Grade10-Kar-Maths-2019

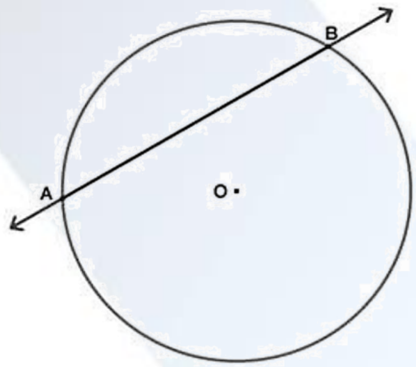
QUESTION PAPER CODE 81-E

SECTION - I

Q1. If the nth term of an arithmetic progression $a_n = 24 - 3n$, then its $2nd$ term is
(A) 18
(B) 15
(C) 0
(D) 2
Solution:
Correct answer: (A)
Given,
$a_n = 24 - 3n$
Substituting $\mathrm{n} = 2$,
$\mathrm{a}_2 = 24 - 3(2)$
$= 24 - 6$
$= 18$
Hence, the 2nd term is 18.

Q2. The lines represented by $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ are
(A) Intersecting lines
(B) Perpendicular lines to each other
(C) Parallel lines
(D) Coincident lines
Solution:
Correct answer: (D)
Given,
$2x + 3y - 9 = 0$
$4x + 6y - 18 = 0$
Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = 2, b_1 = 3, c_1 = -9$
$a_2 = 4, b_2 = 6, c_2 = -18$
$\frac{a_1}{a_2}, = \frac{2}{4} = \frac{1}{2}$
$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$
$\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$
Thus, $\frac{a_1}{a_2} = \frac{\mathrm{b}_1}{\mathrm{b}_2} = \frac{\mathrm{c}_1}{\mathrm{c}_2}$
Therefore, the given equations have infinitely many solutions.
Hence, the lines represented by these lines are coincident lines.

Q3. A straight line which passes through two points on a circle is
(A) a chord
(B) a secant
(C) a tangent
(D) the radius
Solution:
Correct answer: (B)
A straight line which passes through two points on a circle is a secant of the circle.



Here, AB is the secant of the circle with centre O.

Q4. If the area of a circle is 49π sq. units, then its perimeter is
(A) 7π units
(B) 9π units
(C) 14π units
(D) 49π units
Solution:
Correct answer: (C)
Given,
Area of the circle $= 49\pi$ sq. units
Let r be the radius of the circle.
$\pi r^2 = 49\pi$
$r^2 = 49$
r = 7 units
Perimeter of circle $= 2\pi r$
$= 2\pi(7)$
$= 14\pi$ units

Q5. "The product of two consecutive positive integers is 30." This can be expressed algebraically as
(A) $x(x + 2) = 30$
(B) $x(x - 2) = 30$
(C) $x(x - 3) = 30$
(D) $x(x + 1) = 30$

Solution:

Correct answer: (D)
Let x and $(x+1)$ be the two consecutive positive integers.
According to the given,
$x(x+1)=30$

Q6. If a and b are any two positive integers then $\mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b)$ is equal to
(A) $a+b$
(B) $a-b$
(C) $a \times b$
(D) $a \div b$
Solution:

Correct answer: (C)
If a and b are any two positive integers then $\mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b)$ is equal to $a \times b$.
Thus, $\mathrm{HCF} \times \mathrm{LCM} =$ Product of the given two numbers

Q7. The value of $\cos 48^{\circ}-\sin 42^{\circ}$ is
(A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
Solution:

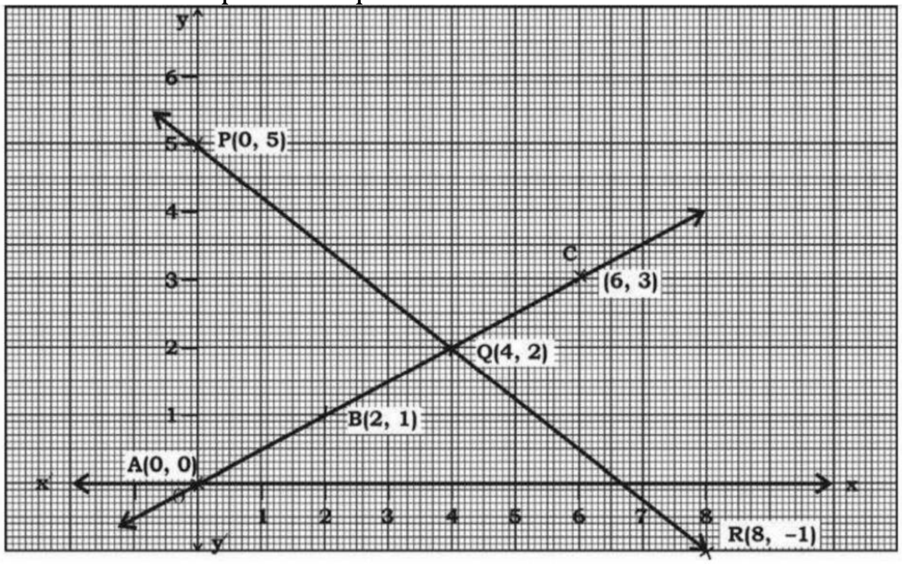
Correct answer: (A)
$\cos 48^{\circ}-\sin 42^{\circ}$
$=\cos \left(90^{\circ}-42^{\circ}\right)-\sin 42^{\circ}$
$=\sin 42^{\circ}-\sin 42^{\circ}$
$=0$

Q8. If $P(A)=0.05$ then $P(\bar{A})$ is
(A) 0.59
(B) 0.95
(C) 1
(D) 1.05
Solution:

Correct answer: (B)
Given,
$\mathrm{P}(\mathrm{A})=0.05$
We know that,
$P(A)+P(\bar{A})=1$
$0.05+P(\bar{A})=1$
$P(\bar{A})=1-0.05=0.95$

SECTION - II

Q9. The given graph represents a pair of linear equations in two variables. Write how many solutions these pairs of equations have.



Solution:

In the given figure,

The lines representing the pair of equations intersect each other at only one point, i.e. at Q(4,2).

Hence, the pair of equations have a unique solution.

Q10. $17 = 6 \times 2 + 5$ is compared with Euclid's Division lemma $a = bq + r$, then which number is representing the remainder?

