Grade 10 Math Karnataka 2016

General Instructions to the Candidate:

1. This Question Paper consists of 40 objective and subjective of questions
2. This questions paper has been sealed by reverse jacket. You have to cut on the right side to open the paper at the time of commencement of the examination . Check whether all the pages of the question paper are intact.
3. Follow the instructions given against both the objective and subjective types of questions.
4. Figures in the right hand margin indicate maximum marks for the questions.
5. The maximum time to answer the paper is given at the top the question paper. It includes 15 for minutes for reading the question paper.

I. Four alternatives are given for each of the following questions / incomplete statements. Only one of them is correct or most appropriate. Choose the correct alternative and write the complete answer along with its letter in the space provided against each question.

Q1. If $T_n = n^2 + 3$ then the value of T_3 is

(A) 6
(B) 9
(C) 12
(D) 27

Solution:

Given $T_n = n^2 + 3$

Substitute $n = 3$ into the formula $T_n = n^2 + 3$:

$T_3 = 3^2 + 3$

$T_3 = 9 + 3$

$T_3 = 12$

Q2. Arithmetic mean of 2 and 8 is

(A) 5
(B) 10
(C) 16
(D) 32

Solution:

The formula for the arithmetic mean of two numbers a and b is:

Arithmetic Mean $= \frac{a+b}{2}$

Here, a = 2 and b = 8. Substituting these values:

Arithmetic Mean $= \frac{2+8}{2}$

Arithmetic Mean$= \frac{10}{2}$

Arithmetic Mean = 5

Q3. If the probability of winning a game is 0.3, then what is the probability of losing it?
(A) 0.1
(B) 0.3
(C) 0.7
(D) 1.3
Solution:
The total probability of winning and losing is 1.
Since P(win)= 0.3, the probability of losing is:
P(lose)= $1 - 0.3 = 0.7$

Q4. The degree of the polynomial $2x^2 - 4x^3 + 3x + 5$ is
(A) 0
(B) 1
(C) 2
(D) 3
Solution:
The degree of a polynomial is the highest power of x in the polynomial.
In $2x^2 - 4x^3 + 3x + 5$, the highest power of x is 3.

Q5. The distance between the origin and the point $(4, -3)$ is
(A) 1 unit
(B) 5 units
(C) 7 units
(D) -12 units.
Solution:
The distance between the origin $(0,0)$ and a point (x, y) is given by:
Distance $= \sqrt{x^2 + y^2}$
Substitute $x = 4$ and $y = -3$y:
Distance $= \sqrt{4^2 + (-3)^2}$
Distance= $16 + 9 = 25 = 5$

Q6. The slope of the straight line whose inclination is 60° is
(A) 0
(B) $\frac{1}{\sqrt{3}}$
(C) $-\sqrt{3}$
(D) $\sqrt{3}$
Solution:
The slope of a straight line is given by slope = tanθ, where θ is the inclination of the line.
For $\theta = 60^\circ$:
Slope $= \tan 60^\circ = \sqrt{3}$.

Q7. If $\sin \theta = \frac{3}{5}$, then the value of cosecθ is

(A) $\frac{4}{5}$

(B) $\frac{5}{3}$

(c) $\frac{4}{3}$

(D) $\frac{5}{4}$

Solution:

The cosec of an angle is the reciprocal of sin:

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Given $\sin\theta = \frac{3}{5}$:

$\operatorname{cosec}\theta = \frac{1}{\frac{3}{5}} = \frac{5}{3}$.

Q8. If the standard deviation of a set of scores is 1.2 and their mean is 10 , then the coefficient of variation of the scores is

(A) 12

(B) 20

(C) 0.12

(D) 120

Solution:

The formula for the coefficient of variation (CV) is:

$CV = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100$

Given standard deviation = 1.2 and mean = 10:

$CV = \frac{1.2}{10} \times 100 = 12$

Q9. If $U = \{1,2,3,4,5\}$ and $A = \{2,4,5\}$ then find A'.

