GRADE10 MATHS MAHARASHTRA 2017

PART-I (ALGEBRA)

Note: -

(1) All questions are compulsory.

(2) Use of calculator is not allowed.

Q1. Attempt any five of the following sub questions:

(i) State whether the following sequence is an Arithmetic Progression or not:

3,6,12,24, … . ..

Solution:

3,6,12,24....

Let, ' a ' be the first term of the given sequence and 'd' be the common difference.

Also, t_2, t_3, t_4 be the 2^{nd}, 3^{rd}, 4^{th} terms respectively.

Consider,

$t_2 - a = 6 - 3$

$= 3$

and

$t_3 - t_2 = 12 - 6$

$= 6$

Here, we can see that difference between two successive terms is not constant.

Hence, it is not an Arithmetic Progression.

(ii) If one root of the quadratic equation is $3 - 2\sqrt{5}$, then write another root of the equation.

Solution:

One root of the quadratic equation is given to be $3 - 2\sqrt{5}$.

The other root will be the conjugate of $3 - 2\sqrt{5}$.

Conjugate of $3 - 2\sqrt{5} = 3 + 2\sqrt{5}$

(iii) There are 15 tickets bearing the numbers from 1 to 15 in a bag and one ticket is drawn from this bag at random. Write the sample space (S) and n(S).

Solution:

Given that there are 15 tickets bearing the numbers from 1 to 15 in a bag.

Hence, sample space can be written as:

$S = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

$n(S) = 15$

(iv) Find the class mark of the class $35 - 39$.

Solution:

Class mark for the given class 35-39 is :

$$\frac{35 + 39}{2} = \frac{74}{2}$$
$$= 37$$

(v) Write the next two terms of the A.P. whose first term is 3 and the common difference is 4.

Solution:

The first term, $a = 3$

common difference, $\mathrm{d} = 4$

So, the next two terms would be $\mathrm{a} + \mathrm{d}, \mathrm{a} + 2\mathrm{~d}$.

That is, the next two terms are 7 and 11 .

(vi) Find the values of a, b, c for the quadratic equation $2x^2 = x + 3$ by comparing with standard form $a^2 + bx + c = 0$

Solution:

Given quadratic equation is $2\mathrm{x}^2 = \mathrm{x} + 3$.

Writing this equation in standard form, we get

$2x^2 - x - 3 = 0$

Comparing with $\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$, we get

$\mathrm{a} = 2, \mathrm{b} = -1, \mathrm{c} = -3$

Q2. Attempt any four of the following sub questions :

(i) Find the first two terms of the sequence for which S_n is given below :

$S_n = n^2(n+2)$

Solution:

To find the first term, substitute $n = 1$ in $S_n = n^2(n+2)$

$\Rightarrow S_1 = 1^2(1+2) = 1(3) = 3$

Now S_1 is the sum of the first term itself, which is also the first term.

So, the first term $= 3$

To find the second term, substitute $n = 2$ in $S_n = n^2(n+2)$

$\Rightarrow S_2 = 2^2(2+2) = 4(4) = 16$

Now S_2 is the sum of the first two terms.

$\Rightarrow$ the first term + the second term $= 16$

$\Rightarrow 3 +$ the second term $= 16$

$\Rightarrow$ the second term $= 13$

So, the first term = 3 and the second term $= 13$.

(ii) Find the value of discriminant (Δ) for the quadratic equation :

$x^2 + 5x + 1 = 0$

Solution:

Given quadratic equation is $\mathrm{x}^2 + 5\mathrm{x} + 1 = 0$.

$a = 1, b = 5, c = 1$

The discriminant $(\Delta) = b^2 - 4\mathrm{ac} = 5^2 - 4(1)(1) = 25 - 4 = 21$

(iii) Write the equation of X-axis. Hence find the point of intersection of the graph of the equation $x + y = 3$ with the X-axis.

Solution:

The equation of the X -axis is $\mathrm{y} = 0$.

To find the point of intersection of the equation $x + y = 3$ with the X - axis, substitute $\mathrm{y} = 0$ in $\mathrm{x} + \mathrm{y} = 3$.

$\Rightarrow x + 0 = 3$

$\Rightarrow x = 3$

So, the point of intersection will be (3,0).

(iv) For a certain frequency distribution the values of Assumed mean $(A) = 1200$, $\sum f_i d_i = 700$ and $\sum f_i$. Find the value of mean $\overline{(X)}$.

