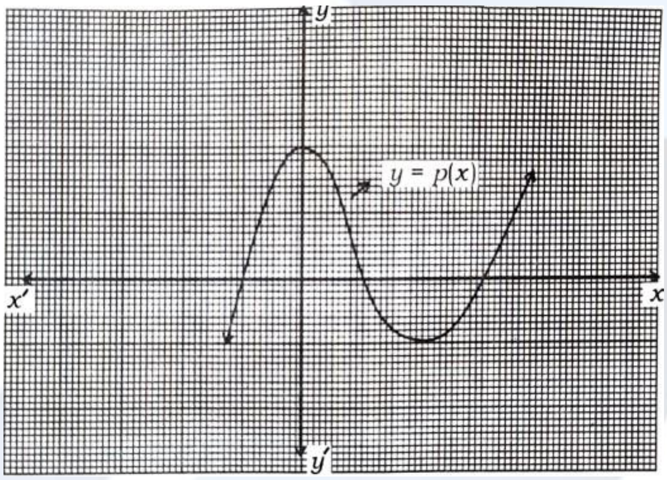
Grade 10 Karnataka Math 2023

General Instructions for Candidates:

1. This question paper consists of objective and subjective types of 38 questions.
2. This question paper has been sealed by a reverse jacket. You have to cut on the right side to open the paper at the time of commencement of the examination. Check whether all the pages of the question paper are intact.
3. Follow the instructions given against both the objective and subjective types of questions.
4. Figures in the right-hand margin indicate maximum marks for the questions.
5. The maximum time to answer the paper is given at the top of the question paper. It includes 15 minutes for reading the question paper.
   Score

I. Four alternatives are given for each of the following questions / incomplete statements. Choose the correct alternative and write the complete answer along with its letter of alphabet. $8 \times 1 = 8$

Q1. The number of zeroes of the polynomial y = p(x) in the given graph is



(A) 3
(B) 2
(C) 1
(D) 4

Solution:

Number of zeros of polynomial = No. of times graph of polynomial cuts x axis
Number of zeros of polynomial = 3
Hence, the correct option is A.

Q2. For an event ' E ', if P(E) = 0.75, then $P(\bar{E})$ is
(A) 2.5
(B) 0.25
(C) 0.025
(D) 1.25

Solution:

Given, $P(E) = 0.75$
We know that $P(E) + P(\bar{E}) = 1$
So, $P(\bar{E}) = 1 - 0.75$
$P(\bar{E}) = 0.25$
Hence, the correct option is B.

Q3. The total surface area of a right circular cylinder having radius ' r ' and height ' h ' is
(A) $\pi r(r + h)$
(B) $2\pi rh$
(C) $2\pi r(r - h)$
(D) $2\pi r(r + h)$

Solution:

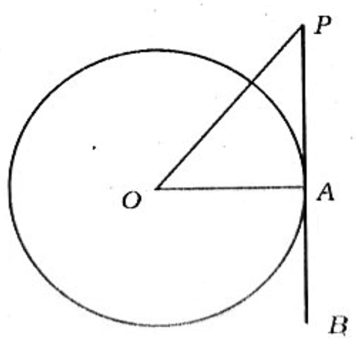
A cylinder has two circular faces, one at the top and other at the bottom, in addition to its curved region.
Let us assume the height of the cylinder be h and the radius of the cylinder be r
Therefore, total surface area = Curved surface area + sum of areas of 2 circular faces
$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
Therefore, the total surface area of a right circular cylinder having radius ' r ' and height 'h' is $2\pi r(h + r)$
Hence, the correct option is D.

Q4. The number that represents the remainder when $19 = 6 \times 3 + 1$ is compared with Euclid's division lemma $a = bq + r$ is
(A) 3
(B) 6
(C) 1
(D) 19

Solution:

If we compare $19 = 6 \times 3 + 1$ with Euclid's division lemma $a = bq + r$
Reminder $r = 1$
Hence, the correct option is C.

Q5. In the given figure, PB is a tangent drawn at the point A to the circle with centre 'O'. If $\angle AOP = 45°$, then the measure of $\angle OPA$ is



(A) 45°
(B) 90°
(C) 35°
(D) 65°

Solution:

Now, In $\triangle$ APO,

As $\angle OAP = 90°$

So,

$\angle OAP + \angle AOP + \angle APO = 180°$

$\Rightarrow 90° + 45° + \angle APO = 180°$

$\Rightarrow \angle APO = 180 - 135 = 45°$

Hence, the correct option is A.

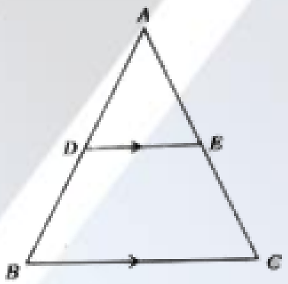
Q6. In the figure, if DE ∥ BC, then the correct relation among the following is

(A) $\frac{AD}{AB} = \frac{AE}{EC}$

(B) $\frac{AD}{DB} = \frac{EC}{AE}$

(C) $\frac{AD}{DB} = \frac{AE}{EC}$

(D) $\frac{DB}{AD} = \frac{AE}{EC}$



Solution:

Basic Proportionality Theorem says that if DE ∥ BC, the ratio in which point D divides AB is the same as the ratio in which point E divides AC.

The ratio in which D divides AB is given by $\frac{AD}{DB}$. Similarly, the ratio in which E divides

AC is given by $\frac{AE}{EC}$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Hence, the correct option is C.

Q7. The lines represented by the equations $4x + 5y - 10 = 0$ and $8x + 10y + 20 = 0$ are

(A) intersecting lines
(B) perpendicular lines to each other
(C) coincident lines

(D) parallel lines

Solution:

For $4x + 5y - 10 = 0$ and $8x + 10y + 20 = 0$

$a_1 = 4, b_1 = 5$ and $c_1 = -10$ and $a_2 = 8, b_2 = 10$ and $c_2 = 20$

$\frac{a_1}{a_2} = \frac{4}{8} = \frac{1}{2}$

$\frac{b_1}{2} = \frac{5}{10} = \frac{1}{2}$

$\frac{c_1}{c_2} = -\frac{10}{20} = -\frac{1}{2}$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, given pairs of linear equations are parallel lines.

