Grade10 Maths Maharashtra 2016

PART – I(ALGEBRA)

Note: -

(1) All questions are compulsory.

(2) Use of calculator is not allowed.

Q1. Attempt any five question from the following:

i. Write the first two terms of the sequence whose nth term is $t_n = 3n - 4$.

Solution:

$t_n = 3n - 4$

For $n = 1, t_1 = 3 \times 1 - 4 = 3 - 4 = -1$

For $n = 2, t_2 = 3 \times 2 - 4 = 6 - 4 = 2$

Hence, the first two terms of the sequence are -1 and 2 .

ii. Find the value of a, b, c in the following quadratic equation :

$2x^2 - x - 3 = 0$

Solution:

Given equation is $2x^2 - x - 3 = 0$.

Comparing the given equation with general form of quadratic equation

$ax^2 + bx + c$, we have

$a = 2, b = -1$ and $c = -3$

iii. Write the quadratic equation whose roots are -2 and -3 .

Solution:

Let the roots be $\alpha = -2$ and $\beta = -3$.

$\therefore \alpha + \beta = (-2) + (-3) = -5$ and $\alpha\beta = (-2)(-3) = 6$

Hence, the required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2 - (-5)x + 6 = 0$

i.e. $x^2 + 5x + 6 = 0$

iv. Find the value of determinant:

$$\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = (4 \times 1) - (-2 \times 3) = 4 + 6 = 10$$

v. Write the sample space for selecting a day randomly of the week.

Solution:

The sample space 'S' for selecting a day randomly of the week is given by

S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

vi. Find the class mark of the classes 20-30 and 30-40.

Solution:

$$\text{Class Mark} = \frac{\text{Upper Limit} + \text{Lower Limit}}{2}$$

$$\therefore \text{Class Mark of the class } 20 - 30 = \frac{30+20}{2} = \frac{50}{2} = 25$$

$$\text{Class Mark of the class } 30 - 40 = \frac{40+30}{2} = \frac{70}{2} = 35$$

Q2. Attempt any four sub-questions from the following:

i. Write the first three terms of the A.P. whose common difference is -3 and first term is 4 .

Solution:

$a = 4, d = -3$

Hence,

$t_1 = 4$

$t_2 = t_1 + d = 4 + (-3) = 4 - 3 = 1$

$t_3 = t_2 + d = 1 + (-3) = 1 - 3 = -2$

Thus, the first three terms of the A.P. are 4, 1 and -2 .

ii. Solve the following quadratic equation by Factorisation method:

$x^2 + 7x + 10 = 0$

Solution:

$x^2 + 7x + 10 = 0$

Splitting the middle term $7x$ as $2x + 5x$, we get

$x^2 + 2x + 5x + 10 = 0$

$\therefore x(x+2) + 5(x+2) = 0$

$\therefore (x+2)(x+5) = 0$

$\therefore x + 2 = 0$ or $x + 5 = 0$

$\therefore \mathrm{x} = -2$ or $\mathrm{x} = -5$

iii. If the value of determinant $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix}$ is 31, find the value of m.

Solution:

$\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

$\therefore (m \times 7) - (-5 \times 2) = 31$

$\therefore 7m - (-10) = 31$

$\therefore 7m + 10 = 31$

$\therefore 7m = 21$

$\therefore \mathrm{m} = 3$

iv. A die is thrown, then find the probability of the following events:

A is an Event: getting an odd number on the top upper surface of the die. B is an Event: getting a perfect square on the upper surface of the die.

Solution:

When a die is thrown, the sample space (S) is given by

$S = \{1,2,3,4,5,6\}$

$\therefore \mathrm{n(S)} = 6$

Let A be the event of getting an odd number on the upper surface of the die. Then

$A = \{1,3,5\}$

$\therefore \mathrm{n}(\mathrm{A}) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Let B be the event of getting a perfect square on the upper surface of the die.

Then $B = \{1,4\}$

$\therefore \mathrm{n}(\mathrm{B}) = 2$

$\therefore P(B) = \frac{n(B)}{n(S)}$

$= \frac{2}{6}$

$= \frac{1}{3}$

v. Below is the given frequency distribution of words in an essay:

Number of Words	Number of Candidates
$600 - 800$	8
$800 - 1000$	22
$1000 - 1200$	40
$1200 - 1400$	18
$1400 - 1600$	12

Find the mean number of words written.

Solution:

(Number of words) Class intervals	Class Mark x_i	(Number of candidates) Frequency f_i	$f_i x_i$

600 – 800	700	8	5600
800 – 1000	900	22	19800
1000 – 1200	1100	40	44000
1200 – 1400	1300	18	23400
1400 – 1600	1500	12	18000
Total		$\sum f_i = 100$	$\sum f_i x_i = 110800$

Mean $= \overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{110800}{100} = 1108$

$\therefore$ Mean number of words written in an essay is 1108 .

vi. Subjectwise marks obtained by a student in an examination are given below:

Subject	Marks
Marathi	85
Hindi	85
Science	90
Mathematics	100
Total	360

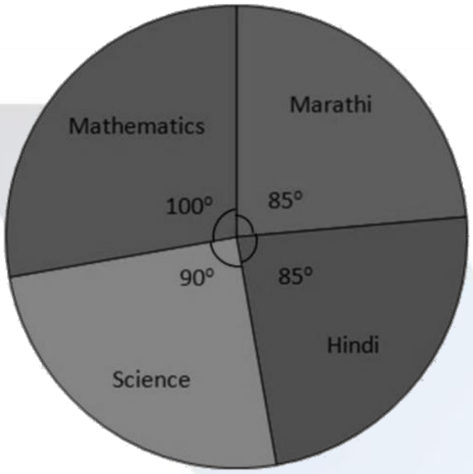
Draw pie diagram.

