Maharashtra Class 10 Mathematics 2015

PART – I(ALGEBRA)

Note:-

(1) All questions are compulsory.

(2) Use of calculator is not allowed.

Q1. Attempt any five question from the following:

i. State whether the following sequence is an A.P. or not? 1,4,7,10,

Solution:

1,4,7,10, ...

Let ' d ' be the common difference, ' a ' be the common ratio, and t_2, t_3, t_4 be the 2 nd, 3 rd and 4^{th} term respectively.

Here $t_2 = 4, t_3 = 7, t_4 = 10$

$d = 4 - 1 = 3$

$\mathrm{a} = 1$

We know $t_2 - a = t_3 - t_2 = 3$

Since the common difference is same throughout the series, therefore the series is an A.P.

ii. A card is drawn from the pack of 25 cards labeled with numbers 1 to 25 . Write the sample space for this random experiment.

Solution:

Sample Space for this experiment

{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25}

iii. Find the value of $x + y$, if $12x + 13y = 29$ and $13x + 12y = 21$

Solution:

$12x + 13y = 29.$

$13x + 12y = 21$

Adding (a) and (b), we have,

$25x + 25y = 50$

$\Rightarrow x + y = \frac{50}{25} = 2$

$\therefore x + y = 2$

iv. For a sequence if $S_n = \frac{n}{n+1}$ then find the value of S_{10}.

Solution:

$S_n = \frac{n}{n+1}$

For $n = 10$,

$S_n = \frac{n}{n+1}$

$= \frac{10}{10+1} = \frac{10}{11}$

v. Verify whether 1 is the root of the quadratic equation :

$x^2 + 3x - 4 = 0$

Solution:

$x^2 + 3x - 4 = 0$

If $x = 1$ satisfies the quadratic equation, then 1 is a root of the quadratic equation.

$1^2 + 3 \times 1 - 4 = 0$

Hence Proved.

vi. If $x + y = 5$ and $x = 3$, then find the value of y.

Solution:

$x + y = 5$ ...(a)

If $x = 3$ is substituted in the equation (a), then

$\Rightarrow 3 + y = 5$

$\Rightarrow y = 5 - 3$

$\therefore y = 2$

Q2. Attempt any four sub-questions from the following:

i. Solve the following quadratic equation by factorization method:

$x^2 - 5x + 6 = 0.$

Solution:

The given quadratic equation is $x^2 - 5x + 6 = 0$

Rewriting the above equation, we have,

$x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x-3) - 2(x-3) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x - 2 = 0$ or $x - 3 = 0$

$\Rightarrow x = 2$ or $x = 3$

ii. Find the term t_{11} of an A.P. :

4,9,14,..

Solution:

We need to find the 11^{th} term of the A.P., 4,9,14

Here, the initial term is, $a = 4$.

Common difference $= 14 - 9 = 9 - 4 = 5$

The general term of an A.P is given by the formula,

$t_n = a + (n-1)d$

$\Rightarrow t_{11} = 4 + (11-1) \times 5$

$\Rightarrow t_{11} = 4 + 10 \times 5$

$\Rightarrow t_{11} = 4 + 50$

$\Rightarrow t_{11} = 54$

iii. If the point $A(3,2)$ lies on the graph of the equation $5x + ay = 19$, then find a.

Solution:

Given that the point $A(3,2)$ lies on the graph of the equation $5x + ay = 19$.

Thus, the point satisfies the equation of the graph.

Substituting the values $x = 3$ and $y = 2$, we have,

$5 \times 3 + a \times 2 = 19$

$\Rightarrow 15 + 2a = 19$

$\Rightarrow 2a = 19 - 15$

$\Rightarrow 2a = 4$

$\Rightarrow a = \frac{4}{2}$

$\Rightarrow a = 2$

Therefore the value of a is 2 .

iv. A die is thrown. If A is the event that the number on upper face is less than 5 , then write sample space and event A in set notation.

Solution:

The sample space (S) is

$S = \{1,2,3,4,5,6\}$

No. of sample points $= n(S) = 6$

Let A be the event of getting numbers less than 5 .

$\therefore A = \{1,2,3,4\}$

$\Rightarrow n(A) = 4$

v. For a certain frequency distribution, the value of Mean is 101 and Median is 100. Find the value of M

Solution:

The interrelation between the measures of central tendency is given by

Mean - Mode = 3(Mean - Median)

Given that Mean = 101 and Median = 100

Thus from the above relation, we have,

⇒ 101 - Mode = 3 (101-100)

⇒ 101 - Mode = 3

⇒ 101-3 = Mode

⇒ Mode = 98

vi. If one root of the quadratic equation $kx^2 - 7x + 12 = 0$ is 3 , then find the value of k.

Solution:

The given quadratic equation is $kx^2 - 7x + 12 = 0$.

Let α and β be the roots of the given equation.

Comparing the given equation with the standard equation,

$ax^2 + bx + c = 0$, we have,

$a = k, b = -7$ and $c = 12$.