Hence, the correct option is D.

Q8. The distance of the point $(-8, 3)$ from the x-axis is

(A) -8 units

(B) 3 units

(C) -3 units

(D) 8 units

Solution:

For the point $(-8, 3)$, x - coordinate $= -8$ and y -coordinate $= 3$.

The point will lie 3 units above the x-axis and 8 units to the left of y-axis. Thus, the distance between the point from the x-axis is 3 units.

Hence, the correct option is B.

II. Answer the following questions:

Q9. Express the denominator of $\frac{7}{80}$ in the form of $2^n \times 5^m$.

Solution:

$$\frac{7}{80} = \frac{7}{2 \times 2 \times 2 \times 2 \times 5} = \frac{7}{2^4 \times 5^1}$$

Q10. If the pair of lines represented by the linear equations $x + 2y - 4 = 0$ and $x + by - 12 = 0$ are coincident lines, then find the values of ' a ' and ' b '

Solution:

The given equations are.

$x + 2y - 4 = 0$

$ax + by - 12 = 0$ for coincident lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{1}{a} = \frac{2}{b} = \frac{-4}{-12}$ we get $\{a = 3, b = 6\}$

Q11. $\triangle$ ABC $\sim\triangle$ PQR. Area of the $\triangle$ ABC is 64 cm^2 and the area of the $\triangle$ PQR is 100 cm^2. If AB $= 8$ cm, then find the length of PQ.

Solution:

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides

Given, $\triangle ABC \backsim \triangle PQR$,

Therefore, $\frac{ar\, \triangle ABC}{ar\, \Delta PQR} = \frac{(AB)^2}{(PQ)^2} \Rightarrow \frac{64\text{ cm}^2}{100\text{ cm}^2} = \frac{(AB)^2}{(PQ)^2}$ corresponding sides, $\frac{64\text{ cm}^2}{100\text{ cm}^2} = \frac{(8\text{ cm})^2}{(PQ)^2}$

This implies $PQ = 10$ cm.

Q12. Express the equation $x(2+x) = 3$ in the standard form of a quadratic equation.

Solution:

$2x + x^2 = 3$

or, $x^2 + 2x - 3 = 0$

This is the standard form of quadratic equation.

Q13. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$.

Solution:

The discriminant of quadratic equation

$2x^2 - 4x + 3 = 0$

is given by $D = b^2 - 4ac$ compared with the standard form of quadratic equation.

$ax^2 + bx + c = 0$

We get $a = 2$

$b = -4, c = 3$ So, $D = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Q14. Find the coordinates of the midpoint of the line segment joining the points (6,3) and (4,7).

Solution:

Let A(6,3) & B(4,7) be the end points

Let $C(x, y)$ be the coordinates of midpoints

Then x co-ordinate: $\frac{6+4}{2} = \frac{10}{2} = 5$, y co-ordinate $= \frac{7+3}{2} = \frac{10}{2} = 5$

So mid points are C(5,5).

Q15. Write the degree of the polynomial $P(x) = 3x^3 - x^4 + 2x^2 + 5x + 2$

Solution:

The highest power of the variable in a polynomial in one variable is called the degree of the polynomial.

Here, in the given polynomial,

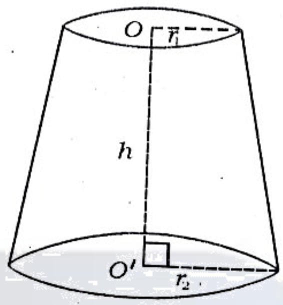
$P(x) = 3x^3 - x^4 + 2x^2 + 5x + 2$

it can be written as, $-x^4 + 3x^3 + 2x^2 + 5x + 2$

Then, the highest power of the variable x is 4.

$\therefore$ The degree of the given polynomial is 4.

Q16. Write the formula to find the volume of the frustum of a cone given in the figure.



Solution:

The Volume of Frustum of Cone $= \frac{1}{3}\pi h\ [(r_2\ ^2 + r_1\ ^2 + r_1 r_2]$

III. Answer the following questions:

Q17. Show that $5 + \sqrt{3}$ is an irrational number.

Solution:

Let us assume on the contrary that $5 + \sqrt{3}$ is rational. Then, there exist prime positive integers a and b such that $5 + \sqrt{3} = \frac{a}{b}$

$\Rightarrow \sqrt{3} = \frac{a}{b} - 5$

$\Rightarrow \sqrt{3} = \frac{a - 5b}{b}$

Here, $\frac{a-5b}{b}$ is a rational number

But $\sqrt{3}$ is irrational

Since, a rational number cannot be equal to an irrational number

This contradicts the fact that $\sqrt{3}$ is irrational.

So, our assumption is incorrect.

Hence, $5 + \sqrt{3}$ is an irrational number and it's proved.

OR

Find the H.C.F. of 72 and 120 by using Euclid's division algorithm.

Solution:

We have to find HCF of 72 and 120 using the Euclidean division algorithm.

Here 120 is greater than 72

Now, consider the largest number as 'a' from the given number i.e., 120 and now satisfy Euclid's division lemma statement $a = bq + r$ where $0 \leq r < b$

Step 1: Since $120 > 72$, we apply the division lemma to 120 and 72, to get

$120 = 72 \times 1 + 48$

Step 2: Since the reminder $72 \neq 0$, we apply division lemma to 48 and 72, to get

$72 = 48 \times 1 + 24$

Step 3: We consider the new divisor 48 and the new remainder 24, and apply the division lemma to get

$48 = 24 \times 2 + 0$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 24, the HCF of 120 and 72 is 24

Notice that $24 = \text{HCF}(48, 24) = \text{HCF}(72, 48) = \text{HCF}(120, 72)$.

Therefore, HCF of 72 and 120 using Euclid's division algorithm is 24.

