
(D) $3a^2+a=28$

Solution:

Let the number be x.

According to the given condition:

$x^2 + 3x = 28$

Thus, the equation form is $x^2 + 3x - 28 = 0$

Q12. The product of the roots of the equation $x^2 - 5x = 0$ is

(A) 0

(B) 5

(C) -5

(D) 1

Solution:

The given equation is:

$x^2 - 5x = 0$

Factorizing:

$x(x - 5) = 0$

The roots are $x = 0$ and $x = 5$.

The product of the roots is:

$0 \times 5 = 0$

Thus, the product of the roots is 0.

Q13. The nature of the roots of the equation $x^2 - 4x + 4 = 0$ is

(A) Real and equal

(B) real and rational

(C) Real and irrational

(D) imaginary.

Solution:

For the quadratic equation $x^2 - 4x + 4 = 0$, the discriminant is:

$D = b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

Since $D = 0$, the equation has real and equal roots.

Q14. Circles having same centre but different radii are called

(A) Concentric circles

(B) congruent circles

(C) Intersecting circles

(D) touching circles.

Solution:

Circles that have the same center but different radii are called concentric circles.

They do not intersect and are always equidistant from the common center.

Q15. A straight line drawn parallel to the side of a triangle divides the other two sides proportionally. This statement was enunciated by

(A) Thales
(B) Pythagoras
(C) Baudhayana
(D) Appolonius.

Solution:

This statement is known as the Basic Proportionality Theorem (Thales' Theorem).

Q16. Angle between radii at the centre of a circle is 120°. Then the angle between tangents drawn at the ends of the radii is

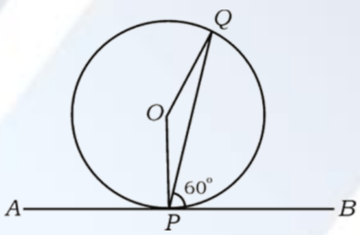
(A) 60°
(B) 120°
(C) 90°
(D) 100°.

Solution:

The angle between the tangents is given by:

$\theta = 180° - central\ angle = 180° - 120° = 60°$

Q17. In the figure APB is a tangent at P to the circle with centre O. If ΔQPB = 60° then ∠POQ is



(A) 60°
(B) 100°
(C) 120°
(D) 150°

Solution:

The angle between the tangent and the chord is equal to half the central angle:

$\angle POQ = 2 \times \angle QPB = 2 \times 60° = 120°$

Q18. The circumference of the base of a right circular cylinder is 44 cm and height is 10 cm. Then its lateral surface area is

(A) 44 cm^2

(B) 400 cm^2

(C) 54 cm^2

(D) 440 cm^2

Solution:

The lateral surface area of a cylinder is given by the formula:

Lateral Surface Area= $2\pi rh$

Given:

Circumference = $2\pi r = 44$, so $r = \frac{44}{2\pi} = 7$

Height $h = 10$ cm

Substituting values:

Lateral Surface Area= $2\pi \times 7 \times 10 = 140\pi \approx 439.82 \text{ cm}^2 = 440 \text{ cm}^2$(approximately)

Q19. The formula used to find the total surface area of a cone is

(A) πrl

(B) $\pi r(r + l)$

(C) 2πrh

(D) $2\pi r(r + h)$

Solution:

The formula to find the total surface area of a cone is:

Total Surface Area= $\pi r(r + l)$)

Where:

r is the radius of the base,

l is the slant height of the cone.

Q20. The shape of each face of a cube is

(A) square

(B) triangular

(C) pentagonal

(D) hexagonal.

Solution:

The shape of each face of a cube is a square.

II. Fill in the blanks with suitable answers:

Q21. If 2,4 and 8 are in geometric progression then the common ratio is.

Solution:

In a geometric progression, the common ratio rr is found by dividing any term by the previous term.

Here, the terms are 2, 4, and 8.

$r = \frac{4}{2} = 2 \ or \ \ r = \frac{8}{4} = 2$

Q22. If A, G, H are Arithmetic mean, Geometric mean and Harmonic mean respectively then $\sqrt{\mathrm{AH}}$ is

Solution:

Since $A \times H = G^2$, we have:

$\sqrt{AH} = G$

Q23. In a right angled triangle, the side opposite to right angle is called.