Thus, $\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{k}$

and $\alpha\beta = \frac{c}{a} = \frac{12}{k}$

Since one of the roots is 3 , we have,

$3 + \beta = \frac{7}{k}$ and $3\beta = \frac{12}{k}$

$\Rightarrow 3 + \beta = \frac{7}{k}$ and $\beta = \frac{4}{k}$

Substituting the value of $\beta = \frac{4}{k}$ in $3 + \beta = \frac{7}{k}$, we have,

$$3 + \frac{4}{k} = \frac{7}{k}$$

$$\Rightarrow \frac{3k + 4}{k} = \frac{7}{k}$$

$\Rightarrow 3k + 4 = 7$

$\Rightarrow 3k = 7 - 4$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Q3. Attempt any three of the following sub questions:

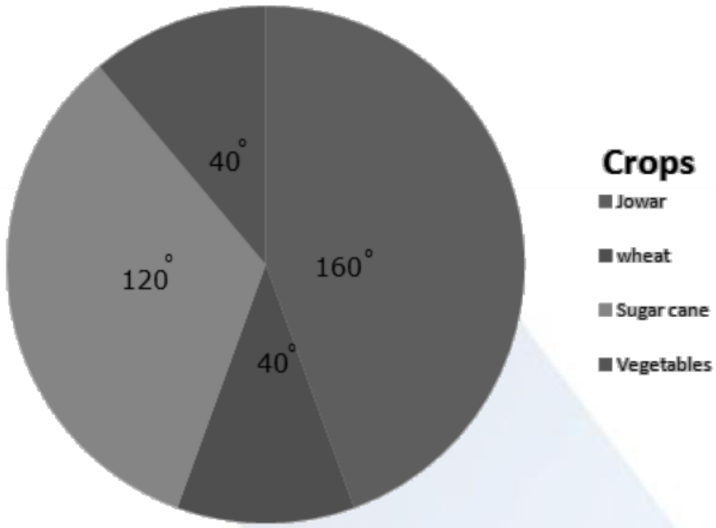
i. Area under different crops in a certain village is given below. Represent it by a pie diagram :

Corps	Area (in Hectares)
Jowar	80
Wheat	20
Sugarcane	60
Vegetables	20

Solution:

We compute the central angle for each crop as shown in the following table.

Crops	Area (in Hectares)	Measure of central angle
Jowar	80	$\frac{80}{180} \times 360^\circ = 160^\circ$
wheat	20	$\frac{20}{180} \times 360^\circ = 40^\circ$
Sugar cane	60	$\frac{60}{180} \times 360^\circ = 120^\circ$
Vegetables	20	$\frac{20}{180} \times 360^\circ = 40^\circ$
Total	180	360°



ii. If two coins are tossed, then find the probability of the event that at least one head turns up.

Solution:

Let S be the sample space.

Then $S = \{HH, HT, TH, TT\}$

$\therefore n(S) = 4$

Let A be the event where at least one tail turns up.

At least one head means 1 head or more than 1 heads.

If $S = \{\underline{HH}, \underline{HT}, \underline{TH}, TT\}$

$\therefore A = \{HT, TH, HH\}$

$\therefore n(A) = 3$

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

iii. Solve the following simultaneous equations by using graphical method :

$x + y = 6;$

$x - y = 4.$

Solution:

First we find three solutions for each equation and make tables.

$x + y = 6 \Rightarrow y = 6 - x$

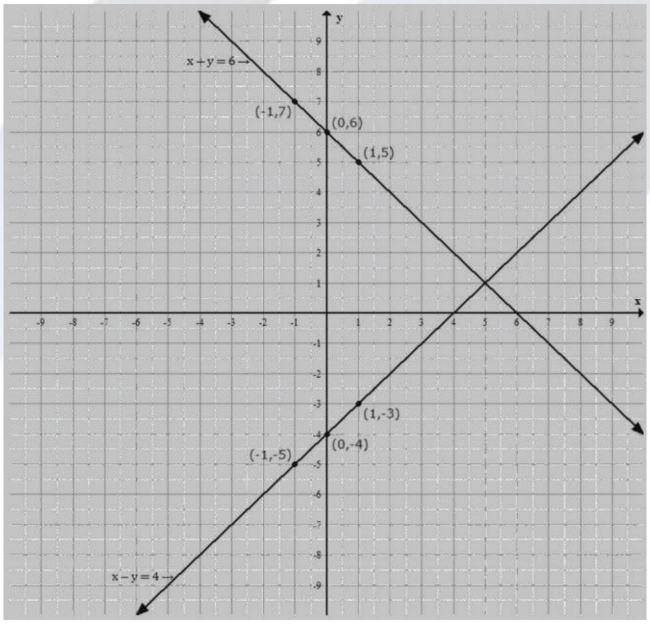
$x - y = 4 \Rightarrow y = x - 4$

$x + y = 6$			
x	1	-1	0
y	5	7	6

Now, plot the points $(1,5), (-1,7)$ and $(0,6)$ and draw the line passing through them.

