Grade10 Maths Maharashtra 2014

PART – I(ALGEBRA)

Note: -

(1) All questions are compulsory.

(2) Use of calculator is not allowed.

Q1. Attempt any five questions from the following:

i. For an A.P. $t_3 = 8$ and $t_4 = 12$, find the common difference d.

Solution:

Given:

$t_3 = 8$ and $t_4 = 12$

Since, $t_n = a + (n-1)d$

$t_3 = a + (3-1)d$

$8 = a + 2d$... (i)

$t_4 = a + (4-1)d$

$12 = a + 3d$ ... (ii)

Subtracting (i) from (ii), we get

$d = 4$

The common difference is 4.

ii. $(x+5)(x-2) = 0$, find the roots of this quadratic equation.

Solution:

$(x+5)(x-2) = 0$

$x + 5 = 0$ and $x - 2 = 0$

$x = -5$ and $x = 2$

-5 and 2 are the roots of the quadratic equation $(x+5)(x-2) = 0$

iii. The following data shows the number of students using different modes of transport.

Modes of Transport	Number of Students
Bicycle	140
Bus	100
Walk	70
Train	40
Car	10

From this table, find the central angle (θ) for the Mode of Transport 'Bus'.

Solution:

Total number of students $= 140 + 100 + 70 + 40 + 10 = 360$

Central angle (θ) for the mode of Transport 'Bus' =

$$\frac{\text{Number of students using Bus}}{\text{Total number of students}} \times 360^\circ = \frac{100}{360} \times 360^\circ = 100^\circ$$

Central angle $(\theta) = 100^\circ$

iv. 'A coin is tossed'. Write the sample space ' S '.

Solution:

Sample Space $S = \{H, T\}$

$n(S) = 2$

v. If $\sum f_i x_i = 75$ and $\sum f_i = 15$, then find the mean $\bar{x}$.

Solution:

Mean $= \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{75}{15} = 5$$

Mean $= \bar{x} = 5$

vi. Write the following quadratic equation in a standard form:

$3x^2 = 10x + 7$.

Solution:

$3x^2 - 10 - 7 = 0$ is in the standard form.

Q2. Attempt any four sub questions from the following:

i. State whether the following sequence is an AP or not:

1,3,6,10 … …..

Solution:

The given sequence is 1,3,6,10, … … . . ...

Here $t_1 = 1, t_2 = 3, t_3 = 6, t_4 = 10$

Then,

$t_2 - t_1 = 3 - 1 = 2$

$t_2 - t_3 = 6 - 3 = 3$

$t_4 - t_3 = 10 - 6 = 4$

$t_2 - t_1 \neq t_2 - t_3 \neq t_4 - t_3$

Since the difference between two consecutive terms is not constant.

Therefore the given sequence is not an A.P

ii. Solve the following quadratic equation by factorization method:
$9x^2 - 25 = 0$

Solution:

$9x^2 - 25 = 0$
$(3x)^2 - (5)^2 = 0$
$(3x - 5)(3x + 5) = 0 \ldots\ldots..[a^2 - b^2 = (a - b)(a + b)]$
$(3x - 5) = 0$ or $(3x + 5) = 0$
$3x = 5$ or $3x = -5$
$x = \frac{5}{3}$ or $x = -\frac{5}{3}$
$\therefore \left\{\frac{5}{3}, -\frac{5}{3}\right\}$ is the solution set of the given equation.

iii. If the point (3,2) lies on the graph of the equation $5x + ay = 19$, then find a.

Solution:

Given:
$5x + ay = 19$
$(x, y) = (3,2)$
The point (x, y) lies on the graph of the equation; hence it satisfies the equation.
Substitute $x = 3$ and $y = 2$ in the given equation,
We get
$5(3) + a(2) = 19$
$\Rightarrow 15 + 2a = 19$
$\Rightarrow 2a = 19 - 15$
$\Rightarrow 2a = 4$
$\Rightarrow a = \frac{4}{2} = 2$
Therefore the value of a is 2

iv. If $12x + 13y = 29$ and $13x + 12y = 21$, find $x + y$..

Solution:

The given equations are
$12x + 13y = 29$ ... (i)
$13x + 12y = 21$ ... (ii)
Add (i) and (ii), we get
$25x + 25y = 50$
$25(x + y) = 50$
$x + y = 2$

v. A die is thrown. Write the sample space (S) and number of sample points $n(S)$ and also write event A of getting even number on the upper surface and write $n(A)$.

Solution:

The sample space (S) is

$S=\{1,2,3,4,5,6\}$

No. of sample points $= n(S)=6$

Let A be the event of getting an even number.

$A=\{2,4,6\}$

$\Rightarrow n(A)=3$

vi. For a certain frequency distribution, the value of mean is 20 and mode is 11. Find the value of median.

Solution:

The inter-relation between the measures of central tendency is given by

Mean - Mode = 3(Mean−Median)

20-11 = 3 (20−Median)

9 = 3 (20−Median)

$\frac{9}{3}=20-\text{Median}$

3 = 20−Median

Median = 20−3

Median =17

Q3. Attempt any three of the following sub questions:

i. Solve the equation by using the formula method.

$3y^2+7y+4=0$

Solution:

The given quadratic equation is $3y^2+7y+4=0$.

Comparing the given equation with $ax^2+bx+c=0$ we get,

$a=3, b=7 \text{ and } c=4.$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(7)\pm\sqrt{(7)^2-4(3)(4)}}{2(3)}$$

$$y=\frac{-(7)\pm\sqrt{49-48}}{6}$$

$$y=\frac{-(7)\pm\sqrt{1}}{6}$$

$$y=\frac{-(7)\pm 1}{6}$$

$$y=\frac{-(7)+1}{6} \text{ or } y=\frac{-(7)-1}{6}$$

$$y=\frac{-6}{6}=-1 \text{ or } y=\frac{-8}{6}=\frac{-4}{3}$$

$y = -1$ *or* $y = -\frac{4}{3}$

Therefore -1 and $-\frac{4}{3}$ are the roots of given equation.

ii. Solve the following simultaneous equations by using Cramers's rule:

$3x - y = 7$

$x + 4y = 11$

Solution:

The given equations are

$3x - y = 7$ ... (i)

$x + 4y = 11$ ... (ii)

Equation (i) and (ii) are in standard form.

$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13 \neq 0$

$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$

$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$

By Cramer's rule, we get

$x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$

$x = \frac{39}{13}$ and $y = \frac{26}{13}$

$x = 3$ and $y = 2$

$(x, y) = (3,2)$

$x = 3$ and $y = 2$ is the solution to the given equation.

iii. Two coins are tossed simultaneously. Write the sample space ' S ' and the number of sample points $n(S)$. Write the following events using set notation and mention the number of elements in each of them:

(a) A is the event of getting at least one head.

(b) B is the event of getting exactly one head.

Solution:

The sample space (S) is

$S = \{TT, HT, TH, HH\}$

$n(s) = 4$

a) Let A be the event of getting at least one head.

