Grade 10 Karnataka Math 2024
QUESTION PAPER CODE 81-E

General Instructions to the Candidates:

1. This question paper consists of 38 questions in all.
2. This question paper has been sealed by reverse Jacket You have to cut on the right side to open the paper at the time of commencement of the examination (Follow the arrow). Do not cut the left side to open the paper. Check whether all the pages of the question paper are intact.
3. Follow the instructions given against the questions
4. Figures in the right hand margin indicate maximum marks for the questions
5. The maximum time to answer the paper is given at the top of the question paper It include 15 minutes for reading the question paper.
6. Ensure that the Version of the question paper distributed to you and the Version printed on your admission ticket is the same

.I. Four alternatives are given for each of the following questions / incomplete statements. Choose the correct alternative and write the complete answer along with its letter of alphabet. $8 \times 1 = 8$

Q1. The product of HCF and LCM of two numbers 15 and 20 is

(A) 15
(B) 20
(C) 300
(D) 35

Solution:

Correct Answer: (C)
Given, HCF of 15 and 20 is 5, and the LCM is 60.
HCF $\times$ LCM $= 15 \times 20 = 300$
Thus, the product of HCF and LCM is 300.

Q2. If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then $\alpha\beta$ is

(A) $\frac{b}{a}$
(B) $\frac{-b}{a}$
(C) $\frac{-c}{a}$
(D) $\frac{c}{a}$

Solution:

Correct answer: (D)
For a quadratic polynomial of the form:
$p(x) = ax^2 + bx + c$
The product of its zeroes α and β is given by the formula:

$\alpha\beta = \frac{c}{a}$

Thus, the product of the zeroes of $p(x)$ is $\frac{c}{a}$.

Q3. If $\sin\theta = \frac{4}{5}$, then the value of $\sqrt{1-\cos^2\theta}$ is

(A) $\frac{16}{25}$

(B) $\frac{4}{5}$

(C) $\frac{5}{4}$

(D) $\frac{9}{25}$

Solution:

Correct answer: (B)

Given $\sin\theta = \frac{4}{5}$

Usking Pythagorean identities

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$

So, $\sqrt{1-\cos^2\theta} = \frac{4}{5}$

Q4. The probability of a sure event is

(A) 1

(B) 0

(C) -1

(D) 1.5

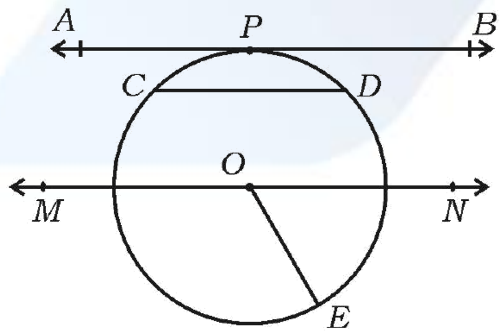
Solution:

Correct answer: (A)

In probability theory, a sure event is an event that is certain to occur.

The probability of a sure event is always 1 or 100%.

Q5. The secant of the circle in the figure, is



(A) MN

(B) OE

(C) CD

(D) AB

Solution:

Correct answer: (A)

Secant is a line which intersect a circle at two distinct points

So, MN is a secant in this given figure.

Q6. The volume of the frustum of a cone whose base radii are r_1 and r_2 and height ' h ', is

(A) $\frac{1}{3}\pi(r_1 + r_2 + r_1 \cdot r_2)h$

(B) $\frac{1}{3}\pi({r_1}^2 + {r_2}^2 - r_1 \cdot r_2)h$

(C) $\frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 \cdot r_2)h$

(D) $\frac{1}{3}\pi({r_1}^2 - {r_2}^2 - r_1 \cdot r_2)h$

Solution:

Correct answer: (C)

Volume of frustum = $\frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 \cdot r_2)h$

Q7. If $2, x, 26$ are in Arithmetic progression, then the value of x is

(A) 12

(B) 14

(C) 28

(D) 24

Solution:

Correct answer: (B)

Given that 2, x, and 26 are in arithmetic progression A.P, the common difference d between consecutive terms is constant.

Therefore, the difference between the second and first terms equals the difference between the third and second terms:

$x - 2 = 26 - x$

$2x = 28$

$x = 14$

Thus, the value of x is 14.

Q8. If $\tan(90° - \theta) = \sqrt{3}$, then the value of cotθ is

(A) $\frac{1}{\sqrt{3}}$

(B) 1

(C) 0

(D) $\sqrt{3}$

Solution:

Correct answer: (D)

Given that $\tan(90° - \theta) = \sqrt{3}$

we can use the co-function identity $\tan(90° - \theta) = \cot\theta$.

Therefore, $\cot\theta = \sqrt{3}$.

II. Answer the following questions $\mathbf{8 \times 1 = 8}$

Q9. In the figure, $\triangle ADE \sim \triangle ABC$ and $DE:BC = 2:3$. Find $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC}$.

Solution:

Given that $\triangle ADE \sim \triangle ABC$ and the ratio of corresponding sides $\frac{DE}{BC} = \frac{2}{3}$

For similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding sides.

Therefore, the ratio of the areas is:

$$\frac{\text{Area of } \triangle \text{ADE}}{\text{Area of } \triangle \text{ABC}} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Thus, the ratio of the area of $\triangle$ADE to the area of $\triangle$ABC is $\frac{4}{9}$.

Q10. The radii of the base and the height of a cylinder and a cone are same. If the volume of the cylinder is 27 cubic units, then find the volume of the cone.

Solution:

Given that the radii of the base and the height of a cylinder and a cone are the same, and the volume of the cylinder is 27 cubic units,

The volume of a cone is one-third the volume of a cylinder with the same base radius and height.

Therefore, the volume of the cone is:

$\frac{1}{3} \times 27 = 9$ cubic units

Thus, the volume of the cone is 9 cubic units.

Q11. If $200 = 2^m \times 5^n$, then find the values of m and n.

Solution:

To express 200 in the form $2^m \times 5^n$, by prime factorization:

$200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$

Therefore, $m = 3$ and $n = 2$.

Q12. Find the number of solutions of the pair of linear equations $2x - 3y + 4 = 0$ and $3x + 5y + 8 = 0$.