Solution:

Hypotenuse

Q24. A solid obtained by the rotation of a rectangle about one of its fixed sides is called.

Solution:

Cylinder

Q25. If $\mathrm{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then $\mathrm{A} - \mathrm{A}'$ is.

Solution:

Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, the transpose $A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

So, $A - A^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Q26. If $\sum_{a,b,c} a = 0$ then the value of $\sum_{a,b,c} (a)^3$ is.

Solution:

Since $\sum_{a,b,c} a = 0$, the positive and negative values of aa cancel each other out.

Therefore, $\sum_{a,b,c} a^3 = 0$.

Q27. If the HCF of ab^2 and a^2b is ab then the product of HCF and LCM is. .. .

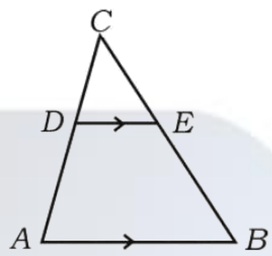
Solution:

Given the HCF of ab^2 and a^2b is ab, the product of the two numbers is

$ab^2 \times a^2b = a^3b^3$.

So, the product of HCF and LCM is a^3b^3.

In the figure DE||AB. If AD = 7, CD = 5 and BC = 18 then CE is.



Solution:

Using the Basic Proportionality Theorem (Thales' Theorem), we have:

$\frac{CE}{BC} = \frac{CD}{AC} \implies \frac{CE}{18} = \frac{5}{12} \implies CE = \frac{15}{2} = 7.5.$

Q28. Two circles of radii 5 cm and 3 cm touch each other internally. Then the distance between their centres is

Solution:

Since the circles touch internally, the distance between their centers is the difference in their radii.

Distance= $5 - 3 = 2$ cm

Q29. A florist has certain number of garlands. 120 of them have Champak, 60 have Jasmine flowers and 30 garlands have both the flowers. Find the total number of garlands with him.

Solution:

Let A be the set having garlands having Champak, therefore $n(A) = 120$.

Let B be the set of garlands having Jasmine, therefore $n(B) = 60$.

A ∩ B is the set of garlands which has both these flowers.

Therefore $(A \cap B) = 30$.

$A \cup B$ is the set of garlands the florist has.

We know,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 60 - 30$

$= 150$

Therefore, the florist has 150 garlands.

Q30. Three terms are in geometric progression. Their product is 8 and common ratio is 2, then find the first term.

Solution:

Let the number in geometric progression be $\frac{a}{r}$, a and ar.
Given, common ratio $(r) = 2$
According to question,
Product of numbers $= 8$
$\Rightarrow \frac{a}{r} \times a \times ar = 8$
$\Rightarrow a^3 = 8$
$\therefore a = 2$
Hence, the first term is $\frac{a}{r} = \frac{2}{2} = 1$

Q31. Find T_{10} in the harmonic progression $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots \dots \dots .$

Solution:

Given sequence is $\frac{1}{2}, \frac{1}{5}, \frac{1}{8},$
Here, first term $= \frac{1}{a} = \frac{1}{2},$
$\therefore a = 2$
Let d be the common difference, then
Second term $= \frac{1}{5}$
$$\Rightarrow \frac{1}{a+d} = \frac{1}{5}$$
$$\Rightarrow \frac{1}{2+d} = \frac{1}{5}$$
$\Rightarrow 5 = 2 + d$
$\therefore d = 3$
Now,
10^{th} term $(T_{10}) =$
$$\frac{1}{[a + (n-1)d]}$$
$$= \frac{1}{[2 + (10-1)3]}$$
$$= \frac{1}{2 + 27}$$
$$= \frac{1}{29}$$

Q32. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$, then find AB.