$A = \{HT, HH, TH\}$

$n(A) = 3$

b) Let B be the event of getting exactly one head.

$B = \{HT, TH\}$

$n(B) = 2$

iv. The following table gives the frequency distribution of trees planted by different Housing Societies in a particular locality:

No. of Trees	No. of Housing Societies
10 – 15	2
15 – 20	7
20 – 25	9
25 – 30	8
30 – 35	6
35 – 40	4

Find the mean number of trees planted by Housing Societies by using 'Assumed Means Method'

Solution:

By Assumed Mean Method

No. of trees	Class mark x_i	$d_i = x_i - A$	No of housing societies(f_i)	$f_i d_i$
10 – 15	12.5	-10	2	-20
15 – 20	17.5	-5	7	-35
20 – 25	22.5 → A	0	9	0
25 – 30	27.5	5	8	40
30 – 35	32.5	10	6	60
35 – 40	37.5	15	4	60
Total	-	-	$\sum f_i = 36$	$\sum f_i d_i = 105$

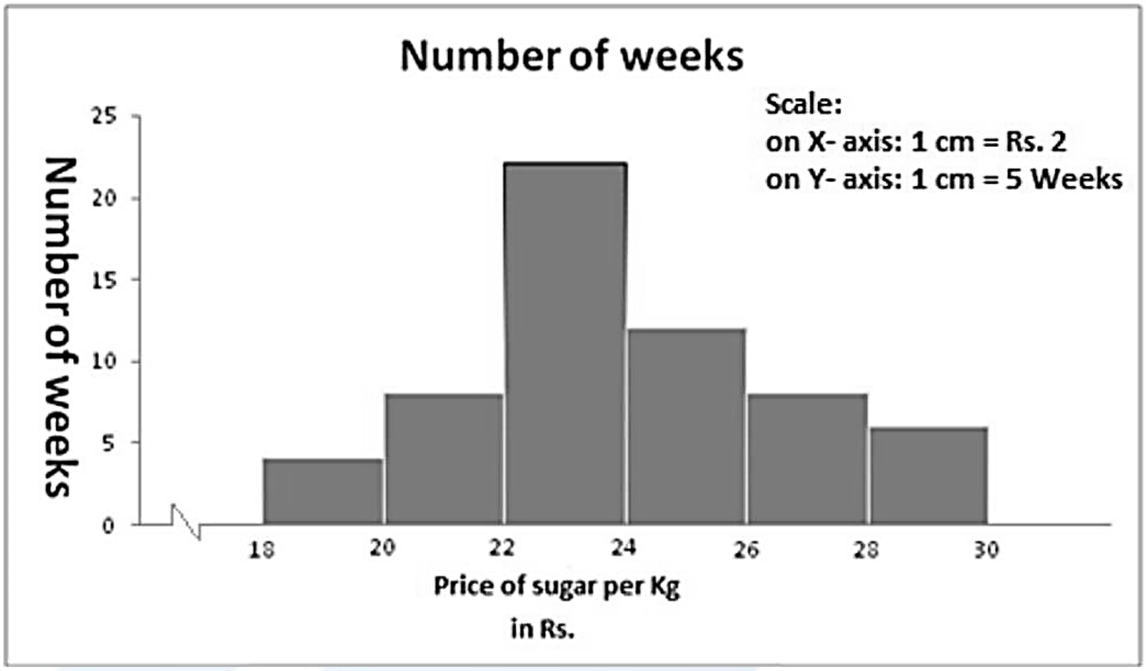
$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{105}{36} = 2.916$$

Mean $= \bar{x} = \text{A} + \bar{d} = 22.5 + 2.916 = 25.42$

v. Represent the following data by Histogram:

Price of Sugar per kg (in Rs.)	Number of Weeks
18 – 20	4
20 – 22	8
22 – 24	22
24 – 26	12
26 – 28	8
28 – 30	6

Solution:



Q4. Attempt any two sub-questions from the following:

i. A farmer borrows Rs.1,000 and agrees to repay with a total interest of Rs. 140 in 12 installments, each installment being less that the preceding installment by Rs. 10. What should be his first installment?

Solution:

As each installment being less that the preceding installment by Rs. 10 the installments are in A.P.

$S_{12} = 1000 + 140 = 1140$
$n = 12, d = -10$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{12} = \frac{12}{2}[2a + (12-1)(-10)]$
$1140 = 6[2a + (11)(-10)]$
$1140 = 6[2a - 110]$
$\frac{1140}{6} = [2a - 110]$
$190 = 2a - 110$
$2a = 190 + 110$
$2a = 300$
$a = \frac{300}{2}$
$a = 150$
The first installment = Rs. 150.

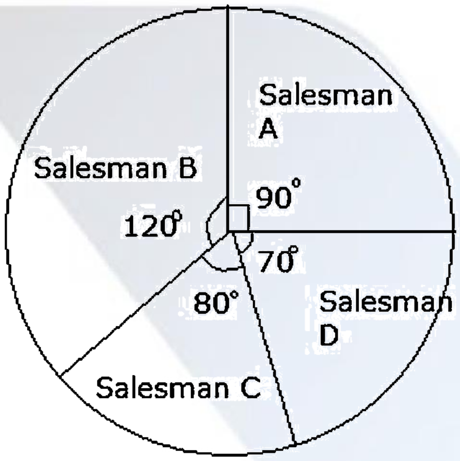
ii. There are three boys and two girls. A committee of two is to be formed. Find the probability of events that the committee contains:
a) At least one girl.
b) One boy and one girl
c) Only boys.

Solution:

Let the three boys be b_1, b_2, b_3 and the three girls be g_1 and g_2.
The Sample space(S) is
$S = \{b_1b_2, b_3b_1, b_2b_3, b_1g_1, b_1g_2, b_2g_1, b_2g_2, b_3g_1, b_3g_2, g_1g_2\}$
$\Rightarrow n(S) = 10$
a) Let B be event that the committee contains only one girls.
$B = \{b_1\ g_1,\ b_1\ g_2,\ b_2\ g_1,\ b_2\ g_2,\ b_3\ g_1,\ b_3\ g_2,\ g_1\ g_2\}$
$\Rightarrow n(\ B) = 7$
$P(B) = \frac{n(B)}{n(S)} = \frac{7}{10}$
b) Let C be the event that the committee contains one boy and one girl.
$C = \{b_1\ g_1,\ b_1\ g_2,\ b_2\ g_1,\ b_2\ g_2,\ b_3\ g_1,\ b_3\ g_2\}$
$\Rightarrow n(C) = 7$
$P(C) = \frac{n(C)}{n(S)} = \frac{6}{10}$
c) Let D be the event that the committee contains only boys.
$D = \{b_1\ b_2,\ b_3\ b_1,\ b_2\ b_3\}$
$\Rightarrow n(D) = 3$

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{10}$$

iii. The sales of salesmen in a week are given in the pie diagram. Study the diagram and answer the following questions. If the total sale due to salesmen A is Rs. 18,000, then



a) Find the total sale.

b) Find the sale of each salesman.

c) Find the salesman with the highest sale.

d) Find the difference between the highest sale and the lowest sale.