$x - y = 4$			
x	1	-1	0
y	-3	-5	-4

Now, plot the points $(1,-3), (-1,-5)$ and $(0,-4)$ and draw the line passing through them.



Thus the point of intersection of two lines is $(5,1)$.

Hence $x = 5$ and $y = 1$ is the solution of the given equations.

iv. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the twenty-first row.

Solution:

There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. Find the number of seats in the twenty-fifth row.

Number of seats in the first row $= 20$

$\therefore a = 20$

Increase in the number of seats in consecutive rows $= 2$

$\therefore d = 2$

To find the number of seats in the 25^{th} row, find t_{25}

$t_n = a + (n-1)d$

$\therefore t_{25} = 20 + (25-1) \times 2$

$= 20 + (24 \times 2)$

$= 20 + 48$

$\therefore t_{25} = 68$

Thus, the number of seats in the twenty-fifth row is 68.

v. Solve the following quadratic equation by completing square method :

$x^2 + 10x + 21 = 0$

Solution:

$x^2 + 10x + 21 = 0$

$\therefore x^2 + 10x = -21$

To complete the square, find the third term.

Third term $= \left(\frac{1}{2} \text{ coefficient of } x\right)^2$

Coefficient of $x = 10$

$\therefore$ Third term $= \left(\frac{1}{2} \times 10\right)^2 = (5)^2 = 25$

Add 25 on both the sides,

$x^2 + 10x + 25 = -21 + 25$

$\therefore (x + 5)^2 = 4$

$x + 5 = \pm 2 \ldots$ (Taking square root on both sides)

$\therefore x + 5 = 2$ or $x + 5 = -2$

$\therefore x = -3$ or $x = -7$

Q4. Attempt any two sub-questions from the following:

i. Two digit numbers are formed using the digits 0,1,2,3,4,5 where digits are not repeated.

P is the event that the number so formed is even.

Q is the event that the number so formed is greater than 50 .

R is the event that the number so formed is divisible by 3.

Then write the sample space S and events P, Q, R using set notation.

Solution:

As we have to form two-digit numbers, 0 cannot be at the tens place.

The sample space is

$S = \{10,12,13,14,15,20,21,23,24,25,30,31,32,34,35,40,41,$
$42,43,45,50,51,52,53,54\}$

Let P be the event that the number formed is an even number.

$\therefore P = \{10,12,14,20,24,30,32,34,40,42,50,52,54\}$

Let Q be the event that the number formed is greater than 50 .

$\therefore Q = \{51,52,53,54\}$

Let R be the event that the number formed is divisible by 3 .

$\therefore R = \{12,15,21,24,30,42,45,51,54\}$

ii. The following table shows ages of 3000 patients getting medical treatment in a hospital on a particular day :

Age (in years)	No. of Patients
10 – 20	60
20 – 30	42
30 – 40	55
40 – 50	70
50 – 60	53
60 – 70	20

Find the median age of the patients.

Solution:

Age (in years)	Number of patients (f)	c.f. (less than type)
10 – 20	60	60
20 – 30	42	102 → c.f.
30 – 40 Median class	55 → f	157
40 – 50	70	227
50 – 60	53	280
60 – 70	20	300

Here, $N = 300$

$\therefore \frac{N}{2} = 150.$

Cumulative frequency just greater than 150 is 157.

∴ Corresponding class (30 − 40) is the median class.

L = 30, f = 55, c.f. = 102, h = 10

Median $= L + \left(\frac{N}{2} - \text{c.f.}\right)\frac{h}{f}$

$= 30 + (150 - 102) \times \frac{10}{55}$

$= 30 + 48 \times \frac{10}{55}$

$= 30 + 8.73$

$= 38.73$

Thus, the median age of patients is 38.73 years.

iii. If $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$, find the quadratic equation whose roots are α and β.

Solution:

$\alpha + \beta = 5$

$\alpha^3 + \beta^3 = 35$ ....

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (5)^3 - 3\alpha\beta(5)$

$= 125 - 15\alpha\beta$

$\therefore 125 - 15\alpha\beta = 35$[from (2)]

$\therefore 15\alpha\beta = 125 - 35$

$\therefore 15\alpha\beta = 90$

$\therefore \alpha\beta = 6$

The required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

Q5. Attempt any two of the following sub questions:

i. Babubhai borrows Rs. 4,000 and agrees to repay with a total interest of Rs. 500 in 10 installments, each instalment being less than the preceding instalment by Rs. 10. What should be the first and the last instalments?

Solution:

Each instalment is Rs. 10 less than the preceding one.

