Grade 10 Karnataka Maths 2017

SECTION – I

Q1. If $U = \{1,2,3,4,5,6,7,8\}$, $A = \{1,2,3\}$ and $B = \{2,3,4,5\}$, then $(A \cup B)'$ is

(A) $\{5,6,7\}$
(B) $\{6,7,8\}$
(C) $\{3,4,5\}$
(D) $\{1,2,3\}$

Solution:

Correct answer: (B)

Given,

$U = \{1,2,3,4,5,6,7,8\}$

$A = \{1,2,3\}$

$B = \{2,3,4,5\}$

$A \cup B = \{1,2,3\} \cup \{2,3,4,5\}$

$= \{1,2,3,4,5\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4,5,6,7,8\} - \{1,2,3,4,5\}$

$= \{6,7,8\}$

Q2. LCM of 18 and 45 is

(A) 9
(B) 45
(C) 90
(D) 81

Solution:

Correct answer: (C)

Prime factorization of 18:

$18 = 2 \times 3 \times 3$

Prime factorization of 45:

$45 = 3 \times 3 \times 5$

$\text{LCM}(18,45) = 2 \times 3 \times 3 \times 5 = 90$

Q3. The mean $(\bar{x})$ and the standard deviation (σ) of certain scores are 60 and 3 respectively. Then the coefficient of variation is

(A) 5
(B) 6

(C) 7
(D) 8
Solution:
Correct answer: (A)
Given,
Mean = 60
Standard deviation = 3
Coefficient of variation = $\left(\frac{standard\ deviation}{mean}\right) \times 100$
$= \left(\frac{3}{60}\right) \times 100$
= 5

Q4. Rationalising factor of $\sqrt{x-y}$ is
(A) $x-y$
(B) $\sqrt{x}$
(C) $\sqrt{x+y}$
(D) $\sqrt{x-y}$
Solution:
Correct answer: (D)
Rationalising factor of $\sqrt{\boldsymbol{x-y}}$ is $\sqrt{\boldsymbol{x-y}}$.
Since $(\sqrt{\boldsymbol{x-y}})(\sqrt{\boldsymbol{x-y}})$
$= [\sqrt{(\boldsymbol{x-y})}]^2$
$= \boldsymbol{x-y}$

Q5. If $f(x) = x^2 - 2x + 15$ then $f(-1)$ is
(A) 14
(B) 18
(C) 15
(D) 13
Solution:
Correct answer: (B)
Given,
$f(x) = x^2 - 2x + 15$
$f(-1) = (-1)^2 - 2(-1) + 15$
$= 1 + 2 + 15$
$= 18$

Q6. In a circle, the angle subtended by a chord in the major segment is

(A) a straight angle

(B) a right angle

(C) an acute angle

(D) an obtuse angle

Solution:

Correct answer: (C)

In a circle, the angle subtended by a chord in the major segment is an acute angle.

Q7. The length of the diagonal of a square of side 12 cm is

(A) $5\sqrt{2}$ cm

(B) 144 cm

(C) 24 cm

(D) $12\sqrt{2}$ cm

Solution:

Correct answer: (D)

Given,

Side of a square $=$ a $=$ 12 cm

Diagonal of square $= a\sqrt{2} = 12\sqrt{2}$ cm

Q8. The distance between the origin and the point $(-12,5)$ is

(A) 13 units

(B) -12 units

(C) 10 units

(D) 5 units

Solution:

Correct answer: (A)

The distance of a point (x, y) from the origin is $\sqrt{(x^2 + y^2)}$.

The distance between the origin and the point $(-12,5) = \sqrt{[(-12)^2 + 5^2]}$

$= \sqrt{(144 + 25)}$

$= \sqrt{169}$

$= 13$ units

SECTION - II

Q9. Write the value of ${}^{100}\mathrm{P}_0$.

Solution:

We know that,

${}^{n}\mathrm{P}_0 = 1$

Therefore, ${}^{100}P_0 = 1$

(or)

$${}^{100}P_0 = \frac{100!}{(100-0)!} = \frac{100!}{100!} = 1$$

Q10. What is the probability of a certain event?

Solution:

Probability of a certain event or sure event is 1.

Q11. Find the midpoint of the class-interval 5 - 15.

Solution:

Given,

The Learning App

Class interval is $5-15$.

Upper limit $= 15$

Lower limit = 5

Midpoint of the class interval $= \frac{(\,lower\ limit + upper\ limit)}{2}$

$= (5+15)/2$

$= \frac{20}{2}$

$= 10$

Q12. Find the value of $\cos\ 48° - \sin\ 42°$.