Solution:

Given: Sales of salesman A = Rs. 18000

a) Sales of salesman A = Rs. 18000

Sales of salesman A $= \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$

$18000 = \frac{90^\circ}{360^\circ} \times \text{Total sales}$

Total sales $= 18000 \times 4 =$ Rs. 72000

b) Sales of salesman B $= \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$

Sales of salesman $B = \frac{120^\circ}{360^\circ} \times 72000$

Sales of salesman B = Rs. 24000

Sales of salesman C $= \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$

Sales of salesman C $= \frac{80^\circ}{360^\circ} \times 72000$

Sales of salesman C = Rs. 16000

Sales of salesman D $= \frac{\text{Central angle}}{360^\circ} \times \text{Total sales}$

Sales of salesman $D = \frac{70^\circ}{360^\circ} \times 72000$

Sales of salesman D = Rs. 14000

c) Salesman B is the salesman with the highest sale.
d) Difference between the highest sale and the lowest sale
= Sales of salesman B – Sales of salesman D
= Rs. 24000 –Rs. 14000
= Rs. 10000

Q5. Attempt any three of the following sub questions:

i. If m times m th term of an A.P. is equal to n times its nth term, then show that $(m+n)$ th term of the A.P. is zero.

Solution:

Given

$t_m = [a+(m-1)d]$

$t_n = [a+(n-1)d]$

$m(t_m) = n(t_n)$

$m[a+(m-1)d] = n[a+(n-1)d]$

$\Rightarrow m[a+md-d] = n[a+nd-d]$

$\Rightarrow am+m^2d-md = an+n^2d-nd$

$\Rightarrow am+m^2d-md-an-n^2d+nd = 0$

$\Rightarrow am-an+m^2\,d-n^2\,d-md+nd = 0$

$\Rightarrow a(m-n)+d(m^2-n^2)-d(m-n) = 0$

$\Rightarrow (m-n)[a+d(m+n)-d] = 0$... [Divide by $(m-n)$]

$\Rightarrow [a+d(m+n)-d] = 0$

$\Rightarrow a+(m+n-1)d = 0$

$\Rightarrow t_{m+n} = 0$

ii. The product of four consecutive natural numbers, which are multiples of fives, is Rs. 15,000. Find those natural numbers.

Solution:

Let $5x$, $5(x+1)$,$5(x+2)$ and $5(x+3)$ be four consecutive natural numbers, which are multiple of fives.

It is given that product of these consecutive numbers is 15000 .

$5x \times 5(x+1) \times 5(x+2) \times 5(x+3) = 15000$

$625 \times x(x+1)(x+2)(x+3) = 15000$

$x(x+1)(x+2)(x+3) = \frac{15000}{625}$

$x(x+1)(x+2)(x+3) = 24$

When $x = 1$

Then,

$x(x+1)(x+2)(x+3)$

$1(1+1)(1+2)(1+3)$

$1(2)(3)(4) = 24$

Hence, the four consecutive natural numbers are

$5x = 5 \times 1 = 5$

$5(x+1) = 5 \times (1+1) = 5 \times 2 = 10$

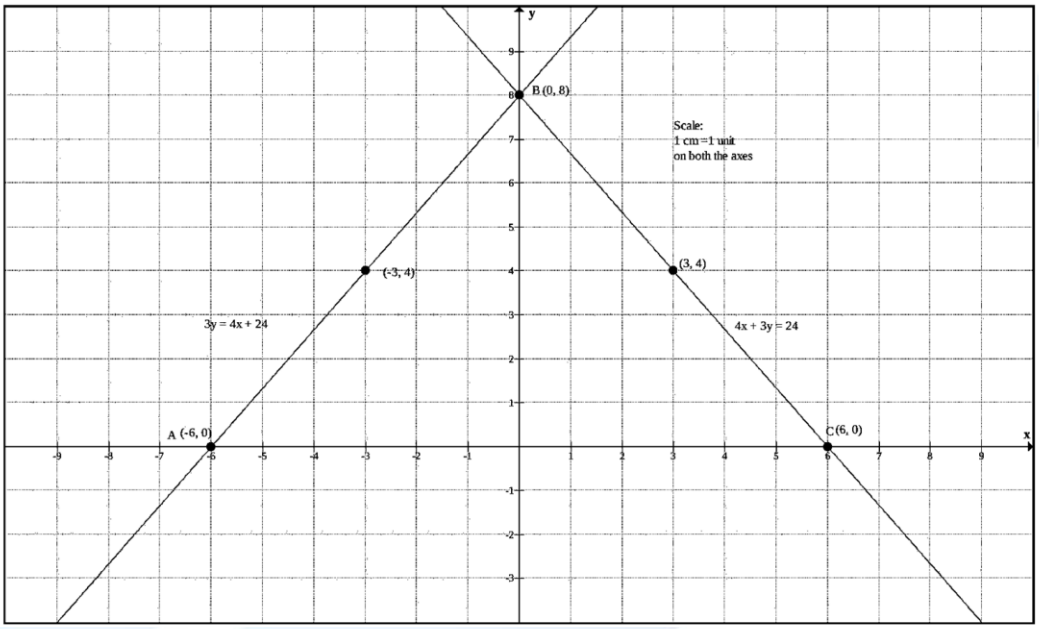
$5(x+2) = 5 \times (1+2) = 5 \times 3 = 15$
$5(x+3) = 5 \times (1+3) = 5 \times 4 = 20$
Therefore four consecutive natural numbers are 5, 10, 15 and 20.

iii. Draw the graphs representing the equations $4x + 3y = 24$ and $3y = 4x + 24$ on the same graph paper. Write the co-ordinates of the point of intersection of these lines and find the area of triangle formed by these lines and the X -axis.

Solution:

The given simultaneous equations are $4x + 3y = 24$ and $4x - 3y = -24$.

$4x + 3y = 24\,(i)$ $y = \frac{24-4x}{3}$				$4x - 3y = -24\,(ii)$ $y = \frac{4x+24}{3}$			
x	*0*	*3*	*6*	x	*0*	*-3*	*-6*
y	*8*	*4*	*0*	y	*8*	*4*	*0*
(x, y)	(0,8)	(3,4)	(6,0)	(x, y)	(2,8)	(−3,4)	(−6,0)

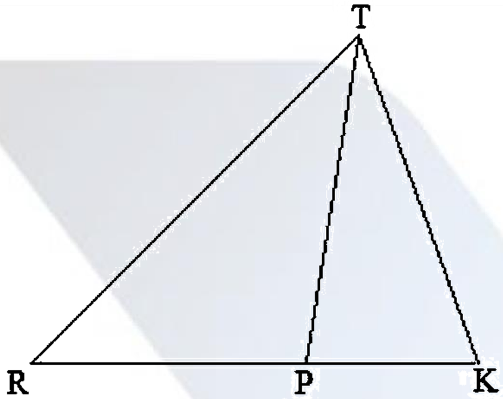


From the graph A$(-6,0)$, B$(0,8)$C$(6,0)$ and AC $= 12$ units.
Height = h=8 units
Base $= b = 12$ units
Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height
$A(\triangle ABC) = \frac{1}{2} \times 12 \times 8$
$A(\triangle ABC) = 48$ sq units

PART – II(GEOMETRY)

Q1. Solve any five sub-questions:

i. In the following figure RP: PK $= 3:2$, then find the value of A($\triangle TRP$): A($\triangle TPK$).