∴ The instalments are in A.P. with common difference -10

Babubhai repays Rs. 4000 with interest of Rs. 500 in 12 instalments

i.e. $S_{10} = 4000 + 500 = 4500$

Here $n = 12$, $d = -10$ and $S_{10} = 4500$.

we have to find the 1st installment i.e. a,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = 4500 = \frac{10}{2}[2a + (10-1)(-10)]$$

$$\therefore 4500 = 5[2a + 9 \times (-10)]$$

$$\therefore \frac{4500}{5} = 2a - 90$$

$$\therefore 900 = 2a - 90$$

$$\therefore 2a = 900 + 90 = 990$$

$$\therefore a = \frac{990}{2} = 495$$

$$\therefore a = 495$$

$t_n =$ last installment.

$$t_n = a + (n-1)d$$

$$t_{10} = 495 + (10-1) \times (-10)$$

$$t_{10} = 495 + 9 \times (-10)$$

$$t_{10} = 495 - 90$$

$$\therefore t_{10} = 405$$

∴ The first installment is Rs. 495 and the last installment is Rs. 405.

ii. On the first day of the sale of tickets of a drama, in all 35 tickets were sold. If the rates tickets were Rs. 20 and Rs. 40 per ticket and the total collection was Rs, 900, find the number of tickets sold of each rate.

Solution:

Let x tickets be sold at the rate of Rs. 20 and y be tickets at the rate of Rs.40.

Therefore, by first condition,

$x + y = 35$..(1)

By second condition,

$20x + 40y = 900$

$\therefore x + 2y = 45$..(2)

Subtracting equation (1) from the equation (2),

$x + 2y = 45$
$x + y = 35$
$- \quad - \quad -$
$y = 10$

Substituting $y = 10$ in equation (1),

$x + 10 = 35$

$x = 35 - 10$

$x = 25$

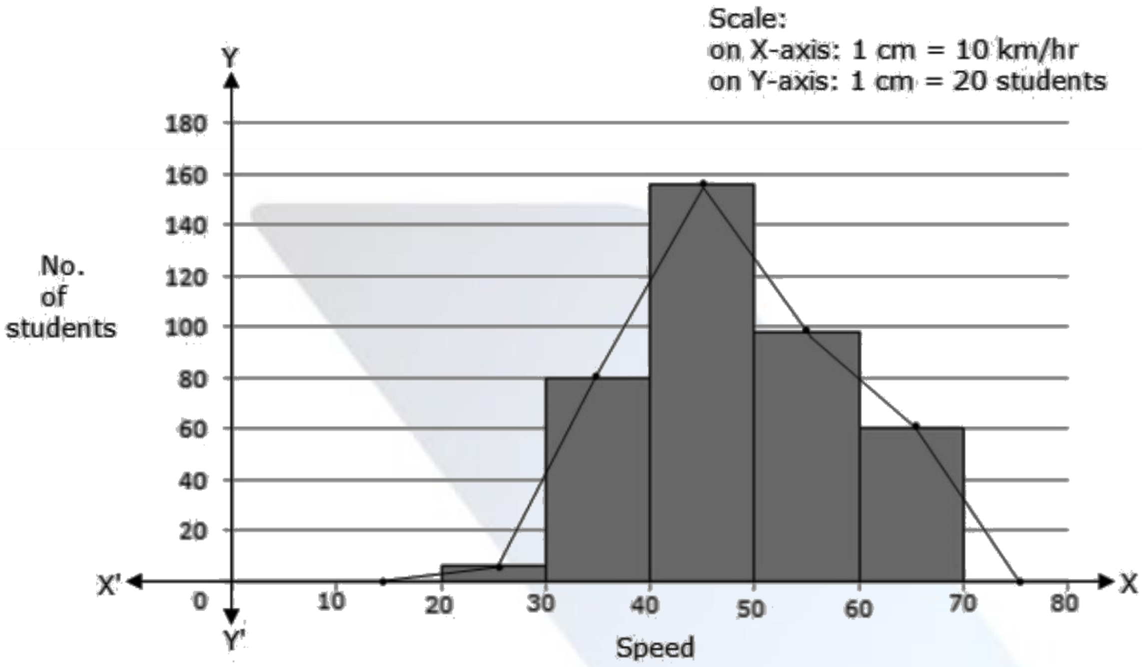
Therefore, 25 tickets at the rate of Rs. 20 each and 10 tickets at the rate of Rs. 40 were sold.

iii. Given below is the frequency distribution of driving speeds (in km/hour) of the vehicles of 400 college students:

Speed (in km/hr)	No. of Students
$20 - 30$	6
$30 - 40$	80
$40 - 50$	156
$50 - 60$	98
$60 - 70$	60

Draw Histogram and hence the frequency polygon for the above data.

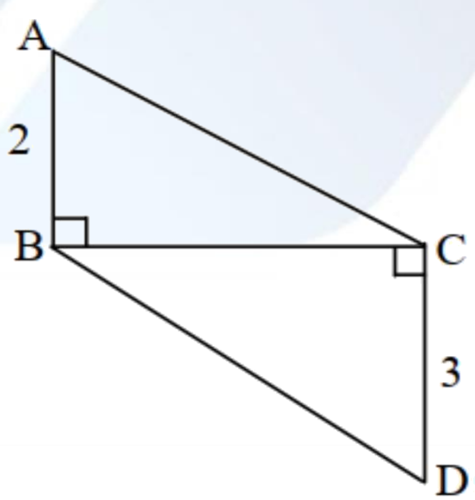
Solution:



PART – II(GEOMETRY)

Q1. Solve any five sub-questions:

(i) In the following figure, seg AB ⊥ segBC, seg DC ⊥ segBC. If AB = 2 and DC = 3, find $\frac{ar(\triangle ABC)}{ar(\triangle DCB)}$.