Solution:

Given,

$2x - 3y + 4 = 0$ and $3x + 5y + 8 = 0$

These given equations are in the form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Where

$a_1 = 2, b_1 = -3$ and $c_1 = 4$

$a_2 = 3, b_2 = 5$ and $c_2 = 8$

$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-3}{5}$ and $\frac{c_1}{c_2} = \frac{4}{8}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
System has unique solution

Q13. In an Arithmetic progression, sum of the first six terms and sum of the first five terms are 78 and 55 respectively. Then find the sixth term of the progression.

Solution:

Let the first term be a and the common difference be d

The sum of the first six terms is given by $S_6 = 78$, so we have the equation:

$S_6 = \frac{6}{2}[2a + 5d] = 78$

$\Rightarrow 2a + 5d = 26$ 1

The sum of the first five terms is given by $S_5 = 55$, so we have the equation:

$S_5 = \frac{5}{2}[2a + 4d] = 55$

$\Rightarrow 2a + 4d = 22$ 2

From equation 1 and 2 we get

$d = 4, \& \ a = 3$

The sixth term of an A.P. $a_6 = a + 5d = 3 + 5 \times 4 = 23$.

Q14. Write the degree of the polynomial $p(x) = x(x^2 + 3) + 5x^2 + 7$.

Solution:

Given

$p(x) = x(x^2 + 3) + 5x^2 + 7$

$p(x) = x^3 + 3x + 5x^2 + 7$

$p(x) = x^3 + 5x^2 + 3x + 7$

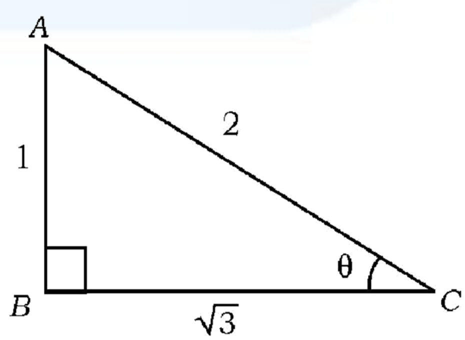
The highest degree term is x^3, so the degree of the polynomial is 3.

Q15. If the value of discriminant of a quadratic equation is zero, then write the nature of roots of the quadratic equation.

Solution:

If the discriminant of a quadratic equation is zero, the equation has real and equal roots.

Q16. Find the value of θ in the figure.



Solution:

In $\triangle \boldsymbol{ABC}$

$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30°$

so, $\theta = 30°$

III. Answer the following questions $\mathbf{8 \times 2 = 16}$

Q17. Prove that $3 + \sqrt{2}$ is an irrational number.

Solution:

Let $3 + \sqrt{2}$ be a rational number.

Thus, $3 + \sqrt{2} = \frac{a}{b}$, where a, b are coprime integers and $b \neq 0$.

$\Rightarrow \sqrt{2} = \left(\frac{a}{b}\right) - 3$

$\Rightarrow \sqrt{2} = \frac{a - 3\text{ b}}{\text{b}}$

Since a and b are integers, $\frac{a-3b}{b}$ is a rational number.

$\Rightarrow \sqrt{2}$ is also a rational number.

This is the contradiction to the fact that $\sqrt{2}$ is an irrational number.

Hence, our assumption that $3 + \sqrt{2}$ is a rational number is wrong.

Therefore, $3 + \sqrt{2}$ is an irrational number.

Hence proved.

Q18. Solve the given pair of linear equations by Elimination method

$2x + y = 8$

$3x - y = 7$

Solution:

Given pair of linear equations are

$2x + y = 8$ ………..1

$3x - y = 7$ ………..2

Adding eq 1and 2 we get

$\Rightarrow 5x = 15$

$\Rightarrow x = 3$

Substitute the value of x in eq (ii)

$\Rightarrow 3(3) - y = 7$

$\Rightarrow y = 2$

$\therefore x = 3$ and $y = 2$

Q19. Find the sum of first 20 terms of the Arithmetic progression 1,5,9, … using formula.

Solution:

Given,

1, 5, 9, …

Here,

First term = $a = 1$

Common difference $= d = 5 - 1 = 4$

Sum of first n terms is

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2} \times [2(1) + (20-1)\,4]$

$= 10 \times [2 + 19(4)]$

$= 10 \times [2 + 76]$

$= 10 \times 78$

$= 780$

Hence, the sum of the first twenty terms of the given arithmetic series is 780.

Q20. Find the roots of the quadratic equation $2x^2 - 3x - 1 = 0$ using quadratic formula

Solution:

Given quadratic equation is $2x^2 - 3x - 1 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 2, b = -3$ and $c = -1$

Discriminant = D

$= b^2 - 4ac$

$= (-3)^2 - 4(2)(-1)$

$= 9 + 8$

$= 17$

Using quadratic formula,

$$x = \frac{\left(-b \pm \sqrt{D}\right)}{2a}$$

$$= \frac{\left(-(-3) \pm \sqrt{17}\right)}{2(2)}$$

$$= \frac{3 \pm \sqrt{17}}{4}$$

$$x = \frac{3 + \sqrt{17}}{4}, \qquad x = \frac{3 - \sqrt{17}}{4}$$

Q21. Prove that $\frac{\cos\ \theta-\sin\ \theta\cdot\cos\ \theta}{\cos\ \theta+\sin\ \theta\cdot\cos\ \theta}=\frac{\text{cosec}\theta-1}{\text{cosec}\theta+1}$.

Solution:

LHS

$$\frac{\cos\ \theta-\sin\ \theta\cdot\cos\ \theta}{\cos\ \theta+\sin\ \theta\cdot\cos\ \theta}$$

$$\Rightarrow\frac{cos\theta(1-sin\theta)}{cos\theta(1+sin\theta)}$$

$$\Rightarrow\frac{(1-sin\theta)}{(1+sin\theta)}$$

RHS

$$\frac{\text{cosec}\theta-1}{\text{cosec}\theta+1}$$

$$\Rightarrow\frac{\left(\frac{1}{sin\theta}-1\right)}{\left(\frac{1}{sin\theta}+1\right)}$$

$$\Rightarrow\frac{\left(\frac{1-sin\theta}{sin\theta}\right)}{\left(\frac{1+sin\theta}{sin\theta}\right)}$$

$$\Rightarrow\frac{(1-sin\theta)}{(1+sin\theta)}$$

LHS = RHS

Hence proved

Or

Prove that $\frac{\sin\ 30^{\circ}+\cos\ 60^{\circ}}{\text{cosec}30^{\circ}-\cot\ 45^{\circ}}=\sin\ 90^{\circ}$.