Solution:

Given,

$17 = 6 \times 2 + 5$

Comparing with Euclid's division lemma, $a = bq + r$, where r is the remainder.

Hence, the number 5 represents the remainder.

Q11. Find the zeroes of the polynomial $p(x) = x^2 - 3$.

Solution:

Given polynomial is:

$p(x) = x^2 - 3$

$= x^2 - (\sqrt{3})^2$

$= (x + \sqrt{3})(x - \sqrt{3})$

Let $p(x) - 0$

$(x + \sqrt{3})(x - \sqrt{3}) = 0$

$x + \sqrt{3} = 0$, $\quad x - \sqrt{3} = 0$

$x = -\sqrt{3}$, $\quad x = \sqrt{3}$

Hence, the zeroes of the given polynomial are $\sqrt{3}$ and $-\sqrt{3}$.

Q12. Write the degree of the polynomial $p(x) = 2x^2 - x^3 + 5$.

Solution:

Given polynomial is:

$p(x) = 2x^2 - x^3 + 5$

$= -x^3 + 2x^2 + 5$

Highest power of variable x is 3.

Hence, the degree of the given polynomial is 3.

Q13. Find the value of the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$.

Solution:

Given quadratic equation is:

$2x^2 - 4\mathrm{x} + 3 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 2$, $b = -4$ and $c = 3$

Discriminant $=$ D

$= b^2 - 4ac$

$= (-4)^2 - 4(2)(3)$

$= 16 - 24$

$= -8$

Therefore, the value of discriminant is -8.

Q14. Write the formula to calculate the curved surface area of the frustum of a cone.

Solution:

Curves surface area of the frustum of a cone $= \pi(r_1 + r_2)\,\mathrm{l}$

Here,

$r_1, r_2 =$ Radii of circular ends

$l =$ Slant height

SECTION - III

Q15. Find the sum of the first twenty terms of Arithmetic series $2 + 7 + 12 + \cdots$ using suitable formulas.

Solution:

Given,

$2 + 7 + 12 + \cdots$

Here,

First term = a = 2

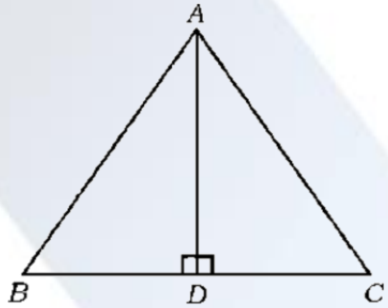
Common difference $= d = 7 - 2 = 5$

Sum of first n terms is

$S_n = \frac{n}{2}[2a + (n-1)d]$

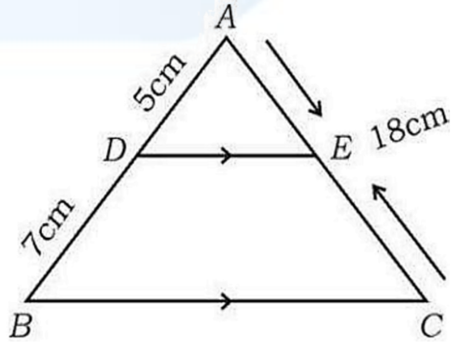
$S_{20} = \frac{20}{2} \times [2(2) + (20 - 1)\ 5]$
$= 10 \times [4 + 19(5)]$
$= 10 \times [4 + 95]$
$= 10 \times 99$
$= 990$
Hence, the sum of the first twenty terms of the given arithmetic series is 990.

Q16. In $\triangle ABC, AD \perp BC$ and $AD^2 = BD \times CD$. Prove that $AB^2 + AC^2 = (BD + CD)^2$.



Solution:
Given,
In triangle ABC ,
$AD \perp BC$ and $AD^2 = BD \times CD$
In $\triangle$ ADB and $\triangle$ ADC,
By Pythagoras theorem,
Then we have,
$AB^2 = AD^2 + BD^2$ ......(i)
$AC^2 = AD^2 + DC^2$ ......(ii)
Adding (i) and (ii),
$AB^2 + AC^2 = AD^2 + BD^2 + AD^2 + DC^2$
$= 2AD^2 + BD^2 + DC^2$
$= 2(BD \times CD) + BD^2 + CD^2$ [given $AD^2 = BD \times CD$]
$= (BD + CD)^2$
Hence proved.

Q17. In $\triangle$ ABC, DE||BC. If AD = 5 cm, BD = 7 cm and AC = 18 cm, find the length of AE.



Solution:

Given that, DE| | BC in triangle ABC .

AD = 5 cm, BD = 7 cm and AC = 18 cm

By Basic proportionality theorem,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{AE}{AC}$$

$$\Rightarrow \frac{5}{5+7} = \frac{AE}{18}$$

$$\Rightarrow \frac{5}{12} = \frac{AE}{18}$$

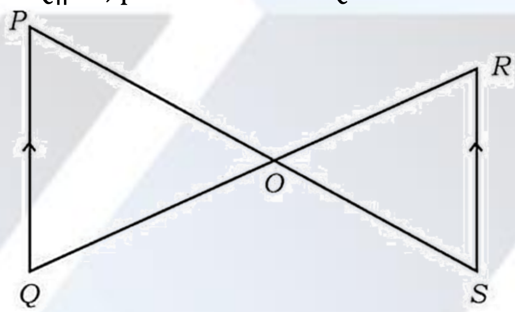
$$\Rightarrow AE = \left(\frac{5}{12}\right) \times 18$$

$$\Rightarrow AE = \frac{15}{2}$$

$$\Rightarrow AE = 7.5 \text{ cm}$$

OR

In the given figure if PQ||RS, prove that ΔPOQ ~ ΔSOR.



Solution:

Given,

PQ || RS

∠P = ∠S (alternate interior angles)

$\angle Q = \angle R$ (alternate interior angles)

∠POQ = ∠ROS (vertically opposite angles)

By AAA similarity criterion,

ΔPOQ ~ ΔSOR

Hence proved.