Solution:

Given,

AB $= 2$ and DC $= 3$

We know that the ratio of areas of two triangles lie on the same base is equal to the ratio of their corresponding heights.

$$\frac{ar(\Delta ABC)}{ar(\Delta DCB)} = \frac{AB}{DC} = \frac{2}{3}$$

(ii) Find the slope and y-intercept of the line $y = -2x + 3$.

Solution:

Given,

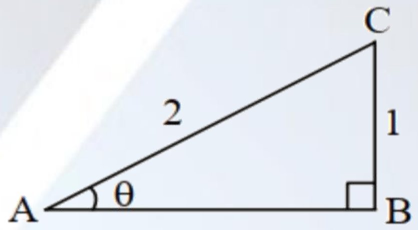
$y = -2x + 3$

Comparing with the equation of line in slope-intercept form $y = mx + c$

Slope $= m = -2$

y −intercept $= c = 3$

(iii) In the following figure, in $\triangle$ ABC, BC $= 1$, AC $= 2$, $\angle B = 90°$. Find the value of $\sin\ \theta$.



Solution:

Given,

BC $= 1$, AC $= 2$, $\angle B = 90°$

$$\sin\theta = \frac{\text{Side opposite to angle}\theta}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{1}{2}$$

(iv) Find the diagonal of a square whose side is 10 cm.

Solution:

Given,

Side of a square $= a = 10$ cm

Diagonal of the square $= \sqrt{2} \times a$

$= \sqrt{2} \times 10$

$= 10\sqrt{2}$ cm

(v) The volume of a cube is 1000 cm^3. Find the side of a cube.

Solution:

Given,

Volume of cube $= 1000\ cm^3$

$\Rightarrow (side)^3 = (10)^3$

$\Rightarrow$ Side $= 10$ cm

Therefore, the side of the cube is 10 cm.

(vi) If two circles with radii 5 cm and 3 cm respectively touch internally, find the distance between their centres.

Solution:

Given,

Two circles with radii 5 cm and 3 cm respectively touch internally.

Distance between their centres = Difference of the radii

$= 5 - 3$

$= 2$ cm

Q2. Solve any four sub-questions:

(i) If $\sin\theta = \frac{5}{13}$, where θ is an acute angle, find the value of cosθ.

Solution:

Given,

$\sin\theta = \frac{5}{13}$

$\cos\theta = \sqrt{(1 - \sin^2\theta)}$

$= \sqrt{\left[1 - \left(\frac{5}{13}\right)^2\right]}$

$= \sqrt{\left[1 - \left(\frac{25}{169}\right)\right]}$

$= \sqrt{\left[\frac{169 - 25}{169}\right]}$

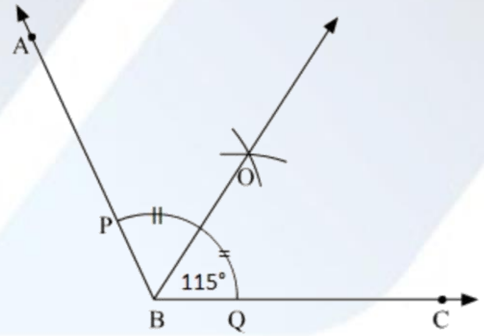
$= \sqrt{\left(\frac{144}{169}\right)}$

$= \frac{12}{13}$

Therefore, $\cos\theta = \frac{12}{13}$

(ii) Draw ∠ABC of measure 115° and bisect it.

Solution:



(iii) Find the slope of the line passing through the points C(3,5) and D(−2, −3).

Solution:

Let the given points be:

$C(3,5) = (x_1, y_1)$

$D(-2, -3) = (x_2, y_2)$

Slope of the line passing through the points (x_1, y_1) and (x_2, y_2)

$= \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \frac{-3 - 5}{-2 - 3}$

$= \frac{-8}{-5}$

$= \frac{8}{5}$

Therefore, the slope is $\frac{8}{5}$.

(iv) Find the area of the sector whose arc length and radius are 10 cm and 5 cm respectively.

Solution:

Given,

Length of the arc $= 10$ cm

Radius $= r = 5$ cm

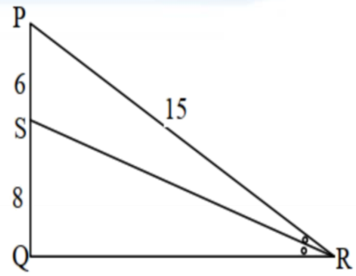
Area of sector $= \left(\frac{r}{2}\right) \times$ Length of arc

$= \left(\frac{5}{2}\right) \times 10$

$= 5 \times 5$

$= 25\ \text{cm}^2$

(v) In the following figure, in $\triangle$ PQR, seg RS is the bisector of $\angle$PRQ, PS $= 6$, SQ $= 8$, PR $= 15$. Find QR.



