Grade 10 Karnataka Math 2020
QUESTION PAPER CODE 81-E

General Instructions to the Candidates:

1. This question paper consists of 38 questions in all.
2. This question paper has been sealed by reverse Jacket You have to cut on the right side to open the paper at the time of commencement of the examination (Follow the arrow). Do not cut the left side to open the paper. Check whether all the pages of the question paper are intact.
3. Follow the instructions given against the questions
4. Figures in the right hand margin indicate maximum marks for the questions
5. The maximum time to answer the paper is given at the top of the question paper It include 15 minutes for reading the question paper.
6. Ensure that the Version of the question paper distributed to you and the Version printed on your admission ticket is the same

I. Four alternatives are given for each of the following questions / incomplete statements. Choose the correct alternative and write the complete answer along with its letter of alphabet. $8 \times 1 = 8$

Q1. In the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the

(A) equations have no solution
(B) equations have unique solution
(C) equation have three solutions
(D) equation have infinitely many solution

Solution:

Correct Answer: (B)

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then , the pair of linear equations has a unique solution

Q2. In an arithmetic progression, if $a_n = 2n + 1$, then the common difference of the given progression is.

(A) 0
(B) 1
(C) 2
(D) 3

Solution:

Given if $a_n = 2n + 1$,, the common difference $d = a_{n+1} - a_n = 2$

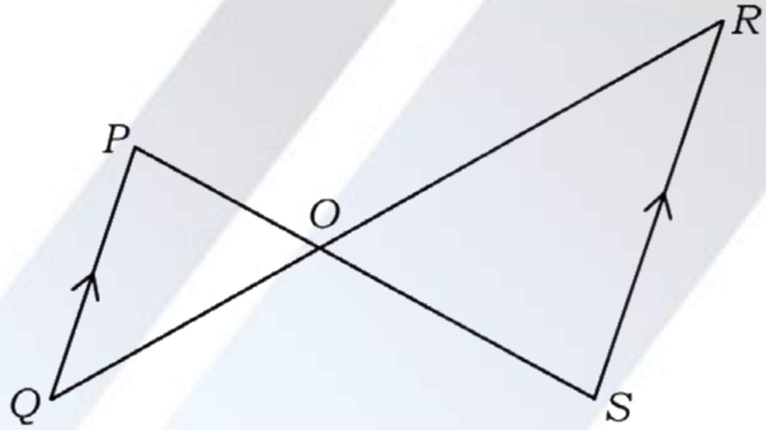
Q3. The degree of a linear polynomial is

(A) 0
(B) 1

(C) 2
(D) 3
Solution:
Correct answer: (B)
The degree of a linear polynomial is 1.

Q4. If $13 \sin \theta = 12$, then the value of $\operatorname{cosec}\theta$ is
(A) $\frac{12}{5}$
(B) $\frac{13}{5}$
(C) $\frac{12}{13}$
(D) $\frac{13}{12}$.
Solution:
Correct answer: (D)
Given $13\sin\theta = 12$,
So, $\sin\theta = \frac{12}{13}$
Therefore, $\csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$.

Q5. In the figure, if $\triangle POQ \sim \triangle SOR$ and $PQ : RS = 1:2$, then $OP : OS$ is



(A) $\mathbf{1:2}$
(B) $\mathbf{2:1}$
(C) $\mathbf{3:1}$
(D) $\mathbf{1:3}$.
Solution:
Correct answer: (A)
Since $\Delta POQ \sim \Delta SOR$, the ratio of corresponding sides is equal.
Given $PQ : RS = 1:2$, so $OP : OS = 1:2$.

Q6. A straight line passing through a point on a circle is
(A) a tangent
(B) a secant

(C) a radius
(D) a transversal.
Solution:
Correct Answer (B)
A straight line passing through a point on a circle is a secant