Solution:

We have,

$A = 1200, \sum f_i d_i = 700$ and $\sum f_i = 100 = N$

$$\overline{X} = A + \frac{1}{N} \sum_{i=1}^{n} f_i d_i$$

$$\Rightarrow \overline{X} = 1200 + \frac{1}{100}(700)$$

$$\Rightarrow \overline{X} = 1200 + 7$$

$$\Rightarrow \overline{X} = 1207$$

(v) Two coins are tossed simultaneously. Write the sample space (S). n(S), the following event A using set notation and $n(A)$, Where ' A ' is the event of getting at the most one tail.'

Solution:

Since two coins are tossed simultaneously,

$S = \{HT, TH, HH, TT\}$

$\Rightarrow n(S) = 4$

$A = \{HT, TH, HH\}$

$\Rightarrow n(A) = 3$

(vi) Find the value of k for which the given simultaneous equation have infinity many solution :

$kx + 2y = 6$

$9x + 6y = 18$

Solution:

The given equations are

$kx + 2y = 6..(i)$

and $9x + 6y = 18$

$\Rightarrow 3(3x + 2y) = 18$

$\Rightarrow 3x + 2y = 6..(ii)$

From (i) and (ii), we get $k = 3$

Q3. Attempt any three of the following sub questions :

(i) How many three digit natural numbers are divisible by 2?

Solution:

The first three digit natural number divisible by 2 is 100.

Common difference $d = 2$

Last three digit natural number divisible by 2 is 998

We know that,

$t_n = a + (n - 1)d$

$\Rightarrow 998 = 100 + (n - 1)2$

$\Rightarrow 898 = 2(n - 1)$

$\Rightarrow 449 = n - 1$

$\Rightarrow n = 450$

Hence, there are 450 three digit natural numbers divisible by 2 .

(ii) Solve the following quadratic equation by factorization method:

$3x^2 - 22x + 40 = 0$

Solution:

$3x^2 - 22x + 40 = 0$

$\Rightarrow 3x^2 - 12x - 10x + 40 = 0$
$\Rightarrow 3x(x - 12) - 10(x - 12) = 0$
$\Rightarrow (x - 12)(3x - 10) = 0$
$\Rightarrow x - 12 = 0 \text{ or } 3x - 10 = 0$
$\Rightarrow x = 12 \text{ or } x = \frac{10}{3}$

(iii) Solve the following simultaneous equation by using Cramer's rule:

$x + 2y = 4$

$3x + 4y = 6$

Solution:

Consider,

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \neq 0$$

$$x = \frac{\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{16 - 12}{-2} = -2$$

$$y = \frac{\begin{vmatrix} 1 & 4 \\ 3 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{6 - 12}{-2} = 3$$

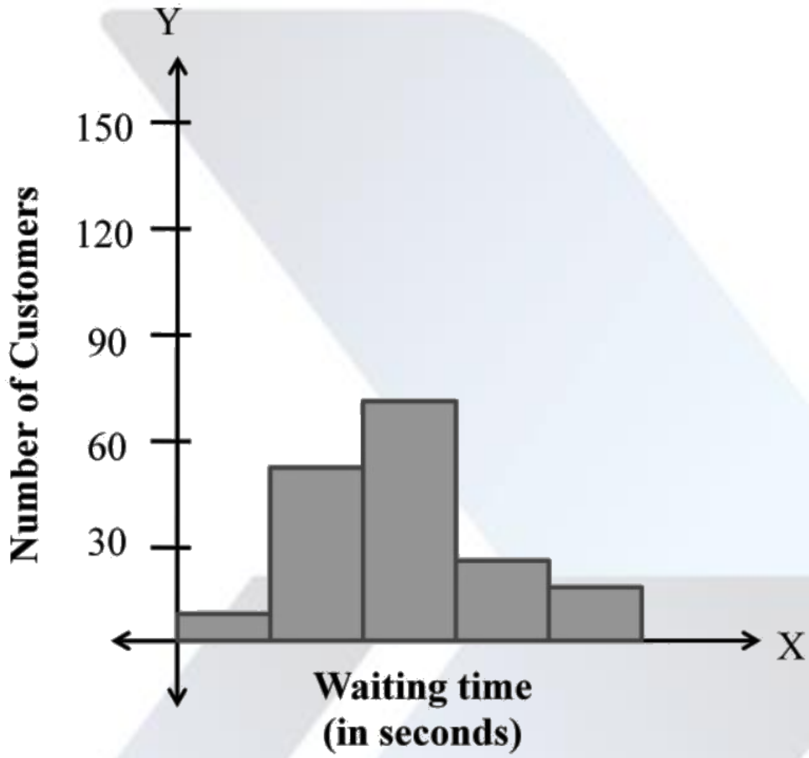
So, $x = -2$ and $y = 3$

(iv) The following is the frequency distribution of waiting time at ATM centre; draw histogram to represent the data :