Solution:

Ratio of the areas of two triangles with common or equal heights is equal to the ratio of their corresponding bases.

$$\frac{A(\Delta TRP)}{A(\triangle TPK)} = \frac{RP}{PK} = \frac{3}{2}$$

ii. If two circles with radii 8 cm and 3 cm, respectively, touch internally, then find the distance between their centers.

Solution:

If two circles touch internally, then distance between their centers is the difference of their radii.

The distance between their centers $= 8 - 3 = 5$ cm

iii. If the angle $\theta = -60°$, find the value of $\sin\ \theta$.

Solution:

We know that, for any angle θ, $\sin(-\theta) = -\sin\ \theta$

$$\therefore \sin(-60°) = -\sin\ 60° = -\frac{\sqrt{3}}{2}$$

iv. Find the slope of the line passing through the points A(2,3) and B(4,7).

Solution:

$A \equiv (2,3) \equiv (x_1, y_1)$ and $B \equiv (4,7) \equiv (x_2, y_2)$

Slope of line $AB = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{7-3}{4-2} = \frac{4}{2} = 2$

v . The radius of a circle is 7 cm. find the circumference of the circle.

Solution:

Given radius of a circle r = 7 cm.

Circumference of the circle $= 2\pi r = 2 \times \frac{22}{7} \times 7 =$44 cm

vi. If the sides of a triangle are 6 cm, 8 cm and 10 cm, respectively, then determine whether the triangle is a right angle triangle or not.

Solution:

The sides of the triangle are 6 cm, 8 cm and 10 cm.

The longest side is 10 cm.

$(10)^2 = 100$ ..... (i)

Now, the sum of the squares of the other two sides will be,

$(6)^2 + (8)^2 = 36 + 64 = 100$ ....(ii)

$(10)^2 = (6)^2 + (8)^2$

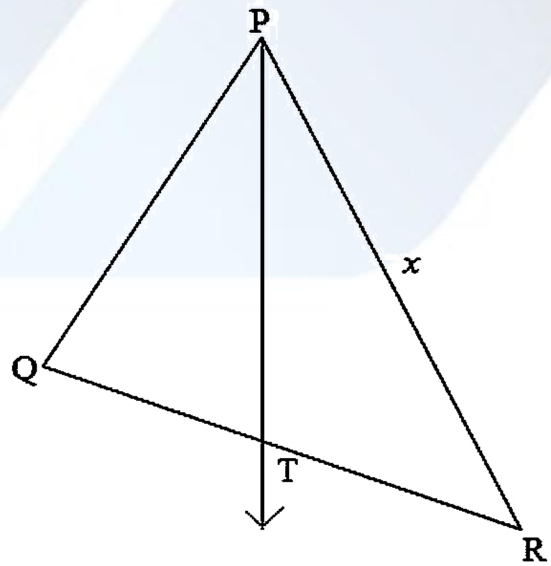
from (i) and (ii)

By the converse of Pythagoras theorem:

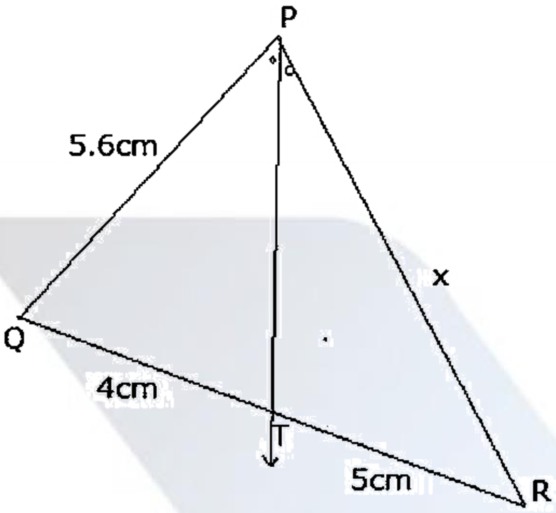
The given sides form a right angled triangle.

Q2. Solve any four sub-questions:

i. In the figure given below, Ray PT is bisector of $\angle QPR$. If PQ = 5.6 cm, QT = 4 cm and $TR = 5$ cm, find the value of x.



Solution:



Given: ray PT is bisector of $\angle QPR$.

$\mathrm{PQ} = 5.6$ cm, $\mathrm{QT} = 4$ cm and $\mathrm{TR} = 5$ cm.

In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.

$$\frac{QT}{TR} = \frac{PQ}{PR}$$

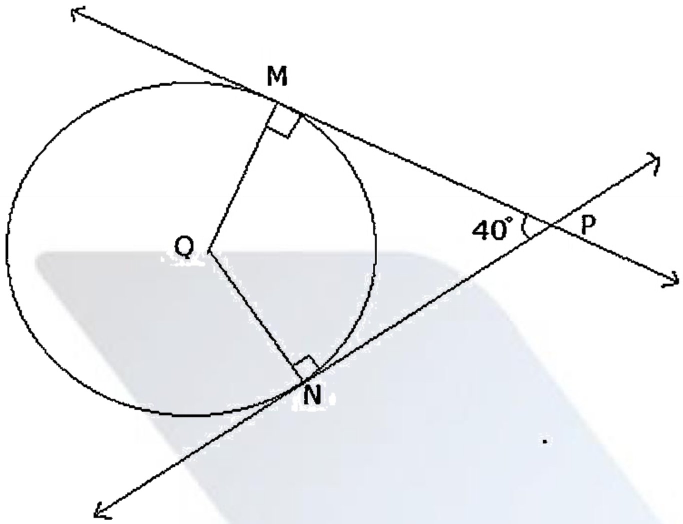
$$\frac{4}{5} = \frac{5.6}{x}$$

$$x = \frac{5.6 \times 5}{4}$$

$$x = \frac{28}{4} = 7$$

$PR = 7$ cm

ii. In the following figure, Q is the center of the circle. PM and PN are tangents to the circle. If $\angle MPN = 40^\circ$, find $\angle MQN$.



Solution:

Given: $\angle MPN = 40°$

The line perpendicular to a radius of a circle at its outer end is a tangent to the circle.

$\angle PMQ = 90°$ and $\angle QNP = 90°$

The sum of the measures of the angles of a quadrilateral is $360°$.