Solution:

LHS

$$\frac{\sin\ 30^{\circ}+\cos\ 60^{\circ}}{\text{cosec}30^{\circ}-\cot\ 45^{\circ}}$$

Putting all the value

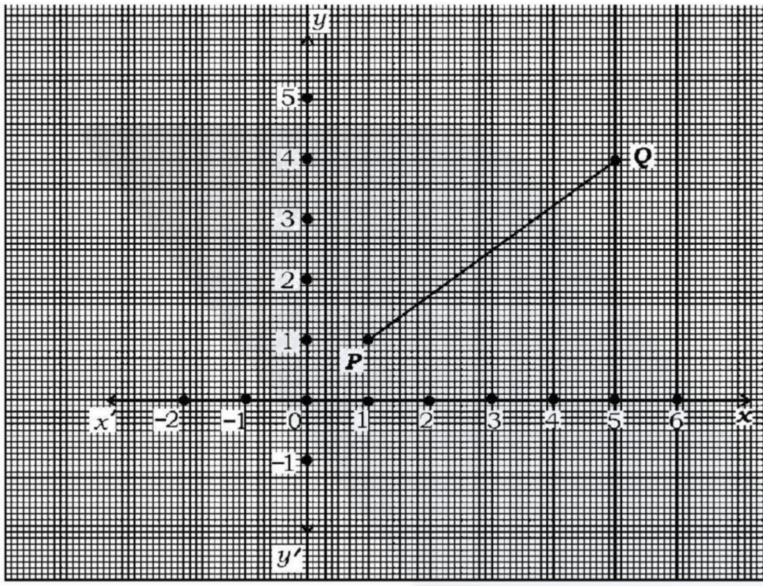
$$\Rightarrow\frac{\left(\frac{1}{2}+\frac{1}{2}\right)}{(2-1)}$$

$$\Rightarrow\frac{\left(\frac{1+1}{2}\right)}{1}$$

$$\Rightarrow\frac{\frac{2}{2}}{1}$$

$$\Rightarrow 1$$

Q22. Find the coordinates of the point P and Q in the given graph and hence find the length of PQ using distance formula.



Solution:

In this given graph,

$P(1,1)$ and $Q(5,4)$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(5-1)^2 + (4-1)^2}$

$PQ = \sqrt{4^2 + 3^2}$

$PQ = \sqrt{16+9}$

$PQ = \sqrt{25}$

$PQ = 5\ units$

Or

Find the coordinates of the point which divides the line segment joining the points $(4,-3)$ and $(8,5)$ in the ratio $3:1$ internally.

Solution:

Given

$A(4,-3)\ , B(8,5)\ \ m_1 : m_2 = \ 3 : \ 1$

Using section formula

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Where

$x_1 = 4, \quad y_1 = -3, \quad x_2 = 8, \quad y_2 = 5$

$$x = \frac{3(8) + 1(4)}{3+1}, y = \frac{3(5) + 1(-3)}{3+1}$$

$$x = \frac{24+4}{4}, y = \frac{15-3}{4}$$

$$x = \frac{28}{4}, y = \frac{12}{4}$$
$x = 7, y = 3$
The co-ordinates of the required point $P(x, y)$ is $(7, 3)$

Q23. A basket contains 36 mangoes. $\frac{1}{4}$ th of them are rotten and others are good. If one mango is drawn at random from the basket, then find the probability of getting a good mango..

Solution:

$n(S) = 36$

$n(A) = \text{Good Mangoes} = \frac{3}{4} \times 36 = 27$

$$\therefore\ P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{27}{36}$$

$$P(A) = \frac{3}{4}$$

Q24. Draw a circle of radius 3.5 cm and construct a pair of tangents to the circle such that the angle between the tangents is 60°.

Solution:

Given, radius

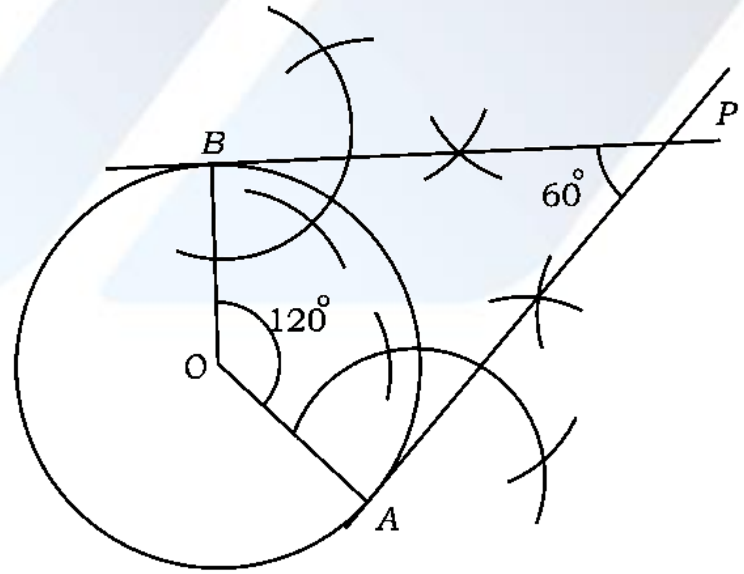
$r = 3.5$ cm

Angle between the radii $= 180° - 60° = 120°$

Construction of circle of radius 3.5 cm

Construction of two arcs

Construction of two tangents



IV. Answer the following questions **$9 \times 3 = 27$**

Q25. Divide $p(x) = x^3 + 3x^2 + 4x + 5$ by $g(x) = x^2 - x + 1$ and find the quotient $[q(x)]$ and remainder $[r(x)]$.