Waiting time (In seconds)	Number of Customers
$0 - 30$	10
$30 - 60$	54
$60 - 90$	68
$90 - 120$	28

120 – 150	20

Solution:



Q4. Attempt any two of the following sub questions :

(i) Three horses A, B and C are in a race, A is twice as likely to win as B and B is twice as likely to win as C. What are their probabilities of winning?

Solution:

$\Pr(A)=2\Pr(B)$, and $\Pr(B)=3\Pr(C)$

Hence, $\Pr(A)=6\Pr(C)$

But $\Pr(A)+\Pr(B)+\Pr(C)=1$

Consequently, $6\Pr(C)+3\Pr(C)+\Pr(C)=1$

So, $\Pr(C)=\frac{1}{10}$;

Since $\Pr(B)=3\Pr(C)$

$$\Rightarrow \Pr(B)=\frac{3}{10}$$

and since $\Pr(A) = 6\Pr(C)$

$$\Rightarrow \Pr(\mathrm{A}) = \frac{6}{10}$$

(ii) The following is the distribution of the size of certain farms from a taluka (tehasil) :

Size of Farms (in acres)	Number of Farms
$5-15$	7
$15-25$	12
$25-35$	17
$35-45$	25
$45-55$	31
$55-65$	5
$65-75$	3

Find the median size of farms.

Solution:

Calculation of the Median size of farms.

Size of Farms	f	c.f
$5-15$	7	7
$15-25$	12	19
$25-35$	17	36
$35-45$	25	61

$45-55$	31	92
$55-65$	5	97
$65-75$	3	100

We have $N = 100 \Rightarrow \frac{N}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 61 and the corresponding class is 35-45.

Thus, $35-45$ is the median class such that $l = 35, f = 25, cf = 36, h = 10$.

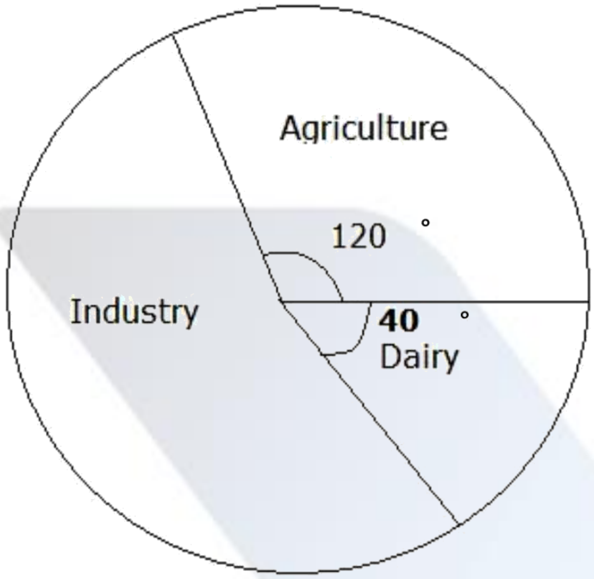
$$\text{Median} = 1 + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 35 + \frac{50-36}{25} \times 10$$

$$= 35 + 5.6$$

$$= 40.6$$

(iii) The following pie diagram represents the sectorwise loan amount in crores of rupees distributed by a bank. From the information answer the following questions:



(a) If the dairy sector receives ʿ20 crores, then find the total loan disbursed.

(b) Find the loan amount for agriculture sector and also for industrial sector.

(c) How much additional amount did industrial sector receive than agriculture sector?

Solution:

We compute the central angle for each crop as shown in the following table.

Sector	Measure of central angle	Amount (in crores)
Agriculture	120°	$\frac{120^\circ}{360^\circ} \times 180 = \text{Rs. } 60$
Dairy	40°	$\frac{40^\circ}{360^\circ} \times 180 = \text{Rs. } 20$
Industry	$360 - (120^\circ + 40^\circ) = 200^\circ$	$\frac{200^\circ}{360^\circ} \times 180 = \text{Rs. } 100$
Total	360°	Rs. 180

(a)

$$\frac{40^\circ}{360^\circ} \times \text{ Total } = 20$$

$$\Rightarrow \text{ Total } = \frac{20 \times 360}{40}$$

$\Rightarrow$ Total $=$ Rs. 180 crores

Hence, the total loan disbursed is Rs. 180 crores.