$\angle MPN + \angle PMQ + \angle QNP + \angle MQN = 360°$

$40° + 90° + 90° + \angle MQN = 360°$

$220° + \angle MQN = 360°$

$\angle MQN = 360° - 220°$

$\angle MQN = 140°$

iii. Write the equation $2x - 3y - 4 = 0$ in the slope intercept form. Hence, write the slope and y-intercept of the line.

Solution:

Given equation is $2x - 3y - 4 = 0$

$-3y = -2x + 4$

$$y = \frac{2}{3}x - \frac{4}{3}$$

Comparing given equation with $y = mx + c$, we get

slope $= m = \frac{2}{3}$ and y -intercept $= c = -\frac{4}{3}$

iv. If $\cos\ \theta = \frac{1}{\sqrt{2}}$, where θ is an acute angle, then find the value of $\sin\ \theta$.

Solution:

Given $\cos\ \theta = \frac{1}{\sqrt{2}}$

$\cos^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{1}{2} + \sin^2\theta = 1$

$\sin^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$

$\sin\theta = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

v. If $(4, -3)$ is a point on the line AB and slope of the line is (-2), write the equation of the line AB.

Solution:

$(4, -3) \equiv (x_1, y_1)$ is a point on the line AB and slope $= \mathrm{m} = -2$

Equation of line AB in point slope form is

$y - y_1 = m(x - x_1)$

$y - (-3) = -2(x - 4)$

$y + 3 = -2x + 8$

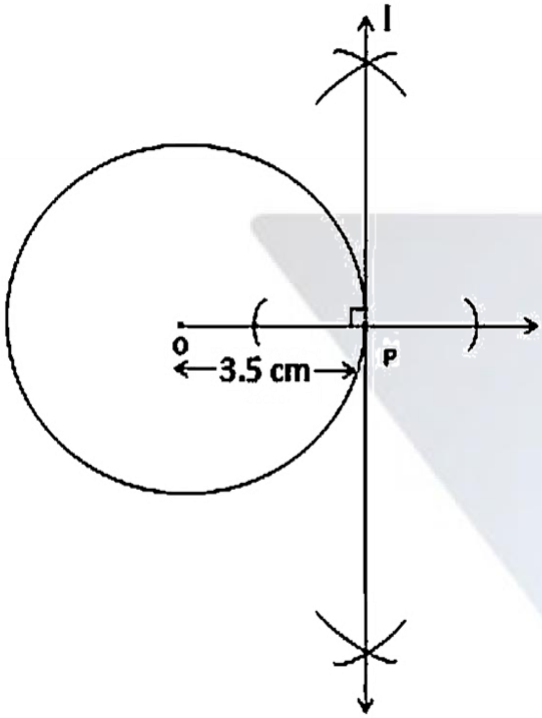
$2x + y - 5 = 0$

The equation of the line AB is $2x + y - 5 = 0$.

vi. Draw a tangent at any point ' P ' on the circle of radius 3.5 cm and centre O

Solution:

Construction:

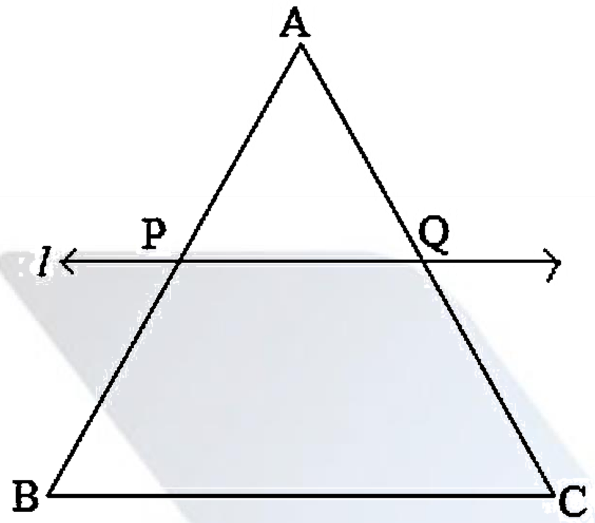


Steps of construction:

1. Draw a circle with centre 0 and radius 3.5 cm .
2. Take any point ' P ' on the circle and draw ray OP.
3. Draw a line perpendicular to ray OP at the point P. Name that line as ' l ' which is tangent to the circle.

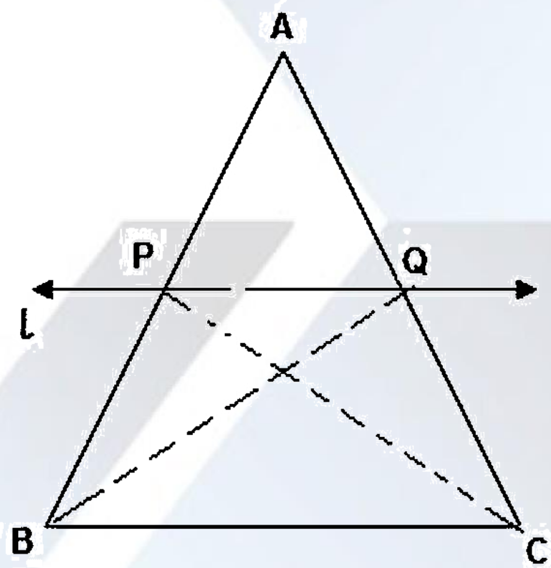
Q3. Solve any three sub-questions:

i. In a triangle ABC , line $l \|$ Side BC and line l intersects side AB and AC in points P and Q, respectively. Prove that: $\frac{AP}{BP}=\frac{AQ}{QC}$.



Solution:

Construction: Join seg PC and seg BQ.



Given: In $\triangle ABC$, line $l||BC$

Line l intersects side AB and side AC in points P and Q, respectively.

To prove: $\frac{AP}{BP} = \frac{AQ}{QC}$

Construction: Join seg BQ and seg CP

In $\triangle APQ$ and $\triangle BPQ$,

$\frac{A(\triangle APQ)}{A(\triangle BPQ)} = \frac{AP}{BP}$ (i) ...(Triangles having equal height)

In $\triangle APQ$ and $\triangle CPQ$,

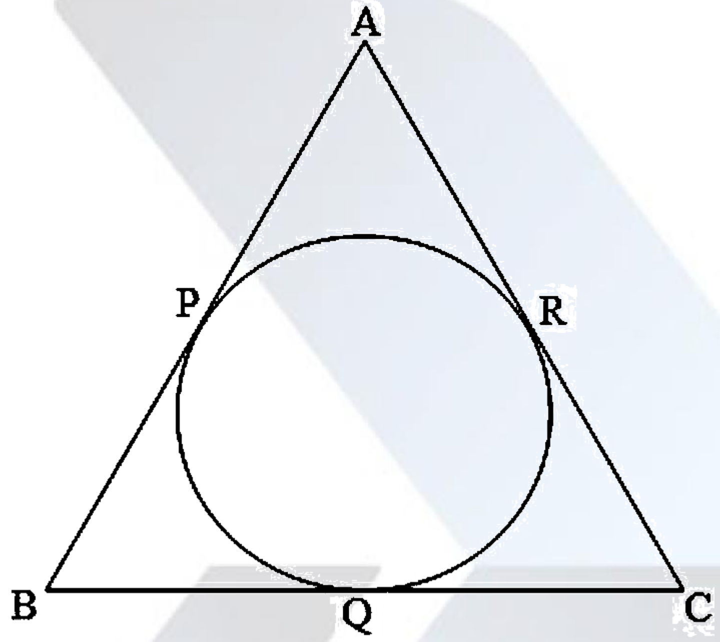
$\frac{A(\triangle APQ)}{A(\triangle CPQ)} = \frac{AQ}{CQ}$ (ii) ...(Triangles having equal height)