Solution:

Given, $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$B = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{bmatrix} (1 \times 2 + 2 \times -1) & (1 \times 0 + 2 \times 3) \\ (3 \times 2 + 4 \times -1) & (3 \times 0 + 4 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 0+6 \\ 6-4 & 0+12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 \\ 2 & 12 \end{bmatrix}$$

Q33. Find the value of

(a) ${}^{n}P_0$

(b) ${}^{10}P_1$

(c) ${}^{100}C_0$

(d) ${}^{2015}C_{2015}$

Solution:

a) ${}^{n}P_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

b) ${}^{10}P_1 = \frac{10!}{(10-1)!} = \frac{10!}{9!} = 10$

c) ${}^{100}C_0 = \frac{100!}{(100-0)!0!} = \frac{100!}{100!} = 1$

d) ${}^{2015}C_{2015} = \frac{2015!}{(2015-2015)!2015!} = \frac{2015!}{2015!} = 1$

Q34. Write $\sqrt[3]{5}$ in its exponential form and mention the rational factor, order and radicand.

Solution:

The exponent form of $\sqrt[3]{5}$ is $5^{\frac{1}{3}}$

The rational factor of $\sqrt[3]{5}$ is 125. Its order is third and radicand is 5.

Q35. Rationalise the denominator and simplify: $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution:

Ans. $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{\sqrt{2}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$= \frac{\sqrt{10}+\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{10}+\sqrt{6}}{5-3}$

$= \frac{\sqrt{10}+\sqrt{6}}{2}$

Q36. Express $2x^2 = 5x - 3$ in the standard form of quadratic equation and write the values of a, b and c.

Solution:

Given equation is $2x^2 = 5x - 3$

The standard form of given quadratic equation is $2x^2 - 5x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$ we get

$a = 2, b = -5$ and $c = 3$.

Q37. The product of two consecutive integers is 12. Find the numbers.

Solution:

Let the numbers be x and x+1.

According to question,

$x(x+1) = 12$

$\Rightarrow x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x+4) - 3(x+4) = 0$

$\Rightarrow (x+4)(x-3) = 0$

Either,

$x + 4 = 0$

$\therefore x = -4$

Or

$x - 3 = 0$

$\therefore x = 3$

Hence, the required numbers are 3 and 4.

Q38. Solve $x^2 + 2x - 1 = 0$ by using formula.

Solution:

Given equation is $x^2 + 2x - 1 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 2$ and $c = -1$

We know,

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2 \pm \sqrt{2^2-4\times1\times-1}}{2\times1}$

$= \frac{-2 \pm \sqrt{4+4}}{2}$

$= \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{-2\pm2\sqrt{2}}{2}$

$= \frac{-2+2\sqrt{2}}{2}$ or $\frac{-2-2\sqrt{2}}{2}$

Q39. Form a quadratic equation whose roots are 3 and 2 .

Solution:

Given, roots of a quadratic equation are 3 and 2.

Now,

Sum of roots $= 3 + 2 = 5$

Product of roots $= 2 \times 3 = 6$

Hence,

The Required equation will be $x^2 - 5x + 6 = 0$

Q40. Construct Cayley's table $(Z_4, \oplus_4)$ under addition modulo 4.

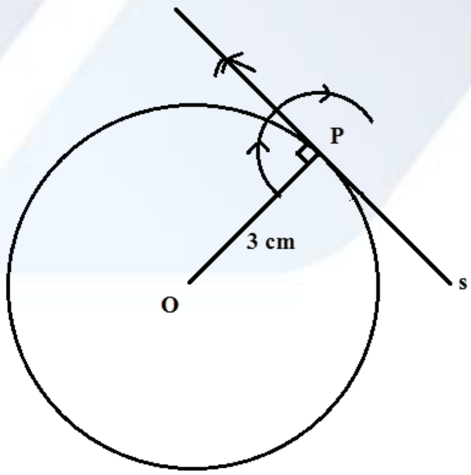
Solution:

The Cayley's table for $(Z_4, \oplus_4)$ under addition modulo 4 is:

$\oplus_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Q41. Construct a tangent to a circle of radius 3 cm at any point P on it.

Solution:



Step of construction:

1. Draw a circle of radius 3 cm with centre O.
2. Take a point P on the circle and Join OP.

3. Make a perpendicular lines at point P such that $AB \perp OP$.
4. Line AB would be tangent of circle at point P.

Q42. Write the formula for
(a) Total surface area of cylinder
(b) Volume of hemisphere.