Solution:

$$x + 4$$

$$x^2 - x + 1 \,\big)\, x^3 + 3x^2 + 4x + 5$$

$$x^3 - x^2 + x$$

$$(-) \quad (+) \quad (-)$$

$$4x^2 + 3x + 5$$

$$4x^2 - 4x + 4$$

$$(-) \quad (+) \quad (-)$$

$$7x + 1$$

$\therefore$ $q(x) = x + 4$

$r(x) = 7x + 1$

Or

When the polynomial $p(x) = x^3 + 4x^2 + 5x - 2$ is divided by the polynomial $g(x)$, the quotient $[q(x)]$ and remainder $[r(x)]$ are $x^2 - x + 2$ and 4 respectively. Find $g(x)$

Solution:

$p(x) = x^3 + 4x^2 + 5x - 2$

$q(x) = x^2 - x + 2$

$r(x) = 4$

$g(x) = ?$

$p(x) = g(x) \times q(x) + r(x)$

$g(x) \times q(x) = p(x) - r(x)$

$$\therefore g(x) = \frac{p(x) - r(x)}{q(x)}$$

$$g(x) = \frac{x^3 + 4x^2 + 5x - 2 - 4}{x^2 - x + 2}$$

$$g(x) = \frac{x^3 + 4x^2 + 5x - 6}{x^2 - x + 2}$$

Q26. Find the mean for the following data :

Class-interval	*Frequency*
2 – 6	2
7 – 11	4
12 – 16	5
17 – 21	3
22 – 26	1

Solution:

Class interval	frequency (f_i)	Mid point x_i	$x_i f_i$
$2-6$	2	4	08
$7-11$	4	9	36
$12-16$	5	14	70
$17-21$	3	19	57
$22-26$	1	24	24
	$\sum f_i = 15$		$\sum f_i x_i = 195$

Mean $= \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{195}{15}$

Mean$(\bar{x}) = 13$

Or

Find the mode for the following data:

Class-interval	Frequency
$1-5$	1
$5-9$	3
$9-13$	7

$13-17$	10
$17-21$	9

In the given frequency distribution
$f_0=7, f_1=10, f_2=9, h=4, l=1$

$\text{Mode}=l+\left[\frac{f_1-f_0}{2f_1-f_0-f_2}\right]\times h$

$=13+\left[\frac{10-7}{2\times 10-7-9}\right]\times 4$

$=13+\left[\frac{3}{20-16}\right]\times 4$

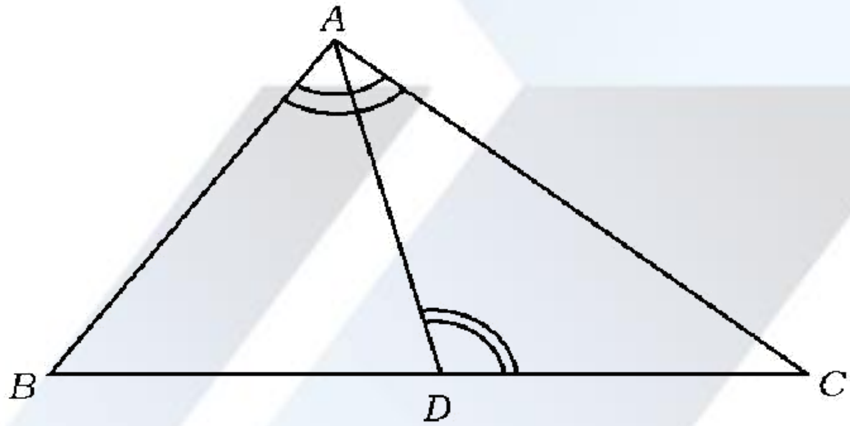
$=3+\left[\frac{3}{4}\times 4\right]$

$=13+3$

$=16$

Q27. D ' is a point on the side BC of a $\triangle ABC$ such that $\lfloor ADC=\lfloor BAC$. Then prove that $AC^2=BC.CD$

Solution:



In $\triangle ABC$ and $\triangle ADC$

$\angle BAC=\angle ADC$ [Given]

$\angle ACB=\angle ACD$ [common angle]

$\therefore \angle ABC=\angle DAC$

By AAA criterion of similarity

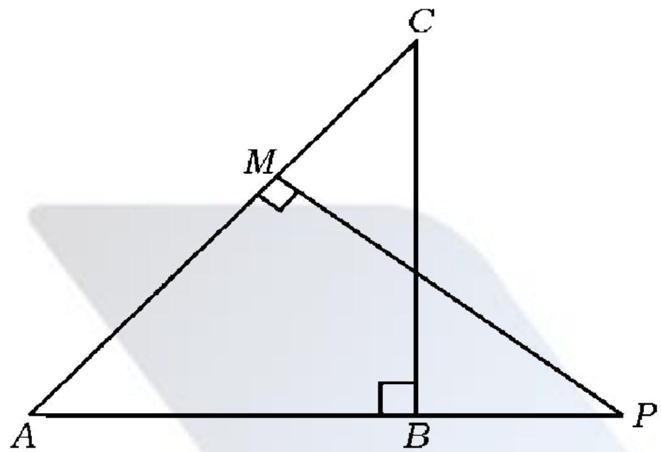
$\therefore \triangle ABC \sim \triangle ADC$

$\therefore \frac{AC}{CD}=\frac{BC}{AC}$

$\therefore AC^2=BC.CD$

Or

In the figure, $\triangle ABC$ and $\triangle AMP$ are right angled triangles, right angled at B and M respectively. Then prove that $\frac{CA}{PA}=\frac{BC}{MP}$.



Solution:

In $\triangle ABC$ and $\triangle AMP$

$\angle ABC = \angle AMP = 90^\circ$ [Given]

$\angle BAC = \angle MAP$ [common angle]

$\therefore \angle ACB = \angle APM$

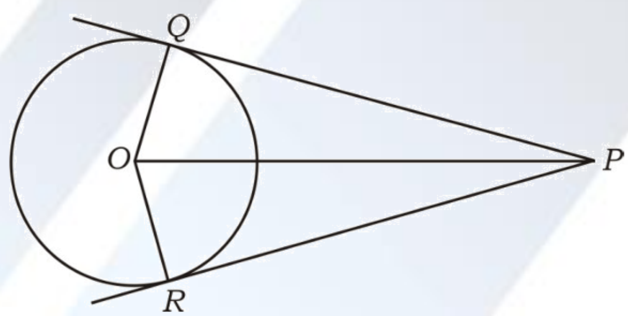
By AAA criteria similarity

$\therefore \triangle ABC \sim \triangle AMP$

$\therefore \frac{CA}{PA} = \frac{BC}{MP}$

Q28. Prove that "The lengths of tangents drawn from an external point to a circle are equal".