(b)

Total loan for the agriculaure sector:

$\frac{120^\circ}{360^\circ} \times 180 =$ Rs. 60 crores

(c)

Total loan for industrial sector :

$\frac{\mathbf{200}}{\mathbf{360}} \times \mathbf{180} =$ Rs. 100 crores

The additional amount the industrial sector received than the agriculture sector

= Rs. 100 - Rs. 60 crores

= Rs. 40 crores

Q5. Attempt any two of the following sub questions :

(i) If the cost of bananas in increased by 10 per dozen, one can get 3 dozen less for 600. Find the original cost of one dozen of bananas.

Solution:

Let x be theoriginal cost of a dozen bananas.

For Rs. 600 let us one gets y dozens.

$xy = 600$

$$\Rightarrow y = \frac{600}{x} \ldots (1)$$

$(x + 10)(y - 3) = 600..(2)$

Substituting the y value in (2), we get,

$$(x+10)\left(\frac{600}{x}-3\right)=600$$
$$\Rightarrow (x+10)\left(\frac{600-3x}{x}\right)=600$$
$$\Rightarrow (10+x)(600-3x)=600x$$
$$\Rightarrow 6000+570x-3x^2=600x$$
$$\Rightarrow 6000-30x-3x^2=0$$
$$\Rightarrow 2000-10x-x^2=0$$
$$\Rightarrow x^2+10x-2000=0$$
$$\Rightarrow (x+50)(x-40)=0$$
$$\Rightarrow x=-50 \text{ or } 40$$

Since cost of bananas cannot be negative, $x=40$.

So, the original cost of one dozen of bananas is Rs. 40.

(ii) If the sum of first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first $(p+q)$ terms is zero where $p \neq q$.

Solution:

To show: $S_{p+q}=0$

that is, to show: $\frac{p+q}{2}(2a+(p+q-1)d)=0$

Given that $S_p=S_q$

Let a be the first term of the AP and d be the common difference.

$$\Rightarrow \frac{p}{2}(2a+(p-1)d)=\frac{q}{2}(2a+(q-1)d)$$
$$\Rightarrow p(2a+(p-1)d)=q(2a+(q-1)d)$$
$$\Rightarrow 2ap+(p-1)dp=2aq+(q-1)dq$$
$$\Rightarrow 2ap-2aq+(p-1)dp-(q-1)dq=0$$
$$\Rightarrow 2ap-2aq+(p-1)dp-(q-1)dq=0$$
$$\Rightarrow 2a(p-q)+d[p^2-p-q^2+q]=0$$
$$\Rightarrow 2a(p-q)+d[p^2-q^2-p+q]=0$$
$$\Rightarrow 2a(p-q)+d[(p-q)(p+q)-(p-q)]=0$$
$$\Rightarrow 2a(p-q)+d[(p-q)[(p+q)-1]]=0$$

Dividing throughout by $p-q$, since $\boldsymbol{p \neq q}$.

$$\Rightarrow 2a+((p+q)-1)d=0$$

$$\Rightarrow 2a+(p+q-1)d=0$$

$$S_{p+q}=\frac{p+q}{2}(2a+(p+q-1)d)=\frac{p+q}{2}(0)=0$$

Hence proved.

(iii) Solve the following simultaneous equations:

$$\frac{1}{3x}-\frac{1}{4y}+1=0$$
$$\frac{1}{5x}+\frac{1}{2y}=\frac{4}{15}$$

Solution:

$\frac{1}{3x}-\frac{1}{4y}+1=0$ and $\frac{1}{5x}+\frac{1}{2y}=\frac{4}{15}$

$\frac{1}{3x}-\frac{1}{4y}=-1$ and $\frac{1}{5x}+\frac{1}{2y}=\frac{4}{15}$

Let $\frac{1}{x}=\mathrm{a}$ and $\frac{1}{\mathrm{y}}=\mathrm{b}$

$\Rightarrow \frac{\mathrm{a}}{3}-\frac{\mathrm{b}}{4}=-1$ and $\frac{\mathrm{a}}{5}+\frac{\mathrm{b}}{2}=\frac{4}{15}$

$\Rightarrow 4\mathrm{a}-3\,\mathrm{b}=-12$ and $2\mathrm{a}+5\,\mathrm{b}=\frac{40}{15}$

$\Rightarrow 4\mathrm{a}-3\,\mathrm{b}=-12$ and $2\mathrm{a}+5\,\mathrm{b}=\frac{8}{3}$

$\Rightarrow 4\mathrm{a}-3\,\mathrm{b}=-12$ and $3\mathrm{a}+15\,\mathrm{b}=8$

Solving the two equations, we get $\mathrm{a}=-2$ and $\mathrm{b}=\frac{4}{3}$.