Solution:

$A' = U - A$

$A' = \{1,2,3,4,5\} - \{2,4,5\}$

$\therefore A' = \{1,3\}$

Q10. The H.C.F. of 12 and 18 is 6. Find their L.C.M.

Solution:

Given two numbers are 12 and 8

H.C.F $= 6$

We know,

$\text{H.C.F} \times \text{L.C.M} = \text{Product of two numbers}$

$\Rightarrow 6 \times \text{L.C.M} = 12 \times 8$

$\Rightarrow \text{L.C.M} = \frac{12\times8}{6}$

$\therefore \text{L.C.M} = 16$

Q11. If $f(x) = 2x^2 + 3x + 2$, then find then find the value of $f(2)$.

Solution:

Given $f(x) = 2x^2 + 3x + 2$

$\therefore f(2) = 2(2)^2 + 3 \times 2 + 2$

$= 8 + 6 + 2$

$= 16$

Q12. Two circles of diameters 10 cm and 4 cm, touch each other externally. Find the distance between their centres.

Solution:

Given,

Diameter of circle (d) = 10 cm

$\therefore$ Radius of circle (R) $= \frac{10}{2} = 5$ cm

Diameter of another circle (d) = 10 cm

$\therefore$ Radius of circle (r) $= \frac{4}{2} = 2$ cm

Using formula, we get

Distance between centres = R + r

$= 5 + 2$

$= 7$ cm

Q13. State Pythagoras theorem.

Solution:

In a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

Q14. Write the formula to find the total surface area of a cylinder.

Solution:

Total surface area of cylinder $= 2\pi r(r + h)$ sq. units

Where, r = radius of the base of cylinder

h = height of the cylinder

Q15. Calculate the maximum number of diagonals that can be drawn in an octagon using the suitable formula.

Solution:

For a diagonal, we need 2 points.

Number of vertices in octagon (n) = 8

$\therefore$ Number of diagonals $= {}^8C_2 - 8$

$= \frac{8!}{2!6!} - 8$

$= \frac{8 \times 7}{2} - 8$

$= 28 - 8$

$= 20$

Q16. Prove that $2+\sqrt{5}$ is an irrational number.

Solution:

Let us assume that $2+\sqrt{5}$ is a rational number.

$2+\sqrt{5}=\frac{p}{q}$ where $p, q \in z, q \neq 0$,

$$\Rightarrow 2-\frac{p}{q}=-\sqrt{5}$$

$$\Rightarrow \frac{2q-p}{q}=-\sqrt{5}$$

Here, $-\sqrt{5}$ must be a rational number

$\because \frac{2q-p}{q}$ is a rational number

But, $-\sqrt{5}$ is not a rational number.

$\therefore$ Our assumption that $2+\sqrt{5}$ is a rational number is wrong.

Hence, $2+\sqrt{5}$ is an irrational number.

Q17. There are 500 wrist watches in a box. Out of these 50 wrist watches are found defective. One watch is drawn randomly from the box. Find the probability that a wristwatch chosen is a defective watch.

Solution:

Total number of watches $n(S)=500$

Number of defective watches $n(A)=50$

$\therefore$ Probability of watches to be defective $=\frac{n(A)}{n(S)}$

$$=\frac{50}{500}=\frac{1}{10}$$

Q18. Find the product of $\sqrt{3}$ and $\sqrt[3]{2}$.

Solution:

Given numbers are $\sqrt{3}$ and $\sqrt[3]{2}$.