Solution:



Given : PQ and PR are tangents drawn from an external point P to a circle of centre O.

To prove that: $PQ = PR$

Construction: Join OP, OQ and OR

Proof: In the figure

$\angle OQP = \angle ORP = 90^\circ$ [$OP \perp PQ,\ OR \perp PR$]

$OQ = OR$ [Radii of same]

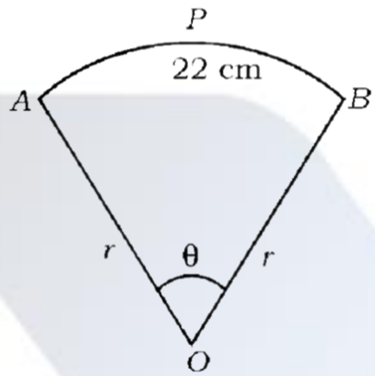
$\therefore OP = OP$ [common side]

By RHS – postdated

$\therefore \triangle OQP \cong \triangle ORP$

$\therefore PQ = PR$ [CPCT]

Q29. In the figure area of sector $AOBPA$ of radius ' r ' is 231 cm^2 and the length of the arc APB is 22 cm. Find the radius of the sector and angle θ.



Solution:

Length of an arc of a sector of angle θ

$= \frac{\theta}{360^\circ} \times 2\pi r$

$\therefore \frac{\theta}{360^\circ} \times 2\pi r = 22$

$\frac{\theta}{360^\circ} \times \pi r = 11 \quad \ldots \ldots . .1$

Area of the sector of angle θ

$= \frac{\theta}{360^\circ} \times 2\pi r^2$

$\therefore \frac{\theta}{360^\circ} \times \pi r \times r = 231 \qquad \ldots \ldots \ldots . \text{from } 2$

$\therefore 11r = 231$

$r = \frac{231}{11} = 21$

$\frac{\theta}{360^\circ} \times 2\pi r = 22$

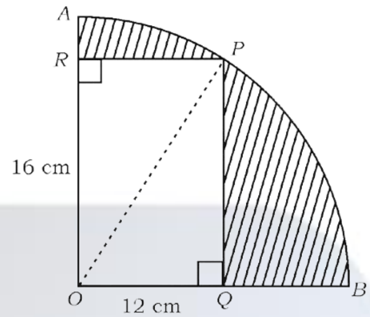
$\frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 21 = 22$

$6\theta = 360^\circ$

$\theta = \frac{360^\circ}{6} = 60^\circ$

Or

In the figure a rectangle $ROQP$ is inscribed in the quadrant of a circle. If the length and breadth of the rectangle are 16 cm and 12 cm respectively, find the area of the shaded region.



Solution:

In the figure ROQP is a rectangle.

$\therefore$ OQP is a right angle triangle, right angled at Q.

$\therefore OP^2 = OQ^2 + PQ^2$ [Pythagoras theorem]

$= 12^2 + 16^2$

$= 144 + 256$

$= 400$

$\therefore$ $OP = \sqrt{400} = 20$ cm

$r = 20$ cm

Area of shaded region = Area of quadrant – Area of rectangle

$= \frac{1}{4} \times \pi r^2 - (\text{length} \times \text{breadth})$

$= \frac{1}{4} \times \pi \times 20^2 - (16 \times 12)$

$= \frac{1}{4} \times \pi \times 400 - 192$

$= 100\pi - 192$

$= 100 \times 3.1428 - 192$

$= 314.28 - 192$

$= 122.28$ cm²

Q30. Age of mother is twice the square of age of her son. After 8 years mother's age becomes 4 years more than the thrice of age of her son. Find their present ages.

Solution:

Let the present age of mother of mother be x years and age of son be y years

Then $x = 2y^2$ ……………..1

After 8 years

Age of mother is $(x + 8)$ years

Age of son is $(y + 8)$ years

According to given question

$\Rightarrow x + 8 = 3(y + 8) + 4$

From (1)

$\Rightarrow 2y^2 + 8 = 3y + 24 + 4$

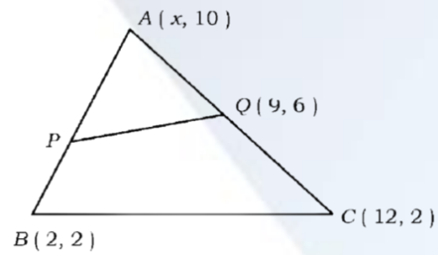
$\Rightarrow 2y^2 + 8 = 3y + 28$

$\Rightarrow 2y^2 - 3y + 8 - 28 = 0$

$\Rightarrow 2y^2 - 3y - 20 = 0$

$\Rightarrow 2y^2 - 8y + 5y - 20 = 0$
$\Rightarrow 2y(y-4) + 5(y-4) = 0$
$\Rightarrow y - 4 = 0$ or $2y + 5 = 0$
$\Rightarrow y = 4$ or $y = -\frac{5}{2}$
Since the age of a person cannot be negative, ignore the value of $y = -\frac{5}{2}$.
Present age of son $= y = 4$ years
Present age of mother $= x = 2y^2 = 2 \times 4^2 =$ 32 years

Q31. In the figure, ABC is a triangle whose vertices are $A(x, 10)$, $B(2,2)$ and $C(12,2)$. If $Q(9,6)$ is the mid-point of AC and area of $\triangle APQ$ is 12 cm^2, then find the area of quadrilateral $PBCQ$.