Solution:

Given,

$PS = 6, SQ = 8, PR = 15$

seg RS is the bisector of $\angle PRQ$.

By the angle bisector property,

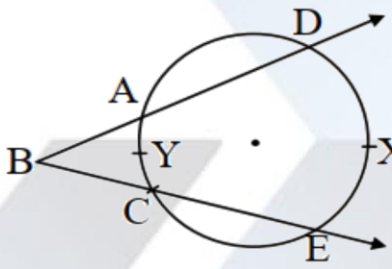
$\frac{PR}{QR} = \frac{PS}{SQ}$

$\frac{15}{QR} = \frac{6}{8}$

$\Rightarrow QR = \frac{15 \times 8}{6}$

$\Rightarrow QR = 20$

(vi) In the following figure, if $m(\text{arcDXE}) = 100°$ and $m(\text{arcAYC}) = 40°$, find $\angle DBE$.



Solution:

Given,

$m(\text{arcDXE}) = 100°$

$m(\text{arcAYC}) = 40°$

By the inscribed angle theorem,

$m\angle AEB = \left(\frac{1}{2}\right) \times m\angle AYC$

$= \left(\frac{1}{2}\right) \times 40°$

$= 20°$

And

$m\angle EAD = \left(\frac{1}{2}\right) \times m\angle DXE$

$= \left(\frac{1}{2}\right) \times 100^\circ$

$= 50^\circ$

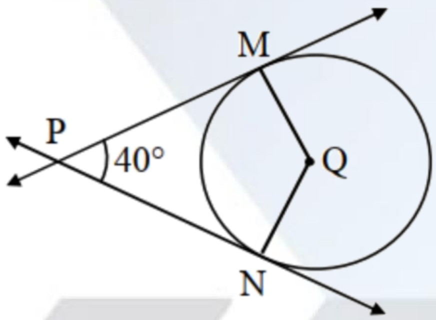
Now, by the exterior angle theorem:

$m\angle DBE + 20^\circ = 50^\circ$

$m\angle DBE = 50^\circ - 20^\circ = 30^\circ$

Q3. Solve any three sub-questions:

(i) In the following figure, Q is the centre of a circle and PM, PN are tangent segments to the circle. If $\angle MPN = 40^\circ$, find $\angle MQN$.



Solution:

Given, $\angle MPN = 40^\circ$

We know that the radius is perpendicular to the tangent through the point of contact.

$\angle PMQ = \angle PNQ = 90^\circ$

In quadrilateral PMQN,

$\angle MPN + \angle PNQ + \angle MQN + \angle PMQ = 360^\circ$

$40^\circ + 90^\circ + \angle MQN + 90^\circ = 360^\circ$

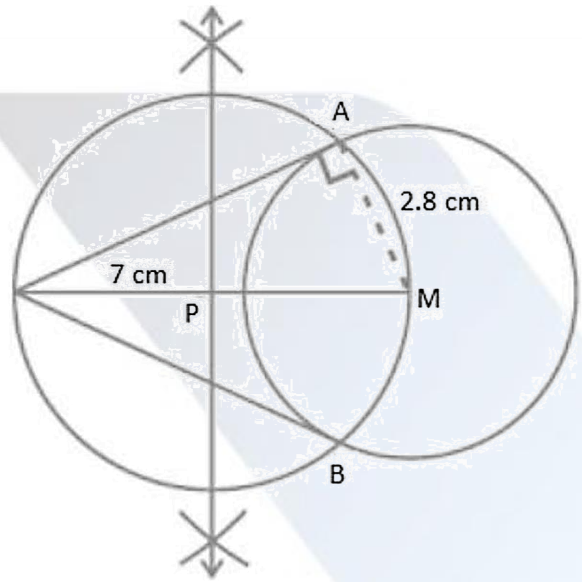
$\angle MQN + 220^\circ = 360^\circ$

$\angle MQN = 360^\circ - 220^\circ$

$\angle MQN = 140^\circ$

(ii) Draw the tangents to the circle from point L with radius 2.8 cm. The point, 'L' is at a distance 7 cm from the centre 'M'.

Solution:



(iii) The ratio of the areas of two triangles with the common base is 6: 5. The height of the larger triangle is 9 cm, then find the corresponding height of the smaller triangle.

Solution:

Let H be the height of the larger triangle and h be the height of the smaller triangle. We know that the ratio of the areas of two triangles with a common base is equal to the ratio of their corresponding heights.