Here,

L.C.M of 2 and 3 is 6

$\sqrt{3}=3^{\frac{1}{2}\times\frac{6}{6}}=3^{\frac{3}{6}}=\sqrt[6]{3^3}=\sqrt[6]{27}$

$\sqrt[3]{2}=2^{\frac{1}{2}\times\frac{6}{6}}=2^{\frac{2}{6}}=\sqrt[6]{2^2}=\sqrt[6]{4}$

$\therefore \sqrt{3}\times\sqrt[3]{2}=\sqrt[6]{27\times 4}$

$=\sqrt[6]{108}$

Q19. Rationalise the denominator and simplify : $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Solution:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{3+2\sqrt{3}\cdot\sqrt{2}+2}{(\sqrt{3})^2-(\sqrt{2})^2} \quad \left[\begin{array}{l}\because (a+b) = a^2+2ab+b^2 \\ \& \ (a+b)(a-b) = a^2-b^2\end{array}\right]$$

$$= \frac{5+2\sqrt{6}}{3-2}$$

$$= 5+2\sqrt{6}$$

Q20. Find out the quotient and the remainder when P(x) = x^3+4x^2-5x+6 is divided by $g(x) = x+1$

Solution:

$$\begin{array}{r|l} & x^2+3x-8 \\ \hline x+1 & x^3+4x^2-5x+6 \\ & x^3+1x^2 \\ & (-)\quad(-) \\ \hline & +3x^2-5x+6 \\ & +3x^2+3x \\ & (-)\quad(-) \\ \hline & -8x+6 \\ & -8x-8 \\ & (+)(+) \\ \hline \end{array}$$

$q(x) = x^2+3x-8$

$R(x) = 14$

OR

Find the polynomial which is to be added to $x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is exactly divisible by x^2+2x-3.

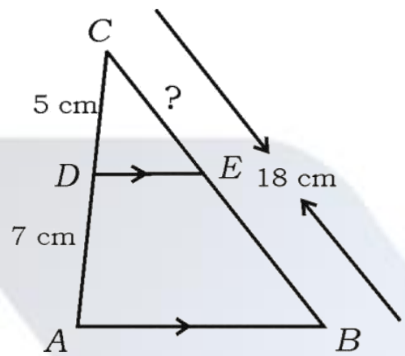
$$\begin{array}{r|l} & x^2+1 \\ \hline x^2+2x-3 & x^4+2x^3-2x^2+x-1 \\ & x^4+2x^3-3x^2 \\ & (-)\quad(-)\quad(+) \\ \hline & x^2+x-1 \\ & x^2+2x-3 \\ & (-)\quad(-)\quad(+) \\ \hline & -x+2 \end{array}$$

$r(x) = -x+2 \quad \Rightarrow \{-r(x)\} = x-2$

Hence, we should add ($x-2$) to $P(x)$

so that the resulting polynomial is exactly divisible by g(x).

Q21. In the following figure, $DE \| AB$. If $AD = 7$ cm, $CD = 5$ cm and $BC = 18$ cm, find CE.



Solution:

In $\triangle$ ABC, DE∥AB

Given, AD = 7 cm, CD = 5 cm and BC = 18 cm

$\frac{CD}{CA} = \frac{CE}{CB}$ [Corollary to B.P.T]

$\Rightarrow \frac{5}{12} = \frac{CE}{18}$

$\Rightarrow CE = \frac{5 \times 18}{12}$

$\therefore CE = 7.5$ cm

Q22. Given $\sqrt{3}\tan\ \theta = 1$ and θ is an acute angle. Find the value of $\sin\ 3\theta$.

Solution:

Given,

$\sqrt{3}\tan\ \theta = 1$

$\Rightarrow \tan\ \theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\ \theta = \tan\ 30^\circ$

$\therefore \theta = 30^\circ$

Putting the value of θ,

$\sin 3\theta = \sin\ (3 \times 30^\circ)$

$= \sin 90^\circ$

$= 1$

Q23. Find the coordinates of the mid-point of the line segment joining the points (2, 3) and (4, 7).