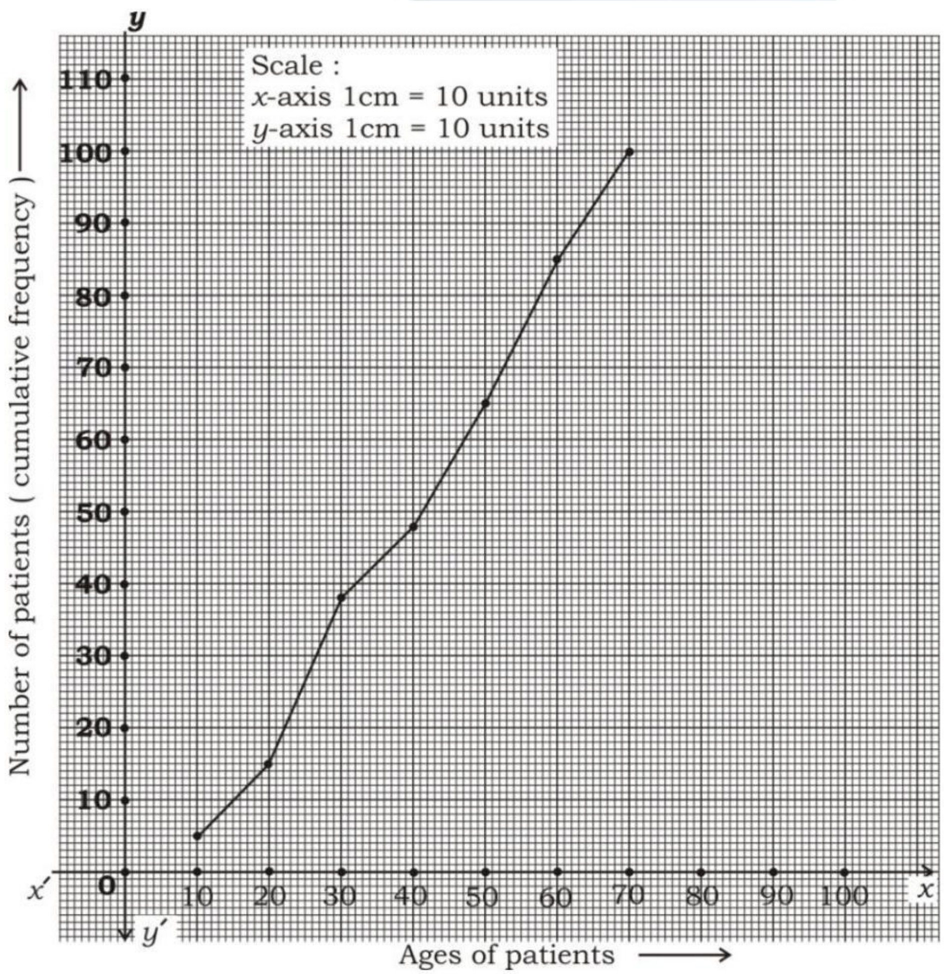


Solution:
Given
$A(x, 10)$ $B(2,2)$ $C(12,2)$
Q is the mid-point of $AC = \frac{x+12}{2} = 9$
$x = 18 - 12 = 6$
Using Area of the triangle
$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
Where
$x_1 = 6,\quad x_2 = 2,\quad x_3 = 12,\quad y_1 = 10,\quad y_2 = 2,\quad y_3 = 2$
$= \frac{1}{2}[6(2-2) + 2(2-10) + 12(10-2)]$
$= \frac{1}{2}[6(0) + 2(-8) + 12(8)]$
$= \frac{1}{2}[0 - 16 + 96]$
$= \frac{1}{2} \times 80$
$= 40 \text{ cm}^2$

Q32. The ages of 100 patients admitted in a hospital are as follows. Draw a "less than type ogive" for the given data:

Age (in years)	Number of patients (cumulative frequency)
Less than 10	6
Less than 20	15
Less than 30	38
Less than 40	46
Less than 50	65
Less than 60	84
Less than 70	100

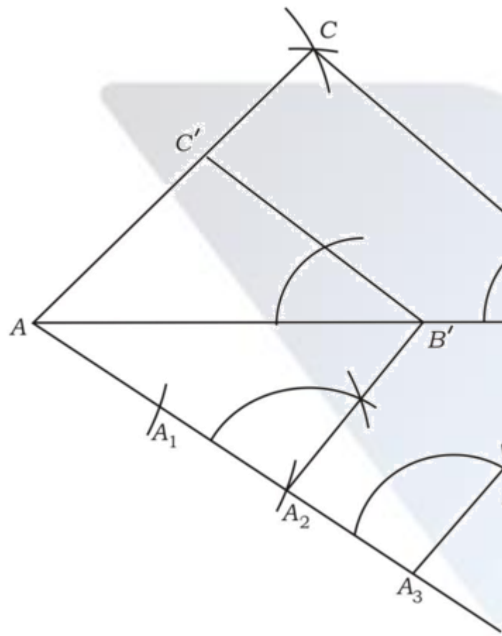
Solution:



Drawing axes and writing scale
Marking points 1
Drawing ogive

Q33. Construct a triangle with sides 6 cm, 8 cm and 9 cm and then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle

Solution:



Construction of given triangle
Construction of acute angle with division
Drawing parallel lines
Obtaining of required triangle

V. Answer the following questions $\mathbf{4 \times 4 = 16}$

Q34. Find the solution of the given pair of linear equations by graphical method:
$2x + y = 8$
$x + y = 5$

Solution:

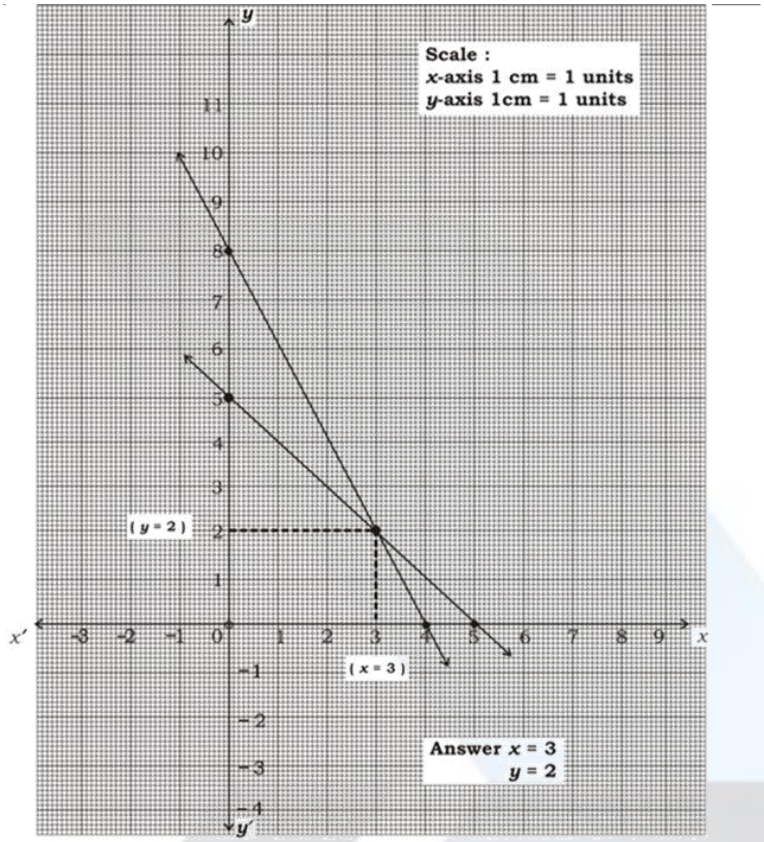
Given

$2x + y = 8$

x	0	4	3
y	8	0	2

$x + y = 5$

x	0	5	3
y	5	0	2



For table construction
Drawing two lines
Marking point of intersection and writing values of x and y