Solution:

Central angle for each subject is computed in the following table:

Subject	Marks	Measure of central angle
Marathi	85	$\frac{85}{360} \times 360^\circ = 85^\circ$
Hindi	85	$\frac{85}{360} \times 360^\circ = 85^\circ$
Science	90	$\frac{90}{360} \times 360^\circ = 90^\circ$

Mathematics	100	$\frac{100}{360} \times 360^\circ = 100^\circ$
Total	360	360°



Q3. Attempt any three of the following sub questions:

i. Solve the following quadratic equation by using formula method:

$5m^2 + 5m - 1 = 0$

Solution:

Given quadratic equation is $5m^2 + 5m - 1 = 0$

Comparing with general form $ax^2 + bx + c = 0$, we have $a = 5, b = 5$ and $c = -1$

$$\therefore x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(5)(-1)}}{2 \times 5}$$

$$= \frac{-5 \pm \sqrt{25 + 20}}{10}$$

$$= \frac{-5 \pm \sqrt{45}}{10}$$

$$= \frac{-5 \pm 3\sqrt{5}}{10}$$

$\therefore \frac{-5+3\sqrt{5}}{10}$ and $\frac{-5-3\sqrt{5}}{10}$ are the roots of the given equation.

ii. There are three boys and two girls. A committee of two is to be formed.

Find the probability of the following events:

Event A: The committee contains at least one girl

Event B: The committee contains one boy and one girl

Solution:

Here, there are three boys B_1, B_2, B_3 and two girls G_1, G_2.

A committee of two is to be formed.

Thus, the sample space (S) is given by

$S = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore \mathrm{n(S)} = 10$

A is the event that the committee contains at least one girl.

Then, $A = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore n(A) = 7$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7}{10}$$

B is the event that the committee contains one boy and one girl.

Then, $B = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$

$\therefore \mathrm{n(B)} = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

iii. The measurements (in mm) of the diameters of the head of the screws are given below:

Diameter (in mm)	No. of Screws
$33 - 35$	10
$36 - 38$	19
$39 - 41$	23

$42 - 44$	21
$45 - 47$	27

Calculate mean diameter of head of a screw by 'Assumed Mean Method'.

Solution:

Let A be the assumed mean.

A is taken as the class mark of the middle class.

Hence, let us take 40 as the assumed mean.

Then $A = 40$ and deviation $d_i = x_i - A = x_i - 40$

Diameter (in mm)	Class marks x_i	Deviations $d_i = x_i - A$ $d_i = x_i - 40$	Number of Screws f_i	$f_i d_i$
$33 - 35$	34	-6	10	-60
$36 - 38$	37	-3	19	-57
$39 - 41$	$40 = A$	0	23	0
$42 - 44$	43	3	21	63
$45 - 47$	46	6	27	162
Total	...	...	$\Sigma f_i = 100$	$\Sigma f_i d_i = 108$

Here, $\Sigma f_i d_i = 108, \Sigma f_i = 100$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{108}{100} = 1.08$$

$$\begin{aligned}\bar{x} &= A + \bar{d} \\ &= 40 + 1.08 \\ &= 41.08\end{aligned}$$

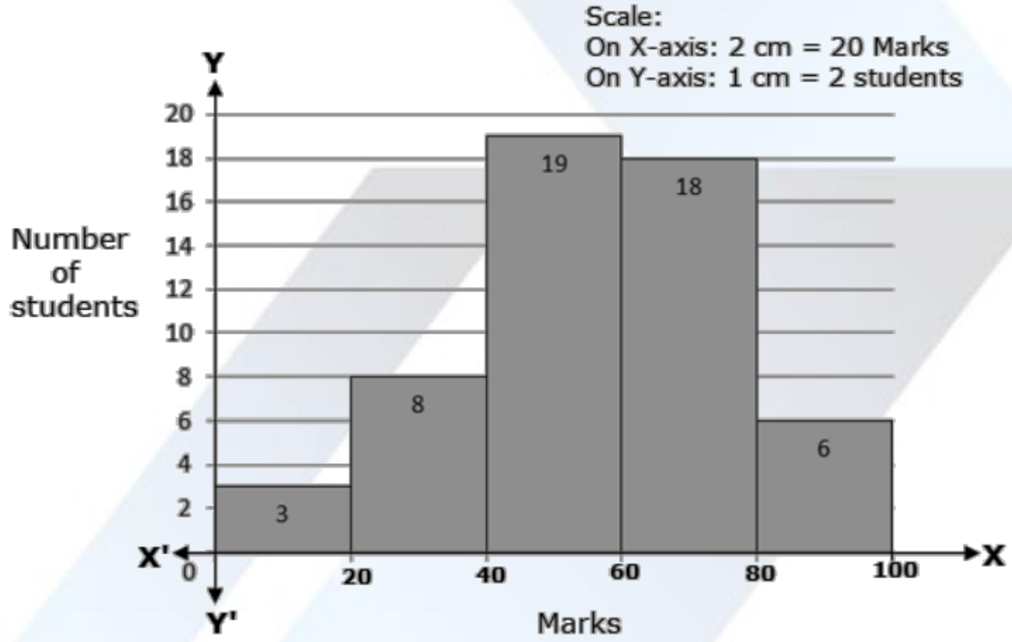
Thus, the mean diameter of the screw heads is 41.08 mm .

iv. The marks scored by students in Mathematics in a certain Examination are given below:

Marks Scored	Number of Students
0 – 20	3
20 – 40	8
40 – 60	19
60 – 80	18
80 – 100	6

Draw histogram for the above data.