Q18. Solve the following pair of linear equations by any suitable method:

$x + y = 5$

$2x - 3y = 5$

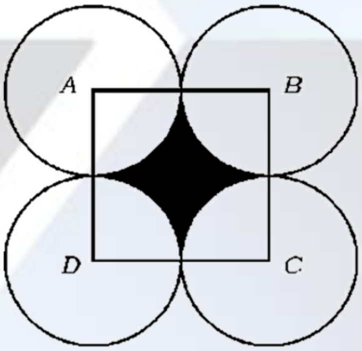
Solution:

Given,

$x + y = 5$ ......(i)

$2x - 3y = 5$ ......(ii)
Using Substitution method:
From (i),
$x = 5 - y$... (iii)
Substituting (iii) in (ii),
$2(5 - y) - 3y = 5$
$10 - 2y - 3y = 5$
$10 - 5 = 2y + 3y$
$\Rightarrow 5y = 5$
$\Rightarrow y = \dfrac{5}{5}$
$\Rightarrow y = 1$
Substituting $y = 1$ in (iii),
$x = 5 - 1$
$x = 4$
Therefore, $x = 4$ and $y = 1$.

Q19. In the figure, $ABCD$ is a square of side 14 cm. A, B, C and D are the centres of four congruent circles such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



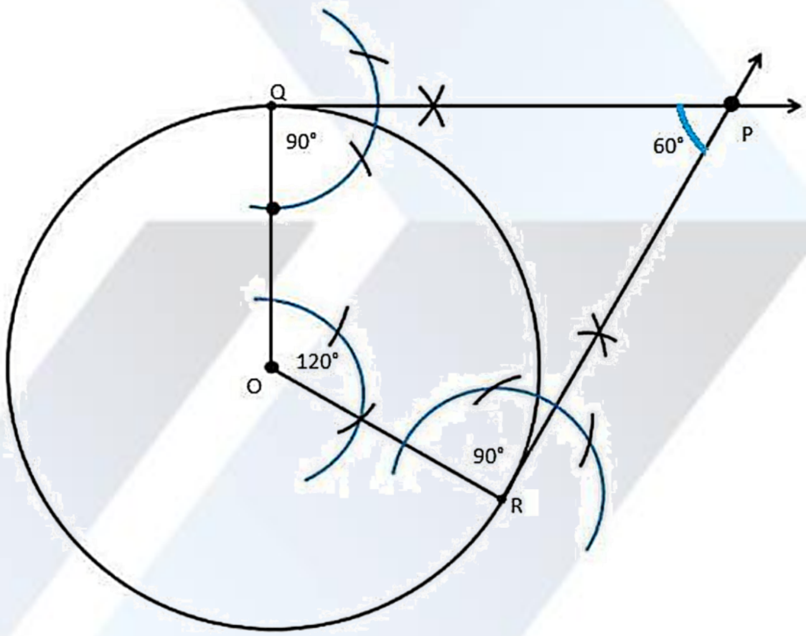
Solution:
Given,
Side of the square $ABCD = 14$ cm
Area of square ABCD $= (14)^2 = 196 \text{ cm}^2$
Radius of each quadrant of the circle $= r = \frac{14}{2} = 7$ cm
Area of one quadrant of circle $= \frac{1}{4}\pi r^2$
$= \left(\dfrac{1}{4}\right) \times \left(\dfrac{22}{7}\right) \times (7)^2$
$= \left(\dfrac{1}{4}\right) \times 22 \times 7$
$= \dfrac{154}{4} \text{ cm}^2$
Area of 4 quadrants $= 4\left(\frac{154}{4}\right) = 154 \text{ cm}^2$
Area of the shaded region = Area of square ABCD – Area of 4 quadrants with

centres A, B, C and C
$= 196 - 154$
$= 42 \text{ cm}^2$

Q20. Draw a circle of radius 4 cm and construct a pair of tangents such that the angle between them is 60°.

Solution:

Steps of construction:
(i) Draw a circle with centre O and radius 4 cm .
(ii) Take a point Q on the circumference of the circle and join OQ.
(iii) Draw a perpendicular to QP at point Q.
(iv) Draw a radius OR, making an angle of $120^\circ(180^\circ - 60^\circ)$ with OQ .
(v) Draw a perpendicular to RP at point R, which intersects the previous perpendicular at P .
Therefore, PQ and PR are the required tangents inclined at an angle of 60°.



Q21. Find the coordinates of the point which divides the line segment joining the points $A(4, -3)$ and $B(8, 5)$ in the ratio 3:1 internally.

Solution:

Let $P(x, y)$be the point which intersects the line segment joining the points A$(4, -3)$ and B$(8, 5)$ internally in the ratio 3:1 .
Here,
$(x_1, y_1) = (4, -3)$
$(x_2, y_2) = (8, 5)$
$m_1 : m_2 = 3 : 1$
Using the section formula,
x − coordinate:

$$x=\frac{m_1\,x_2+m_2\,x_1}{m_1+m_2}$$
$$=\frac{[3(8)+1(4)]}{3+1}$$
$$=\frac{24+4}{4}$$
$=\frac{28}{4}$
$=7$
y − coordinate
$$y=\frac{m_1\,y_2+m_2\,y_1}{m_1+m_2}$$
$$=\frac{[3(5)+1(-3)]}{3+1}$$
$$=\frac{15-3}{4}$$
$$=\frac{12}{4}$$
$=3$
Hence, the coordinates of the required points are (7,3).

Q22. Prove that $3+\sqrt{5}$ is an irrational number.
Solution:
Let $3+\sqrt{5}$ be a rational number.
Thus, $3+\sqrt{5}=\frac{a}{b}$, where a, b are coprime integers and $b \neq 0$.
$\Rightarrow \sqrt{5}=\left(\frac{a}{b}\right)-3$
$$\Rightarrow \sqrt{5}=\frac{a-3\,\mathrm{b}}{\mathrm{b}}$$
Since a and b are integers, $\frac{a-3b}{b}$ is a rational number.
$\Rightarrow \sqrt{5}$ is also a rational number.
This is the contradiction to the fact that $\sqrt{5}$ is an irrational number.
Hence, our assumption that $3+\sqrt{5}$ is a rational number is wrong.
Therefore, $3+\sqrt{5}$ is an irrational number.
Hence proved.