Q35. In an Arithmetic progression the sum of first n terms is 210 and the sum of first $(n-1)$ terms is 171. If the first term of the Arithmetic progression is 3, then find the Arithmetic progression and find its 20^{th} term.

Solution:

Given

$S_n = 210, \; S_{n-1} = 171, \; a_n = S_n - S_{n-1} = 210 - 171 = 39, \quad a = 3, \; n = ?$

$S_n = \frac{n}{2}(a + a_n)$

$210 = \frac{n}{2}(3 + 39)$

$210 = \frac{n}{2} \times 42$

$21n = 210$

$n = \frac{210}{21} = 10$

$a = 3, \quad n = 10, \quad a_n = 39, \quad d = ?$

$a_n = a + (n-1)d$

$39 = 3 + (10-1)d$

$9d = 36$

$d=4$
Required A.P. is $a, a+d, a+2d$ … ….
$3, 3+4, 3+8$ … … ..
$3, 7, 11$……
$a=3, d=4, n=20, a_{20}=?$
$a_n=a+(n-1)d$
$a_{20}=3+(20-1)4$
$=3+19\times 4$
$=3+76=79$

OR

The sum of interior angles of a polygon of ' n ' sides is $(n-2)180^\circ$. If the interior angles of a pentagon are in Arithmetic progression and its least angle is 72°, then find all the interior angles of the pentagon.

Solution:

The sum of interior angles of a polygon of n sides $=(n-2)180^\circ$
The sum of interior angles of a pentagon $=(5-2)180^\circ=3\times 180^\circ=540^\circ$
$a=72, n=5, S_n=540, d=?$

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$540=\frac{5}{2}[2\times 72+(5-1)d]$$

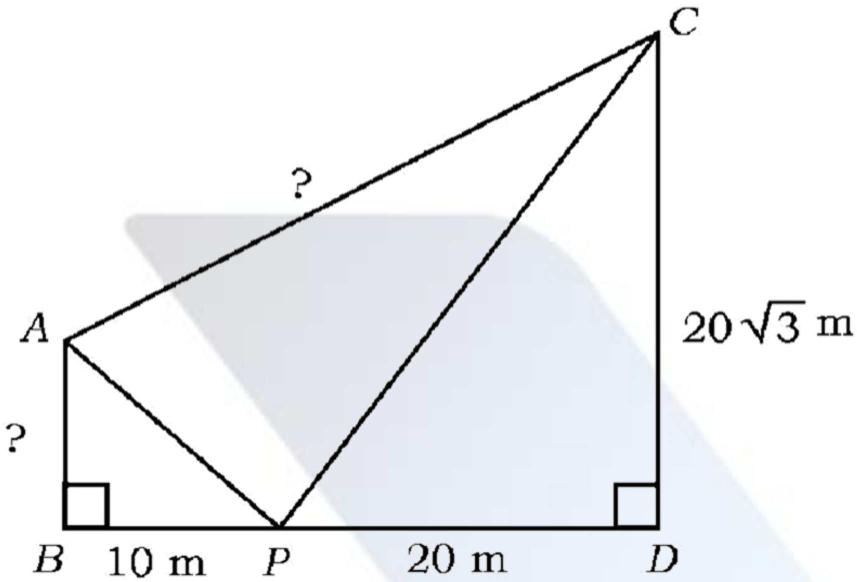
$$540=\frac{5}{2}[144+4d]$$

$108=72+2d$
$2d=108-72$
$2d=36$

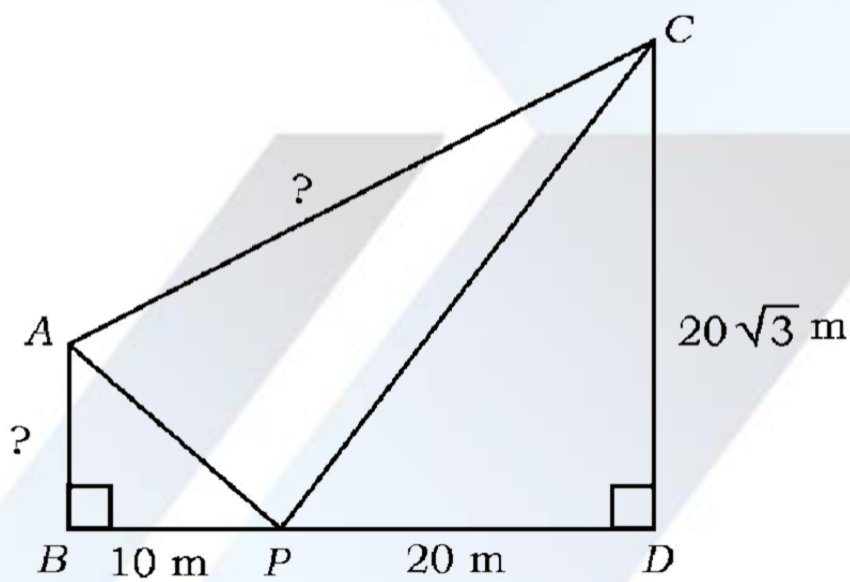
$$d=\frac{36}{2}=18$$

The interior angles of the pentagon are:
$a, a+d, a+2d, a+3d, a+4d$
$72, 72+18, 72+2\times 18, 72+3\times 18, 72+4\times 18$
$72^\circ, \quad 90^\circ, \quad 108^\circ, \quad 126^\circ, \quad 144^\circ$

Q36. In the figure the poles AB and CD of different heights are standing vertically on a level ground. From a point P on the line joining the foots of the poles on the level ground, the angles of elevation to the tops of the poles are found to be complementary. The height of CD and the distance PD are $20\sqrt{3}$ m and 20 m respectively. If BP is 10 m , then find the length of the pole AB and the distance AC between the tops of the poles.