Solution:



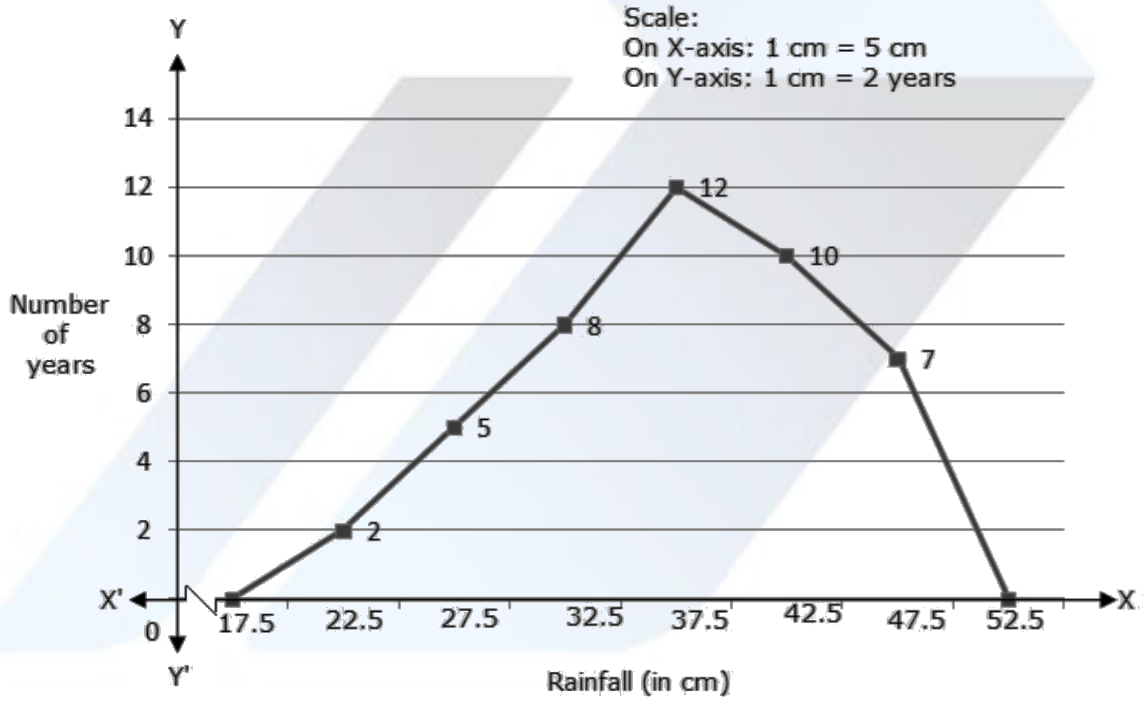
v. Draw the frequency polygon for the following frequency distribution:

Rainfall (in cm)	No. of Years
20 – 25	2
25 – 30	5
30 – 35	8

35 – 40	12
40 – 45	10
45 – 50	7

Solution:

Class mark	No. of Years
17.5	0
22.5	2
27.5	5
32.5	8
37.5	12
42.5	10
47.5	7
52.5	0



Q4. Attempt any two of the following sub questions:

i. The 11^{th} term and the 21^{st} term of an A.P. are 16 and 29 respectively, then find:

(a) The first term and common difference

(b) The 34^{th} term

(c) ' n ' such that $t_n = 55$

Solution:

(a) Let ' a ' be the first term and ' d ' the common difference of the given A.P.

For $\mathrm{t}_{11} = 16, \mathrm{n} = 11$, we have

$\mathrm{t}_{11} = \mathrm{a} + (11 - 1)\mathrm{d}$

$\therefore 16 = \mathrm{a} + 10\ \mathrm{d}$..(1)

For $t_{21} = 29, n = 21$, we have

$t_{21} = a + (21 - 1)d$

$\therefore 29 = \mathrm{a} + 20\ \mathrm{d}$....(2)

Subtracting (1) from (2), we get

$13 = 10d \Rightarrow d = 1.3$

Substituting d=1.3 in (1), we get

$16 = a + 10(1.3)$

$\therefore 16 = a + 13 \Rightarrow a = 3$

Thus, the first term is 3 and the common difference is 1.3.

(b) For 34^{th} term, $\mathrm{n} = 34, \mathrm{a} = 3, \mathrm{d} = 1.3$

$$
\begin{aligned}
\mathrm{t}_{\mathrm{n}} &= \mathrm{a} + (\mathrm{n} - 1)\mathrm{d} \\
\therefore \mathrm{t}_{34} &= 3 + (34 - 1)(1.3) \\
&= 3 + 33 \times 1.3 \\
&= 3 + 42.9 \\
&= 45.9
\end{aligned}
$$

Thus, the 34^{th} term is 45.9.

(c) $\mathrm{t}_{\mathrm{n}} = 55, \mathrm{a} = 3, \mathrm{d} = 1.3$

$\therefore \mathrm{t}_{\mathrm{n}} = \mathrm{a} + (\mathrm{n} - 1)\mathrm{d}$

$\therefore 55 = 3 + (n - 1)(1.3)$

$\therefore 55 - 3 = (n - 1)(1.3)$

$\therefore (\mathrm{n} - 1)(1.3) = 52$

$\therefore \mathrm{n} - 1 = \dfrac{52}{1.3}$

$\therefore \mathrm{n} - 1 = 40$

$\therefore n = 40 + 1$

$\therefore$ n = 41

ii. Solve the following simultaneous equations:

$\frac{7}{2x+1}+\frac{13}{y+2}=27, \frac{13}{2x+1}+\frac{7}{y+2}=33.$

Solution:

$$\frac{7}{2x+1}+\frac{13}{y+2}=27 \ldots (1)$$

$$\frac{13}{2x+1}+\frac{7}{y+2}=33..(2)$$

Substituting $\frac{1}{2x+1}=m$ and $\frac{1}{y+2}=n$ in equations (1) and (2), we get

$7m+13n=27..(3)$

and $13m+7n=33...(4)$

Adding equations (3) and (4), we get

$20\text{ m}+20\text{n}=60$

$\therefore \text{m}+\text{n}=3...(5)$

Subtracting equation (3) from equation (4), we get

$6m-6n=6$

$\therefore \text{m}-\text{n}=1...(6)$

Adding equations (5) and (6), we get

$2\text{ m}=4$

$\therefore \text{m}=2$

Substituting $m=2$ in equation (5), we get

$2+\text{n}=3$

$\therefore \text{n}=1$

Resubstituting the values of m and n, we get

$\frac{1}{2x+1}=2$ and $\frac{1}{y+2}=1$

$\therefore 4\text{x}+2=1$ and $\text{y}+2=1$

$\therefore 4x=-1$ and $y=1-2$

$\therefore \text{x}=-\frac{1}{4}$ and $\text{y}=-1$

iii. In a certain race, there are three boys A, B, C. The winning probability of A is twice than B and the winning probability of B is twice than C. If $P(A)+P(B)+P(C)=1$, then find the probability of win for each boy.