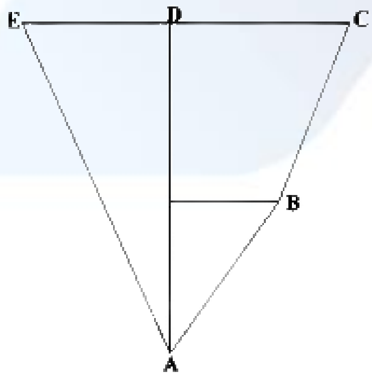
Solution:

a) Total Surface Area of cylinder $= 2\pi$rh
Where, r = radius of the base of cylinder
h $=$ height of the cylinder
b) Volume of hemisphere $= \frac{2}{3}\pi r^3$
Where, r = radius of hemisphere

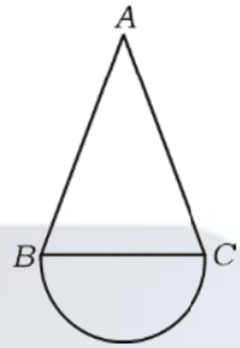
Q43. Draw a plan using the information given below:
[Scale: 50 m $=$ 1 cm]

	To D (in mts)	
To E 100	300 200 150 50	150 to C 100 to B
	From A	

Solution:



Q44. Verify Euler's formula for the graph:



Solution:

In the given figure,

Number of vertices (V) $= 3$

Number of Faces (F) $= 1$

Number of Edges $(E) = 3$

According to Euler's formula,

$V + F - E = 2$

Substituting the values in the formula, we get

$3 + 1 - 3 = 2$

$1 \neq 2$

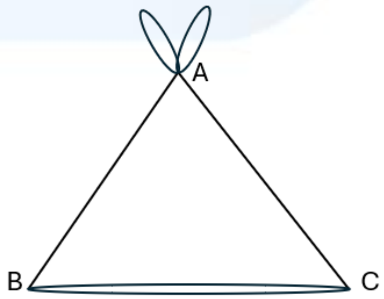
Hence, Euler's formula is not verified.

Q45. Draw a graph (network) for the matrix:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Solution:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$



Q46. There are 7 non-collinear points. How many (a) straight lines and (b) triangles can be drawn by joining these points?

Solution:

a) We have 7 non collinear points.

So,

Number of lines that can be drawn

$= {}^{n}C_2$

$= {}^{15}C_2$

$= 15 \times \frac{14}{2}$

$= 105$

b) A triangle is formed by joining any three non-collinear points in pairs.

There are 7 non-collinear points.

The number of triangles formed,

$= {}^{7}C_3$

$= \frac{\{7 \times (7-1) \times (7-2)\}}{3!}$

$= \frac{(7 \times 6 \times 5)}{(3 \times 2 \times 1)}$

$= 7 \times 5$

$= 35.$

Q47. Calculate the standard deviation for the given frequency distribution table :

Class-interval	Frequency
$1-5$	2
$6-10$	3
$11-15$	4
$16-20$	1

Solution:

Class - interval	Frequency(f)	Mid-point(x)	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
$1-5$	2	3	6	$(3-10)^2 = 49$	98
$6-10$	3	8	24	$(8-10)^2 = 4$	12

11 – 15	4	13	52	$(13-10)^2 = 9$	36
16 – 20	1	18	18	$(18-10)^2 = 64$	64
	$\sum f = 10$		$\sum fx = 100$		$\sum f(x-\bar{x})^2 = 210$

Here, Mean $(\bar{x}) = \frac{\sum fx}{\sum f} = \frac{100}{10} = 10$

$\therefore$ Standard deviation $= \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{210}{10}} = 21$

Q48. Find the H.C.F of $4x^3 - 3x^2 - 24x - 9$ and $8x^3 - 2x^2 - 53x - 39$.

Solution:

Given,

First expression $= 4x^3 - 3x^2 - 24x - 9$

$= 4x^2(x-3) + 9x(x-3) + 3(x-3)$

$= (x-3)(4x^2 + 9x + 3)$

Second expression $= 8x^3 - 2x^2 - 53x - 39$.

$= 8x^2(x-3) + 22x(x-3) + 13(x-3)$

$= (x-3)(8x^2 + 22x + 13)$

Hence, the H.C.F is (x – 3).

Q49. Prove that —tangents drawn to a circle from an external point are equal.

Solution:

Given:-PQ and TQ are two tangent drawn from an external point T to the circle C

To prove:- PT = TQ

∠OTP = ∠OTQ

Construction: join OT

Proof:-

We know that, a tangent to circle is perpendicular to the radius through the point of contact.