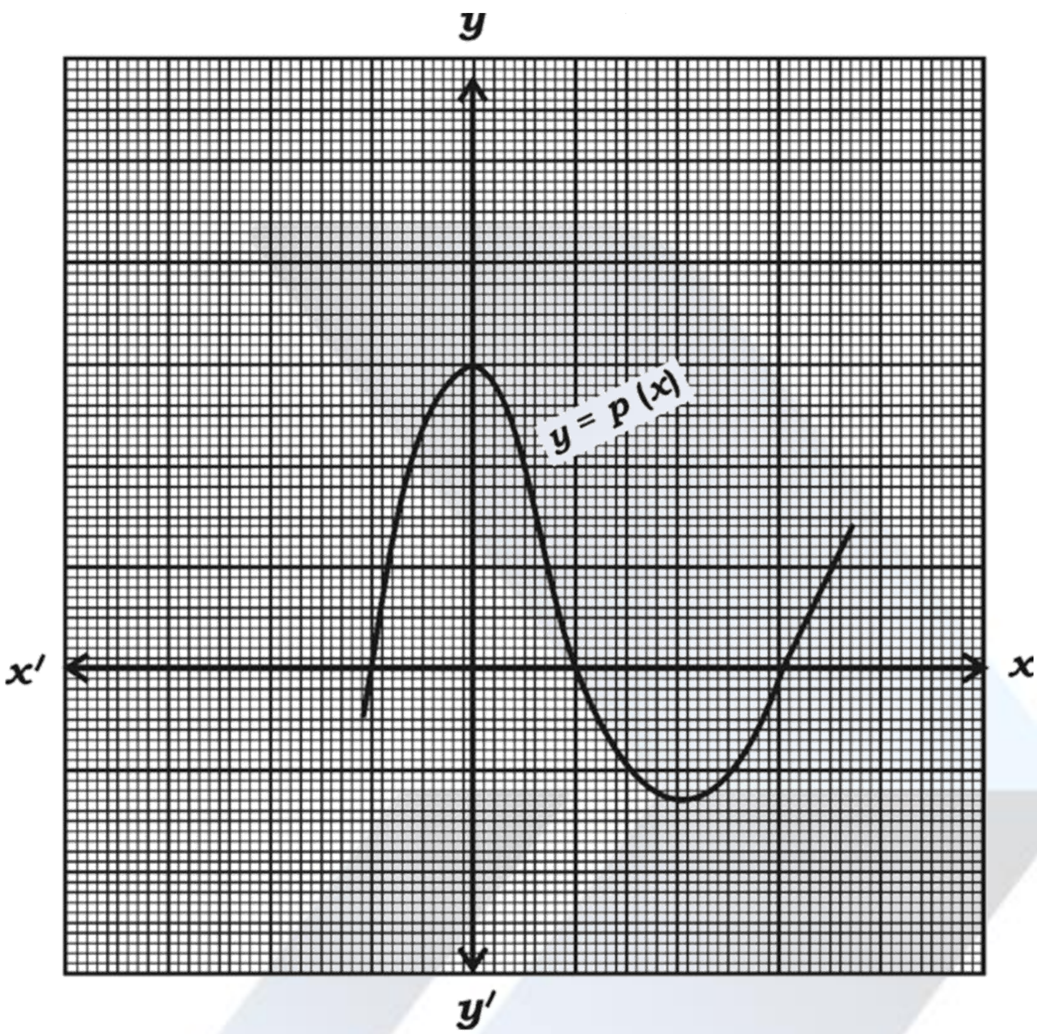
Q7. Length of an arc of a sector of a circle of radius r and angle θ is
(A) $\frac{\theta}{360°} \times \pi r^2$
(B) $\frac{\theta}{360°} \times 2\pi r^2$
(C) $\frac{\theta}{180°} \times 2\pi r$
(D) $\frac{\theta}{360°} \times 2\pi r$.
Solution:
Correct Answer (D)
The length of an arc is given by $\frac{\theta}{360°} \times 2\pi r$.

Q8. If the area of the circular base of a cylinder is 22 cm^2 and its height is 10 cm, then the volume of the cylinder is
(A) 2200 cm^2
(B) 2200 cm^3
(C) 220 cm^3
(D) 220 cm^2
Solution:
Correct Answer (D)
Volume of cylinder = Area of base × Height = $22 \times 10 = 220\ \text{cm}^3$

II. Answer the following questions $\mathbf{8 \times 1 = 8}$

Q9. Express the denominator of $\frac{23}{20}$ in the form of $2^n \times 5^m$ and state whether the given fraction is terminating or non-terminating repeating decimal.
Solution:
The denominator $20 = 2^2 \times 5^1$.
Since the denominator is of the form $2^n \times 5^m$, the fraction is a terminating decimal.

Q10. The following graph represents the polynomial $y = p(x)$. Write the number of zeroes that $p(x)$ has.



Solution:
The number of zeroes of a polynomial is equal to the number of times the graph intersects the x-axis.
So, number of zero $p(x) = 3$

Q11. Find the value of $\tan 45^\circ + \cot 45^\circ$.
Solution:
$\tan 45^\circ = 1$ and $\cot 45^\circ = 1$.
So, $\tan 45^\circ + \cot 45^\circ = 1 + 1 = 2$.

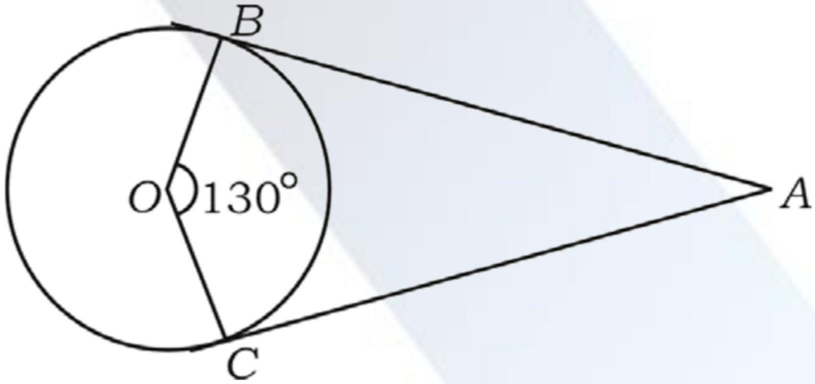
Q12. Find the coordinates of the mid-point of the line joining the points (x_1, y_1) and (x_2, y_2).
Solution:
The mid-point is given by
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Q13. State "Basic proportionality theorem".

Solution:

The Basic Proportionality Theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Q14. In the figure AB and AC are the two tangents drawn from the point A to the circle with centre O. If $\angle BOC = 130°$ then find $\angle BAC$.



Solution:

Since AB and AC are tangents, $\angle OBA = \angle OCA = 90°$.

In quadrilateral $OBAC$, the sum of angles is $360°$.

So, $\angle BAC = 360° - (90° + 90° + 130°) = 50°$.

Q15. Write, $\frac{x+1}{2} = \frac{1}{x}$ in the standard form of a quadratic equation.

Solution:

Given

$$\frac{x+1}{2} = \frac{1}{x}$$

$\Rightarrow x(x+1) = 2$

$\Rightarrow x^2 + x = 2$

$\Rightarrow x^2 + x - 2 = 0$

Q16. Write the formula to find the total surface area of the cone whose radius is ' r ' units and slant height is ' l ' units.