Solution:

$\cos\ 48° - \sin\ 42°$

$= \cos(90° - 42°) - \sin\ 42°$

$= \sin\ 42° - \sin\ 42°$

$= 0$

(or)

$\cos\ 48° - \sin\ 42°$

$= \cos\ 48° - \sin(90° - 48°)$

$= \cos\ 48° - \sin\ 48°$

$= 0$

Q13. Write the slope and y-intercept of the line $y = 3x$.

Solution:

Given,

Equation of line is $y = 3x$

Comparing with $y = mx + c$

Here,

Slope $= m = 3$
y −intercept $= c = 0$

Q14. Write the formula used to find the total surface area of a solid hemisphere.
Solution:
Total surface area of a solid hemi-sphere $= 3\pi r^2$ sq.units
Here,
$r =$ Radius of the solid hemisphere

SECTION - III

Q15. If A and B are the sets such that $n(A) = 37, n(B) = 26$ and $n(A \cup B) = 51$, then find $n(A \cap B)$.
Solution:
Given,
$\boldsymbol{n(A) = 37}$
$\boldsymbol{n(B) = 26}$
$\mathbf{n(A \cup B) = 51}$
We know that,
$\boldsymbol{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$
$\boldsymbol{51 = 37 + 26 - n(A \cap B)}$
$\boldsymbol{n(A \cap B) = 63 - 51}$
$\boldsymbol{n(A \cap B) = 12}$

Q16. Write the formula used to find
a) arithmetic mean between a and $b (a > b)$
b) harmonic mean between a and $b (a > b)$.
Solution:
a) Arithmetic mean A.M. $= \frac{a+b}{2}$ (where $a > b$)
b) Harmonic mean H.M. $= \frac{2ab}{a+b}$ (where $a > b$)

Q17. Find the sum to infinity of the geometric series $2 + \left(\frac{2}{3}\right) + \left(\frac{2}{9}\right) + \cdots$
Solution:
Given,
$$2 + \left(\frac{2}{3}\right) + \left(\frac{2}{9}\right) + \cdots$$
Here,
$a = 2$

$$r = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{2}{\left[1 - \left(\frac{1}{3}\right)\right]}$$

$$= \frac{2}{\left[\frac{3-1}{3}\right]}$$

$$= \frac{2}{\frac{2}{3}} = 3$$

Therefore, the sum to infinity of the given geometric series is 3.

Q18. Prove that $3 + \sqrt{5}$ is an irrational number.

Solution:

Let $3 + \sqrt{5}$ be a rational number.

Thus, $3 + \sqrt{5} = \frac{p}{q}$, where p, q are coprime integers and $q \neq 0$.

$$\Rightarrow \sqrt{5} = \left(\frac{p}{q}\right) - 3$$

$$\Rightarrow \sqrt{5} = \frac{p - 3q}{q}$$

Since p and q are integers, $\frac{p-3q}{q}$ is a rational number.

$\Rightarrow \sqrt{5}$ is also a rational number.

This is the contradiction to the fact that $\sqrt{5}$ is an irrational number.

Hence, our assumption that $3 + \sqrt{5}$ is a rational number is wrong.

Therefore, $3 + \sqrt{5}$ is an irrational number.

Hence proved.

Q19. Find how many triangles can be drawn through 8 points on a circle.

Solution:

We know that a triangle is formed by joining 3 non-collinear points.

$\therefore$ Total number of triangles that can be drawn out of 8 non-collinear points = ${}^{8}C_3$

$${}^{n}C_r = \frac{n!}{(n-r)!r!}$$

Here, $n = 8$ and $r = 3$

$${}^{8}C_3 = \frac{8!}{(8-3)!\,3!}$$

$= \frac{8 \times 7 \times 6 \times 5!}{5!\,(3 \times 2)}$

$= \frac{8 \times 7 \times 6}{6}$

$= 56$

Hence, the required number of triangles is 56.

Q20. If $\left(\frac{1}{8!}\right) + \left(\frac{1}{9!}\right) = \frac{x}{10!}$, then find the value of x.

Solution:

$\left(\frac{1}{8!}\right) + \left(\frac{1}{9!}\right) = \frac{x}{10!}!$

$(1/8!) + (1/9 \times 8!) = x/(10 \times 9 \times 8!)$

$\left(\frac{1}{8!}\right)[1 + \left(\frac{1}{9}\right)] = \frac{x}{10 \times 9 \times 8!}$

$\frac{9+1}{9} = \frac{x}{90}$

$\frac{10}{9} = \frac{x}{90}$

$\Rightarrow \text{x} = \frac{900}{9}$

$\Rightarrow x = 100$

Q21. A box has 4 red and 3 black marbles. Four marbles are picked up randomly. Find the probability that two marbles are red.

Solution:

Given,

A box has 4 red and 3 black marbles.