Q23. The sum and product of the zeroes of a quadratic polynomial $p(x)=ax^2+bx+c$ are -3 and 2 respectively. Show that $b+c=5a$.
Solution:
Given quadratic polynomial is:
$p(x)=ax^2+bx+c$
Sum of the zeroes $=\frac{-\text{coefficient of x}}{\text{coefficient of x}^2}$
$-3=-\frac{b}{a}$

$\Rightarrow \frac{b}{a} = 3$
$\Rightarrow b = 3a$ ......(i)
Product of the zeroes $= \frac{\text{constant term}}{\text{coefficient of } x^2}$
$2 = \frac{c}{a}$
$\Rightarrow \frac{c}{a} = 2$
$\Rightarrow c = 2a$
Adding (i) and (ii),
$b + c = 3a + 2a$
$b + c = 5a$

Q24. Find the quotient and the remainder when $p(x) = 3x^3 + x^2 + 2x + 5$ is divided by $g(x) = x^2 + 2x + 1$.

Solution:

Given,
$p(x) = 3x^3 + x^2 + 2x + 5$
$g(x) = x^2 + 2x + 1$
Therefore,
Quotient $= q(x) = 3x - 5$
Remainder $= r(x) = 9x + 10$

Q25. Solve $2x^2 - 5x + 3 = 0$ by using formula.

Solution:

Given quadratic equation is $2x^2 - 5x + 3 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
$a = 2, b = -5$ and $c = 3$
Discriminant = D
$= b^2 - 4ac$
$= (-5)^2 - 4(2)(3)$
$= 25 - 24$
$= 1$
Using quadratic formula,

$$x = \frac{(-b \pm \sqrt{D})}{2a}$$
$$= \frac{(-(-5) \pm \sqrt{1})}{2(2)}$$
$$= \frac{5 \pm 1}{4}$$
$$x = \frac{5+1}{4}, \quad x = \frac{5-1}{4}$$
$$x = \frac{6}{4}, \quad x = \frac{4}{4}$$
$$x = \frac{3}{2}, \quad x = 1$$

Q26. The length of a rectangular field is 3 times its breadth. If the area of the field is 147 m^2, find its length and breadth.

Solution:

Let x be the breadth and $3x$ be the length of the rectangular field.

According to the given,

$x(3x) = 147 \text{ m}^2$

$3x^2 = 147$

$x^2 = \frac{147}{3}$

$x^2 = 49$

$x = \sqrt{49}$

$x = 7 \text{ m}$

Therefore, breadth $= x = 7$ m

Length $= 3x = 3(7) = 21$ m

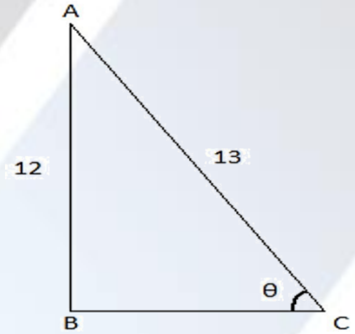
Q27. If $\sin\ \theta = \frac{12}{13}$, find the values of $\cos\theta$ and $\tan\theta$.

Solution:

Given,

$\sin\ \theta = \frac{12}{13}$

$\sin\ \theta = \frac{Opposite}{Hypotenuse} = \frac{AB}{AC}$



In right triangle ABC ,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$= (13)^2 - (12)^2$

$= 169 - 144$

$= 25$

$BC = \sqrt{25} = 5$

$\cos\ \theta = \frac{BC}{AC} = \frac{5}{13}$

$\tan\ \theta = \frac{AB}{BC} = \frac{12}{5}$

OR

If $\sqrt{3}\tan\ \theta = 1$ and θ is acute, find the value of $\sin 3\theta + \cos 2\theta$.

Solution:

Given,

$\sqrt{3}\tan\ \theta = 1$ and θ is acute

$\tan\ \theta = \dfrac{1}{\sqrt{3}}$

$\tan\ \theta = \tan\ 30^\circ$

$\theta = 30^\circ$

Now,

$\sin\ 3\theta + \cos\ 2\theta = \sin\ 3(30^\circ) + \cos\ 2(30^\circ)$

$= \sin\ 90^\circ + \cos\ 60^\circ$

$= 1 + \left(\dfrac{1}{2}\right)$

$= \dfrac{3}{2}$

Q28. Prove that $\frac{1+\cos\theta}{1-\cos\theta} = (\mathrm{cosec}\theta + \cot\ \theta)^2$.

Solution:

$\text{LHS} = \frac{1+\cos\theta}{1-\cos\theta}$

$= \left(\dfrac{1+\cos\theta}{1-\cos\theta}\right) \times \left(\dfrac{1+\cos\theta}{1+\cos\theta}\right)$

$= \left(1 + \dfrac{\cos\theta)^2}{(1-\cos^2\theta)}\right)$

Using the identity $\sin^2\ A + \cos^2\ A = 1$,

$= \dfrac{(1+\cos^2\theta + 2\cos\theta)}{\sin^2\theta}$

$= \left(\dfrac{1}{\sin^2\theta}\right) + \left(\dfrac{\cos^2\theta}{\sin^2\theta}\right) + \left(\dfrac{2\cos\theta}{\sin^2\theta}\right)$

$= \mathrm{cosec}^2\theta + \cot^2\ \theta + 2 \times \left(\dfrac{1}{\sin\theta}\right) \times \left(\dfrac{\cos\theta}{\sin\theta}\right)$

$= \mathrm{cosec}^2\theta + \cot^2\ \theta + 2\mathrm{cosec}\theta\cot\ \theta$

$= (\mathrm{cosec}\theta + \cot\ \theta)^2$

= RHS

Hence proved.

Q29. A cubical die numbered from 1 to 6 is rolled twice. Find the probability of getting the sum of numbers on its faces is 10.

Solution:

Total number of possible outcomes $= 6^2 = 36$

i.e. $n(S) = 36$

Let E be the event that the sum of numbers on dices is 10.
$E = \{(4,6),(6,4),(5,5)\}$
Number of outcomes favourable to $E = n(E) = 3$
$P(E) = \frac{n(E)}{n(S)}$
$= \frac{3}{36}$
$= \frac{1}{12}$
Therefore, the required probability is $\frac{1}{12}$.

Q30. The radii of two circular ends of a frustum of a cone shaped dustbin are 15 cm and 8 cm. If its depth is 63 cm, find the volume of the dustbin.

Solution:

Given,

The radii of two circular ends of a frustum of a cone shaped dustbin are 15 cm and 8 cm.