Solution:

Given: $P(A)=2P(B)$ and $P(B)=2P(C)$

$\therefore P(A)=2[2P(C)]=4P(C)$

Now, $P(A)+P(B)+P(C)=1$

$\therefore 4\mathrm{P(C)}+2\mathrm{P(C)}+\mathrm{P(C)}=1$

$\therefore 7P(C)=1$

$\therefore \mathrm{P(C)}=\dfrac{1}{7}$

$\therefore P(A)=4P(C)=4\times\frac{1}{7}=\frac{4}{7}$ and $P(B)=2P(C)=2\times\frac{1}{7}=\frac{2}{7}$

Q5. Attempt any two of the following sub questions:

i. The divisor and quotient of the number 6123 are same and the remainder is half the divisor. Find the divisor.

Solution:

Dividend $=6123$

Now, divisor and quotient are same

Let divisor = quotient $=$ d

Now, remainder $=\frac{\text{divisor}}{2}=\frac{d}{2}$

Since dividend $=$ divisor $\times$ quotient + remainder, we have

$6123=d^2+\dfrac{d}{2}$

$\therefore 12246=2\,\mathrm{d}^2+\mathrm{d}$

$\therefore 2d^2+d-12246=0$

Comparing with ax^2+bx+c, we get

$a=2, b=1, c=-12246$

$\therefore d=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-12246)}}{4}$$

$$= \frac{-1 \pm \sqrt{97969}}{4}$$

$$= \frac{-1 \pm 313}{4}$$

$\therefore \mathrm{d} = \frac{-1+313}{4}$ or $\mathrm{d} = \frac{-1-313}{4}$

$\therefore \mathrm{d} = 78$ or $\mathrm{d} = -78.5$

Ignoring the negative value, the divisor is 78.

ii. Find the sum of all numbers from 50 to 350 which are divisible by 6 . Hence find the 15^{th} term of that A.P.

Solution:

The numbers from 50 to 150 which are divisible by 6 are 54,60,66, … … ,348.

$\therefore$ First term $= a = t_1 = 54, d = 6$ and $t_n = 348$

$\mathrm{t_n} = \mathrm{a} + (\mathrm{n} - 1)\mathrm{d}$

$\therefore 348 = 54 + (n - 1)6$

$\therefore 294 = (n - 1)6$

$\therefore 49 = n - 1$

$\therefore \mathrm{n} = 50$

$$\mathrm{S_n} = \frac{\mathrm{n}}{2}(\mathrm{t_1} + \mathrm{t_n})$$

$$\therefore \mathrm{S_{50}} = \frac{50}{2}(54 + 348)$$

$= 25 \times 402$

$= 10050$

$\mathrm{t_{15}} = 54 + 14(6) = 54 + 84 = 138$

Thus, the sum of all numbers from 50 to 350 , which are divisible by 6 , is 10050 and the 15^{th} term of this A.P. is 138.

iii. A three digit number is equal to 17 times the sum of its digits. If 198 is added to the number, the digits are interchanged. The addition of first and third digit is 1 less than middle digit. Find the number.

Solution:

Let the three-digit number be xyz.

Its numerical value $= 100x + 10y + z$

According to first information provided in the question,

$100x + 10y + z = 17(x + y + z)$

$\therefore 100x + 10y + z = 17x + 17y + 17z$

$\therefore 83x - 7y - 16z = 0$

Number obtained by reversing the digits: zyx

Its numerical value $= 100z + 10y + x$

According to second information provided in the question,

$(100x + 10y + z) + 198 = 100z + 10y + x$

$\therefore 99z - 99x = 198$

$\therefore z - x = 2$

$\therefore z = x + 2$

According to third information provided in the question,

$x + z = y - 1$

$\therefore \mathrm{x + x + 2 = y - 1}$ ….[from (2)]

$\therefore y = 2x + 3$

Substituting the values of z and y in equation (1),

$83x - 7(2x + 3) - 16(x + 2) = 0$

$\therefore 83x - 14x - 21 - 16x - 32 = 0$

$\therefore 53x - 53 = 0$

$\therefore 53x = 53$

$\therefore x = 1$

$\therefore y = 2x + 3 = 2(1) + 3 = 2 + 3 = 5$

$\therefore z = x + 2 = 1 + 2 = 3$

Thus, the three-digit number is 153 .

PART – II(GEOMETRY)

Q1. Solve any five sub-questions:

(i) $\triangle$ DEF $\sim \Delta$MNK. If DE $= 2$, MN $= 5$, then find the value of $\left(\frac{A(\triangle DEF)}{A(\triangle MNK)}\right)$.

Solution:

Given,

$\triangle$ DEF $\sim \triangle$ MNK

DE $= 2$, MN $= 5$

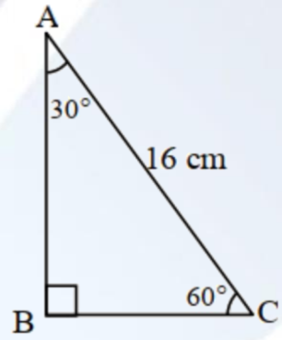
We know that the ratio of areas of similar triangles is equal to squares of the ratio of their corresponding sides.

$$\frac{A(\triangle DEF)}{A(\triangle MNK)} = \frac{DE^2}{MN^2}$$

$$= \frac{(2)^2}{(5)^2}$$

$$= \left(\frac{4}{25}\right)$$

(ii) In the following figure, in $\triangle$ ABC, $\angle B = 90^\circ, \angle C = 60^\circ, \angle A = 30^\circ$, AC $= 16$ cm. Find BC.