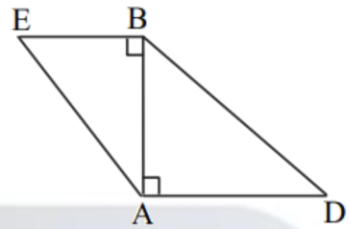
Resubstituting $\frac{1}{x}=\mathrm{a}$ and $\frac{1}{\mathrm{y}}=\mathrm{b}$,

$\Rightarrow \mathrm{x}=-\frac{1}{2}$ and $\mathrm{y}=\frac{3}{4}$

PART – II (GEOMETRY)

Q1. Solve any five sub-questions:

(i) In the following figure, seg BE ⊥ seg AB and seg BA ⊥ seg AD. If BE $=6$ and AD $=9$, find $\frac{\mathrm{A}(\triangle \mathrm{ABE})}{\mathrm{A}(\triangle \mathrm{BAD})}$.



Solution:

Given,

BE = 6 and AD = 9

$$\left(\frac{A(\triangle\ ABE)}{A(\Delta BAD)}\right) = \left(\frac{\left[\left(\frac{1}{2}\right) \times BE \times AB\right]}{\left[\left(\frac{1}{2}\right) \times AB \times AD\right]}\right)$$

$$= \left(\frac{BE}{AD}\right)$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

(ii) If two circles with radii 8 cm and 3 cm respectively touch internally, then find the distance between their centres.

Solution:

Given,

Two circles with radii 8 cm and 3 cm respectively touch internally.

Distance between their centres = Difference of the radii

= 8 - 3

= 5 cm

(iii) Find the height of an equilateral triangle whose side is 6 units.

Solution:

Given,

Side of an equilateral triangle = 6 units

Height (Altitude) of an equilateral triangle = $\left(\frac{\sqrt{3}}{2}\right) \times$ side

$= (\frac{\sqrt{3}}{2}) \times 6$

$= 3\sqrt{3}$ units

(iv) If the angle $\theta = -45°$, find the value of $\tan\ \theta$.

Solution:

Given,

$\theta = -45°$

We know that:

$\tan(-\theta) = -\tan\theta$

Now, $\tan\ \theta = \tan(-45°)$

$= -\tan\ 45°$

$= -1$

(v) Find the slope and y-intercept of the line $y = 3x - 5$.

Solution:

Given,

$y = 3x - 5$

Comparing with the equation of line having slope m and y-intercept c: $y = mx + c$

$m = 3, c = -5$

Therefore, slope $= 3$ and y-intercept $= -5$

(vi) Find the circumference of a circle whose radius is 7 cm.

Solution:

Given,

Radius of the circle $= r = 7$ cm

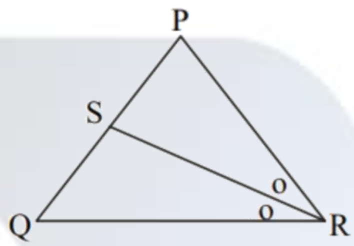
Circumference of the circle $= 2\pi r$

$= 2 \times \left(\frac{22}{7}\right) \times 7$

$= 44$ cm

Q2. Solve any four sub-questions:

(i) In $\triangle$ PQR, seg RS is the bisector of $\angle PRQ$, $PS = 6, SQ = 8, PR = 15$. Find QR.



Solution:

Given,

$PS = 6, SQ = 8, PR = 15$

seg RS is the bisector of $\angle PRQ$.