Out of 7 marbles, 4 marbles can be drawn in ${}^{7}C_{4} = 35$ ways

Thus, $n(S) = 35$

Two red marbles can be drawn in ${}^{4}C_{2} = 6$ ways

The remaining 2 marbles must be black and they can be drawn in ${}^{3}C_{2} = 3$ ways

$n(A) = 6 \times 3 = 18$

$\text{P(A)} = \frac{n(A)}{n(S)}$

$= \frac{18}{35}$

Q22. Calculate standard deviation for the following scores: 5, 6, 7, 8, 9.

Solution:

Using direct method:

x	x^2
5	25
6	36
7	49
8	64
9	81
$\sum x = 35$	$\sum x^2 = 255$

, N = 5

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{255}{5} - \left(\frac{35}{5}\right)^2}$$

$$= \sqrt{51 - 49}$$

$$= \sqrt{2}$$

$$\sigma = 1.414$$

Q23. Solve $x^2 - 2x - 4 = 0$ by using formula.

Solution:

Given quadratic equation is:

$x^2 - 2x - 4 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$, a = 1, b = −2, c = −4

Using the quadratic formula,

$$x = \frac{\left[-b \pm \sqrt{(b^2 - 4ac)}\right]}{2a}$$

$$x = \frac{\left[-(-2) \pm \sqrt{\{(-2)^2 - 4(1)(-4)\}}\right]}{2(1)}$$

$$= \frac{[2 \pm \sqrt{(4+16)}]}{2}$$

$$= \frac{(2 \pm 2\sqrt{5})}{2}$$

$$= \frac{2(1 \pm \sqrt{5})}{2}$$

$$= 1 \pm \sqrt{5}$$

Hence, the roots of the given equation are $(1 + \sqrt{5})$ and $(1 - \sqrt{5})$.

OR

Determine the nature of the roots of the equation $x^2 - 2x - 3 = 0$.

Solution:

Given quadratic equation is:

$x^2 - 2x - 3 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 1, b = -2, c = -3$

$D = b^2 - 4ac$

$= (-2)^2 - 4(1)(-3)$

$= 4 + 12$

$= 16$

$D > 0$

Discriminant is greater than 0 .

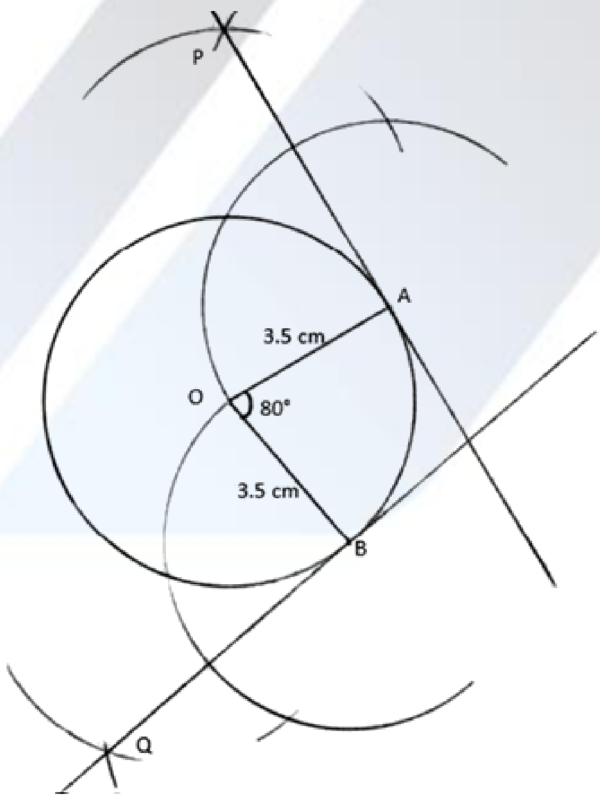
Therefore, the roots of the equation are real and distinct.

Q24. In a circle of radius 3.5 cm, draw two radii such that the angle between them is 80°. Construct tangents to the circle at the non-centre ends of the radii.

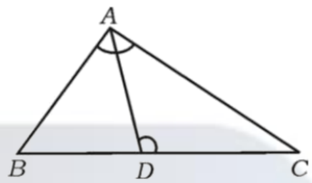
Solution:

Given, radius $= 3.5$ cm

Angle between two radii $= 80^\circ$



Q25. In $\triangle ABC, D$ is a point on BC such that $\angle BAC = \angle ADC$. Prove that $AC^2 = BC \times DC$.