$\Rightarrow \frac{6}{5} = \frac{\mathrm{H}}{\mathrm{h}}$

$\Rightarrow \frac{6}{5} = \frac{9}{\mathrm{h}}$ (given height of the larger triangle is 9 cm)

$\Rightarrow \mathrm{h} = \frac{9 \times 5}{6}$

$\Rightarrow \mathrm{h} = \frac{15}{2} = 7.5 \text{ cm}$

Hence, the corresponding height of the smaller triangle is 7.5 cm.

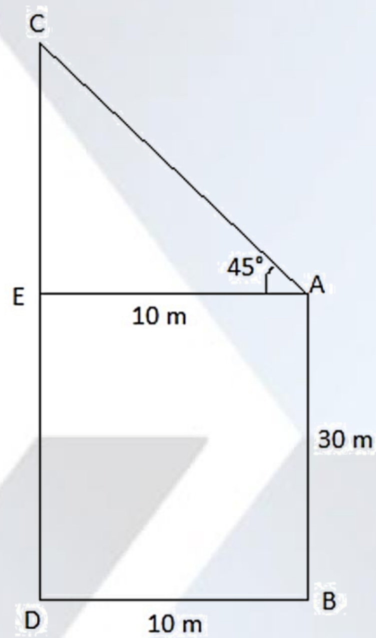
(iv) Two buildings are in front of each other on either side of a road of width 10 metres. From the top of the first building which is 30 metres high, the angle of elevation to the top of the second is 45°. What is the height of the second building?

Solution:

Let AB be the first building and CD be the second building.

BD = Width of the road = 10 m

AB = 30 m



BD = AE = 10 m

AB = ED = 30 m

In the right triangle AEC, $\tan 45° = \frac{CE}{AE}$

$1 = \frac{CE}{10}$

CE = 10 m

CD = CE + ED

= 10 + 30

= 40 m

Therefore, the height of the second building is 40 m.

(v) Find the volume and surface area of a sphere of radius 4.2 cm. ($\pi = \frac{22}{7}$)

Solution:

Given,

Radius of the sphere $= r = 4.2$ cm

Volume of sphere $= \left(\frac{4}{3}\right)\pi r^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 4.2 \times 4.2 \times 4.2$

$= 310.464\ \text{cm}^3$

The surface area of the sphere $= 4\pi r^2$

$= 4 \times \left(\frac{22}{7}\right) \times 4.2 \times 4.2$

$= 221.76\ \text{cm}^2$

Therefore, the volume of the sphere is $310.464\ \text{cm}^3$ and the surface area of the sphere is $221.76\ \text{cm}^2$.

Q4. Solve any two sub-questions:

(i) Prove that "the opposite angles of a cyclic quadrilateral are supplementary".

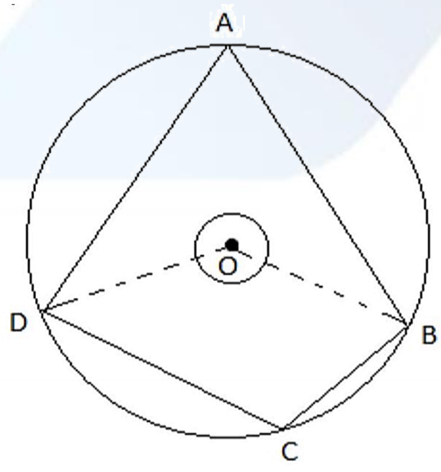
Solution:

Given,

ABCD is a cyclic quadrilateral of a circle with centre O.

Construction: Join OB and OD.

To prove: $\angle BAD + \angle BCD = 180°$



Proof:

We know that the angle subtended by the arc at the centre is twice the angle subtended by it at the remaining part of the circle.

$\angle BOD = 2\angle BAD$ ......(i)

Also,

reflex $\angle BOD = 2\angle BCD$ ... (ii)

Adding (i) and (ii),

$2\angle BAD + 2\angle BCD = \angle BOD + \text{reflex } \angle BOD$

$2(\angle BAD + \angle BCD) = 360^\circ$

$\angle BAD + \angle BCD = \dfrac{360^\circ}{2}$

$\angle BAD + \angle BCD = 180^\circ$

Hence proved.

(ii) Prove that $\sin^6 \theta + \cos^6 \theta = 1 - 3\sin^2 \theta \cdot \cos^2 \theta$.

Solution:

$\text{LHS} = \sin^6 \theta + \cos^6 \theta$

$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$

$= (\sin^2 \theta + \cos^2 \theta) + [(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta]$

$= (1)[(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta]$

$= (1)^2 - 3\sin^2 \theta \cos^2 \theta$

$= 1 - 3\sin^2 \theta \cdot \cos^2 \theta$

= RHS

Hence proved.