Solution:

Given,

$\triangle$ ABC is a $30^\circ - 60^\circ - 90^\circ$ triangle.

Side opposite to $30^\circ =$ BC

BC $= \left(\frac{1}{2}\right) \times$ Hypotenuse

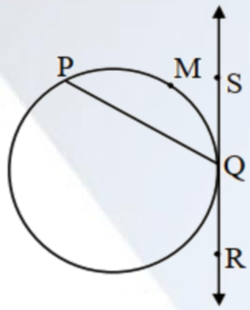
$= \left(\frac{1}{2}\right) \times AC$

$= \left(\frac{1}{2}\right) \times 16$

$= 8$ cm

Therefore, $BC = 8$ cm

(iii) In the following figure, $m(arcPMQ) = 110^\circ$, find $\angle PQS$.



Solution:

Given,

$m(arcPMQ) = 110^\circ$

We know that the measure of the angle formed by the intersection of a chord and tangent of the circle equal to the half the angle made by the arc with the chord.

$\angle PQS = \left(\frac{1}{2}\right) \times m(arcPMQ)$

$= \left(\frac{1}{2}\right) \times 110^\circ$

$= 55^\circ$

(iv) If the angle $\theta = -30^\circ$, find the value of $\cos\ \theta$.

Solution:

Given,

$\theta = -30^\circ$

We know that, $\cos(-\theta) = \cos\theta$

$\cos\theta = \cos(-30^\circ)$

$= \cos 30°$

$= \frac{\sqrt{3}}{2}$

(v) Find the slope of the line with inclination $60°$.

Solution:

Given,

Inclination of line $= \theta = 60°$

The slope of the line $= \tan\ \theta$

$= \tan\ 60°$

$= \sqrt{3}$

(vi) Using Euler's formula, find V if $E = 10, F = 6$.

Solution:

Given,

$E = 10, F = 6$

Using Euler's formula,

$F + V = E + 2$

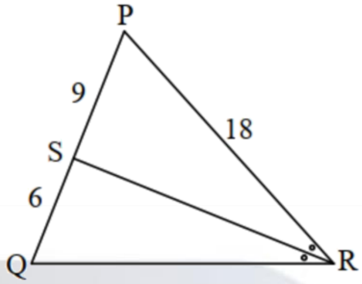
$6 + V = 10 + 2$

$V = 12 - 6$

$V = 6$

Q2. Solve any four sub-questions:

(i) In the following figure, in $\triangle$ PQR, seg RS is the bisector of $\angle PRQ$. If $PS = 9, SQ = 6, PR = 18$, find QR.



Solution:

Given that, in triangle PQR, seg RS is the bisector of ∠PRQ.

$PS = 9, SQ = 6, PR = 18$

By the angle bisector property,

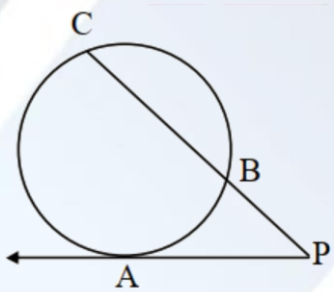
$$\frac{PR}{QR} = \frac{PS}{SQ}$$

$$\frac{18}{QR} = \frac{9}{6}$$

$$\Rightarrow QR = \frac{(18 \times 6)}{9}$$

$$\Rightarrow QR = 12$$

(ii) In the following figure, a tangent segment PA touching a circle in A and a secant PBC are shown. If $AP = 12$, $BP = 9$, find BC.



Solution:

Given,

$PA =$ tangent

$PBC =$ secant

By the tangent - secant theorem,

$PB \times PC = PA^2$

$9 \times \mathrm{PC} = (12)^2$ [given $\mathrm{AP} = 12, \mathrm{BP} = 9$]

$\mathrm{PC} = (\frac{144}{9})$

$\mathrm{PC} = 16$

Now,

$\mathrm{PC} = \mathrm{PB} + \mathrm{BC}$

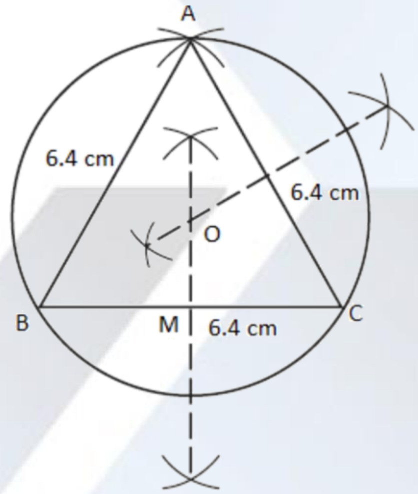
$16 = 9 + \mathrm{BC}$

$\Rightarrow \mathrm{BC} = 16 - 9$

$\Rightarrow \mathrm{BC} = 7$

(iii) Draw an equilateral $\triangle$ ABC with side 6.4 cm and construct its circumcircle.

Solution:



(iv) For the angle in standard position if the initial arm rotates 130° in an anticlockwise direction, then state the quadrant in which the terminal arm lies. (Draw the figure and write the answer.)

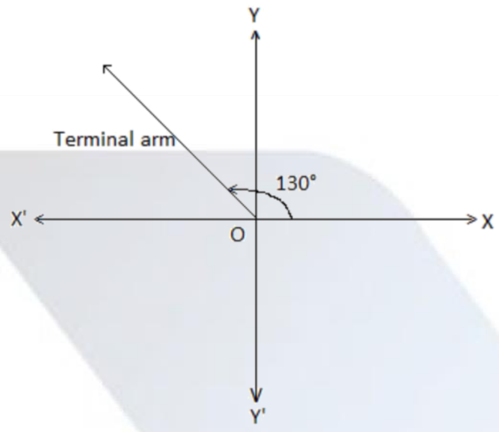
Solution:

Given,

The initial arm rotates 130° in the anticlockwise direction from the standard position.

The measure of angle 130° lies between 90° and 180°.