Solution:



Let angle CPD be θ

Then $\tan\ \theta = \frac{CD}{PD} = \frac{20\sqrt{3}}{20} = \sqrt{3}$

$\therefore \theta = 60^{\circ}$

$\therefore \angle APB = 90^{\circ} - \theta = 90^{\circ} - 60^{\circ} = 30^{\circ}$

In right $\triangle ABP$

$\tan\ 30^{\circ} = \frac{AB}{BP}$

$\frac{1}{\sqrt{3}} = \frac{AB}{10}$

$AB \times \sqrt{3} = 10$

$AB = \frac{10}{\sqrt{3}}$ m

In right $\triangle PDC, PC^2 = PD^2 + DC^2$

$= 20^2 + (20\sqrt{3})^2$

$= 400 + (400 \times 3)$

$= 400 + 1200$

$PC^2 = 1600$ 1

In right $\triangle ABP, AP^2 = AB^2 + BP^2$

$= \left(\frac{10}{\sqrt{3}}\right)^2 + 10^2$

$= \frac{100}{3} + 100$

$= \frac{100 + 300}{3}$

$AP^2 = \frac{400}{3}$ 2

In right $\triangle APC, AC^2 = AP^2 + PC^2$

$= \frac{400}{3} + 1600$

$= \frac{400 + 4800}{3}$

$= \frac{5200}{3}$

$= \frac{400 \times 13}{3}$

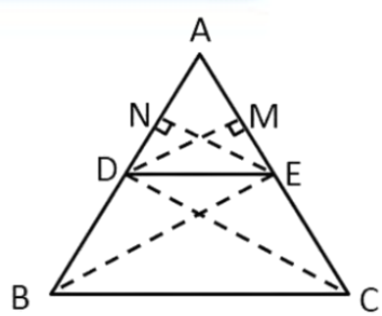
$\therefore AC = \sqrt{\frac{400 \times 13}{3}}$

$AC = \frac{20 \times \sqrt{13}}{\sqrt{3}} = \frac{20\sqrt{39}}{3}$ m

Q37. Prove : "Basic proportionality theorem" or "Thales theorem".

Solution:

Given: In $\triangle$ ABC, DE||BC

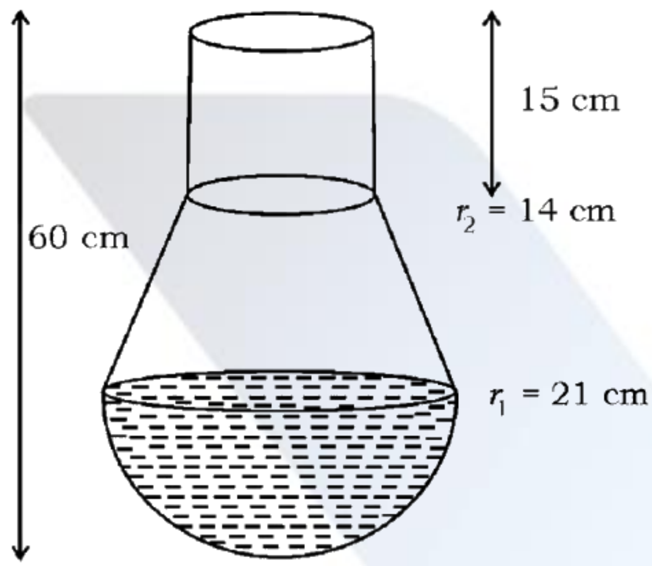


To Prove: $\frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$
Construction: Join BE, DC
Draw DM ⊥ AC and EN ⊥ AB

$$\frac{\text{ar}\ (\triangle\ ADE)}{\text{ar}\ (\triangle\ BDE)}=\frac{\frac{1}{2}\times AD\times EN}{\frac{1}{2}\times DB\times EN}$$
$$\frac{\text{ar}(\triangle\ ADE)}{\text{ar}(\triangle\ BDE)}=\frac{AD}{DB}$$
and $\frac{\text{ar}(\triangle ADE)}{\text{ar}(\Delta CDE)}=\frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM}$
$$\frac{\text{ar}(\triangle\ ADE)}{\text{ar}(\triangle\ CDE)}=\frac{AE}{EC}$$
ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC .
∴ ar(ΔBDE) = ar(ΔCDE)
So,
$$\frac{AD}{DB}=\frac{AE}{EC}$$
Hence proved

VI. Answer the following questions **$1\times 5=5$**

Q38. An insect control device made of a cylinder, a frustum of a cone and a hemisphere attached to each other is as shown in the figure. Sticky liquid is completely filled in the hemispherical part. If the radii of hemisphere and cylinder are 21 cm and 14 cm respectively and total height of the device is 60 cm and height of the cylinder is 15 cm , then calculate the curved surface area of the device and also find the quantity of the sticky liquid in the hemisphere.



Solution:
Given
$r = 14$ cm, $h = 15$cm
Outer surface area of the device = CSA of cylinder + CSA of frustum of cone + CSA of hemisphere.
CSA of cylinder $= 2\pi rh = 2 \times \frac{22}{7} \times 14^2 \times 15 = 88 \times 15 = 1320$ cm^2
Height of frustum $= 60 - (15 + 21) = 24$ cm
$r_1 = 21, r_2 = 14, h = 24, l = ?$
$\therefore l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + (21 - 14)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ cm
CSA of frustum $= \pi(r_1 + r_2)l$
$= \frac{22}{7} \times (21 + 14) \times 25$
$= \frac{22}{7} \times 35 \times 25$
$= 2750$ cm^2
CSA of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = 44 \times 63 = 2772$ cm^2

$\therefore$ Outer surface area of the device= CSA of (Cylinder + frustum + hemisphere)
$= 1320 + 2750 + 2772$
$= 6842$ cm^2

Quantity of the liquid in hemisphere = Volume of hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 44 \times 441 = 19404 \text{ cm}^3$$