Solution:

In $\triangle ABC$ and $\triangle ADC$,

$\angle BAC = \angle ADC$ (given)

$\angle ACB = \angle ACD$ (common)

By AA similarity criterion,

$\triangle$ ACB $\sim \triangle$ DCA

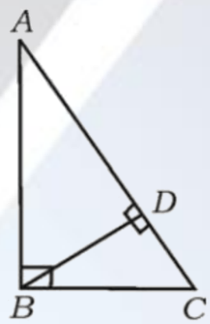
$\Rightarrow \dfrac{AC}{DC} = \dfrac{CB}{CA}$

$\Rightarrow AC^2 = BC \times DC$

Hence proved.

OR

In the right triangle $ABC, \angle ABC = 90°$ and $BD \perp AC$. Prove that: $\frac{AB^2}{BC^2} = \frac{AD}{CD}$



Solution:

Given that, in right triangle $ABC, \angle ABC = 90°$ and $BD \perp AC$.

By the corollary of right angle theorem,

$\mathrm{AB}^2 = \mathrm{AD} \times \mathrm{AC}$....(i)

And

$\mathrm{BC}^2 = \mathrm{CD} \times \mathrm{AC}$ ......(ii)

Dividing (i) by (ii),

$\dfrac{\mathrm{AB}^2}{\mathrm{BC}^2} = \dfrac{\mathrm{AD} \times \mathrm{AC}}{\mathrm{CD} \times \mathrm{AC}}$

$\dfrac{\mathrm{AB}^2}{\mathrm{BC}^2} = \dfrac{\mathrm{AD}}{\mathrm{CD}}$

Hence proved.

Q26. Find the value of $\sin 30^\circ \times \cos\ 60^\circ - \tan^2\ 45^\circ$.

Solution:

$\sin\ 30^\circ = \frac{1}{2}$

$\cos\ 60^\circ = \frac{1}{2}$

$\tan\ 45^\circ = 1$

$\sin 30^\circ \times \cos\ 60^\circ - \tan^2\ 45^\circ$

$= \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) - (1)^2$

$= \left(\frac{1}{4}\right) - 1 = \frac{1-4}{4}$

$= -\frac{3}{4}$

Q27. Find the radius of a circle whose centre is $(-5,4)$ and which passes through the point $(-7,1)$.

Solution:

Given,

Centre $C = (-5,4)$

Circle passes through the point $A = (-7,1)$

Radius of the circle = Distance between A and C

Let, $(x_1, y_1) = (-5,4)$

$(x_2, y_2) = (-7,1)$

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[-7 - (-5)]^2 + (1 - 4)^2}$

$= \sqrt{(-7 + 5)^2 + (-3)^2}$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

Therefore, the radius of the circle is $\sqrt{13}$ units.

Q28. The radii of two right circular cylinders are in the ratio $2:3$ and the ratio of their curved surface areas is $5:6$. Find the ratio of their heights.

Solution:

Let h_1 and h_2 be the heights of two right circular cylinders.

Given,

Ratio of the radii of two right circular cylinders $= r_1 : r_2 = 2:3$

Ratio of the curved surface areas = $S_1 : S_2 = 5:6$

$$\frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{5}{6}$$

$$\frac{2h_1}{3h_2} = \frac{5}{6}$$

$$\frac{h_1}{h_2} = \frac{5 \times 3}{6 \times 2} = \frac{5}{4}$$

Hence, the required ratio is 5: 4.

Q29. The radius of a solid metallic sphere is 10 cm. It is melted and recast into small cones of height 10 cm and base radius 5 cm. Find the number of small cones formed.

Solution:

Given,

Radius of solid sphere = R = 10 cm

Base radius of cone = r = 5 cm

Height of cone = h = 10 cm

Number of small cones = Volume of sphere/Volume of one small cone

$$= \frac{\left(\frac{4}{3}\right)\pi R^3}{\left[\left(\frac{1}{3}\right)\pi r^2 h\right]}$$

$$= \frac{4 \times 10 \times 10 \times 10}{5 \times 5 \times 10}$$

$= 4 \times 2 \times 2$

$= 16$

Hence, the number of small cones formed from the sphere = 16

Q30. Draw a plan by using the information given below:

[Scale: 25 metres = 1 cm]

	Metre To D	
	200	
	125	75 to C
100 to E	75	
	50	25 to B
	From A	

Solution:

Scale:

25 m = 1 cm

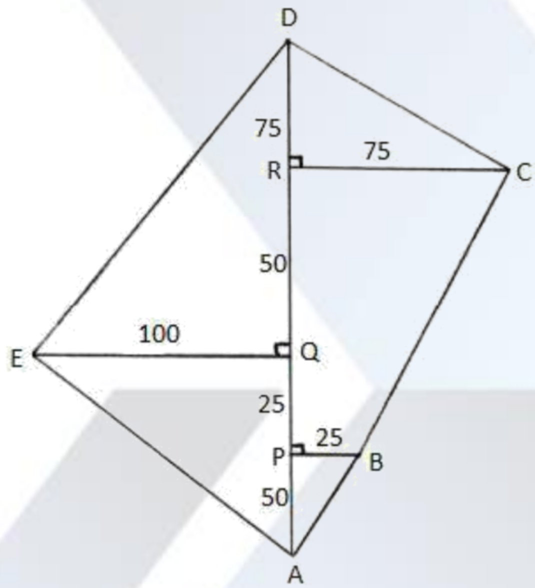
50 m = 2 cm

75 m = 3 cm

100 m = 4 cm

125 m = 5 cm

200 m = 8 cm



SECTION - IV

Q31. Rationalise the denominator and simplify:

$(\sqrt{6}+\sqrt{3})/(\sqrt{6}-\sqrt{3})$

Solution:

$(\sqrt{6}+\sqrt{3})/(\sqrt{6}-\sqrt{3})$

By rationalising the denominator,

$$=\left[\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}}\right]\times\left[\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}\right]$$

$$=\frac{(\sqrt{6}+\sqrt{3})^2}{[(\sqrt{6})^2-(\sqrt{3})^2]}$$

$$=\frac{6+3+2\sqrt{6}\sqrt{3}}{6-3}$$

$$=\frac{9+2\sqrt{18}}{3}$$

$$= \frac{9 + 6\sqrt{2}}{3}$$
$$= \frac{3(3 + 2\sqrt{2})}{3}$$
$$= 3 + 2\sqrt{2}$$

Q32. Find the quotient $q(x)$ and remainder $r(x)$ on dividing $p(x) = x^3 + 4x^2 - 5x + 6$ by $g(x) = x + 1$ and hence verify $p(x) = [g(x) \times q(x)] + r(x)$.

Solution:

Given,

$p(x) = x^3 + 4x^2 - 5x$

$g(x) = x + 1$

$$\begin{array}{r|llll} & x^2 & +3x & -8 & \\ \hline x+1 & x^3 & +4x^2 & -5x & +6 \\ - & x^3 & +x^2 & & \\ \hline & & 3x^2 & -5x & +6 \\ & - & 3x^2 & +3x & \\ \hline & & & -8x & +6 \\ & & - & -8x & -8 \\ \hline & & & & 14 \end{array}$$

Quotient $= q(x) = x^2 + 3x - 8$

Remainder $= r(x) = 14$

Verification:

$[g(x) \times q(x)] + r(x)$

$= (x + 1)(x^2 + 3x - 8) + 14$

$= x^3 + 3x^2 - 8x + x^2 + 3x - 8 + 14$

$= x^3 + 4x^2 - 5x + 6$

$= p(x)$

Therefore, $p(x) = [g(x) \times q(x)] + r(x)$

OR

Find the quotient and remainder by using synthetic division:

$(4x^3 - 16x^2 - 9x - 36) \div (x + 2)$

Solution:

$(4x^3 - 16x^2 - 9x - 36) \div (x + 2)$

Using synthetic division,

-2	4	-16	-9	
		-8	48	$(-2) \times 39 = -78$
	4	-24	39	$(-36) + (-78)$ $= -114$

Therefore,
Quotient $= \mathrm{q(x)} = 4\mathrm{x}^2 - 24\mathrm{x} + 39$
Remainder $= r(x) = -114$

Q33. Find three consecutive positive integers such that the sum of the square of the first integer and the product of the other two is 92.

Solution:

Let x, $(x+1)$ and $(x+2)$ be the three consecutive positive integers.
According to the given,

$x^2 + (x+1)(x+2) = 92$

$x^2 + x^2 + 2x + x + 2 = 92$

$2x^2 + 3x + 2 - 92 = 0$

$2x^2 + 3x - 90 = 0$

$2x^2 - 12x + 15x - 90 = 0$

$2x(x-6) + 15(x-6) = 0$

$(x-6)(2x+15) = 0$

$x - 6 = 0, \qquad 2x + 15 = 0$

$x = 6, x = -\dfrac{15}{2}$

x cannot be negative.
Therefore, $x = 6$
Hence, the required three consecutive positive integers are 6, 7 and 8.

OR

Sum of the squares of any two numbers is 180. If the square of the smaller number is equal to 8 times the bigger number, find the two numbers.