Solution:

The total surface area of a cone is $\boldsymbol{\pi r(r + l)}$.

III. Answer the following questions $\quad 8 \times 2 = 16$

Q17. Solve the given pair of linear equations by Elimination method:

$2x + y = 11$

$x + y = 8$

Solution:

Given equations:
$\boldsymbol{2x + y = 11}$
$x + y = 8$
Subtract the second equation from the first:
$\boldsymbol{(2x + y) - (x + y) = 11 - 8 \Rightarrow x = 3}$
Substitute $\boldsymbol{x = 3}$ into the second equation:
$\boldsymbol{3 + y = 8 \Rightarrow y = 5}$
So, $\boldsymbol{x = 3,\ \ y = 5}$

Q18. Find the sum of $5 + 8 + 11 + \cdots$ to 10 terms using the formula

Solution:

Given AP: 5, 8, 11, ...
First term $a = 5$, common difference $d = 3$

$S_{10} = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2(5) + (10 - 1)3]$

$S_{10} = 5[10 + 27]$

$S_{10} = 5 \times 37 = 185.$

Q19. Find the value of k, if the pair of linear equations $2x - 3y = 8$ and $2(k - 4)x - ky = k + 3$ are inconsistent.

Solution:

Given, the equation to be inconsistent;

$\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}.$

Here,

$\boldsymbol{\frac{2}{2(k - 4)} = \frac{-3}{-k}}$

$\boldsymbol{\Rightarrow \frac{1}{k - 4} = \frac{3}{k}}$

$\boldsymbol{\Rightarrow k = 3(k - 4)}$

$\boldsymbol{\Rightarrow k = 3k - 12}$

$\boldsymbol{\Rightarrow 3k - k = 12}$

$\boldsymbol{\Rightarrow k = \frac{12}{2} = 6}$

Q20. Find the discriminant of the equation $2x^2 - 5x + 3 = 0$ and hence write the nature of the roots

Solution:

Given quadratic equation is $2x^2 - 5x + 3 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
$a = 2, b = -5$ and $c = 3$
Discriminant = D

$= b^2 - 4ac$
$= (-5)^2 - 4(2)(3)$
$= 25 - 24$
$= 1$
Hence, the equation has two distinct real roots.

Q21. If one zero of the polynomial $p(x) = x^2 - 6x + k$ is twice the other then find the value of k.
Solution:
Let the zero be α and 2α.
Sum of zeros:
$\alpha + 2\alpha = \frac{6}{1}$
$\Rightarrow 3\alpha = 6$
$\Rightarrow \alpha = 2$
Product of zeros:
$\alpha \times 2\alpha = k$
$\Rightarrow 2(2)^2 = k$
$\Rightarrow k = 8.$

Or

Find the polynomial of least degree that should be subtracted from $p(x) = x^3 - 2x^2 + 3x + 4$ so that it is exactly divisible by $g(x) = x^2 - 3x + 1$
Solution:
We need to find the polynomial $q(x)$ that should be subtracted from $p(x) = x^3 - 2x^2 + 3x + 4$ so that it becomes exactly divisible by $g(x) = x^2 - 3x + 1$
Perform polynomial division:
Divide x^3 by x^2 to get x, multiply and subtract:
$(x^3 - 2x^2 + 3x + 4) - (x^3 - 3x^2 + x) = x^2 + 2x + 4$
Divide x^2 by x^2 to get 1, multiply and subtract:
$(x^2 + 2x + 4) - (x^2 - 3x + 1) = 5x + 3$
Since the remainder is $5x + 3$, subtracting it from $p(x)$ makes it exactly divisible by $g(x)$.

Q22. Find the distance between the points $P(-5,7)$ and $Q(-1,3)$.
Solution:
Given,
$P(-5, 7)$and $Q(-1, 3)$
$PQ = \sqrt{(-1-(-5))^2 + (3-7)^2}$
$PQ = \sqrt{(-1+5)^2 + (-4)^2}$
$PQ = \sqrt{4^2 + 4^2}$
$PQ = \sqrt{16 + 16}$
$PQ = \sqrt{32}$
$PQ = 4\sqrt{2}$ units

Or

Find the coordinates of the point which divides the line joining the points $(1,6)$ and $(4,3)$ in the ratio $1:2$.

Solution:

Given

$P(1,6), Q(4,3)$

Using section formula

$$(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

Where

$x_1=1, \quad y_1=6, \quad x_2=4, \quad y_2=3, \quad m=1, \quad n=2$

$$x=\frac{1\times 6+2\times 1}{1+2}, y=\frac{1\times 4+2\times 3}{1+2}$$

$$x=\frac{8}{3}, y=\frac{10}{3}$$

$x=4, y=5$

The co-ordinates of the required point $P(x,y)$ is $\left(\frac{8}{3},\frac{10}{3}\right)$

Q23. The points $A(1,1), B(3,2)$ and $C(5,3)$ cannot be the vertices of the triangle ABC. Justify.