Hence, the terminal arm lies in quadrant II.



(v) Find the area of the sector whose arc length and radius are 16 cm and 9 cm respectively.

Solution:

Given,

Length of the arc = 16 cm

Radius = r = 9 cm

Area of sector = $\left(\frac{r}{2}\right) \times$ Length of arc

$= \left(\frac{9}{2}\right) \times 16$

$= 9 \times 8$

$= 72 \text{ cm}^2$

(vi) Find the surface area of a sphere of radius 1.4 cm. $\left(\pi = \frac{22}{7}\right)$

Solution:

Given,

Radius of the sphere = r = 1.4 cm

The surface area of the sphere = $4\pi r^2$

$= 4 \times (\frac{22}{7}) \times 1.4 \times 1.4$

$= 24.64 \text{ cm}^2$

Therefore, the surface area of the sphere is 24.64 cm^2.

Q3. Solve any three sub-questions:

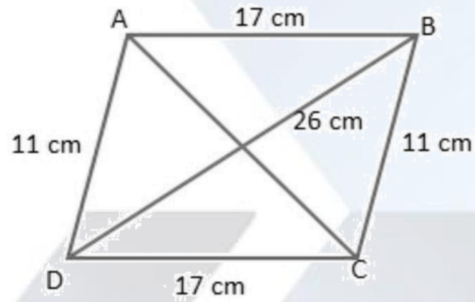
(i) Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm, find the length of the other.

Solution:

Given,

Adjacent sides of a parallelogram are 11 cm and 17 cm.

The length of one diagonal is 26 cm.



$AB = CD = 17 \text{ cm}$

$BC = AD = 11 \text{ cm}$

$BD = 26 \text{ cm}$

We know that,

Sum of squares of sides of a parallelogram = Sum of squares of its diagonals

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

$(17)^2 + (11)^2 + (17)^2 + (11)^2 = AC^2 + (26)^2$

$289 + 121 + 289 + 121 = AC^2 + 676$

$AC^2 = 820 - 676$

$AC^2 = 144$

$AC = 12 \text{ cm}$

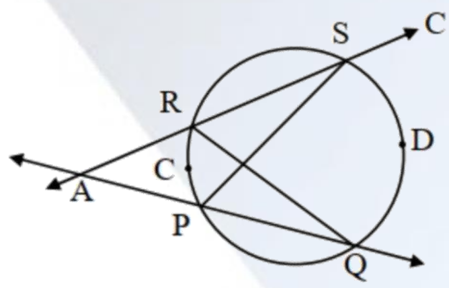
Therefore, the length of the other diagonal is 12 cm.

(ii) In the following figure, secants containing chords RS and PQ of a circle intersect each other in point A in the exterior of a circle. If $m(\text{arcPCR}) = 26°$, $m(\text{arcQDS}) = 48°$, then find:

a. $m\angle PQR$

b. $m\angle SPQ$

c. $m\angle RAQ$



Solution:

Given,

$m(\text{arcPCR}) = 26°$

$m(\text{arcQDS}) = 48°$

By the inscribed angle theorem,

a. $\angle PQR = \frac{1}{2} \times m(\text{arcPCR})$

$= \left(\frac{1}{2}\right) \times 26°$

$= 13°$

$\angle PQR = \angle AQR = 13°. \dots (i)$

b. $\angle SPQ = \frac{1}{2} \times m(\text{arcQDS})$

$= \left(\frac{1}{2}\right) \times 48°$

$= 24°$

$\angle SPQ = 24°$

c. In triangle AQR,

$\angle RAQ + \angle AQR = \angle SRQ$ (remote interior angle theorem)

$\angle SRQ = \angle SPQ$ (angles subtended by the same arc)

Therefore,

$\angle RAQ + \angle AQR = \angle SPQ$

$m\angle RAQ = m\angle SPQ - m\angle AQR$

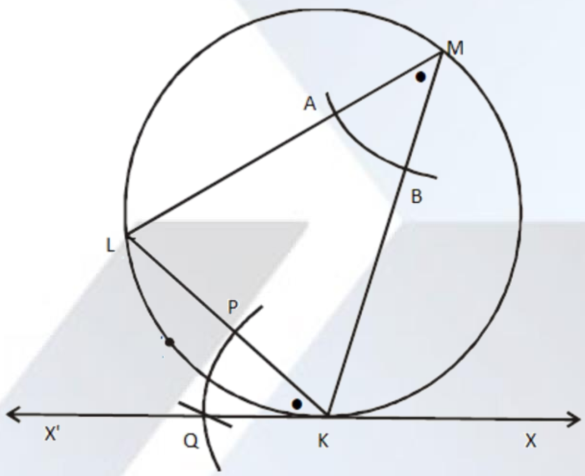
$= 24^\circ - 13^\circ$ [From (i) and (ii)]

$= 11^\circ$

$m\angle RAQ = 11^\circ$

(iii) Draw a circle of radius 3.5 cm. Take any point K on it. Draw a tangent to the circle at K without using centre of the circle.

Solution:



Therefore, XKX' is the required tangent to the circle with a radius of 3.5 cm.

(iv) If $\sec\ \alpha = \frac{2}{\sqrt{3}}$, then find the value of $\frac{(1-\operatorname{cosec}\alpha)}{(1+\operatorname{cosec}\alpha)}$, where α is in IV quadrant.

Solution:

Given,

$\sec\alpha = \left(\frac{2}{\sqrt{3}}\right)$

Thus, $\sec\alpha = \left(\frac{r}{x}\right) = \frac{2}{\sqrt{3}}$

Let $r = 2k$ and $x = \sqrt{3}k$

We know that,

$r^2 = x^2 + y^2$

$(2k)^2 = (\sqrt{3}k)^2 + y^2$

$y^2 = 4k^2 - 3k^2$

$y^2 = k^2$

$y = \pm k$

Given that α lies in quadrant IV.