Solution:

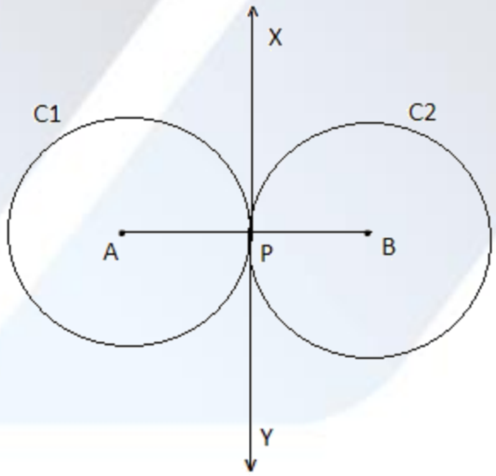
Let x, y be the two numbers and $x > y$.
Sum of the squares of two numbers is 180.
i.e. $x^2 + y^2 = 180$ ......(i)
Given that the square of the smaller number is equal to 8 times the bigger number.
i.e. $y^2 = 8x$.....(ii)

From (i) and (ii),
$x^2 + 8x = 180$
$x^2 + 8x - 180 = 0$
$x^2 + 18x - 10x - 180 = 0$
$x(x + 18) - 10(x + 18) = 0$
$(x + 18)(x - 10) = 0$
$x = -18, \quad x = 10$
$x = -18$ is not possible.
Thus, $x = 10$
Substituting $x = 10$ in (ii),
$y^2 = 8 \times 10$
$= 80$
$y = \sqrt{80} = 4\sqrt{5}$
Hence, the required numbers are 10 and $4\sqrt{5}$.

Q34. Prove that "If two circles touch each other externally, the centres and the point of contact are collinear".

Solution:

Let A be the centre of circle C1 and B be the centre of circle C2.
P be the point of contact.
Draw tangent XY which passes through P.



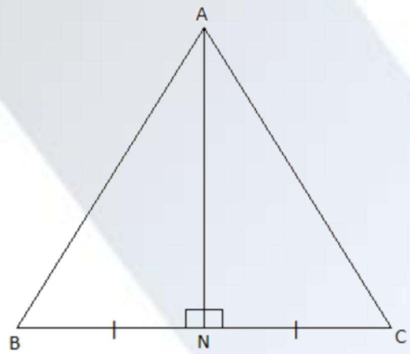
$\angle APX = \angle BPX = 90^\circ$ (radius is perpendicular to the tangent through the point of contact)
Now,
$\angle APX + \angle BPX = 90^\circ + 90^\circ = 180^\circ$
180° is the angle formed by a straight line.
Thus, $\angle APX$ and $\angle BPX$ is a linear pair.

Therefore, A, P and B are collinear.
Hence proved.

Q35. In an equilateral triangle ABC, AN $\perp$ BC, prove that $4AN^2 = 3AB^2$.

Solution:

Given that in an equilateral triangle ABC, AN $\perp$ BC.



Thus, $BN = NC = \left(\frac{1}{2}\right) BC = \left(\frac{1}{2}\right) AB$

In right triangle ANB, $AB^2 = AN^2 + BN^2$

$AN^2 = AB^2 - BN^2$

$= AB^2 - [\left(\frac{1}{2}\right) AB]^2$

$= AB^2 - \left(\frac{AB^2}{4}\right)$

$= \frac{(4AB^2 - AB^2)}{4}$

$4AN^2 = 3AB^2$

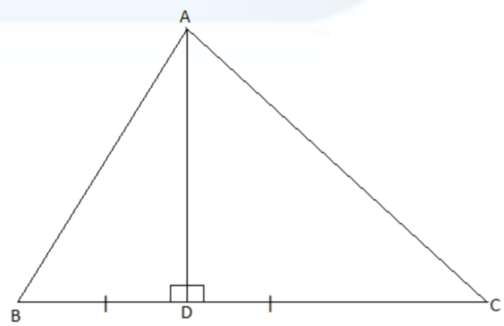
Hence proved.

OR

In $\triangle ABC, AD \perp BC$, prove that $AB^2 + CD^2 = AC^2 + BD^2$.

Solution:

Given that in $\triangle ABC, AD \perp BC$.



In right triangle ADB ,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$ .... (i)

In right triangle ADC ,

$AC^2 = AD^2 + DC^2$

$\Rightarrow AD^2 = AC^2 - DC^2$ .....(ii)

From (i) and (ii),

$AB^2 - BD^2 = AC^2 - DC^2$

$AB^2 + DC^2 = AC^2 + BD^2$

Hence proved.

Q36. Prove that $\tan^2 A - \sin^2 A = \tan^2 A \times \sin^2 A$.