Q18. Solve the given pair of linear equations:

$3x + y = 12, \quad x + y = 6$

Solution:

Given pair of linear equations are $3x + y = 12$ ..... (i)

$x + y = 6 \ldots \ldots \text{(ii)}$

$\Rightarrow y = 6 - x \ldots \ldots \text{(iii)}$

Substitute eq(iii) in eq(i)

$\Rightarrow 3x + 6 - x = 12$

$\Rightarrow 2x = 12 - 6 = 6$

$\Rightarrow x = \frac{6}{2} = 3$

Substitute the value of x in eq (iii)

$\Rightarrow y = 6 - 3 = 3$

$\therefore x = 3$ and $y = 3$

Q19. Find the 20^{th} term of the arithmetic progression 4,7,10, ... by using formulas.

Solution:

The given AP is 4,7,10, ...

First term = 4

Common difference $(d) = 7 - 4 = 3$

nth term of AP is $a_n = a + (n-1)d$

where, a is first term and d is a common difference.

We need to find the 20th term of the given AP.

Substitute $n = 20, a = 4$ and $d = 3$ in the above formula.

$a_{20} = 4 + (20 - 1)3$

$a_{20} = 4 + (19)3$

$a_{20} = 4 + 57$

$a_{20} = 61$

Therefore, the 20th term of the given AP is 61.

Q20. Find the roots of the equation $2x^2 - 5x + 3 = 0$ by using 'quadratic formula'.

Solution:

Given, $2x^2 - 5x + 3 = 0$

On comparing it with $ax^2 + bx + c = 0$ where $a = 2, b = -5$ and $c = 3$

$\therefore b^2 - 4ac = (-5)^2 - 4 \times 2 \times 3$

$b^2 - 4ac = 25 - 24 = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{1}}{2 \times 2}$$

$= \frac{5 \pm \sqrt{1}}{4}$

$x = \frac{5+1}{4}$ and $x = \frac{5-1}{4}$

$x = \frac{6}{4} = \frac{3}{2}$ and $x = \frac{4}{4} = 1$

OR

Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing the square.

Solution:

Given quadratic equation is $5x^2 - 6x - 2 = 0$.

$\Rightarrow x^2 - \frac{6}{5}x - \frac{2}{5} = 0$ (Divide on both sides by 5)

$\Rightarrow x^2 - 2\left(\frac{3}{5}\right)x + \frac{9}{25} = \frac{9}{25} + \frac{2}{5}$ (add $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ on both sides)

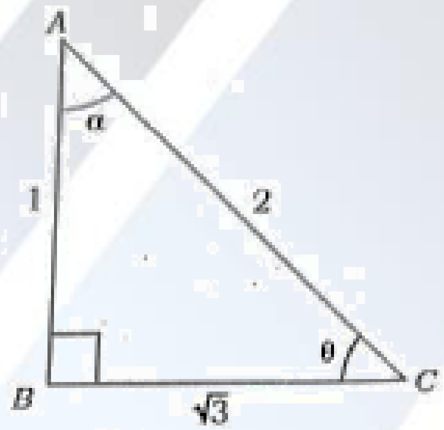
$\Rightarrow \left(x - \frac{3}{5}\right)^2 = \frac{19}{25} \rightarrow x - \frac{3}{5} = \pm\sqrt{\frac{19}{25}}$

(take square root on both sides)

Thus, we have $x = \frac{3}{5} \pm \frac{\sqrt{19}}{5} = \frac{3 \pm \sqrt{19}}{5}$.

Hence, the solution set is $\left\{\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}\right\}$.

Q21. In the given fig, if $\angle ABC = 90°$, then find the value of $\sin\theta$ and $\cos\alpha$.



Solution:

$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

$\therefore \sin\ \theta = \frac{1}{2}$

$\cos\ \alpha = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$\therefore \cos\ \alpha = \frac{1}{2}$

Q22. A box contains cards which are numbered from 9 to 19. If one card is drawn at random from the box, find the probability that it bears a prime number.

Solution:

Let, S be the sample space and A be the event of taking the prime number from 9 to 19.

$S = \{9,10,11,12,13,14,15,16,17,18,19\}$

$n(S) = 11$

$A = \{11,13,17,19\}$

$n(A) = 4$

The probability that it bears a prime number will be,

$$\mathrm{P(A)} = \frac{n(A)}{n(S)}$$

$$\therefore \mathrm{P(A)} = \frac{4}{11}$$

Q23. In the given figure, ABCD is a trapezium in which AB ∥ CD, and BC DC. If AB = 6 cm, CD = 10 cm and AD = 5 cm, then find the distance between the parallel lines.

Solution:

Construction: Draw a line AE ⊥ DC such that AE ∥ BC.

Then, CE will be equal to AB.

So, CE = 6 cm and DE = 4 cm.

In △ AED, ∠E = 90°

By Pythagoras theorem, $\mathrm{Hyp}^2 = \mathrm{Opp}^2 + \mathrm{Adj}^2$

$5^2 = 4^2 + \mathrm{AE}^2$

$25 = 16 + \mathrm{AE}^2$

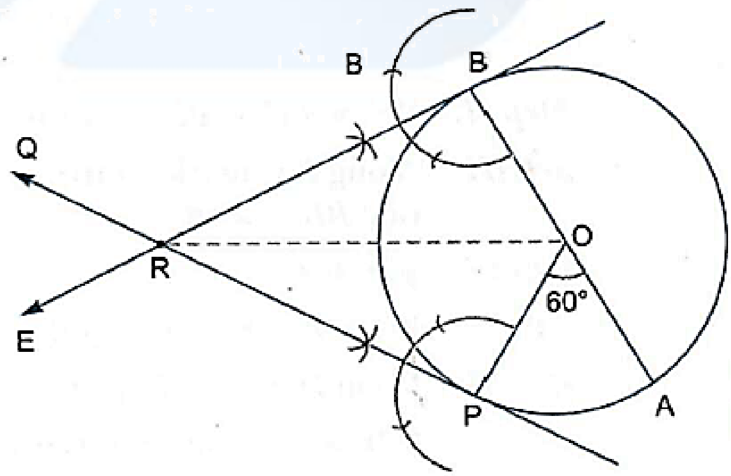
$\mathrm{AE}^2 = 9$

$\therefore$ AE = 3 cm

$\therefore$ The distance between the parallel lines is 3 cm.