Solution:

Given the points A(1,1), B(3,2), and C(5,3) form a triangle,

Use the area formula:

Area $=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

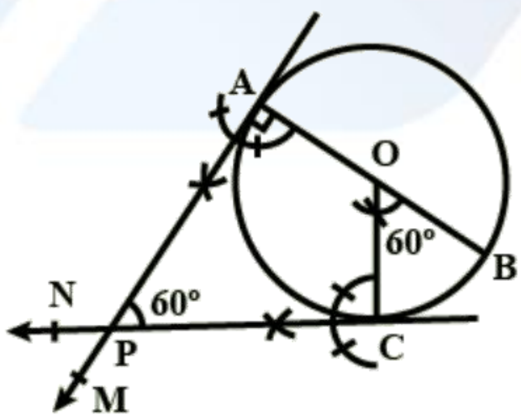
Substituting A(1,1), B(3,2), and C(5,3):

Area $=\frac{1}{2}\mid 1(2-3)+3(3-1)+5(1-2)\mid=\frac{1}{2}\mid -1+6-5\mid=0$

Since the area is 0, the points are collinear and cannot form a triangle.

Q24. Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of 60°

Solution:



Draw a circle with O as centre and radius = 3 cm

Draw any diameter AOB of this circle
Construct $\angle BOC = 60°$ such that radius OC meets the circle
Draw AM⊥AB and CN⊥OC

IV. Answer the following questions $\mathbf{9 \times 3 = 27}$

Q25. Prove that $\sqrt{5}$ is an irrational number.

Solution:

Let us assume $\sqrt{5}$ is rational number
So, it can be written in the form $\frac{a}{b}$ where a and b ($b \neq 0$) are co-prime (no common factor other than 1)
Hence,
$\sqrt{5} = \frac{a}{b}$
$\sqrt{5}b = a$
Squareing both sides
$(\sqrt{5}b)^2 = a^2$
$5b^2 = a^2$
$\frac{a^2}{5} = b^2$
hence, 5 divides a^2
So, 5 shall divide a also ……1
Hence,
$\frac{a}{5} = 5$ where c is some integer
So, $a = 5c$
Now we know that
$5b^2 = a^2$
Putiing $a = 3c$
$5b^2 = (5c)^2$
$5b^2 = 25c^2$
$b^2 = 5c^2$
$\frac{b^2}{5} = c^2$
Hence, 5 divides b^2
So, 5 divides b also …………….2
By 1 an 2
5 divides both a & b
Hence, 5 is a factor of a and b
So, a & b have a factor 5
Therefore, a & b are not co-prime
Hence, our assumption
∴ By contradiction, $\sqrt{5}$ is irrational number.
Hence proved

Or

Find the HCF of 24 and 40 by using Euclid's division algorithm. Hence find the LCM of HCF (24, 40) and 20.

Solution:

Using Euclid's Division Algorithm, we divide 40 by 24:

$40 = 24 \times 1 + 16$

$24 = 16 \times 1 + 8$

$16 = 8 \times 2 + 0$

Since the remainder is 0, HCF(24,40) = 8.

Now, LCM(8,20) is found using prime factorization:

$8 = 2^3$

$20 = 2^2 \times 5$

LCM(8,20)= $2^3 \times 5 = 40$

Thus, HCF = 8 and LCM = 40.

Q26. To save fuel, to avoid air pollution and for good health two persons A and B ride bicycle for a distance of 12 km to reach their office. As the cycling speed of B is 2 km/h more than that of A, B takes 30 minutes less than that of A to reach the office. Find the time taken by A and B to reach the office.

Solution

Let the speed of A be x km/h. Then the speed of B is $x + 2$ km/h.

Time taken by A

$$t_A = \frac{12}{x}$$

Time taken by B

$$t_B = \frac{12}{x + 12}$$

Given $t_A - t_B = 0.5$ hours

$$\frac{12}{x} - \frac{12}{(x + 2)} = 0.5$$

$\Rightarrow 12(x + 2) - 12x = 0.5x(x + 2)$

$\Rightarrow 12x + 24 - 12x = 0.5x^2 + x$

$\Rightarrow x^2 + 2x = 48 = 0$

$\Rightarrow x^2 + 8x - 6x - 48 = 0$

$\Rightarrow x(x + 8) - 6(x + 8) = 0$

$\Rightarrow x = 6, -8$

Since negative value is not possible so $x = 6$

So. Speed of A = 6 km/h

Speed of B = 8 km/h

Time taken by $A = \frac{12}{6} = 2$ hrs

Time taken By $B = \frac{12}{8} = 1.5$ hrs

Or

In a right angled triangle, the length of the hypotenuse is 13 cm. Among the remaining two sides, the length of one side is 7 cm more than the other side. Find the sides of the triangle.

Solution:

Let one of the other sides be x then the other side is $x+7$.

Then according to the problem,

$x^2+(x+7)^2=132$

or, $2x^2+14x-120=0$

or, $x^2+7x-60=0$

or, $x^2+12x-5x-60=0$

or, $(x-5)(x+12)=0$

or, $x=5$. [Since $x \neq -12$]

So the other sides are 5 cm and $5+7=12$ cm.

Q27. If $x=p\tan\theta+q\sec\theta$ and $y=p\sec\theta+q\tan\theta$ then prove that $x^2-y^2=q^2-p^2$

Solution:

Given

$x=p\tan\theta+q\sec\theta, \quad y=p\sec\theta+q\tan\theta$

LHS

$(x^2-y^2)=(ptan\theta+qsec\theta)^2-(psec\theta+qtan\theta)^2$

$\Rightarrow p^2\tan^2\theta+2pqtan\theta+q^2\sec^2\theta-(p^2\sec^2\theta+2pqsec\theta tan\theta+q^2\tan^2\theta)$.