Therefore, $y = -k$

$\text{cosec}\alpha = \left(\frac{r}{y}\right) = \left(\frac{2k}{-k}\right) = -2$

Now,

$\frac{1 - \text{cosec}\alpha}{1 + \text{cosec}\alpha}$

$= \frac{[1 - (-2)]}{[1 + (-2)]}$

$= \frac{1 + 2}{1 - 2}$

$= -3$

(v) Write the equation of the line passing through the pair of points (2,3) and (4,7) in the form of $y = mx + c$.

Solution:

Let the given points be:

$A(2,3) = (x_1, y_1)$

$B(4,7) = (x_2, y_2)$

Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is:

$\frac{(x - x_1)}{(x_2 - x_1)} = \frac{(y - y_1)}{(y_2 - y_1)}$

$\frac{x - 2}{4 - 2} = \frac{y - 3}{7 - 3}$

$\frac{x - 2}{2} = \frac{y - 3}{4}$

$4(x - 2) = 2(y - 3)$

$4x - 8 = 2y - 6$

$4x - 8 - 2y + 6 = 0$

$4x - 2y - 2 = 0$

$2(2x - y - 1) = 0$

$2x - y - 1 = 0$

$y = 2x - 1$

This is of the form $y = mx + c$ $[m = 2, c = -1]$

Hence, the required equation of line is $y = 2x - 1$.

Q4. Solve any two sub-questions:

(i) Prove that "The length of the two tangent segments to a circle drawn from an external point are equal".

Solution:

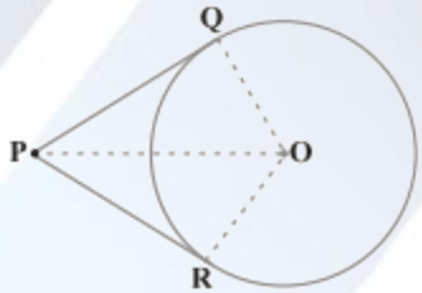
Given,

PQ and PR are the tangents to the circle with centre O from an external point P.

To prove: $PQ = PR$

Construction:

Join OQ, OR and OP.



Proof:

We know that the radius is perpendicular to the tangent through the point of contact.

$\angle OQP = \angle ORP = 90^\circ$

In right $\triangle$ OQP and ORP,

$OQ = OR$ (radii of the same circle)

$OP = OP$ (common)

By RHS congruence criterion,

$\triangle OQP \cong \triangle ORP$

By CPCT,

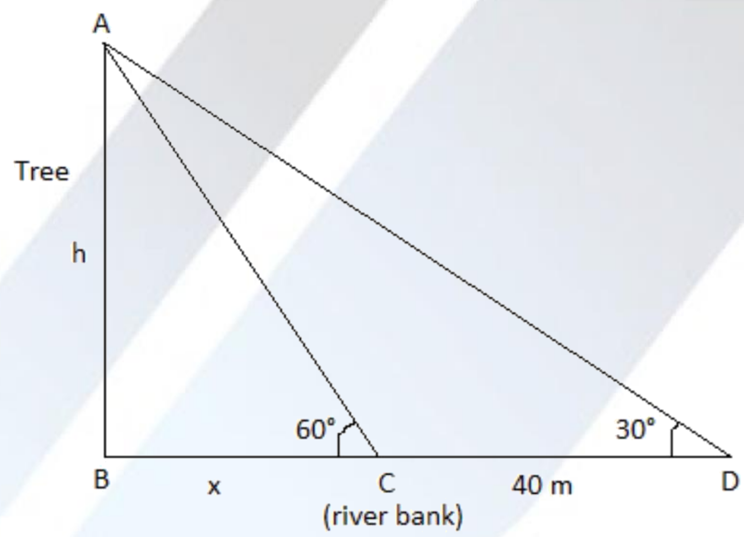
$PQ = PR$

Hence proved.

(ii) A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river. ($\sqrt{3} = 1.73$)

Solution:

Let AB be the tree and BC be the width of the river.



$CD = 40$ m

$BC = x$ and $AB = h$

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}x$ …… (i)

In right triangle ABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$

$x + 40 = h\sqrt{3}$

$x + 40 = (\sqrt{3x}) \times \sqrt{3}$[From (i)]

$x + 40 = 3x$

$2x = 40$

$x = 20$ m

Substituting $x = 20$ in (i),

$h = 20\sqrt{3}$

$= 20 \times 1.73$

$= 34.6$ m

Therefore, the height of the tree is 34.6 m, and the width of the river is 20 m.

(iii) $A(5,4), B(-3,-2)$ and $C(1,-8)$ are the vertices of a triangle ABC. Find the equations of median AD and the line parallel to AC passing through point B.

Solution:

Given,

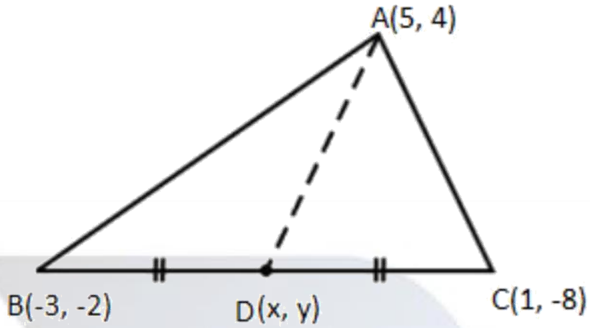
Vertices of a triangle ABC are $A(5,4), B(-3,-2)$ and $C(1,-8)$.

$A\ (5,4) = (x_1, y_1)$

$B(-3,-2) = (x_2, y_2)$

$C(1,-8) = (x_3, y_3)$

Let $D(x, y)$ be the midpoint of BC .



D is the midpoint of BC.