Solution:

$\text{LHS} = \tan^2 A - \sin^2 A$

$= \left(\frac{\sin^2 A}{\cos^2 A}\right) - \sin^2 A$

$= \frac{(\sin^2 A - \sin^2 A \cos^2\ A)}{\cos^2 A}$

$= \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}$

Using the identity $\sin^2\ \theta + \cos^2\ \theta = 1$,

$= \frac{(\sin^2 A \times \sin^2 A)}{\cos^2 A}$

$= \left(\frac{\sin^2 A}{\cos^2 A}\right) \sin^2 A$

$= \tan^2 A \times \sin^2 A$

$= \text{RHS}$

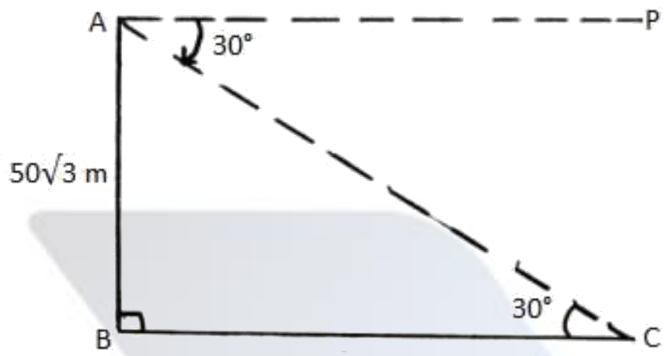
Hence proved.

OR

From the top of a building $50\sqrt{3}$ m high the angle of depression of an object on the ground is observed to be 30°. Find the distance of the object from the foot of the building.

Solution:

Let AB be the building and C be the object.



$AB = 50\sqrt{3}$ m
$\angle PAC = \angle ACB = 30^\circ$
Angle of depression $= \theta = 30^\circ$
In right triangle ABC , $\tan 30^\circ = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$

$BC = (50\sqrt{3}) \times \sqrt{3}$
$= 50 \times 3$
$= 150$
Hence, the distance between the building and the objec $= BC = 150$ m

SECTION - V

Q37. The sum of 3rd and 5th terms of an arithmetic progression is 30 and the sum of 4th and 8th terms of it is 46. Find the arithmetic progression.

Solution:

Let a be the first term and d be the common difference of an AP.
According to the given,
$a_3 + a_5 = 30$
$a + 2d + a + 4d = 30$
$2a + 6d = 30$
$2(a + 3d) = 30$
$a + 3d = 15$
And
$a_4 + a_8 = 46$
$a + 3d + a + 7d = 46$
$2a + 10d = 46$
$2(a + 5d) = 46$
$a + 5d = 23$ ......(ii)
Subtracting (i) from (ii),

$a+5d-(a+3d)=23-15$
$2d=8$
$d=\frac{8}{2}=4$
Substituting $d=4$ in (i),
$a+3(4)=15$
$a+12=15$
$a=15-12$
$a=3$
$a+d=3+4=7$
$a+2d=3+2(4)=11$
$a+3d=3+3(4)=15$
Hence, the required AP is 3,7,11,15, …

OR

If the fourth term of a geometric progression is 8 and its eighth term is 128, find the sum of the first ten terms of the progression.

Solution:

Given that in GP,
$a_4=8$
$ar^3=8$....(i)
And
$a_8=128$
$ar^7=128$....(ii)
Dividing (ii) by (i),
$\frac{ar^7}{ar^3}=\frac{128}{8}$
$r^4=16$
$r^4=(2)^4$
$r=2$
Substituting $r=2$ in (i),
$a(2)^3=8$
$8a=8$
$a=\frac{8}{8}=1$
Sum of the first n term
$S_n=\frac{a(r^n-1)}{r-1}$

$S_{10} = \frac{1(2^{10} - 1)}{2 - 1}$

= $1024 - 1$

$= 1023$

Hence, the sum of the first 10 terms of the GP is 1023.

Q38. Solve $x^2 - 2x - 3 = 0$ graphically.

Solution:

Given,

$x^2 - 2x - 3 = 0$

$x^2 - (2x + 3) = 0$

Thus, the solution will be the intersection of $y = x^2$ and $y = 2x + 3$

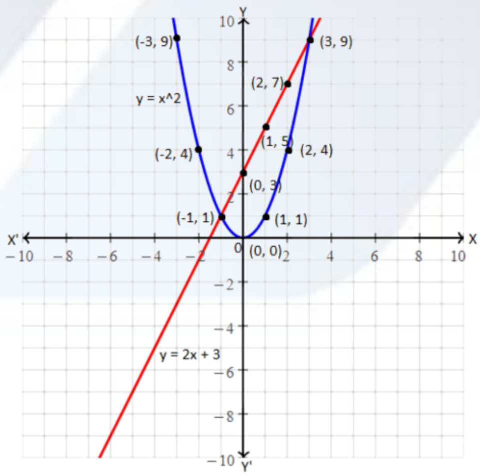
Consider, $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

$y = 2x + 3$

x	-1	0	1	2	3
y	1	3	5	7	9

Graph:

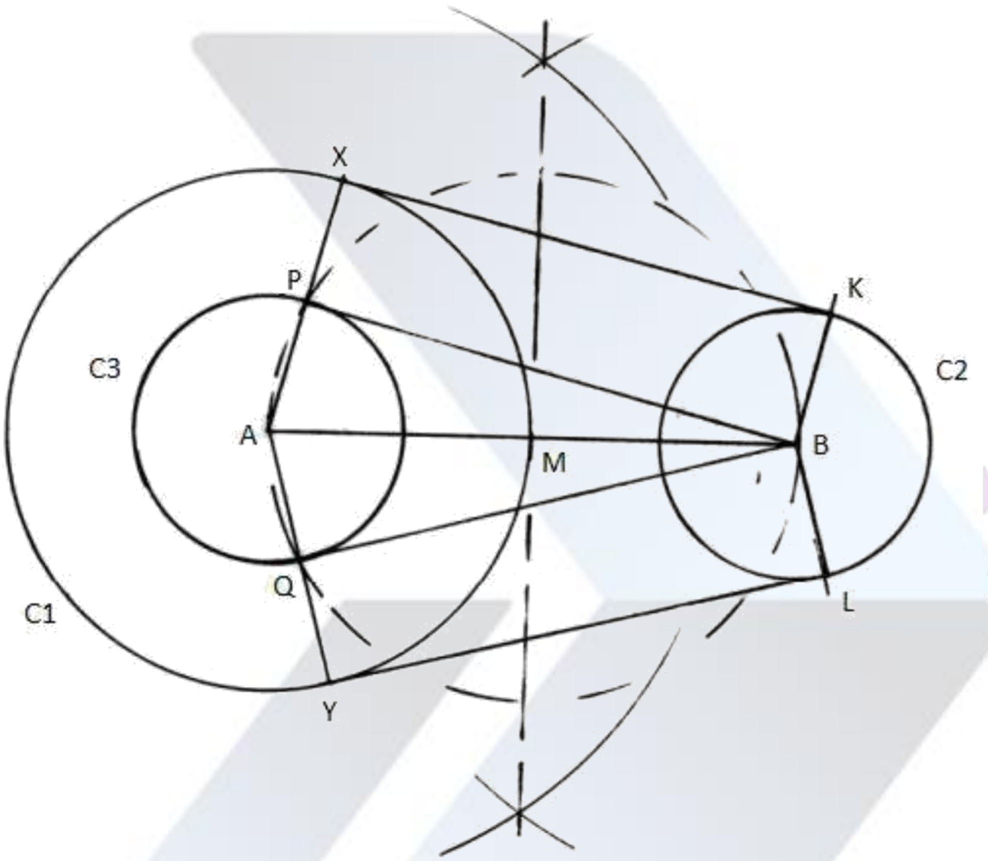


These intersect each other at $x = -1$ and $x = 3$.

Hence, the required solution is $x = -1$ and $x = 3$

Q39. Construct a pair of direct common tangents to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart. Measure and write the length of the direct common tangent.

Solution:



Therefore, KX and LY are the required tangents of length 7.8 cm.

Q40. Prove that "If two triangles are equiangular, then their corresponding sides are in proportion".

Solution:

Given,

In $\triangle$ ABC and $\triangle$ DEF,

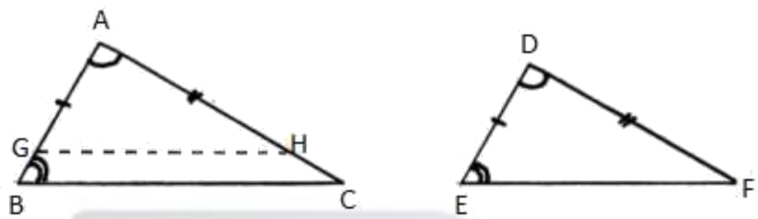
$\angle BAC = \angle EDF$

$\angle ABC = \angle DEF$

To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction:

Points G and H are marked on AB and AC such that $AG = DE$ and $AH = DF$. Join GH.



In △ AGH and △ DEF, AG = DE (by construction) $\angle GAH = \angle EDF$ (given)
AH = DF (by construction)
BY SAS congruence criterion,
ΔAGH ≅△ DEF
Thus, by CPCT,
GH = EF
$\angle AGH = \angle DEF$
Given that, $\angle DEF = \angle ABC$
$\angle AGH = \angle ABC$ (alternate angles)
Therefore, GH || BC
By the corollary of Thales(BPT) theorem,
$\frac{AB}{AG} = \frac{BC}{GH} = \frac{AC}{AH}$
And, $AG = DE, GH = EF, AH = DF$
Therefore,
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
Hence proved.