$\mathrm{R} = 15$ cm

$\mathrm{r} = 8$ cm

Depth $= h = 63$ cm

Volume $= \frac{\pi h}{3}(\mathrm{R}^2 + \mathrm{r}^2 + \mathrm{Rr})$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 63 \times [(15)^2 + (8)^2 + 15 \times 8]$

$= (22 \times 3) \times (225 + 64 + 120)$

$= 66 \times 409$

$= 26994 \text{ cm}^3$

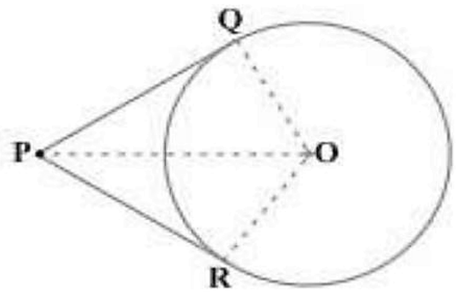
Therefore, the volume of the dustbin is 26994 cm^3.

SECTION - IV

Q31. Prove that "the lengths of tangents drawn from an external point to a circle are equal".

Solution:

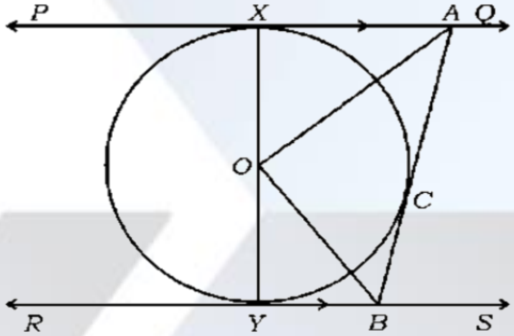
Let O be the centre of the circle.
PQ and PR be the two tangents drawn to the circle from an external point P.
Join OP, OQ and OR.



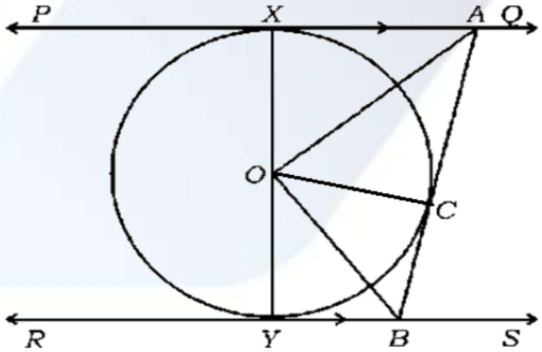
We know that radius is perpendicular to the tangent through the point of contact.
Thus, $\angle OQP = \angle ORP = 90^\circ$
In right $\triangle$ OQP and $\triangle$ ORP,
$OQ = OR$ (radius of the same circle)
OP = OP (common)
By RHS congruence criterion,
$\triangle OQP \cong \triangle ORP$
By CPCT,
$PQ = PR$
Hence proved.

OR

In the given figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90^\circ$.



Solution:
Given,
PQ and RS are two parallel tangents to a circle with centre O.
Join OC



In $\triangle$ OXA and $\triangle$ OCA,
OX = OC (radius of the same circle)
AO = AO (common)
AX = AC (tangents drawn from point A)
By SSS congruence criterion,
$\triangle OXA \cong \triangle OCA$
Similarly,
$\triangle OYB \cong \triangle OCB$

Thus,
∠XOA = ∠COA... (i)
∠YOB = ∠COB... (ii)
From the given figure, XOY is a straight line and is the diameter of the circle.
$\angle XOA + \angle COA + \angle COB + \angle YOB = 180^\circ$
Substituting (i) and (ii) in the above equation,
$2\angle COA + 2\angle COB = 180^\circ$
$\Rightarrow \angle COA + \angle COB = 90^\circ$
$\therefore \angle AOB = 90^\circ$
Hence proved.

Q32. Calculate the median of the following frequency distribution table:

Class-interval	Frequency (f_i)
$1-4$	6
$4-7$	30
$7-10$	40
$10-13$	16
$13-16$	4
$16-19$	4
	$\sum f_i = 100$

Solution:
Cumulative frequency distribution table

Class-interval	Frequency (f_i)	Cumulative frequency
$1-4$	6	6
$4-7$	30	$36 = cf$
$7-10$	$40 = f$	76
$10-13$	16	92
$13-16$	4	96
$16-19$	4	100

$$\frac{\mathrm{N}}{2} = \frac{100}{2} = 50$$
Cumulative frequency greater or equal to $\frac{\mathrm{N}}{2}$ is 76, which lies in the class interval $7-10$
Median class = $7-10$

Lower limit of the median class = 7
Frequency of the median class = f = 40
Cumulative frequency above the median class = cf = 36
Class height = h = 3

$$\text{Median} = 1 + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

$$= 7 + \left[\frac{50 - 36}{40}\right] \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

$$= 7 + \left(\frac{42}{40}\right)$$

$$= 7 + 1.05$$

$$= 8.05$$

OR

Calculate the mode for the following frequency distribution table.

Class-interval	Frequency (f_i)
$10 - 25$	2
$25 - 40$	3
$40 - 55$	7
$55 - 70$	6
$70 - 85$	6
$85 - 100$	6
	$\sum f_i = 30$

Solution:

Highest frequency = 7
Thus, modal class is $40 - 55$
Frequency of the modal class = $f_m = 7$
Lower limit of the modal class = 40
Frequency of the class preceding the modal class = $f_1 = 3$
Frequency of the class succeeding the modal class = $f_2 = 6$
Class height = h = 15

$$\text{Mode} = 1 + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)}\right] \times h$$

$$= 40 + \left[\frac{7 - 3}{2 \times 7 - 3 - 6}\right] \times 15$$

$$= 40 + \left[\frac{4}{14 - 9}\right] \times 15$$

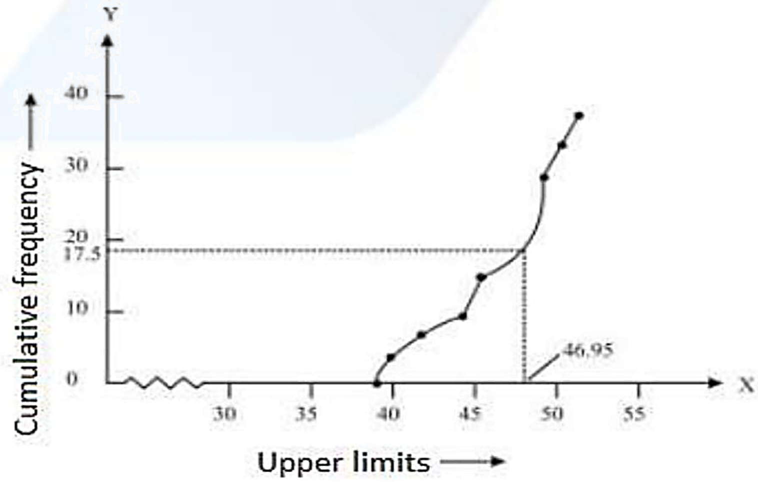
$$= 40 + \frac{4 \times 15}{5} = 40 + 12$$
$$= 52$$