Solution:

Given,

$(2,3) = (x_1, y_1)$

$(4,7) = (x_2, y_2)$

We know, formula for

Co-ordinates of mid-point $= \left\{\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right\}$

$= \left\{\frac{2+4}{2}, \frac{3+7}{2}\right\}$
$= \left\{\frac{6}{2}, \frac{10}{2}\right\}$
$= (3, 5)$

Q24. The radius of a cone is 7 cm and its slant height is 10 cm. Calculate the curved surface area of the cone.

Solution:

Given, radius $(r) = 7$ cm
Slant height $(l) = 10$ cm
$\therefore$ Curved surface area of cone (C.S.A) $= \pi r l$
$= \frac{22}{7} \times 7 \times 10$
$= 220 \text{ cm}^2$

OR

Calculate the volume of a right circular cylinder whose radius is 7 cm and height is 10 cm.

Solution: Given, radius $(r) = 7$ cm
height $(h) = 10$ cm
$\therefore$ Volume of cylinder $= \pi r^2 h$
$= \frac{22}{7} \times 7^2 \times 10$
$= 22 \times 7 \times 10$
$= 1540 \text{ cm}^3$

Q25. Solve the quadratic equation $x^2 - 4x + 2 = 0$ by formula method.

Solution:

Given equation is $x^2 - 4x + 2 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
$a = 1, b = -4 \text{ and } c = 2$
We know,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 2}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{16 - 8}}{2}$$
$$= \frac{4 \pm \sqrt{8}}{2}$$
$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$= \frac{4+2\sqrt{2}}{2}$ or $\frac{4-2\sqrt{2}}{2}$

$= \frac{2(2+\sqrt{2})}{2}$ or $\frac{2(2-\sqrt{2})}{2}$

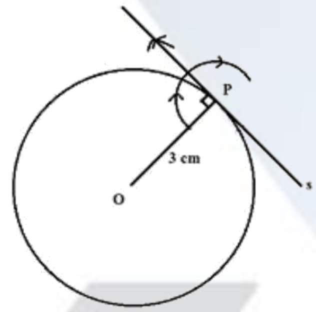
$= 2+\sqrt{2}$ or $2-\sqrt{2}$

Q26. Construct a tangent at any point P on a circle of radius 3 cm .

Solution:

Step of construction:

1. Draw a circle of radius 3 cm with centre O.
2. Take a point P on the circle and Join OP.
3. Make a perpendicular line s at point P such that $AB \perp OP$.
4. Line AB would be tangent of circle at point P.



Q27. Draw a plan using the information given below:

[Scale: 20 m = 1 cm]

	Metre to D	
	160	
	120	60 to C
40 to E	80	
	40	40 to B
	From A	

Solution:

Scale: 20 m = 1 cm

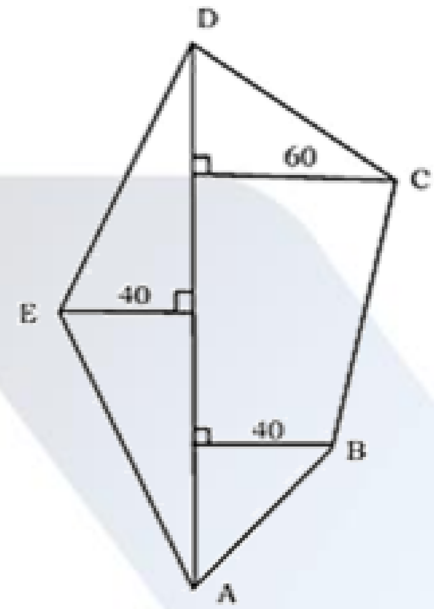
40 m = 2 cm

60 m = 3 cm

80 m = 4 cm

120 m = 5 cm

160 m = 6 cm



Q28. In a group of people, 12 people know music, 15 people know drawing and 7 people know both music and drawing. If people know either music or drawing then calculate the number of people in the group.

Solution:

Let M denote music and D denote drawing,

Then, Given $n(M) = 12, n(D) = 15$ and $n(M \cap D) = 7$

$\therefore n(M \cup D) = n(M) + n(D) - n(M \cap D)$

$= 12 + 15 - 7$

$= 20$

Therefore, total number of people in group is 20.