$A(\triangle BPQ) = A(\triangle CPQ)$ (iii) ...(Triangles with common base PQ and same height)

$\frac{A(\triangle APQ)}{A(\triangle BPQ)} = \frac{A(\triangle APQ)}{A(\triangle CPQ)}$ ......From (i), (ii) and (iii)

$$\frac{AP}{BP} = \frac{AQ}{QC}$$

ii. In figure, $\triangle ABC$ is an isosceles triangle with perimeter 44 cm . The base BC is of length 12 cm . Side AB and side AC are congruent. A circle touches the three sides as shown in the figure below. Find the length of the tangent segment from A to the circle.



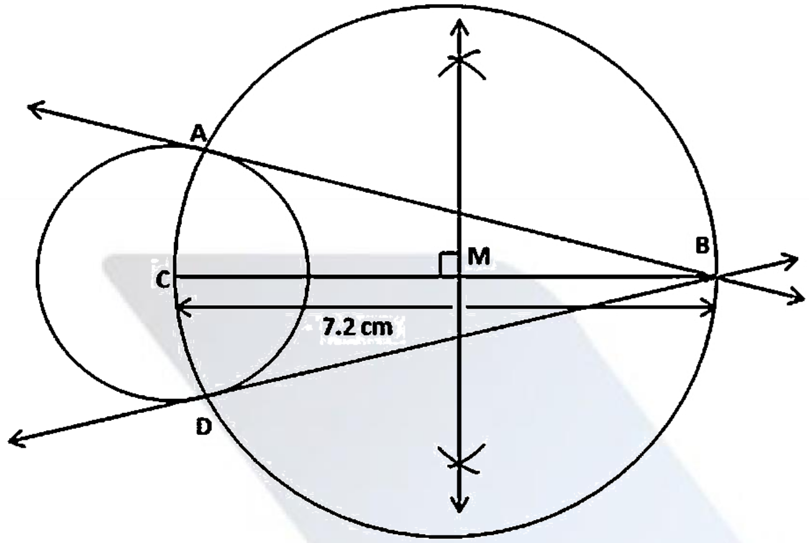
Solution:

Given: In an isosceles triangle $\triangle ABC$ side AB $\cong$ side AC ,
Perimeter $=$ 44 cm and base BC $=$ 12 cm.
Perimeter of $\triangle ABC = 44$ cm
$AB + BC + AC = 44$
$AB + BC + AB = 44$ …..(side $AB \cong$ side AC)
$2AB + 12 = 44$ ….(side $BC = 12$)
$2AB = 44 - 12 = 32$
$AB = \frac{32}{2} = 16$
$AB = AC = 16$

iii. Draw tangents to the circle with center ' C ' and radius 3.6 cm , from a point B at a distance of 7.2 cm from the center of the circle.

Solution:

Construction:



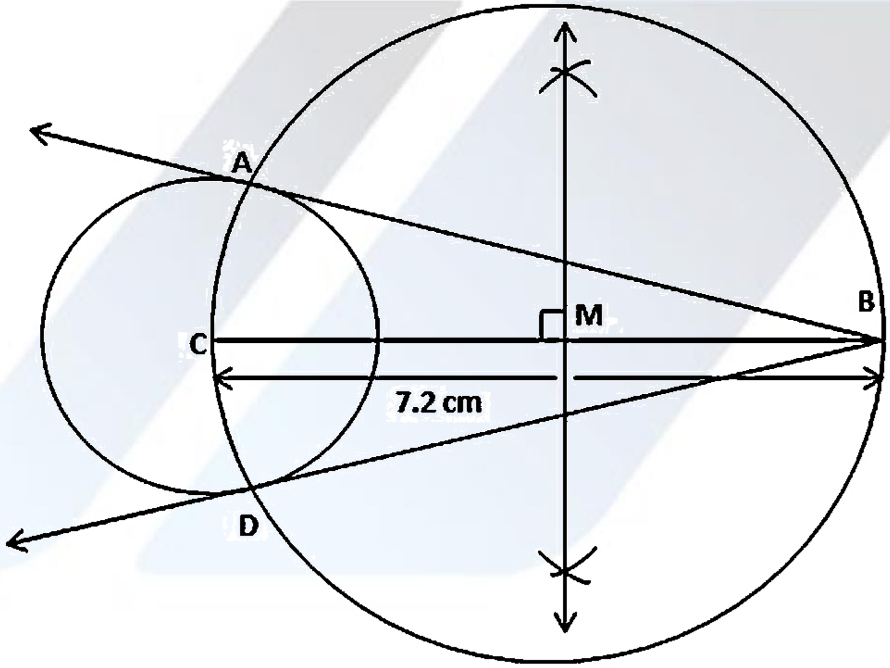
Analytical figure

Steps of Construction:

Step 1. Construct a circle with centre C and radius 3.6 cm. Take point B such that $\mathrm{CB}=7.2$ cm.

Step 2. Obtain Midpoint M of seg CB. Draw a circle with centre M and radius MB.

Step 3. Let A and D be the points of intersection of these two circles. Draw lines BA and BD which are the required tangents.



iv. Prove that:

$\sec^2\ \theta+\operatorname{cosec}^2\theta=\sec^2\ \theta\times\operatorname{cosec}^2\theta$

Solution:

To prove that: $\sec^2\ \theta+\operatorname{cosec}^2\theta=\sec^2\ \theta\times\operatorname{cosec}^2\theta$

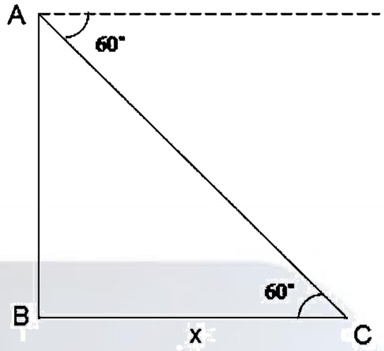
L.H.S $=\sec^2\ \theta+\operatorname{cosec}^2\theta$