(iii) A test tube has a diameter 20 mm, and the height is 15 cm. The lower portion is a hemisphere. Find the capacity of the test tube. ($\pi = 3.14$)

Solution:

Given,

Diameter of the test tube $= 20$ mm

Radius of cylindrical part = Radius of the hemisphere $= r = \frac{20}{2} = 10$ mm $= 1$ cm

Height of the test tube $= 15$ cm

Height of the cylindrical part $= h = 15 - 1 = 14$ cm

Volume (capacity) of the test tube = Volume of cylinder + Volume of hemisphere

$= \pi r^2 h + \left(\frac{2}{3}\right)\pi r^3$

$= 3.14 \times 1 \times 1 \times 14 + \left(\frac{2}{3}\right) \times 3.14 \times 1 \times 1 \times 1$

$= 43.96 + 2.09$

$= 46.05$

Hence, the capacity of the test tube is 46.05 cm^3.

Q5. Solve any two sub-questions:

(i) Prove that the angle bisector of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Solution:

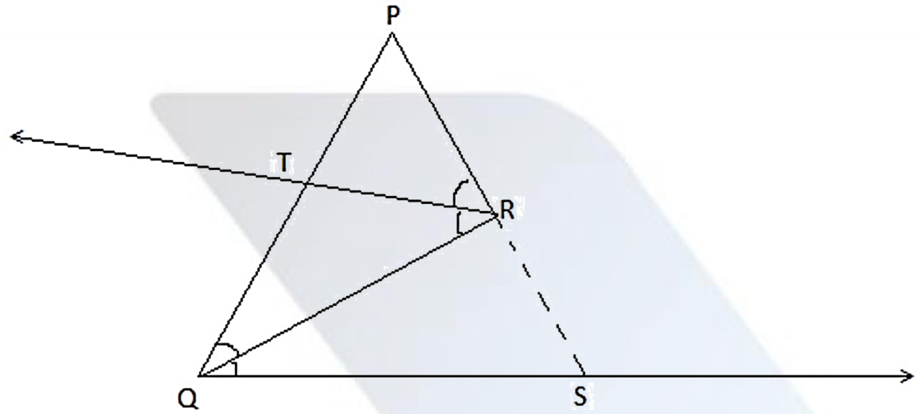
Given,

Triangle PQR in which RT is the angle bisector of $\angle QRP$.

Construction:

Draw angle bisector RT from R which intersects PQ at T.

Extend PR and QR so that they intersect each other at S.



To prove: $\left(\frac{PT}{TQ}\right) = \left(\frac{PR}{QR}\right)$

Proof:

RT || QS and PS is the transversal.

$\angle PRT = \angle RSQ$ (corresponding angles)

Now, BC is the transversal.

$\angle TRQ = \angle RQS$ (alternate angles)

$\angle PRT = \angle TRQ$ (given)

From the all above,

$\angle RQS = \angle RSQ$

In triangle RQS,

$QR = RS$ (sides opposite to equal angles are equal)

In triangle PQS,

RT || QS

$\frac{PT}{TQ} = \frac{PR}{RS}$

$$\frac{PT}{TQ} = \frac{PR}{RQ}$$

Hence proved.

(ii) Write down the equation of a line whose slope is $\left(\frac{3}{2}\right)$ and which passes through point P, where P divides the line segment AB joining $A(-2,6)$ and $B(3,-4)$ in the ratio $2:3$.

Solution:

Given,

P divides the line segment AB joining $A(-2,6)$ and $B(3,-4)$ in the ratio $2:3$.

$A\ (-2,6) = (x_1, y_1)$

$B(3,-4) = (x_2, y_2)$

$m:n = 2:3$

Using the section formula,

$$P = \left[\frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}\right]$$

$$= \left[\frac{6-6}{2+3}, \frac{-8+18}{2+3}\right]$$

$$= \left(\frac{0}{5}, \frac{10}{5}\right)$$

$= (0,2)$

Therefore, $P = (0,2)$

Equation of the line passing through $P(0,2)$ and having slope $\left(\frac{3}{2}\right)$ is:

$$y - 2 = \frac{3}{2}(x - 0)$$
$$2(y-2) = 3x$$
$$2y - 4 = 3x$$
$$3x - 2y + 4 = 0$$

(iii) $\Delta RST \sim \Delta UAY$. In ΔRST, $RS = 6$ cm, $\angle S = 50°$, $ST = 7.5$ cm. The corresponding sides of ΔRST and ΔUAY are in the ratio 5:4. Construct $\triangle$ UAY.

Solution:

Given,

$\Delta\ RST \sim \Delta UAY$

In $\triangle$ RST,

$RS = 6$ cm, $\angle S = 50°$, $ST = 7.5$ cm

ΔRST and $\triangle UAY$ are in the ratio 5: 4. (given)

$\frac{RS}{UA} = \frac{ST}{AY} = \frac{RT}{UY} = \frac{5}{4}$

$\angle S = \angle A = 50°$

Now,

$\frac{RS}{UA} = \frac{5}{4}$

$\frac{6}{UA} = \frac{5}{4}$

$UA = \frac{6 \times 4}{5}$

$UA = 4.8$ cm

Similarly,

$\frac{ST}{AY} = \frac{5}{4}$

$\frac{7.5}{AY} = \frac{5}{4}$

$AY = \frac{7.5 \times 4}{5}$

$AY = 6$ cm