Q29. A solid hemisphere of wax of radius 12 cm is melted and made into a cylinder of its base radius 6 cm. Calculate the height of the cylinder.

Solution:

Given, Radius of hemisphere $(R) = 12$ cm

Radius of cylinder $(r) = 6$ cm

Let h be the height of cylinder.

According to question,

Volume of hemisphere = Volume of cylinder

$\Rightarrow \frac{2}{3}\pi R^3 = \pi r^2 h$

$\Rightarrow \frac{2}{3} \times (12)^3 = (6)^2 \times h$

$\Rightarrow \frac{2}{3} \times 12 \times 12 \times 12 = 6 \times 6 \times h$

$\Rightarrow h = \frac{2 \times 4 \times 12 \times 12}{6 \times 6}$

$\therefore h = 32$ cm

Q30. Find the sum of first 20 terms of the series $4 + 7 + 10 + \dots \dots$

Solution:

Given series is $4 + 7 + 10 + \dots$.

Here, first term $(a) = 4$

Common difference $(d) = 7 - 4 = 3$

We know,

Sum of n terms $= \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of 20 terms $(S_{20}) = \frac{20}{2}[2 \times 4 + (20-1)3]$

$= 10[8 + 19 \times 3]$

$= 10[8 + 57]$

$= 10 \times 65$

$= 650$

Q31. Prove that -If two circles touch each other externally then their centres and the point of contact are collinear.

Solution:

Given: A and B are the centres of touching circles. P is point of contact.

To proof: A, P and B are collinear

Construction: Draw the common tangent XPY

Proof:

$\angle APX = 90^\circ \quad [AP \perp XY]$

$\angle BPX = 90^\circ$

$[BP \perp XY]$

Adding (i) and (ii), we get

$\angle APX + \angle BPX = 180^\circ$

$\therefore \angle APB = 180^\circ$

APB is a straight line.

Hence, the points A, P and B are collinear.

Q32. Calculate the standard deviation for the following data:

Class-interval	Frequency
$1-5$	4
$6-10$	3
$11-15$	2
$16-20$	1
	$N = 10$

Solution:

Class-interval	Frequency (f)	Mid-point (x)	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
$1-5$	4	3	12	$(3-8)^2 = 25$	100
$6-10$	3	8	24	$(8-8)^2 = 0$	0
$11-15$	2	13	26	$(13-8)^2 = 25$	50
$16-20$	1	18	18	$(18-8)^2 = 100$	100
	$\sum f = 10$		$\sum fx = 80$		$\sum f(x-\bar{x})^2 = 250$

Here, Mean $(\bar{x}) = \frac{\sum fx}{\sum f} = \frac{80}{10} = 8$

$\therefore$ Standard deviation $= \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{250}{10}} = \sqrt{25} = 5$

Q33. Find how many 4 -digit numbers can be formed using the digits 1,2,3,4,5,6 without repetition of the digits. Find out how many of these are less than 5000?

Solution:

Given digits are 1, 2, 3, 4, 5, 6

(a) 4-digit number can be formed in ${}^6P_4$ ways

$\therefore\ {}^6P_4 = \frac{6!}{(6-4)!}$

$= \frac{6!}{2!}$

$= 6 \times 5 \times 4 \times 3$

$= 360$

b) Less than 5000

Unit's place can be filled in ${}^3P_1$ ways

Ten's place can be filled in ${}^4P_1$ ways

Hundred's place can be filled in ${}^5P_1$ ways

Thousand's place can be filled in ${}^4P_1$ ways

Total number of ways $= {}^3P_1 \times {}^4P_1 \times {}^5P_1 \times {}^4P_1$

$= 3 \times 4 \times 5 \times 4$

$= 240$

Hence, the number less than 5000 $= 240$

OR

If $16 \times {}^nP_3 = 13 \times {}^{n+1}P_3$ then find n.