Q33. During the medical check-up of 35 students of a class, their weights were recorded as follows. Draw a less than type of ogive for the given data:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Solution:

From the given data, to draw less than ogive, choose the upper limits of the class intervals on the x −axis and cumulative frequencies on the y −axis by choosing the convenient scale.
Plot the points corresponding to the ordered pairs given by:
$(38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32)$ and $(52, 35)$ on a graph paper.
Join all these points to get a smooth curve.
The curve obtained in the graph is known as less than type ogive.



Q34. The seventh term of an Arithmetic progression is four times its second term and twelfth term is 2 more than three times its fourth term. Find the progression.

Solution:

Let a be the first term and d be the common difference of an AP.
nth term of an AP is $a_n = a + (n-1)d$
According to the given,
$a_7 = 4a_2$ .... *(i)*
and $a_{12} = 3a_4 + 2$....(ii)
From (i),
$a + 6d = 4(a + d)$
$a + 6d = 4a + 4d$
$\Rightarrow 4a - a = 6\,d - 4\,d$
$\Rightarrow 3a = 2d$ ....(iii)
From (ii),
$a + 11d = 3(a + 3d) + 2$
$a + 11d = 3a + 9d + 2$
$\Rightarrow 3a - a = 11d - 9d - 2$
$\Rightarrow 2a = 2\,d - 2$
$\Rightarrow 2a = 3a - 2$
$\Rightarrow 3a - 2a = 2$
$\Rightarrow a = 2$
Substituting a = 2 in (iii),
$3(2) = 2\,d$
$\Rightarrow d = \frac{6}{2}$
$\Rightarrow d = 3$
Hence, the required AP is 2,5,8,11, ...

OR

A line segment is divided into four parts forming an Arithmetic progression. The sum of the lengths of 3rd and 4th parts is three times the sum of the lengths of the first two parts. If the length of the fourth part is 14 cm, find the total length of the line segment.

Solution:

Let a_1, a_2, a_3 and a_4 be the lengths of four parts which form an AP.
According to the given,
$a_3 + a_4 = 3(a_1 + a_2)$ .... (i)
$a_4 = 14$ cm....(ii)
From (i),
$a + 2d + a + 3d = 3(a + a + d)$
$2a + 5\,d = 3(2a + d)$
$2a + 5\,d = 6a + 3\,d$
$\Rightarrow 6a - 2a = 5\,d - 3\,d$

$\Rightarrow 4a = 2\,d$
$\Rightarrow 2a = d$
From (ii),
$a + 3d = 14$
Substituting (iii),
$a + 3(2a) = 14$
$a + 6a = 14$
$7a = 14$
$a = \frac{14}{7} = 2$
Substituting a = 2 in (iii),
$d = 2(2) = 4$
Now,
$a_1 = a = 2\,cm$
$a_2 = a + d = 2 + 4 = 6\,cm$
$a_3 = a_2 + d = 6 + 4 = 10\,cm$
Therefore, the total length of the line segment = (2 + 6 + 10 + 14)cm = 32 cm

Q35. The vertices of a ΔABC are $A(-3, 2)$, $B(-1, -4)$ and $C(5, 2)$. If M and N are the mid-points of AB and AC respectively, show that 2MN = BC.

Solution:

Given,

Vertices of a triangle ABC are $A(-3,2)$, $B(-1,-4)$ and $C(5,2)$.

M is the midpoint of AB.

Using midpoint formula,

$$M = \left[\frac{-3-1}{2}, \frac{2-4}{2}\right]$$

$$= \left(-\frac{4}{2}, -\frac{2}{2}\right)$$

$$= (-2, -1)$$

N is the midpoint of AC .

$$N = \left[\frac{-3+5}{2}, \frac{2+2}{2}\right]$$

$$= \left(\frac{2}{2}, \frac{4}{2}\right)$$

$$= (1,2)$$

$$MN = \sqrt{(1-(-2))^2 + (2-(-1))^2}$$
$$= \sqrt{(1+2)^2 + (2+1)^2}$$
$$= \sqrt{3^2 \times 2}$$

$= 3\sqrt{2}$

$BC = \sqrt{(5-(-1))^2 + (2-(-4))^2}$

$= \sqrt{(5+1)^2 + (2+4)^2}$

$= \sqrt{6^2 \times 2}$

$= 6\sqrt{2}$

Therefore, $2MN = BC$

OR

The vertices of a ΔABC are $A(-5,-1), B(3,-5), C(5,2)$. Show that the area of the ΔABC is four times the area of the triangle formed by joining the mid-points of the sides of the triangle ABC.

Solution:

Given,

Vertices of a triangle ABC are A(-5, -1), B(3, -5), C(5, 2).

Let D, E and F be the midpoints of AB, BC and AC of triangle ABC .

Using the midpoint formula,

$D = \left[\frac{-5+3}{2}, \frac{-1-5}{2}\right]$

$= \left(-\frac{2}{2}, -\frac{6}{2}\right)$

$= (-1, -3)$

$E = \left[\frac{3+5}{2}, \frac{-5+2}{2}\right]$

$= \left(\frac{8}{2}, -\frac{3}{2}\right)$

$= \left(4, -\frac{3}{2}\right)$

$F = \left[\frac{-5+5}{2}, \frac{-1+2}{2}\right]$

$= \left(\frac{0}{2}, \frac{1}{2}\right)$

$= \left(0, \frac{1}{2}\right)$

Area of triangle ABC $= \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2+1) + 5(-1+5)]$