$= 1 + \tan^2\ \theta + 1 + \cot^2\ \theta$ [$\because \sec^2\ \theta = 1 + \tan^2\ \theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\ \theta$]
R.H.S $= \sec^2\ \theta \times \operatorname{cosec}^2\theta$
$= (1 + \tan^2\ \theta) \times (1 + \cot^2\ \theta)$ [$\because \sec^2\ \theta = 1 + \tan^2\ \theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\ \theta$]
$= 1 + \cot^2\ \theta + \tan^2\ \theta + \tan^2\ \theta \times \cot^2\ \theta$
$= 1 + \cot^2\ \theta + \tan^2\ \theta + \tan^2\ \theta \times \frac{1}{\tan^2\ \theta}\ \left(\because \cot^2\ \theta = \frac{1}{\tan^2\ \theta}\right)$
$= 2 + \tan^2\ \theta + \cot^2\ \theta$
From (i) and (ii)
$\sec^2\ \theta + \operatorname{cosec}^2\theta = \sec^2\ \theta \times \operatorname{cosec}^2\theta$

v. Write the equation of each of the following lines:
(1) The x-axis and the y-axis.
(2) The line passing through the origin and the point $(-3,5)$.
(3) The line passing through the point $(-3,4)$ and parallel to X -axis.
Solution:

(1) The required equation of x-axis is $y = 0$ and y-axis is $x = 0$.
(2) Let $P \equiv (0,0) \equiv (x_1, y_1)$ and $Q \equiv (-3,5) \equiv (x_2, y_2)$
The required equation is
$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$
$\frac{x - 0}{0 + 3} = \frac{y - 0}{0 - 5}$
$\frac{x}{3} = \frac{y}{-5}$
$5x + 3y = 0$
(3) The equation of x-axis line is $y = 0$
Slope of the line $= 0$
Required line is parallel to X -axis we know that parallel lines have equal slopes.
Slope of the required line $= m = 0$ and point $(-3,$
$4)$ is on the line.
By point slope form of equation,
$y - y_1 = m(x - x_1)$
$y - 4 = 0(x - (-3))$
$y = 4$ is the required equation.

Q4. Solve any two sub-questions:
i. From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 60°. If the height of the lighthouse is 90 meters, then find how far is that ship from the lighthouse? $(\sqrt{3} = 1.73)$
Solution:



As shown in the figure, assume AB as the lighthouse and let A be the position of the observer and C be the position of the ship. Let the distance from the ship to the lighthouse be x .
Given, the height of the lighthouse is 90 m and the angle of depression to be 60°.
For the right-angled triangle ABC

$$\tan\ 60^\circ = \frac{AB}{BC}$$

$$\tan\ 60^\circ = \frac{AB}{x}$$

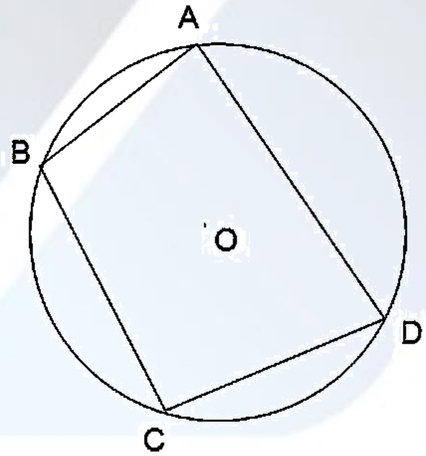
$$x = \frac{AB}{\tan\ 60^\circ} = \frac{90}{\sqrt{3}}$$

$$x = 52.02 \approx 52 \text{ m}$$

The ship is at a distance of 52 m from the lighthouse.

ii. Prove that the "the opposite angles of the cyclic quadrilateral are supplementary".

Solution:



Consider the circle having centre 0 and a cyclic quadrilateral $ABCD$.
To prove: $\angle BAD + \angle BCD = 180^\circ$ and $\angle ABC + \angle ADC = 180^\circ$ arcBCD is intercepted by the inscribed $\angle BAD$.
$\angle BAD = \frac{1}{2}$ m(arcBCD) … (1) (Inscribed angle theorem)
arc BAD is intercepted by the inscribed $\angle BCD$.

$\angle BCD = \frac{1}{2}$ m(arcDAB) ... (2) (Inscribed angle theorem)
From (1) and (2)

$\angle \text{BAD} + \angle \text{BCD} = \frac{1}{2}[\text{ m}(\text{arc}BCD) + \text{m}(\text{arcDAB})]$

$= \frac{1}{2} \times 360^\circ$

$= 180^\circ$

The sum of the measure of angles of a quadrilateral is 360°

$\therefore \angle ADC + \angle ABC = 360^\circ - (\angle BAD + \angle BCD) = 360^\circ - 180^\circ = 180^\circ$

Hence the opposite angles of a cyclic quadrilateral are supplementary.

iii. The sum of length, breadth and height of a cuboid is 38 cm and the length of its diagonal is 22 cm. Find the total surface area of the cuboid.

Solution:

Let length, breadth and height of the cuboid be lcm, bcm and hcm , respectively.
Given: l + b + h = 38 cm and diagonal = 22 cm.
To find: total surface area of the cuboid
Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$

$\sqrt{l^2 + b^2 + h^2} = 22$

As $l + b + h = 38$

$(l + b + h)^2 = 1444$

$l^2 + b^2 + h^2 + 2(lb + bh + lh) = 1444$

$484 + 2(lb + bh + lh) = 1444$

$2(lb + bh + lh) = 960$

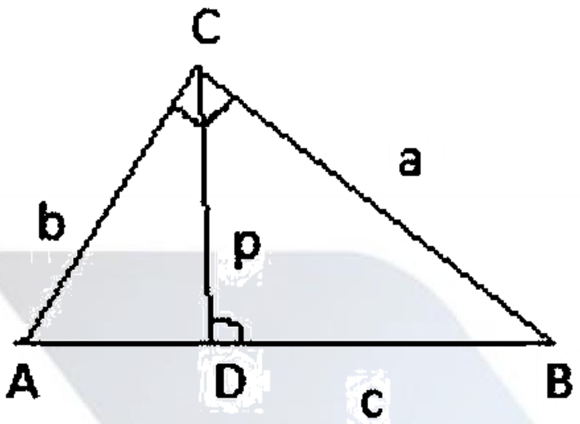
The surface area of the cuboid is 960 cm^2.

Q5. Solve any two sub-questions:
i. In triangle ABC, $\angle C = 90^\circ$. Let BC = a, CA = b, AB = c and let ' p ' be the length of the perpendicular from ' C ' on AB, prove that:
(1) $cp = ab$
(2) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution:

(1) Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height

$\text{A}(\triangle \text{ ABC}) = \frac{1}{2} \times \text{AB} \times \text{CD}$



Area of right angle triangle ABC $= A(\triangle ABC) = \frac{1}{2} \times AC \times BC$

$A(\triangle ABC) = \dfrac{1}{2} \times ba$

From (i) and (ii)

$cp = ba \Rightarrow cp = ab$

(2) We have,

cp $= a$ b...From (iii)

$p = \dfrac{ab}{c}$

Square both sides of the equation.