Solution: $16 \times {}^nP_3 = 13 \times {}^{n+1}P_3$

$16n(n-1)(n-2) = 13(n+1)n(n-1)$

$16(n-2) = 13(n+1)$

$16n-32=13n+13$
$16n-13n=13+32$
$3n=45$
$\therefore n=15$

Q34. Prove that:

$$\frac{\sin(90^\circ-\theta)}{1+\sin\theta}+\frac{\cos\theta}{1-\cos(90^\circ-\theta)}=2\sec\theta$$

Solution:

$\text{L.H.S}=\frac{\sin(90^\circ-\theta)}{1+\sin\theta}+\frac{\cos\theta}{1-\cos(90^\circ-\theta)}$

$=\frac{\cos\theta}{1+\sin\theta}+\frac{\cos\theta}{1-\sin\theta}$

$=\frac{\cos\theta(1-\sin\theta)+\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$=\frac{\cos\theta-\cos\theta\sin\theta+\cos\theta+\cos\theta\sin\theta}{1-\sin^2\theta}$ $\quad[\because (a+b)(a-b)=a^2-b^2]$

$=\frac{2\cos\theta}{\cos^2\theta}$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

$=\frac{2}{\cos\theta}$

$=2\sec\theta$

$=\text{R.H.S}$

OR

If $A=60^\circ, B=30^\circ$ then verify that $\cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B$.

Solution:

Given, $A=60^\circ$ and $B=30^\circ$

Now,

$\cos(A+B)=\cos A\times\cos B-\sin A\times\sin B$

$\Rightarrow \cos(60^\circ+30^\circ)=\cos 60^\circ\times\cos 30^\circ-\sin 60^\circ\times\sin 30^\circ$

$\Rightarrow \cos 90^\circ=\frac{1}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\times\frac{1}{2}$

$\Rightarrow 0=\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}$

$\therefore 0=0$

Hence verified.

Q35. Pupils of X^{th} Standard of a school had arranged for a function at a total cost of Rs. 1,000 which was to be shared equally among them. Since 10 of them failed to join the function each of them had to pay Rs. 5 more. Find the number of pupils in the class.

Solution:

Let the number of pupils in the class be x.

Total Amount = 1000

Cost each student had to pay = $\frac{1000}{x}$

Since 10 students have failed to join;

New amount = previous amount + 5

$$\frac{1000}{(x-10)} = \frac{1000}{x} + 5$$

$$\Rightarrow \frac{1000}{(x-10)} = \frac{1000+5x}{x}$$

$\Rightarrow 1000x = (1000+5x)(x-10)$

$\Rightarrow 1000x = 1000x - 10000 + 5x^2 - 50x$

$\Rightarrow 5x^2 - 50x - 10000 = 0$

$\Rightarrow x^2 - 10x - 2000 = 0$

$\Rightarrow x^2 - 50x + 40x - 2000 = 0$

$\Rightarrow x(x-50) + 40(x-50) = 0$

$\Rightarrow (x-50)(x+40) = 0$

$\therefore x = 50$ or -40

Since, the number of pupils can't be negative,

Therefore, total number of pupils is 50.

OR

If m and n are the roots of the equation $x^2 - 5x + 3 = 0$, find the values of

i. $(m+n)^2 + (m-n)^2$

ii. $(m+n)^3 + 4mn$

Solution:

Given equation is $x^2 - 5x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = -5$ *and* $c = 3$

Roots of the equation are m and n.