$\Rightarrow p^2(\tan^2\theta-\sec^2\theta)+q^2(\sec^2\theta-\tan^2\theta)$

Using the identity $\sec^2\theta-\tan^2\theta=1$

$x^2=y^2=p^2(-1)+q^2(1)$

Hence $x^2-y^2=q^2-p^2$

Or

Prove that $\frac{\cot^2(90°-\theta)}{\tan^2\theta-1}+\frac{\mathrm{cosec}^2\theta}{\sec^2\theta-\mathrm{cosec}^2\theta}=\frac{1}{\sin^2\theta-\cos^2\theta}$.

Solution:

LHS

$$\frac{\cot^2(90°-\theta)}{\tan^2\theta-1}+\frac{\mathrm{cosec}^2\theta}{\sec^2\theta-\mathrm{cosec}^2\theta}$$

$$=\frac{\tan^2\theta}{\tan^2\theta-1}+\frac{cosec^2\theta}{\sec^2\theta-cosec^2\theta}$$

$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}-1}+\frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}-\frac{1}{\sin^2\theta}}$$

$$=\frac{\sin^2\theta}{\sin^2\theta-\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta-\cos^2\theta}$$

$= \dfrac{1}{\sin^2\theta - \cos^2\theta}$

LHS = RHS

Hence proved

Q28. Find the median of the following data:

Class-interval	Frequency
$20-40$	7
$40-60$	15
$60-80$	20
$80-100$	8

Solution:

Class-interval	Frequency (f_i)	Cumulative frequency
$20-40$	7	7
$40-60$	15	$22 = cf$
$60-80$	$20 = f$	42
$80-100$	8	50

$\dfrac{\mathrm{N}}{2} = \dfrac{50}{2} = 25$

Cumulative frequency greater or equal to $\frac{\mathrm{N}}{2}$ is 42, which lies in the class interval $60-80$

Median class $= \ 60-80$

Lower limit of the median class $= 60$

Frequency of the median class $= f = 20$

Cumulative frequency above the median class $= cf = 22$

Class height $= h = 20$

Median $= l + \left[\dfrac{\frac{\mathrm{N}}{2} - cf}{f}\right] \times h$

$= 60 + \left[\dfrac{25-22}{20}\right] \times 20$

$= 60 + \dfrac{3 \times 20}{20}$

$= 60 + 3$

$= 63$

Or

Find the mode of the following data:

Class-interval	Frequency
$1-3$	6
$3-5$	9
$5-7$	15
$7-9$	9
$9-11$	1

Solution:

Highest frequency $= \mathbf{15}$

Thus, modal class is $\mathbf{5-7}$

Frequency of the modal class $= f_1 = \mathbf{15}$

Lower limit of the modal class $= \mathbf{5}$

Frequency of the class preceding the modal class $= f_0 = \mathbf{9}$

Frequency of the class succeeding the modal class $= f_2 = \mathbf{9}$

Class height $= h = \mathbf{2}$

Mode $= l + \left[\frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}\right] \times h$

$= 5 + \left[\frac{15-9}{2 \times 15 - 9 - 9}\right] \times 2$

$= 5 + \left[\frac{6}{30-18}\right] \times 2$

$= 5 + \frac{6 \times 2}{12}$

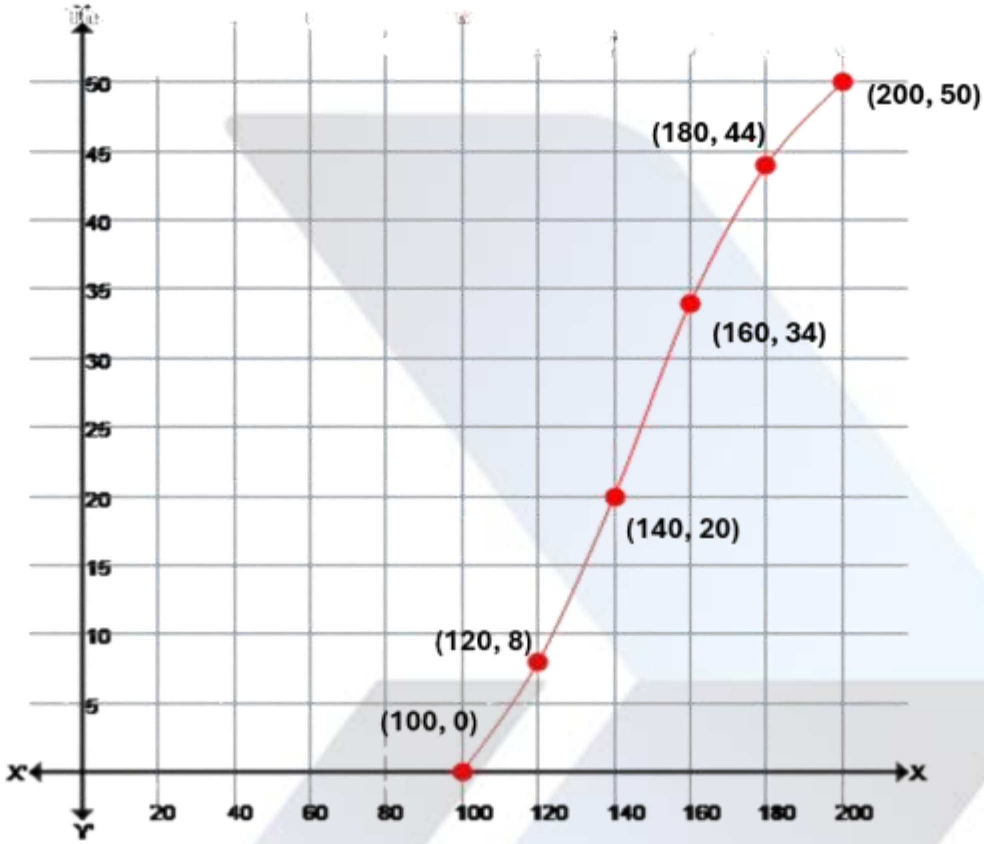
$= 5 + 1$

$= 6$

Q29. The following table gives the information of daily income of 50 workers of a factory. Draw a 'less than type ogive' for the given data:

Daily Income	Number of workers
Less than 100	0
Less than 120	8
Less than 140	20
Less than 160	34
Less than 180	44
Less than 200	50

Solution:



From the given data, to draw less than ogive, choose the upper limits of the class intervals on the x −axis and cumulative frequencies on the y −axis by choosing the convenient scale.

Plot the points corresponding to the ordered pairs given by:

$(100, 0), (120, 8), (140, 20), (160, 34), (180, 44)$, and$(200, 50)$ on a graph paper.

Join all these points to get a smooth curve.

The curve obtained in the graph is known as less than type ogive

Q30. A bag contains 3 red balls, 5 white balls and 8 blue balls. One ball is taken out of the bag at random. Find the probability that the ball taken out is (a) a red ball, (b) not a white ball.

Solution:

Total balls = 3 (red) + 5 (white) + 8 (blue) = 16.

(a) Probability of drawing a red ball:

P(red)= $\frac{3}{16}$

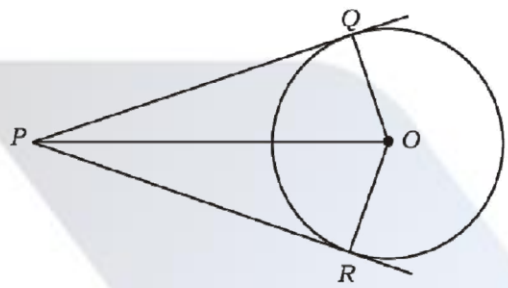
(b) Probability of NOT drawing a white ball (red or blue):

P(not white)= $\frac{3+8}{16} = \frac{11}{16}$

Thus, P(red) $= \frac{3}{16}$ and P(not white) $= \frac{11}{16}$.

Q31. Prove that "the lengths of tangents drawn from an external point to a circle are equal".

Solution:



Given

O is the centre of the circle. PQ and PR are tangents drawn from external point ‘P.

To prove: PQ=PR

Construction: Join OP, OQ and OR.

Proof In the figure

$\angle OQP = \angle ORP = 90^\circ$ $\begin{bmatrix} OQ \perp PQ \\ OR \perp PR \end{bmatrix}$

OQ=OR [radii of same circle]

OP=OP [common side]

$\triangle$ OQP $\cong$ $\triangle$ ORP

PQ=PR [CPCT]

Q32. Construct a triangle ABC with sides $BC = 3$ cm, $AB = 6$ cm and $AC = 4.5$ cm. Then construct a triangle whose sides are $\frac{4}{3}$ of the corresponding sides of the triangle ABC.

Solution:

Steps of construction:

(i) Draw a triangle ABC with side AB = 6 cm , BC = 3 cm and AC = 4.5 cm

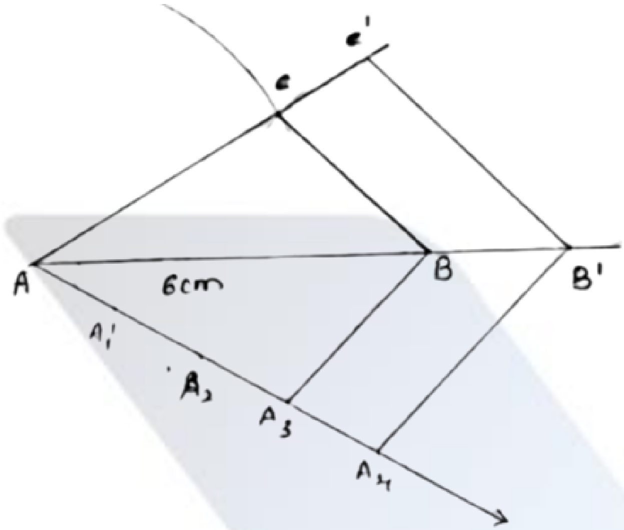
(ii) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C.

(iii) Along AX, mark off 4 points $\boldsymbol{A_1}, \boldsymbol{A_2}, \boldsymbol{A_3}$ and $\boldsymbol{A_4}$ on AX such that $\boldsymbol{AA_1} = \boldsymbol{A_1A_2} = \boldsymbol{A_2A_3} = \boldsymbol{A_3A_4}$.

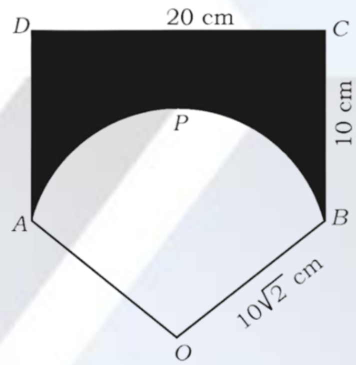
(iv) Join $\boldsymbol{A_3}$ to B and draw a line through $\boldsymbol{A_4}$ parallel to $\boldsymbol{A_3B}$, intersecting the extended line segment AB at B’.