Q24. Draw a circle of radius 4 cm and construct a pair of tangents to the circle such that the angle between them is 60°.

Solution:



Here are the stepwise instructions to construct a pair of tangents to a circle of radius 4 cm , such that the angle between them is 60 degrees:
Step I: Draw a circle with center O and radius 4 cm .
Step II: Draw any diameter AOB.
Step III: Make $\angle AOP = 60^\circ$. OP is radius which intersect the circle at P.
Step IV: Draw $PQ \perp OP$ and $BE \perp OB$. PQ and BE intersect at R.
Step V : Hence, RB and RP are the required tangents.

IV. Answer the following questions:

Q25. Divide $p(x) = 3x^3 + x^2 + 2x + 5$ by $g(x) = x^2 + 2x + 1$ and find the quotient $[q(x)]$ and remainder $[r(x)]$.

Solution:

Given that, dividend $p(x) = 3x^3 + x^2 + 2x + 5$ and divisor $g(x) = x^2 + 2x + 1$ If quotient $[q(x)]$ and remainder $[r(x)]$.
By using division algorithm,
$p(x) = g(x)q(x) + r(x)$
$3x^3 + x^2 + 2x + 5 = (x^2 + 2x + 1)q(x) + r(x)$

$$\begin{array}{r|rrrr} & 3x & -5 & & \\ x^2+2x+1 & 3x^3 & +x^2 & +2x & +5 \\ - & 3x^3 & +6x^2 & +3x & \\ \hline & & -5x^2 & -x & +5 \\ & & - & & \\ & & -5x^2 & -10x & -5 \\ \hline & & & 9x & +10 \end{array}$$

Hence, Quotient $[q(x)] = 3x - 5$ and remainder $[r(x)] = 9x + 10$
The zeroes of the polynomial are the values of x which satisfies the equation and makes the equation equal to zero as whole.

OR

Find the zeroes of the quadratic polynomial $p(x) = x^2 + 7x + 10$, and verify the relationship between zeroes and the coefficients.

Solution:

Let us split the middle term to find the factors of the equation $x^2 + 7x + 10$
$x^2 + 7x + 10 = x^2 + 2x + 5x + 10$
Taking out the common terms, we get
$x(x + 2) + 5(x + 2)(x + 2)(x + 5)$
Put both the factors equal to zero.
$x + 2 = 0$ and $x + 5 = 0$, $\quad x = -2$ and $x = -5$
The zeroes of the polynomial $x^2 + 7x + 10$ are -2 and -5
A quadratic polynomial in the form of $ax^2 + bx + c = 0$ where $a \neq 0$ The

coefficients can be expressed as sum and product of the zeroes.

The sum of the zeroes is expressed as $-\frac{\mathrm{b}}{\mathrm{a}}$ that is $\frac{-\text{coefficient of } x}{\text{coefficent of } x^2}$

$(-2)+(-5)=-7=-\frac{-7}{1}=-\frac{b}{a}\ \left(\because \frac{-\text{coefficient of } x}{\text{coefficent of } x^2}\right)$

The product of the zeroes is expressed as $\frac{c}{a}$ that is $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$-2\times(-5)=10=\frac{10}{1}=\frac{c}{a}\ \left(\because \frac{\text{constant term}}{\text{coefficient of } x^2}\right)$

Q26. Prove that $\sqrt{\frac{1+\cos A}{1-\cos A}}=\operatorname{cosec} A+\cot A$

Solution:

We need to prove $\sqrt{\frac{1+\cos A}{1-\cos A}}=\operatorname{cosec} A+\cot A$

By taking L.H.S. $=\sqrt{\frac{1+\cos A}{1-\cos A}}$

$=\sqrt{\frac{1+\cos A}{1-\cos A}\times\frac{1+\cos A}{1+\cos A}}$

$=\sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$

$=\sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$

$=\frac{1+\cos A}{\sin A}$

$=\frac{1}{\sin A}+\frac{\cos A}{\sin A}$

$=\operatorname{cosec} A+\cot A=$ R.H.S.

Hence, proved.

OR

Prove that $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\operatorname{cosec} A$.

Solution:

LHS $=\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}$

$=\left(\frac{\sin^2 A+(1+\cos A)^2}{(1+\cos A)\sin A}\right)$

$=\frac{\sin^2 A+1+\cos^2 A+2\cos A}{(1+\cos A)\sin A}$

$=\frac{2(1+\cos A)}{(1+\cos A)\sin A}$

$=2\operatorname{cosec} A$

$=$ RHS

Hence proved.

Q27. Find the mean of the following data.

Class-interval	Frequency
$1-5$	4
$6-10$	3
$11-15$	2
$16-20$	1
$21-25$	5

Solution:

Class	frequency (f)	Mid value (x)	$d=\frac{x-A}{h}=\frac{x-13}{5}$ $A=13, h=5$	$f\times d$
$1-5$	4	3	-2	-8
6-10	3	8	-1	-3
11-15	2	$13=A$	0	0
16-20	1	18	1	1
$21-25$	5	23	2	10
---	---	---	---	---
	$n=15$	-----	----	$\Sigma f\times d=0$

Mean $\bar{x}=A+\frac{\Sigma fd}{n}\times h$

$=13+\frac{0}{15}\times 5$

$=13+0=13$

OR

Find the mode of the following data.