By the property of roots,

$$m + n = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$mn = \frac{c}{a} = \frac{3}{1} = 3$$

i. $(m+n)^2 + (m-n)^2$

$= (m+n)^2 + [(m+n)^2 - 4mn]$

$= 5^2 + [5^2 - 4 \times 3]$

$= 25 + [25 - 12]$

$= 25 + 13$

$= 38$

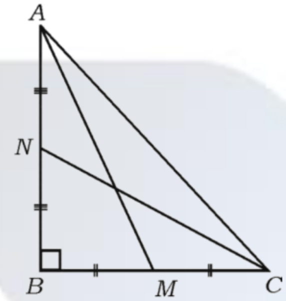
ii. $(m+n)^3 + 4mn$

$= (5)^3 + 4 \times 3$

$= 125 + 12$

$= 137$

Q36. In the right-angled triangle ABC, $\angle ABC = 90°$. AM and CN are the medians drawn from A and C respectively to BC and AB. Show that $4(AM^2 + CN^2) = 5AC^2$.



Solution:

Given the right-angled triangle ABC in which $\angle ABC = 90°$.

According to question,

$\Rightarrow AM^2 + CN^2 = AB^2 + BM^2 + BN^2 + BC^2$

$\Rightarrow AM^2 + CN^2 = AC^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2$

$\Rightarrow AM^2 + CN^2 = AC^2 + \frac{BC^2}{4} + \frac{AB^2}{4}$

$\Rightarrow AM^2 + CN^2 = \frac{4AC^2 + BC^2 + AB^2}{4}$

$\Rightarrow 4(AM^2 + CN^2) = 4AC^2 + BC^2 + AB^2$

$\Rightarrow 4(AM^2 + CN^2) = 4AC^2 + AC^2$

$\therefore 4(AM^2 + CN^2) = 5AC^2$

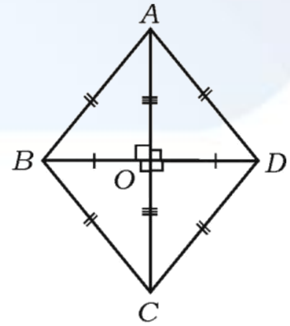
$\Rightarrow AM^2 + CN^2 = AC^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2$ $[AB^2 + BC^2 = AC^2]$

Hence proved.

OR

In the rhombus, show that $4AB^2 = AC^2 + BD^2$

ABCD



Solution:

Here, we see that,

$AO = \frac{1}{2}AC$

$\mathrm{BO} = \frac{1}{2}BD$

In triangle ABO,

According to Pythagoras theorem,

$\mathrm{AO}^2 + \mathrm{BO}^2 = \mathrm{AB}^2$

$\Rightarrow \left(\frac{1}{2}\mathrm{AC}\right)^2 + \left(\frac{1}{2}\mathrm{BD}\right)^2 = \mathrm{AB}^2$

$\Rightarrow \frac{1}{4}\mathrm{AC}^2 + \frac{1}{4}\mathrm{BD}^2 = \mathrm{AB}^2$

$\Rightarrow \frac{1}{4}(\mathrm{AC}^2 + \mathrm{BD}^2) = \mathrm{AB}^2$

$\therefore 4\mathrm{AB}^2 = \mathrm{AC}^2 + \mathrm{BD}^2$

Hence proved.

Q37. Prove that: If two triangles are equiangular then their corresponding sides are in proportion.

Solution:

Construction: Two triangles ABC and DEF are drawn so that their corresponding angles are equal. This means:

$\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

To prove: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Draw a line PQ in the second triangle so that $\mathrm{DP} = \mathrm{AB}$ and $\mathrm{PQ} = \mathrm{AC}$

Proof:

Since corresponding sides of these two triangles are equal,

$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{DEF}$

So,

$\angle B = \angle P = \angle E$ and $\mathrm{PQ} \| \mathrm{EF}$

$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$ ... (i)

Similarly,

$\frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{BC}}{\mathrm{EF}}$ ....(ii)

Therefore,

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [From (i) and (ii)]

Hence proved.

Q38. Draw two direct common tangents to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart. Measure the length of the tangents.