$D(x, y) = \left[\frac{(x_2 + x_3)}{2}, \frac{(y_2 + y_3)}{2}\right]$

$= \left[\frac{-3 + 1}{2}, \frac{-2 - 8}{2}\right]$

$= \left(-\frac{2}{2}, -\frac{10}{2}\right)$

$= (-1, -5)$

$D(-1, -5) = (x_4, y_4)$

Equation of median AD is

$\frac{(x - x_1)}{(x_4 - x_1)} = \frac{(y - y_1)}{(y_4 - y_1)}$

$\frac{x - 5}{-1 - 5} = \frac{y - 4}{-5 - 4}$

$\frac{x - 5}{-6} = (y - 4)(-9)$

$-9(x - 5) = -6(y - 4)$

$-9x + 45 = -6y + 24$

$9x - 45 - 6y + 24 = 0$

$9x - 6y - 21 = 0$

$3(3x - 2y - 7) = 0$

$3x - 2y - 7 = 0$

Hence, the required equation of median AD is $3x - 2y - 7 = 0$.

We know that the line parallel to AC = Slope of AC

Slope of AB $= \frac{(y_3 - y_1)}{(x_3 - x_1)}$

$$= \frac{-8-4}{1-5}$$

$$= -\frac{12}{-4}$$

$$= 3$$

Thus, $m = 3$

Equation of the line parallel to AC and passing through the point $B(-3, -2)$ is

$y - y_2 = m(x - x_2)$

$y - (-2) = 3[x - (-3)]$

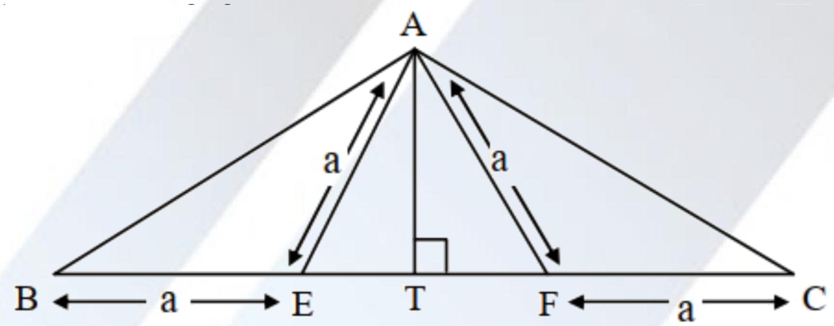
$y + 2 = 3x + 9$

$3x + 9 - y - 2 = 0$

$3x - y + 7 = 0$

Q5. Solve any two sub-questions:

(i) In the following figure, $AE = EF = AF = BE = CF = a$, $AT \perp BC$. Show that $AB = AC = \sqrt{3} \times a$



Solution:

Given,

$AE = EF = AF = BE = CF$

$AT \perp EF$

$\triangle$ AEF is an equilateral triangle.

$ET = TF = \frac{a}{2}$

$BT = CT = a + \left(\frac{a}{2}\right)$ ..... (i)

In right triangle ATB and ATC, AT = AT (common)

$\angle$ATB = $\angle$ATC (right angles)

BT = CT [From (i)]

By SAS congruence criterion,

$\triangle$ ATB $\cong \triangle$ ATC

BY CPCT,

AB = AC

In triangle AEF,

AE = AF = EF (given)

$\triangle$ AEF is an equilateral triangle.

Altitude of an equilateral triangle = AT = $\left(\frac{\sqrt{3}}{2}\right)$ a

In right triangle ATB,

$AB^2 = AT^2 + BT^2$

$= [(\frac{\sqrt{3}}{2})a]^2 + [a + (\frac{a}{2})]^2$

$= \left(\frac{3a^2}{4}\right) + (\frac{3a}{2})^2$

$= \left(\frac{3a^2}{4}\right) + \left(\frac{9a^2}{4}\right)$

$= \frac{12a^2}{4}$

$= 3a^2$

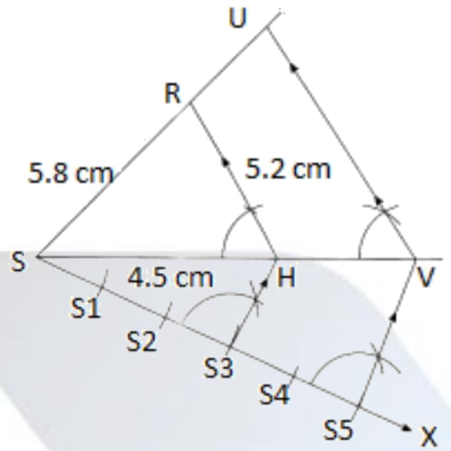
$AB = \sqrt{3}a$

Therefore, $AB = AC = \sqrt{3} \times a$

(ii) ΔSHR ~ Δ SVU. In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$.

Construct $\triangle$ SVU.

Solution:



(iii) Water flows at the rate of 15 m per minute through a cylindrical pipe, having a diameter 20 mm. How much time will it take to fill a conical vessel of base diameter 40 cm and depth 45 cm?

Solution:

Given,

Diameter of cylindrical pipe = 20 mm

Radius of the cylindrical pipe $= r = \left(\frac{20}{2}\right) = 10 \text{ mm} = 1 \text{ cm}$

Speed of water $= h = 15$ m per minute $= 1500 \ \frac{\text{cm}}{\text{min}}$

Volume of cylindrical pipe = Volume of water flowing per minute

$= \pi r^2 h$

$= \left(\frac{22}{7}\right) \times 1 \times 1 \times 1500$

$= \frac{33000}{7} \text{ cm}^3$

Also,

Diameter of conical vessel = 40 cm

Radius of conical vessel $= R = \left(\frac{40}{2}\right) = 20$ cm

Depth $= H = 45$

Capacity of the conical vessel = Volume of cone

$= \left(\frac{1}{3}\right) \pi R^2 H$

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 20 \times 20 \times 45$$

$$= \frac{396000}{21}\ \text{cm}^3$$

Time required to fill the vessel $= \frac{\text{Volume of conical vessel}}{\text{Volume of water flowing per minute}} = \frac{\frac{396000}{21}}{\frac{33000}{7}}$

$$= \frac{396000 \times 7}{21 \times 33000}$$

$$= 4$$

Hence, the time required to fill the conical vessel is 4 min.