We get, $p^2 = \frac{a^2b^2}{c^2}$

$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$...(iv) ...[By invert endo]

In right angled triangle ABC ,

$AB^2 = AC^2 + BC^2$....[By Pythagoras' theorem]

$c^2 = b^2 + a^2$

Substituting the value of c^2 in equation (iv), we get

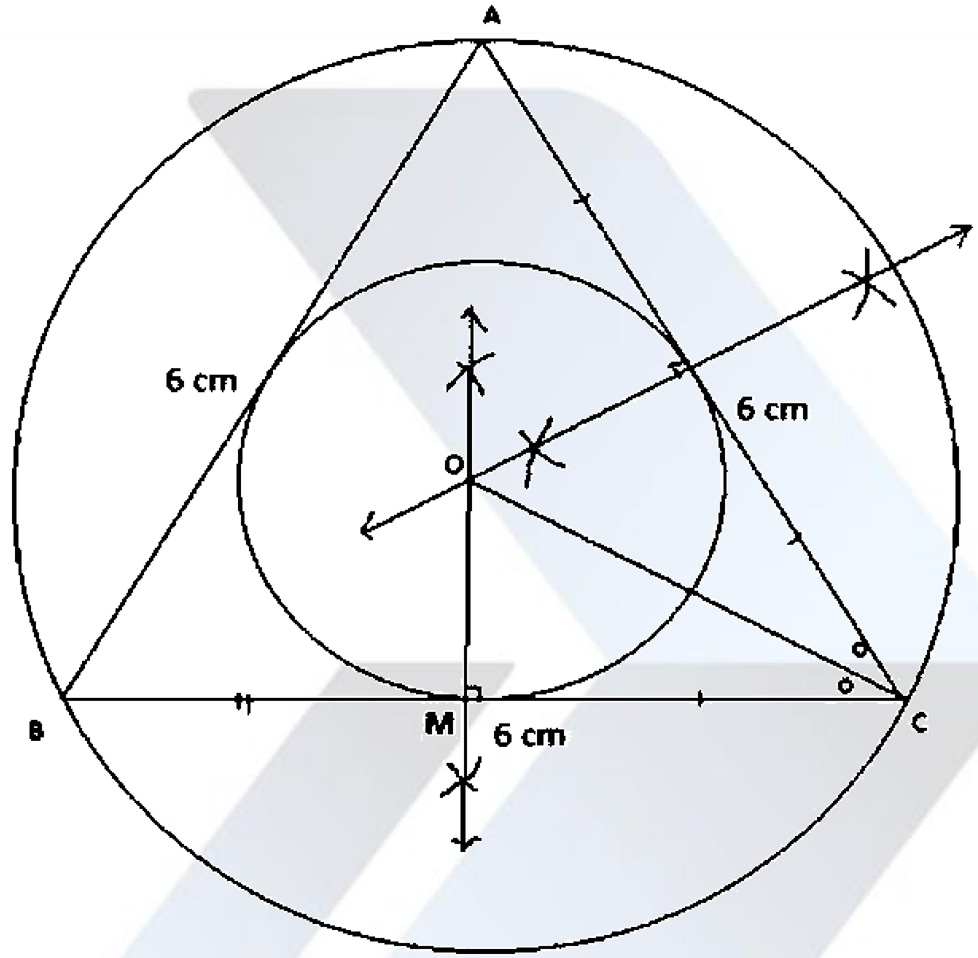
$$\frac{1}{p^2} = \frac{b^2 + a^2}{a^2b^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

ii. Construct the circumcircle and incircle of an equilateral triangle ABC with side 6 cm and centre 0 . Find the ratio of radii of circumcircle and incircle.

Solution:



Steps of Construction:

1 Construct an equilateral triangle △ ABC with side 6 cm.

2 Draw perpendicular bisectors of any two sides. Let O be the point of intersection.

3 Draw a circle with centre O and radius OA or OB or OC. This gives us circumcircle equilateral triangle △ ABC.

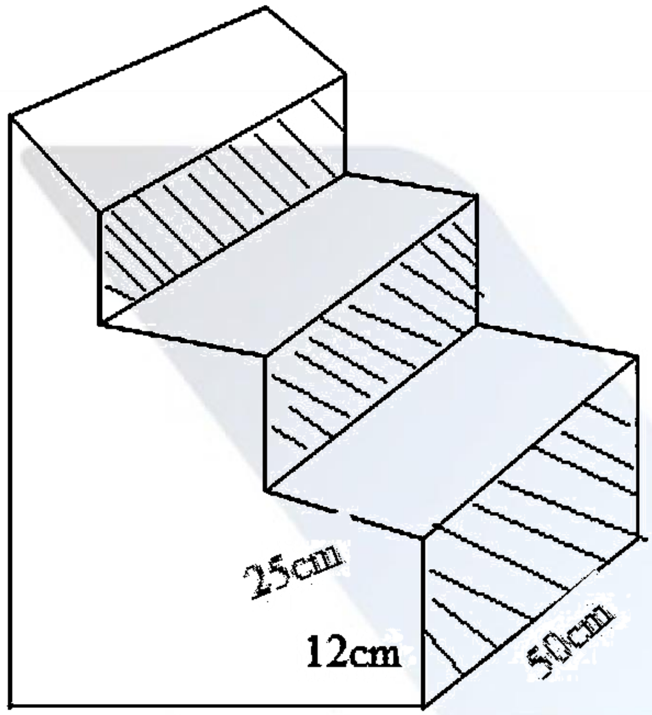
Draw the bisector of $\angle C$. It passes through centre of the circle 0 .

Draw a circle with radius OM . This gives us incircle of the equilateral triangle △ ABC.

In an equilateral triangle angle bisector and median are same, also circumcentre and incentre aresame. For equilateral triangle circumcentre divides the median in 2:1.

So the ratio of radii of circumcircle and incircle is 2: 1.

iii. There are three stair-steps as shown in the figure below. Each stair step has width 25 cm , height 12 cm and length 50 cm . How many bricks have been used in it, if each brick is 12.5 cm × 6.25 cm × 4 cm ?



Solution:

The 1 st stair-step $= h_1 = 12$ cm.
The 2 nd stair-step $= h_2 = 24$ cm.
The 3 rd stair-step $= h_3 = 36$ cm.
The total height $= h = h_1 + h_2 + h_3 = 12 + 24 + 36 = 72$ cm.
Length and width will remain same.
Length$= l = 50$ cm
Width$= w = 25$ cm
The volume of the three stair-step = Length(l) × Width(w) × Height(h)
The volume of the three stair-step $= 50 \times 25 \times 72$ cm³
Given: The volume of 1 brick $= 12.5 \times 6.25 \times 4$ cm³
Let the number of bricks required be n.
$(12.5 \times 6.25 \times 4) \times n = 50 \times 25 \times 72$

$$n = \frac{50 \times 25 \times 72}{12.5 \times 6.25 \times 4} = 288$$

288 bricks are used in three stair-steps.