(v) Draw a line through B’ parallel to BC intersecting the extended line segment AC at C’

Then AB’C’ is the required triangle



Q33. $ABCD$ is a rectangle of length 20 cm and breadth 10 cm. $OAPB$ is a sector of a circle of radius $10\sqrt{2}$ cm. Calculate the area of the shaded region. [Take $\pi = 3.14$]



Solution:

Given

Length of rectangle = 20 cm

Breadth of rectangle = 10 cm

Area of rectangle =20 × 10 = 200 cm^2

Radius of sector = $10\sqrt{2}$ cm

Angle = $90°$

Area of segment $= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 sin\theta$

$$= \frac{3.14 \times 10\sqrt{2} \times 10\sqrt{2} \times 90°}{360°} - \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2} \times sin 90°$$

$= 3.14 \times 50 - 100$

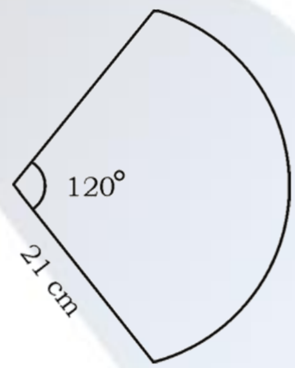
$= 57$ cm^2

Area of shaded region = Area of rectangle−Area of segment

$= 200 - 57 = 143 cm^2$

Or

A hand fan is made up of cloth fixed in between the metallic wires. It is in the shape of a sector of a circle of radius 21 cm and of angle 120° as shown in the figure. Calculate the area of the cloth used and also find the total length of the metallic wire required to make such a fan



Solution:

Given

Radius of sector = 21cm

Angle = 120°

Area of sector $= \frac{\pi r^2 \theta}{360^\circ}$

$= \frac{22 \times 21 \times 21 \times 120^\circ}{7 \times 360^\circ}$

$= 22 \times 21 = 461\text{cm}^2$

Length of wire $= \frac{(2\pi r\theta)}{360^\circ}$

$= \frac{2 \times 22 \times 21 \times 120^\circ}{360^\circ \times 7}$

$= 2 \times 22 = 44$ cm

V. Answer the following questions **$4 \times 4 = 16$**

Q34. Find the solution of the given pair of linear equations by graphical method :

$x + y = 7$

$3x - y = 1$

Solution:

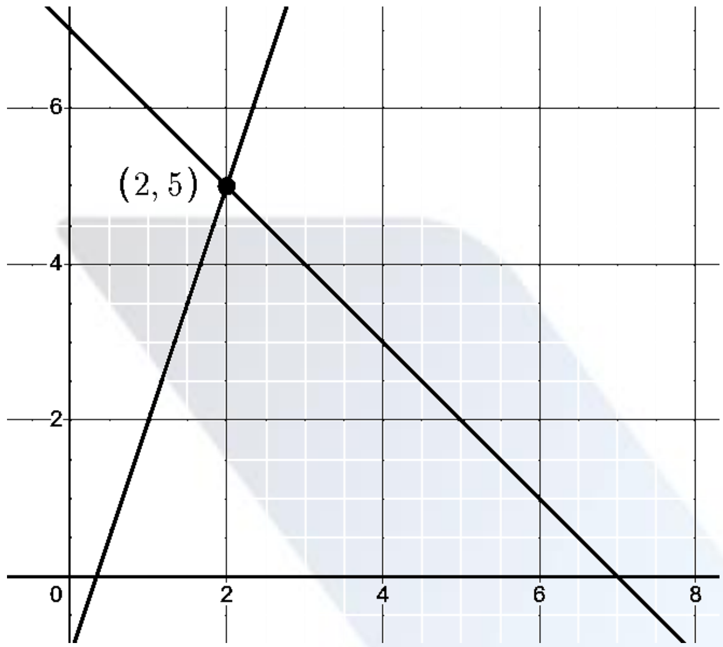
Given

$3x - y = 1$

x	0	1
y	-1	2

$x + y = 7$

x	0	7
y	7	0



Q35. There are five terms in an Arithmetic Progression. The sum of these terms is 55 , and the fourth term is five more than the sum of the first two terms. Find the terms of the Arithmetic progression.

Solution:

Let the first term be a and the common difference be d.

The five terms are:

$a, \quad a+d, \quad a+2d, \quad a+3d, \quad a+4d$

Given, sum of terms = 55:

$a+(a+d)+(a+2d)+(a+3d)+(a+4d)=55$

$5a+10d=55$

$\Rightarrow a+2d=11$ 1

Also, fourth term = sum of first two terms + 5:

$a+3d=\big(a+(a+d)\big)+5$

$a+3d=2a+d+5$

$a-d=5$ 2

From 1

$a=11-2d$

Substituting in 2

$(11-2d)-d=5$

$11-3d=5$

$3d=6$

$d=2$

Substituting $d=2$ in 1

$a+2(2)=11$

$a+4=11$

$a = 7$
So,
$a = 7, d = 2$, so the AP is 7, 9, 11, 13, 15 .

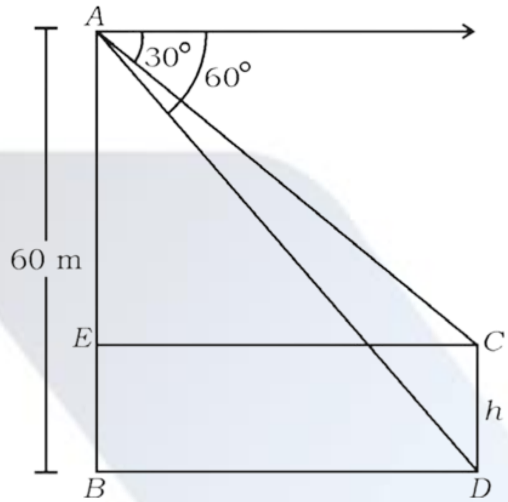
OR

In an Arithmetic Progression sixth term is one more than twice the third term. The sum of the fourth and fifth terms is five times the second term. Find the tenth term of the Arithmetic Progression.