Class-interval	Frequency
$1-3$	6
$3-5$	9
$5-7$	15
$7-9$	9
$9-11$	1

Solution:

Class	Frequency (f)
$1-3$	6
$3-5$	9
$5-7$	15
$7-9$	9
$9-11$	1
-	$n=40$

To find Mode Class Here, maximum frequency is 15.
$\therefore$ The mode class is $5-7$.
$\therefore L=$ lower boundary point of mode class $=5$
$\therefore f_1=$ frequency of the mode class $=15$
$\therefore f_0=$ frequency of the preceding class $=9$
$\therefore f_2=$ frequency of the succeeding class $=9$
$\therefore c=$ class length of mode class $=2$

$$\text{Mode} = L+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times c$$
$$=5+\left(\frac{15-9}{2\times 15-9-9}\right)\times 2$$
$$=5+\left(\frac{6}{12}\right)\times 2$$
$$=5+1=6$$

Q28. Find the ratio in which the line segment joining the points $A(-6,10)$ and $B(3,-8)$ is divided by the point $(-4,6)$.

Solution:

Let the required ratio be $k:1$
According to section formula the coordinates of M are given by.
$-4=\frac{3k-6}{k+1}$ or , $-4k-4=3k-6$ or ; $-7k=-2, \quad k=\frac{2}{7}$ or
The ratio is $2:7$

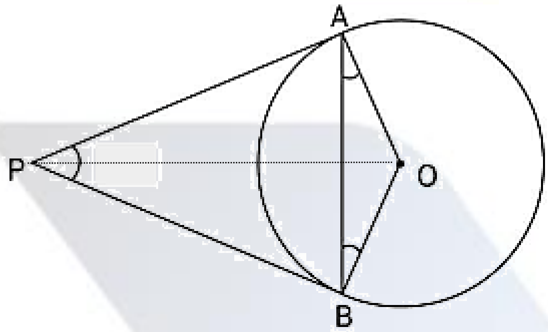
OR

Find the area of a triangle whose vertices are $A(1,-1),\ B(-4,6)$ and $C(-3,-5)$
Putting the values
$\left|\frac{1}{2}\{1(6+5)+-4(-4)+-3(-7)\}\right|$
$=\left|\ \frac{1}{2}\{1(11)+16+21\right|\ \frac{1}{2}\times 48$ sq. cm $|=24\ m^2$.

Q29. Prove that "The lengths of tangents drawn from an external point to a circle are equal".

Solution:

Let PA and PB be two tangents drawn from an external point P to the circle We need to Prove that PA = PB



$\angle PAO = \angle PBO = 90^\circ$ (Radius is perpendicular to the tangent at the point of contact]

In $\triangle$ PAO and $\triangle$ PBO

OP = OP (common hypotenuse to the Triangles)

OA = OB : radii of same circle

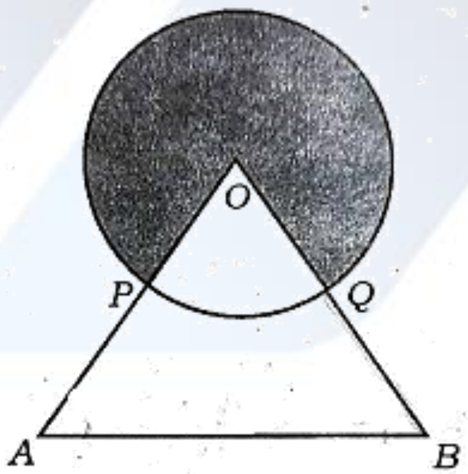
So by RHS congruence,

$\triangle PAO \cong \triangle PBO$

So, PA = PB

Hence the length of tangents drawn from an external point to a circle are equal.

Q30. In the given figure, ' O ' is the centre of a circle and OAB is an equilateral triangle. P and Q are the mid-points of OA and OB respectively. If the area of $\triangle$ OAB is $36\sqrt{3}\ \text{cm}^2$, then find the area of the shaded region.



Solution:

Area of an Equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

ATQ: -

$36\sqrt{3} = \frac{\sqrt{3}}{4} a^2 144 = a^2 a = \pm 12$

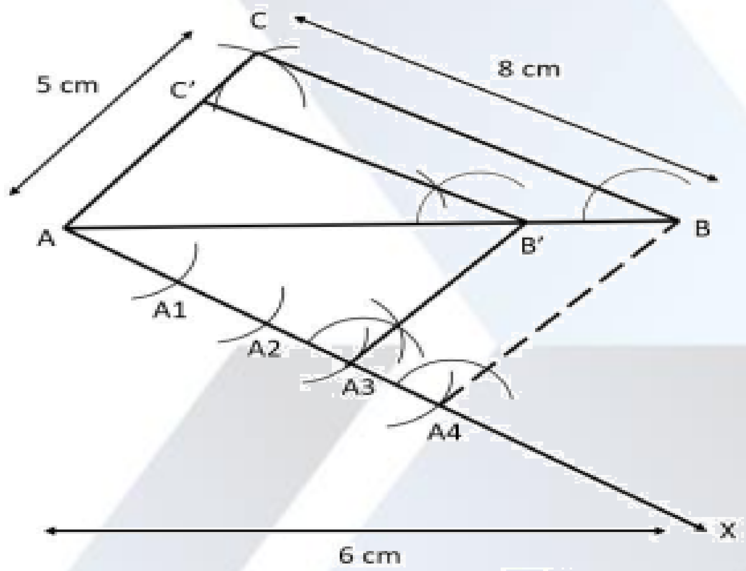
a will be 12 as the side cannot be negative.

$OA = OB = 12$ cm

$OP = 6$ cm as P is the midpoint.
Now radius of the circle will be 6 cm. area of circle will become $\Rightarrow \pi r^2$
$\Rightarrow \pi \times 6^2 \Rightarrow 36\pi\ \text{cm}^2$
area of OPQ will be $\Rightarrow \frac{60}{360} \pi \times r^2$
$\Rightarrow \frac{1}{6} \times \pi \times 36 \Rightarrow 6\pi\ \text{cm}^2$
Area of shaded part $=$ Area of circle - area of OPQ
$= 36\pi - 6\pi = 30\pi\ \text{cm}^2$

Q31. Construct a triangle with sides 5 cm, 6 cm and 8 cm and then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.