$= \frac{1}{2}[-5(-7) + 3(3) + 5(4)]$

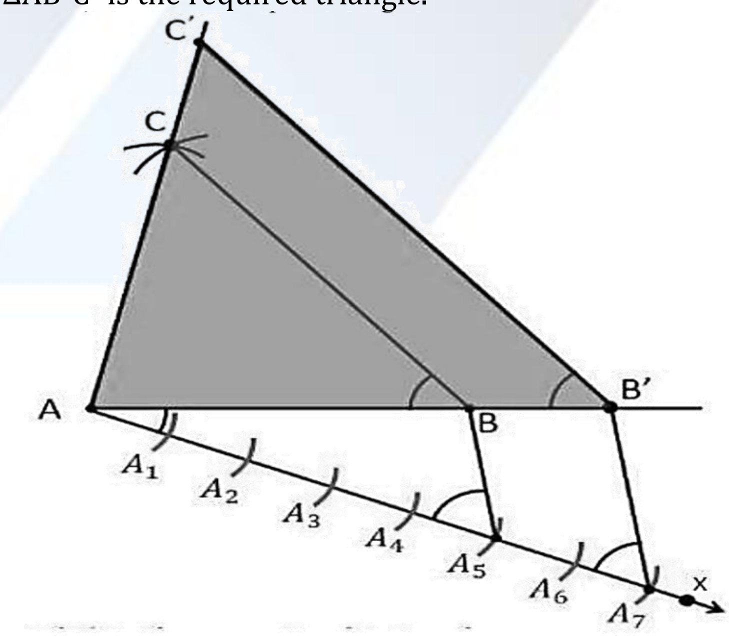
$= \frac{1}{2}(35 + 9 + 20)$

$= \frac{1}{2}(64)$
$= 32$ sq.units
Area of triangle DEF $= \frac{1}{2}[-1\left(-\frac{3}{2}-\frac{1}{2}\right)+4\left(\frac{1}{2}+3\right)+0\left(-3+\frac{3}{2}\right)]$
$= \frac{1}{2}[-1(-2)+4\left(\frac{7}{2}\right)+0]$
$= \frac{1}{2}[2+14] = \frac{1}{2}(16)$
$= 8$
Therefore, Area of triangle ABC $= 4 \times$ Area of triangle DEF

Q36. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Solution:

Steps of construction:
(i) Draw a line segment AB of length 5 cm.
(ii) Taking A and B as centres, draw the arcs of radius 6 cm and 5 cm respectively, which intersect each other at C.
(iii) Join AC and BC and $\triangle ABC$ is obtained.
(iv) Draw a ray AX which makes an acute angle with AB on the opposite side of vertex C.
(v) Locate the 7 points on AX such as $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.
(vi) Join the points BA_5 and draw a line from A_7 to BA_5 which is parallel to the line BA_5 this wil intersect the produced AB at B'.
(vii) Draw a line from B' parallel to AC so that it intersects the produced AC at C'.
Therefore, $\Delta AB'C'$ is the required triangle.



SECTION – V

Q37. Find the solution of the following pairs of linear equation by the graphical method:

$2x + y = 6$

$2x - y = 2$

Solution:

Given pair of equations is:

$2x + y = 6$

$2x - y = 2$

Consider the first equation:

$2x + y = 6$

$y = -2x + 6$

Table for the respective x and y values is:

x	0	1	2	3
y	6	4	2	0

Now, consider the second equation:

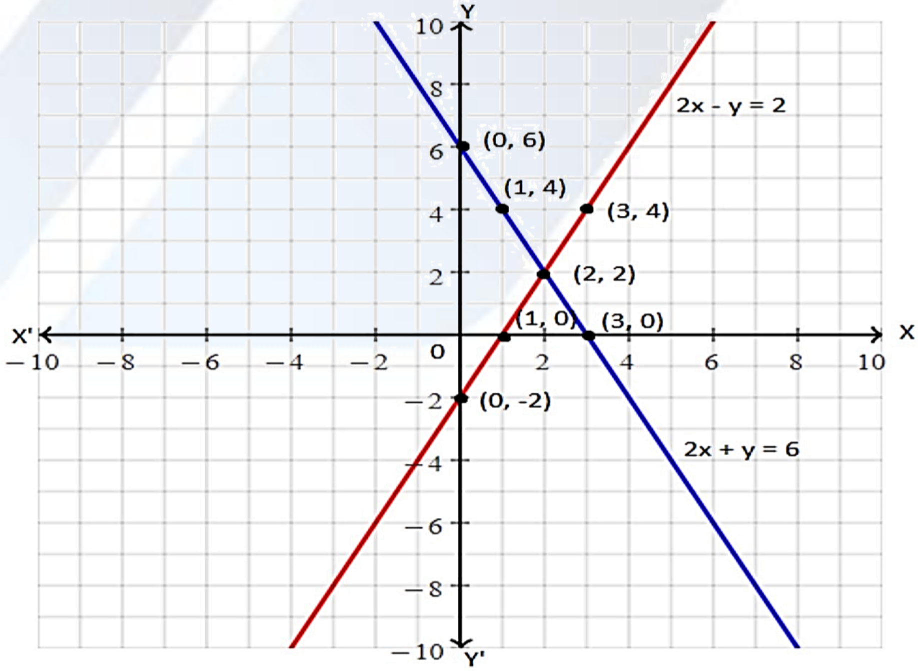
$2x - y = 2$

$y = 2x - 2$

Table for the respective x and y values is:

x	0	1	2	3
y	-2	0	2	4

Graph:

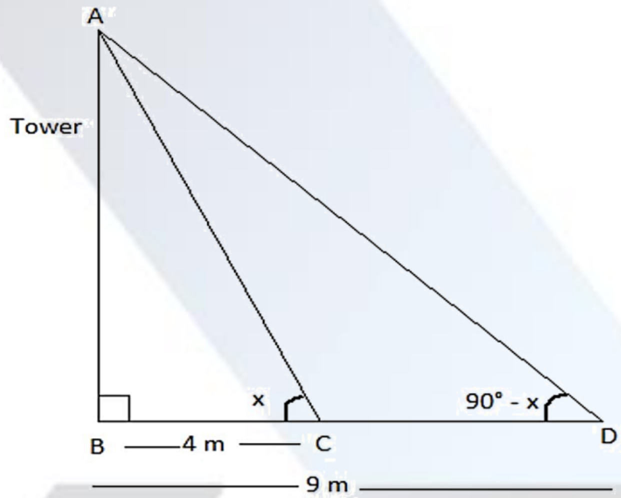


The lines representing the given pair of equations intersecting each other at (2,2) Therefore $x = 2$ and $y = 2$ is the required solution.