Solution:

Given, Distance between centres of two circle = 8 cm

Radius of one circle (R) = 4 cm

Radius of another circle (r) = 2 cm

Now, $\mathrm{R} - \mathrm{r} = 4 - 2 = 2$ cm

Steps of construction:

1. Draw AB and mark mid - point M .

2. Draw circle C_1, C_2, C_3
3. Join BK, BL, PQ and RS
PQ and RS are the required tangents.
Measure and write the length of tangents i.e PQ = RS = 7.8 cm

Q39. In an arithmetic progression, the sum of first term, third term and the fifth term is 39 and the sum of second term, fourth term and the sixth term is 51. Find the tenth term of the sequence.

Solution:

Let the first term of the A.P be a and common difference be d.
Sum of First term, third term and the fifth term = 39 and
Sum of second term, fourth term and the sixth term = 51.
According to question,
Using formula for nth term , $t_n = a + (n-1)d$, we get
$a + a + 2d + a + 4d = 39$
$\Rightarrow 3a + 6d = 39$
And,
$a + d + a + 3d + a + 5d = 51$
Solving (i) and (ii), we get
$3a + 9d - 3a - 6d = 51 - 39$
$\Rightarrow 3d = 12$
$\therefore d = 4$
Now,
Putting value of d in (i), we get
$3a + 6 \times 4 = 39$
$\Rightarrow 3a + 24 = 39$
$\Rightarrow 3a = 15$
$\therefore a = 5$
Therefore,
Tenth term $(t_{10}) = a + (10-1)d$
$= 5 + 9 \times 4$
$= 5 + 36$
$= 41$

OR

In geometric progression, the sum of the first 3 terms is 7 and the sum of the next 3 terms is 56. Find geometric progression.

Solution:

Let the G.P. be $a, ar, ar^2, ar^3, \ldots$..
According to the given condition,
$a + ar + ar^2 = 7$
$\Rightarrow a(1 + r + r^2) = 16$
And
$ar^3 + ar^4 + ar^5 = 56$

$\Rightarrow ar^3(1 + r + r^2) = 56$
$\Rightarrow ar^3 \times \frac{7}{a} = 56$
$\Rightarrow r^3 = \frac{56}{7}$
$\Rightarrow r^3 = 8$
$\Rightarrow r = 2^3$
$\therefore r = 2$
Putting the value of r in (1), we get
$(1 + 2 + 2^2) = \dfrac{7}{a}$
$\Rightarrow 7 = \dfrac{7}{a}$
$\therefore a = 1$
So,
$ar = 1 \times 2 = 2$
$ar^2 = 1 \times (2)^2 = 4$
$ar^3 = 1 \times (2)^3 = 8$

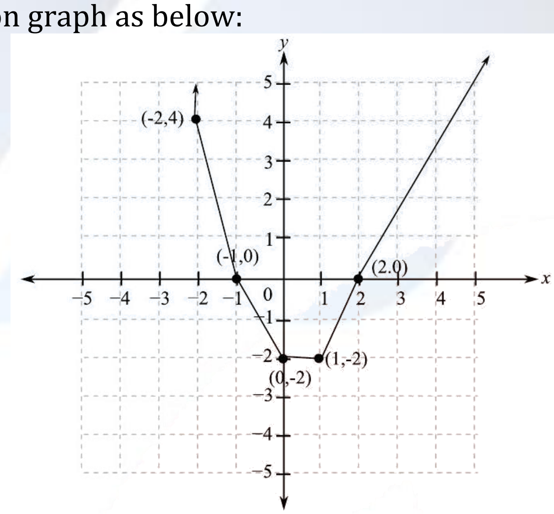
Q40. Therefore, the required geometric progression is 1,2,4,8 ....
Solve the equation graphically:
$x^2 - x - 2 = 0$

Solution:

$y = x^2 - x - 2$
Now for different values of x, we will find the values of y

x	1	2	0	-1	-2
y	-2	0	-2	0	4

Now we will plot it on graph as below:



Therefore, the solution points are -1 and 2