Solution:



Step 1 - Draw a line segment $AB = 6$ cm. taking point A as centre, draw an arc of 5 cm radius.
Similarly, taking point B as its centre, draw an arc of 8 cm radius. These arcs will intersect each other at point C. Now, $AC = 5$ cm and $BC = 8$ cm and $\triangle ABC$ is the required triangle.
Step 2 - Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.
Step 3 - Locate 4 points A_1, A_2, A_3, A_4 (as 4 is greater between 3 and 4) on line AX such that
$AA_1 = A_1 A_2 = A_2 A_3 = A_3 A_4$
Step 4 - Join BA4 and draw a line through $A3$ parallel to $BA4$ to intersect AB at point B'.
Step 5 - Draw a line through B' parallel to the line BC to intersect AC at C'. $\triangle AB'C'$ is the required triangle.

Q32. The distance between two cities ' A ' and ' B ' is 132 km. Flyovers are built to avoid the traffic in the intermediate towns between these cities. Because of this, the average speed of a car traveling in this route through flyovers increases by 11 km/h and hence, the car takes 1 hour less time to travel the same distance than earlier. Find the current average speed of the car.

Solution:

Distance between two cities $= 132$ km

Increased average speed $= 11$ km/h

Time of travel decreased = 1 hr

We know that average speed = $\frac{distance}{time}$

If previously time taken is t,

According to the question,

$\frac{132}{t-1} - \frac{132}{t} = 11$

$132t - 132(t-1) = 11t(t-1)$

$11t^2 - 11t - 132 = 0$

$t^2 - t - 12 = 0$

$t^2 - 4t + 3t - 12 = 0$ (On Dividing both sides by 11)

$t(t-4) + 3(t-4) = 0$

$(t+3)(t-4) = 0$

Hence, $t = 4$hrs ($t = -3$ is rejected as time can't be negative)

Hence, Current average speed of car $= \frac{132}{t-1} = \frac{132}{4-1} = \frac{132}{3} = 44$ km/h

Q33. A life insurance agent found the following data for distribution of ages of 100 policy holders. Draw a "Less than type ogive" for the given data:

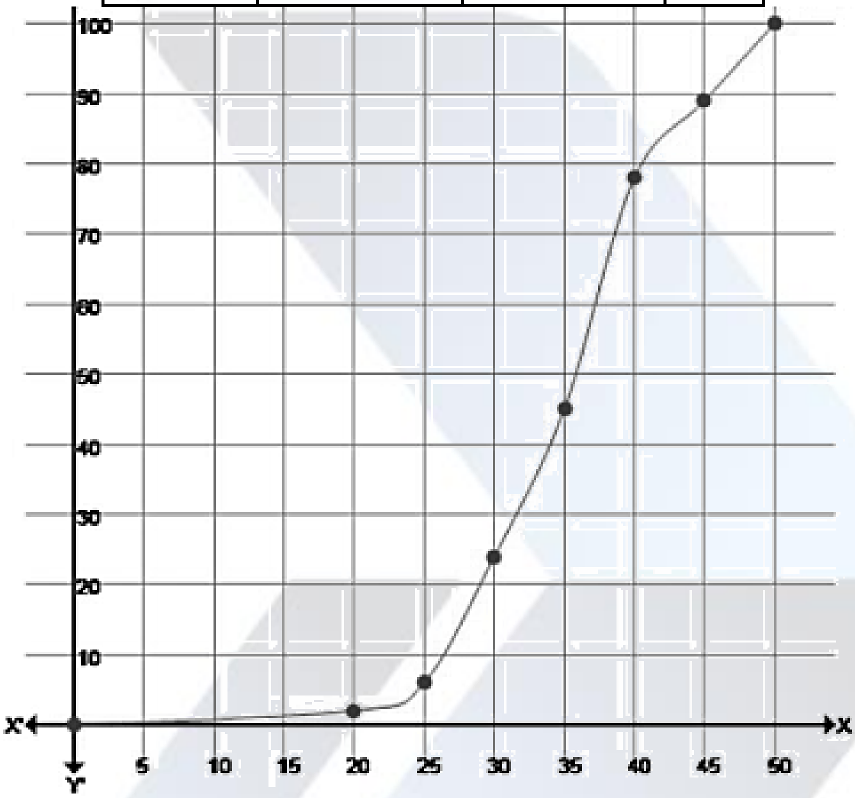
Age (in years)	Number of policyholders (cumulative frequency)
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	100

Solution:

To draw Less than type cumulative frequency curve, we first prepare the following table

Class	Frequency f	< Upper	cf
$0-20$	2	20	2
$20-25$	4	25	6

25 – 30	18	30	24
30 – 35	21	35	45
35 – 40	33	40	78
40 – 45	11	45	89
45 – 50	11	50	100



V. Answer the following questions:

Q34. The sum of 2nd and 4th terms of an AP is 54 and the sum of its first 11 terms is 693. Find the arithmetic progression. Which term of this progression is 132 more than its 54th term?

Solution:

The sum of 2nd and 4th terms of an AP is 54.

$a_2 + a_4 = 54$

$a + d + a + 3d = 54$

$2a + 4d = 54$

$a + 2d = 27$

The sum of its first 11 terms is 693.

$S_{11} = 693$

$\frac{11}{2}(2a + (11 - 1)d) = 693$

$2a + 10d = 693 \times \frac{2}{11}$

$2a + 10d = 126$

$a + 5d = 63 \ldots \text{(ii)}$

Subtract (i) and (ii),
$a+2d=27$
$a+5d=63$
$-3d=-36$
Now, substitute d in (i),
General form of AP is,
$a, a+d, a+2d, a+3d, \ldots$
$3,3+12,3+24,3+36, \ldots$
$3,15,27,39, \ldots$

$\therefore$ The arithmetic progression is $3,15,27,39, \ldots$

' x ' term of this progression is 132 more than its 54^{th} term.

$x_n=132+a_{54}$
$x_n=132+a+53d$
$x_n=132+3+636$
$x_n=771$
$a+(n-1)d=771$
$3+(n-1)12=771$
$12(n-1)=768$
$n-1=64$
$n=65$
$\therefore 65^{\text{th}}$ term of this progression is 132 more than its 54^{th} term.