So, ∠OPT = ∠OQT = 90°

In △ OPT and △ OQT,

OT=OT (common side)

OP = PQ (radius of the circle)

∠OPT = ∠OQT(90°)

So, OPT ≈ OQT (RHS)

So,
PT = TQ and ∠OTP = ∠OTQ
Hence,
PT = TQ [Since, the lengths of the tangent drawn from an external point to a circle are equal.]
∠OTP = ∠OTQ [Since, centre lies on the bisector of the angle between the two tangent.
Hence proved.

Q50. If $x^2 - 3x + 1 = 0$, then find the value of $x^2 + \frac{1}{x^2}$.

Solution:
Given,
$x^2 - 3x + 1 = 0$
Now,
$$x^2 + \frac{1}{x^2}$$
$$= \left(x + \frac{1}{x}\right)^2 - 2$$
$$= \left(\frac{x^2 + 1}{x}\right)^2 - 2$$
$= \left(\frac{3x}{x}\right)^2 - 2$ [From (i)]
$= 3^2 - 2$
$= 7$

Q51. Show that the area of an equilateral triangle is $\frac{\sqrt{3}}{4}a^2$.

Solution:
Let ABC be an equilateral triangle with sides 'a'. Now, draw AD perpendicular to BC.
Let h be the height.
Here, we have △ ABD =△ ADC.
In △ ABD,
Using Pythagoras theorem,
$$a^2 = h^2 + \left(\frac{a}{2}\right)^2$$
$$\Rightarrow h^2 = a^2 - \frac{a^2}{4}$$
$\Rightarrow h^2 = \frac{3a^2}{4}$.

$\therefore h = \frac{\sqrt{3}}{2} a.$

Now,

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a$$

$$= \frac{\sqrt{3}}{4} a^2$$

Hence, the area of an equilateral triangle is $\frac{\sqrt{3}}{4} a^2$.

Q52. Draw direct common tangents to two circles of radii 4 cm and 2 cm with their centres 9 cm apart and measure their lengths.

Solution:

Given, Radii of two circles are

R = 4 cm and r = 2 cm

Distance between centre of the circles (d) = 9 cm

Let t be the length of the common tangent.

We know,

$$t = \sqrt{d^2 - (R - r)^2}$$

Substituting it the given values, we get

$$t = \sqrt{9^2 - (4 - 2)^2}$$

$$\Rightarrow t = \sqrt{81 - 2^2}$$

$$\Rightarrow t = \sqrt{78}$$

$$\therefore t = 8.8 \text{ cm}$$

Hence, the required length of tangent is 8.8 cm

Q53. Prove that —if two triangles are equiangular then their corresponding sides are proportional.

Solution:

Construction: Two triangles ABC and DEF are drawn so that their corresponding angles are equal. This means:

$\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

To prove: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Draw a line PQ in the second triangle so that $DP = AB$ and $PQ = AC$

Proof:

Since corresponding sides of these two triangles are equal,

$\therefore \triangle ABC \cong \triangle DEF$

So,

∠B = ∠P = ∠E and PQ||EF

$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

Similarly,

$\frac{AB}{DE} = \frac{BC}{EF}$ .... (ii)

Therefore,

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [From (i) and (ii)]

Hence proved.

Q54. Find three numbers in Arithmetic Progression, whose sum and product are 6 and 6 respectively.

Solution:

Let the three numbers be a-d ,a, a+d

According to question,

Sum of numbers = 6

$a - d + a + a + d = 6$

$a + a + a = 6$

3a=6

$\therefore$ a = 2

Also,

Product of numbers = 6

$(a - d) \times a \times (a + d) = 6$

$(a^2 - d^2) \times a = 6$ [$\because (a + b)(a - b) = a^2 - b^2$]

Substituting the value of a, we get

$(2^2 - d^2) \times 2 = 6$

$4 - d^2 = 3$

$d^2 = 1$

$\therefore d = \sqrt{1} = +1$ or -1

Therefore,

If,

a = 2 and d = 1

A.P will be 2,3,4,5,6

Or

If $a = 2$ and $d = -1$

A.P will be 2,1,0,-1,-2..........