Q38. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Find the height of the tower.

Solution:

Let AB be the tower and C, D be the points of observation.



In right △ ABC,

$\tan\ x = \frac{AB}{BC}$

$\tan\ x = \frac{\text{AB}}{4}$

$\text{AB} = 4\tan x$... (i)

In right △ ABD,

$\tan\ (90^\circ - x) = \frac{AB}{BD}$

$\cot x = \frac{\text{AB}}{9}$

$AB = 9\cot x$....(ii)

Multiplying (i) and (ii),

$\text{AB2} = 9\cot x \times 4\tan x$

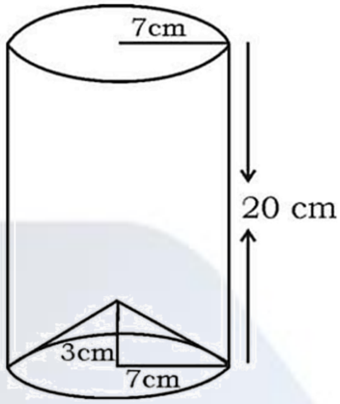
$\Rightarrow AB2 = 36\cot\ x = \frac{1}{\tan x}$

$\Rightarrow AB = \pm 6$

Height cannot be negative.

Therefore, the height of the tower is 6 m.

Q39. The bottom of a right cylindrical shaped vessel made from metallic sheet is closed by a cone shaped vessel as shown in the figure. The radius of the circular base of the cylinder and radius of the circular base of the cone are each equal to 7 cm. If the height of the cylinder is 20 cm and height of cone is 3 cm, calculate the cost of milk to fill completely this vessel at the rate of Rs. 20 per litre.



Solution:
Given,
Height of the cylinder $= H = 20$ cm
Height of the cone $= h = 3$ cm
Radius of cylinder & cone = r=7 cm
Volume of the vessel = Volume of cylinder − Volume of cone

$= \pi r^2 \text{H} - \frac{1}{3}\pi r^2 \text{ h}$

$= \left(\frac{22}{7}\right) \times 7 \times 7 \times 20 - \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 7 \times 7 \times 3$

$= \left(\frac{22}{7}\right) \times 7 \times 7 \; [20 - 1]$

$= 154 \times 19$

$= 2926 \text{ cm}^3$

$= \frac{2926}{1000}$ lit

$= 2.926$ lit

Cost of 1 litre milk = Rs. 20
Total cost of milk which will fill the vessel = Rs. 20 × 2.926
= Rs. 58.52

OR

A hemispherical vessel of radius 14 cm is fully filled with sand. This sand is poured on a level ground. The heap of sand forms a cone shape of height 7 cm. Calculate the area of ground occupied by the circular base of the heap of the sand.

Solution:
Given,
Radius of the hemispherical vessel $=R=$ 14 cm
Height of the cone $= h = 7$ cm
Let r be the radius of conical heap.

Volume of hemispherical vessel = Volume of conical heap

$\left(\frac{2}{3}\right)\pi r^3 = \left(\frac{1}{3}\right)\pi r^2 h$

$2 \times 14 \times 14 \times 14 = r^2 \times 7$

$r^2 = 2 \times 2 \times 14 \times 14$

$r^2 = (2)^2 \times (14)^2$

$r = 2 \times 14$

$r = 28$ cm

Area of the circular base of conical heap = πr^2

$= \left(\frac{22}{7}\right) \times 28 \times 28$

$= 22 \times 4 \times 28$

$= 2464$ cm^2

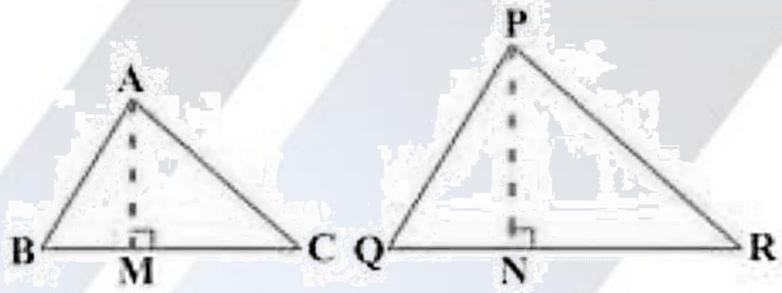
Q40. Prove that "the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides".

Solution:

Given,

Two triangles ABC and PQR such that $\triangle ABC \sim \triangle PQR$

Draw altitudes AM and PN of two triangles ABC and PQR respectively.



$\text{ar}(\triangle \text{ABC}) = \frac{1}{2}\text{BC} \times \text{AM}$

$\text{ar}(\triangle \text{PQR}) = \frac{1}{2}\text{QR} \times \text{PN}$

Now,

$\frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{PQR})} = \frac{\left[\left(\frac{1}{2}\right)\text{BC} \times \text{AM}\right]}{\left[\left(\frac{1}{2}\right)\text{QR} \times \text{PN}\right]}$

$= \frac{\text{BC} \times \text{AM}}{\text{QR} \times \text{PN}} \ldots. \text{(i)}$

In $\triangle ABM$ and $\triangle PQN$,

$\angle B = \angle Q (\triangle ABC \sim \triangle PQR)$

$\angle \text{M} = \angle \text{N}$ $(\textit{each angle } 90°)$

Thus, $\triangle$ ABM $\sim \triangle$ PQN (by AA similarity criterion)

$\Rightarrow \frac{AM}{PN} = \frac{AB}{PQ}$....(ii)

$\triangle$ ABC ~ ΔPQR (given)

$\frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{BC}}{\mathrm{QR}} = \frac{\mathrm{CA}}{\mathrm{RP}}$... (iii)

From (i) and (iii),

$$\frac{\mathrm{ar}(\triangle \mathrm{ABC})}{\mathrm{ar}(\triangle \mathrm{PQR})} = \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right) \times \left(\frac{\mathrm{AM}}{\mathrm{PN}}\right)$$

$= \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right) \times \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)$ [From (ii)]

$$= \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2$$

Similarly,

$$\frac{\mathrm{ar}(\triangle \mathrm{ABC})}{\mathrm{ar}(\triangle \mathrm{PQR})} = \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2 = \left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^2 = \left(\frac{\mathrm{CA}}{\mathrm{RP}}\right)^2$$

Hence proved.