OR

The first and the last terms of an arithmetic progression are 3 and 253 respectively. If the 20^{th} term of the progression is 98, then find the arithmetic progression. Also find the sum of the last 10 terms of this progression.

Solution:

$a=3, l=253$
$a_{20}=98$
$a+19d=98$
$3+19d=98$
$19d=95$
$\therefore d=5$
General form of AP is,
$a, a+d, a+2d, a+3d, \ldots$
$3,3+5,3+10,3+15, \ldots$
$3,8,13,18, \ldots$
$\therefore$ The arithmetic progression is $3,8,13,18, \ldots 253$

The sum of the last 10 terms of the progression is, (Reversing the AP)

$253,248,243, \ldots a_{10}$

$a_{10}=a+9d$

$a_{10}=253-45$

$a_{10}=208$

$$S_{10}=\frac{n}{2}(a+l)$$

$$S_{10}=\frac{10}{2}(253+208)$$

$$S_{10}=5(461)$$

$$S_{10}=2305$$

∴ The sum of the last 10 terms of the progression is 2305.

Q35. Find the solution of the given pair of linear equations by graphical method: $2x+y=8,\ \ x-y=1$

Solution:

Given linear equations

$L_1:\ 2x+y=8,\qquad L_2: x-y=1$

Which are the equations of a line

In order to draw the graph of a line we only need two points satisfying the equation of the line

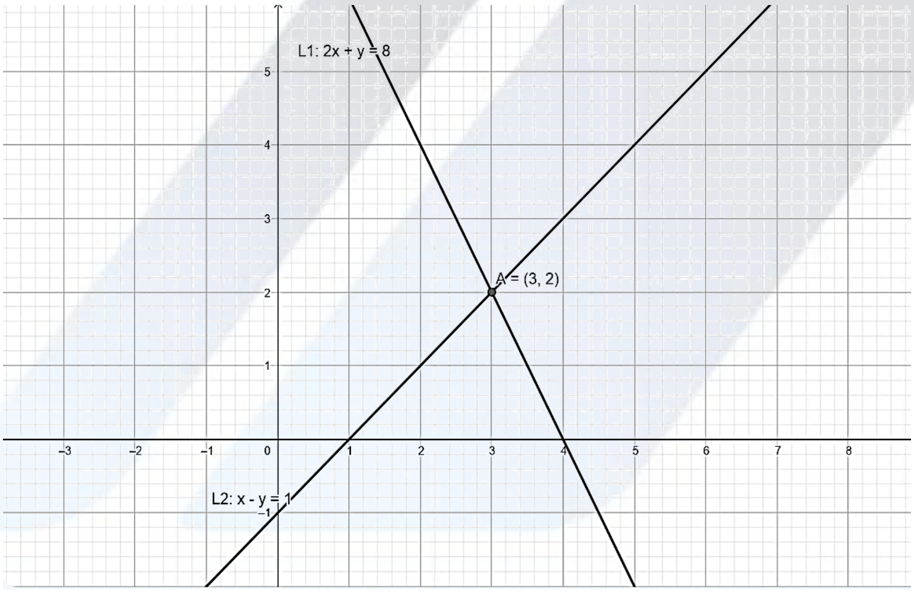
The graphs of lines L_1 and L_2 are drawn

From graph it is clear that the lines intersect at point A

The coordinates of point A are (3,2)

Therefore, the solution of the pair of linear equations is

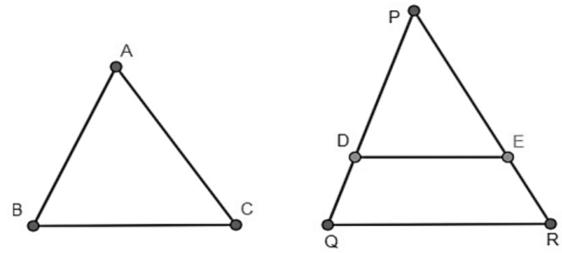
$$x=3, y=2$$



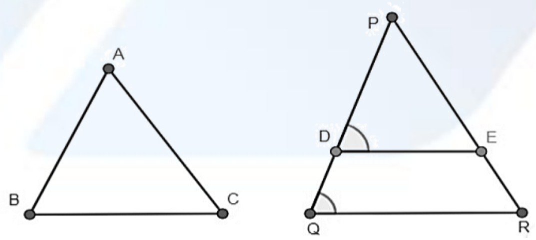
Q36. Prove that "If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar".

Solution:

In the below figure, we have drawn two triangles ABC and PQR and we have drawn DE on PQR as follows:



In the above figure, AB = PD & AC = PE.
It is given that corresponding angles of triangles ABC and PQR are equal so:
$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$
The below sides are equal due to its construction in such a way:
AB = PD & AC = PE
Let us consider $\triangle$ ABC & $\triangle$ PDE,
AB = PD & AC = PE
$\angle A = \angle P$
Then $\triangle ABC \cong \triangle PDE$
by SAS congruence rule.
As $\triangle ABC \cong \triangle PDE$ then the following angles are equal due to CPCT
(Corresponding part of Congruent triangles),
$\angle B = \angle D \ldots \ldots \ldots . . Eq. (1)$
$\angle C = \angle E \ldots \ldots . Eq. (2)$
It is also given that:
$\angle B = \angle Q \ldots \ldots \ldots . . Eq. (3)$
$\angle C = \angle R \ldots \ldots . Eq. (4)$
From eq. (1 and 3) we get,
$\angle D = \angle Q$
From eq. (2 and 4) we get,
$\angle E = \angle R \ldots \ldots \ldots . Eq$
From eq. (5 and 6), we can say that DE is parallel to QR then the above angles are corresponding angles.