Solution:

Let the first term be a and the common difference be d.
The five terms are:
$a, \quad a + d, \quad a + 2d, \quad a + 3d, \quad a + 4d, \quad a + 5d, ...$
The sixth term is one more than twice the third term:
$a + 5d = 2(a + 2d) + 1$
$a + 5d = 2a + 4d + 1$
$a - d = 1$ 1
The sum of the fourth and fifth terms is five times the second term:
$a + 3d) + (a + 4d) = 5(a + d)$
$2a + 7d = 5a + 5d$
$2a + 7d - 5a - 5d = 0$
$-3a + 2d = 0$
$3a = 2d$ 2
From 1
$a = d + 1$
Substituting in 2
$3(d + 1) = 2d$
$3d + 3 = 2d$
$3d - 2d = -3$
$d = -3$
Substituting $d = -3$ in $a = d + 1$:
$a = -3 + 1 = -2$
The ten term is
$a_{10} = a + 9d$
$a_{10} = -2 + 9(-3)$
$a_{10} = -2 - 27 = -29$

Q36. A tower and a pole stand vertically on the same level ground. It is observed that the angles of depression of top and foot of the pole from the top of the tower of height 60 m is 30° and 60° respectively. Find the height of the pole.



Solution:

Let the height of the pole be h meters and the horizontal distance between the tower and the pole be d meters.

Given that the height of the tower is 60 m, the angles of depression to the top and foot of the pole are 30° and 60°, respectively.

Using the tangent function in right-angled triangles:

For the foot of the pole:

$$tan60° = \frac{60}{d}$$

$\Rightarrow d = \frac{60}{\sqrt{3}} = \frac{20}{\sqrt{3}}$. 1

For the top of the pole:

$$tan30° = \frac{60-h}{d} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}}$$

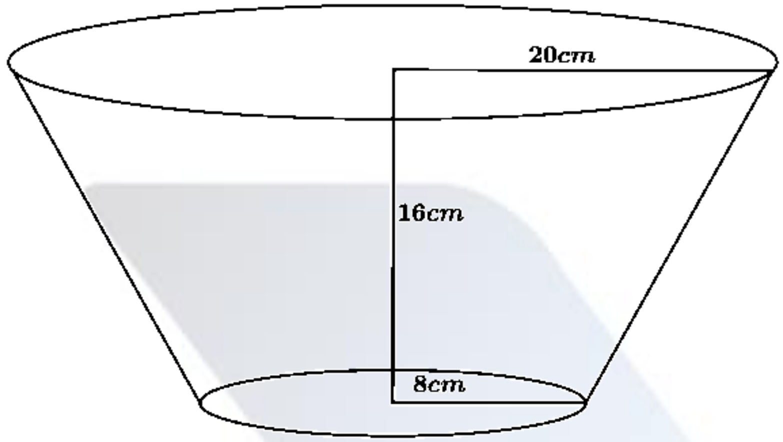
$\Rightarrow 20 = 60 - h$

$\Rightarrow h = 20$

Thus, the height of the pole is 40 m.

Q37. A container opened from the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 20 per litre. [Take $\pi = 3.14$]

Solution:



Radius r_1 of upper end of the container $= 20$ cm
Radius r_2 of lower end of the container $= 8$ cm
Height h of the container $= 16$ cm
Slant height 1 of the frustrum $= \sqrt{(r_1 - r_2)^2 + h^2}$
$= \sqrt{(20-8)^2 + 16^2} = \sqrt{144 + 256} = 20$ cm
Capacity of container = Volume of the frustrum

$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times 3.14 \times 16(20^2 + 8^2 + 20 \times 8)$

$= \frac{1}{3} \times 3.14 \times 16(400 + 64 + 160)$

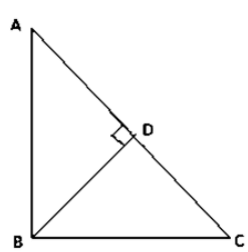
$= \frac{1}{3} \times 3.14 \times 16 \times 624$

$= 10449.92$ cm^3
$= 10.45$ litres
Cost of 1 litre of milk = Rs 20
Cost of 10.45 litre of milk = Rs$20 \times 10.45 =$ Rs209

VI. Answer the following questions **$1 \times 5 = 5$**

Q38. State and prove Pythagoras theorem.

Solution:



Statement:
In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Given: ABC is a triangle in which $\angle ABC = 90^\circ$
Construction: Draw $BD \perp AC$.
Proof:
In $\triangle ADB$ and $\triangle ABC$
$\angle A = \angle A$ [Common angle]
$\angle ADB = \angle ABC$ [$\because$Each 90°]
$\triangle ADB \sim \triangle ABC$ [A − A Criteria]
So,
$$\frac{AD}{AB} = \frac{AB}{AC}$$
Now, $AB^2 = AD \times AC$. (1)
Similarly,
$BC^2 = CD \times AC$
Adding equations (1) and (2) we get,
$AB^2 + BC^2 = AD \times AC + CD \times AC$
$= AC(AD + CD)$
$= AC \times AC$
$\therefore AB^2 + BC^2 = AC$ [hence proved]