There is a theorem that if a line is parallel to one side of the triangle and intersects the other two sides in two distinct points then the other two sides are divided in the same ratio.
Using this theorem in the triangle PQR, DE ∥ QR and the side DE intersects PQ and PR in two distinct points D and E respectively.
Then, the following sides are proportional in the following way:
$\frac{PD}{DQ} = \frac{PE}{ER}$

Taking reciprocal on both the sides of the above equation we get,

$\frac{DQ}{PD} = \frac{ER}{PE}$

Adding 1 on both the sides we get,

$\frac{DQ}{PD} + 1 = \frac{ER}{PE} + 1 \Rightarrow \frac{DQ+PD}{PD} = \frac{ER+PE}{PE} \Rightarrow \frac{PQ}{PD} = \frac{PR}{PE}$

Taking reciprocal on both the sides we get,

$\frac{PD}{PQ} = \frac{PE}{PR}$

We have shown above that:

$AB = PD \;\&\; AC = PE$

So, substituting $AB = PD \;\&\; AC = PE$ in eq. (7) we get,

$\frac{AB}{PQ} = \frac{AC}{PR}$

Similarly, we can show that:

$\frac{AB}{PQ} = \frac{BC}{QR}$

Therefore, assemble all the proportional sides that we have shown above we get,

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

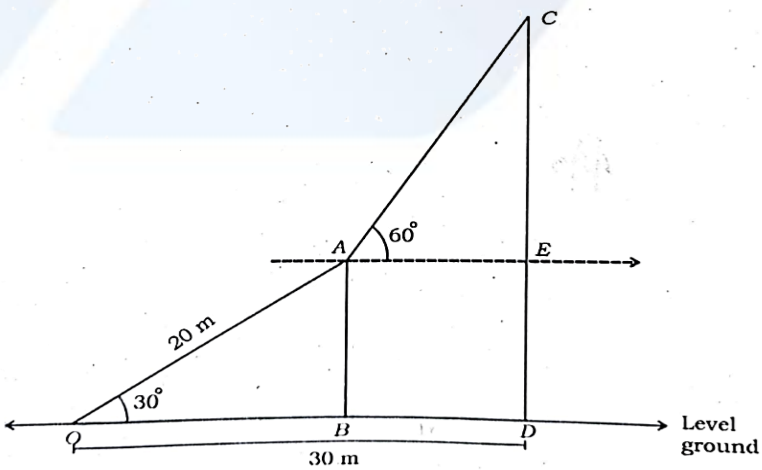
Since, we have shown that all the sides of the triangles ABC and PQR are proportional so:

$\triangle ABC \sim \triangle PQR$

Hence, we have shown that two triangles ABC and PQR are similar.

Q37. In the given figure, a rope is tightly stretched and tied from the top of a vertical pole to a peg on the same level ground such that the length of the rope is 20 m and the angle made by it with the ground is 30°. A circus artist climbs the rope, reaches the top of the pole and from there he observes that the angle of elevation of the top of another pole on the same ground is found to be 60°. If the distance of the foot of the longer pole from the peg is 30m, then find the height of this pole. (Take $\sqrt{3} = 1.73$)

Solution:



In right angle triangle ABO.

$\cos\ 30^\circ = \frac{OB}{OA}$

$\frac{\sqrt{3}}{2} = \frac{OB}{20}$

$OB = 10\sqrt{3}\ \text{m}$

$\sin\ 30^\circ = \frac{AB}{OA}$

$\frac{1}{2} = \frac{AB}{20}$

$AB = 10\ \text{m}$

Now

$BD = OD - OB = 30 - 10\sqrt{3} = 10\sqrt{3}(\sqrt{3} - 1)\text{m}$

Now in right angle $\triangle$ CEA

$\tan 60^\circ = \frac{CE}{AE}$

$\sqrt{3} = \frac{CE}{10\sqrt{3}(\sqrt{3} - 1)}$

$CE = 30(\sqrt{3} - 1)$

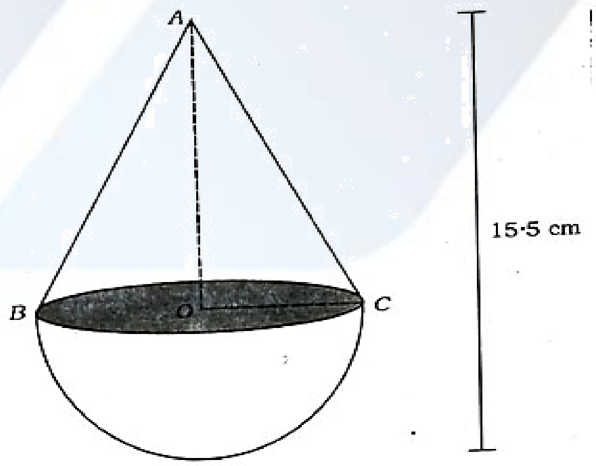
Now height of the pole $CD = CE + ED$

$CD = 30\sqrt{3} - 30 + 10$

$CD = 30\sqrt{3} - 20$

$CD = 10(3\sqrt{3} - 2)\text{m}$

Q38. A wooden solid toy is made by mounting a cone on the circular base of a hemisphere as shown in the figure. If the area of base of the cone is $38.5\ \text{cm}^2$ and the total height of the toy is 15.5cm, then find the total surface area and volume of the toy.



Solution:

If r is the radius of the base of the cone then according to the question,

Area of the base $= 38.5\ \text{cm}^2$

$\frac{22}{7} \times r^2 = 38.5$

$$r^2 = 38.5 \times \frac{7}{22}$$

$$r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$$

Height of hemisphere $= r = 3.5$ cm
Height of the cone (h) will be,
$h =$ Height of toy $-$ Height of hemisphere $= 15.5 - 3.5 = 12$ cm
Slant height of the cone will be,

$$l = \sqrt{h^2 + r^2} = \sqrt{144 + \frac{49}{4}} = \frac{25}{2}$$

$l = 12.5$ cm.
Total surface area of the toy will be,
T.S. $A = \pi r l + 2\pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 137.5 + 77 = 214.5 \text{ cm}^2$$

Volume of the toy will be,

$$V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 + \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 154 + 89.83 = 243.83 \text{ cm}^3$$