By the angle bisector property,

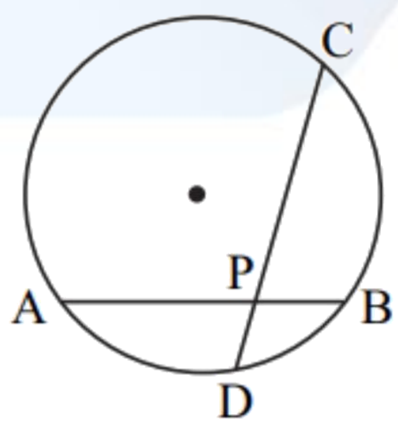
$$\frac{PR}{QR} = \frac{PS}{SQ}$$

$$\frac{15}{QR} = \frac{6}{8}$$

$$\Rightarrow QR = \frac{(15 \times 8)}{6}$$

$$\Rightarrow QR = 20$$

(ii) In the given figure $PA = 6$, $PB = 4$ and $PC = 8$. Find PD.



Solution:

We know that when the two chords of a circle intersect each other inside it then the product of their segments is equal.

Given,

$PA = 6, PB = 4$ and $PC = 8$

AB and CD intersect each other at P.

$PA \times PB = PC \times PD$

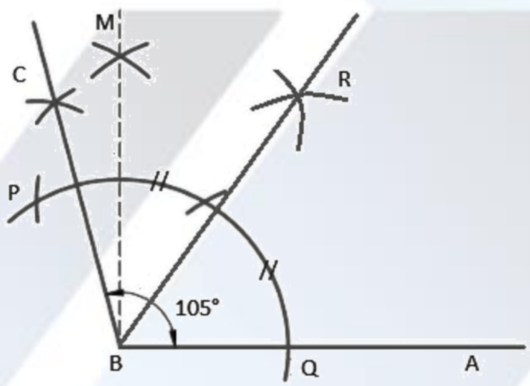
$6 \times 4 = 8 \times PD$

$PD = \frac{(6 \times 4)}{8}$

$PD = 3$

(iii) Draw $\angle ABC$ of measure $105°$ and bisect it.

Solution:



Therefore, $\angle ABC = 105°$ and BR is its bisector.

(iv) Find the sine ratio of θ in a standard position whose terminal arm passes through (4, 3).

Solution:

Given,

Terminal arm passes through the point (3,4)

i.e. $(3,4) = (x, y)$

$r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(3^2 + 4^2)}$

$= \sqrt{(9 + 16)}$

$= \sqrt{25}$

$= 5$

We know that, $y = r\sin\ \theta$

$\sin\ \theta = \frac{y}{r} = \frac{4}{5}$

(v) Find the slope of the line passing through the points $A(6, -2)$ and $B(-3,4)$.

Solution:

Let the given points be:

$A\ (6, -2) = (x_1, y_1)$

$B(-3,4) = (x_2, y_2)$

Slope of the line passing through the points $(\ x_1, y_1\ )$ and (x_2, y_2) is

$\frac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \frac{(4 + 2)}{(-3 - 6)}$

$= \frac{6}{-9}$

$= -\frac{2}{3}$

Therefore, slope is $-\frac{2}{3}$.

(vi) The dimensions of cuboid in cm are $30 \times 18 \times 10$. Find its volume.

Solution:

Given,

Dimensions of the cuboid $= 30\text{ cm} \times 18\text{ cm} \times 10\text{ cm}$

i.e. Length $= 30$ cm

Breadth $= 18$ cm

Height $= 10$ cm

Volume of the cuboid = Length × Breadth × Height

$= 30 \times 18 \times 10$

$= 5400\ \text{cm}^3$

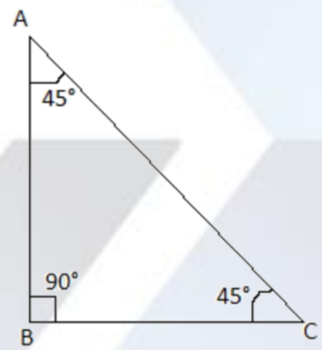
Q3. Solve any three sub-questions:

(i) Prove that "If the angles of a triangle are $45^\circ - 45^\circ - 90^\circ$, then each of the perpendicular sides is $(\frac{1}{\sqrt{2}})$ times the hypotenuse."

Solution:

Given,

ABC is a right triangle in which $\angle B = 90^\circ$ and $\angle A = \angle C = 45^\circ$.



To prove: $AB = BC = (\frac{1}{\sqrt{2}})AC$

Proof:

$\angle A = \angle C = 45^\circ$

$AB = BC$ (sides opposite to equal angles are equal)

In right triangle ABC ,

By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$AB^2 + AB^2 = AC^2$

$2AB^2 = AC^2$

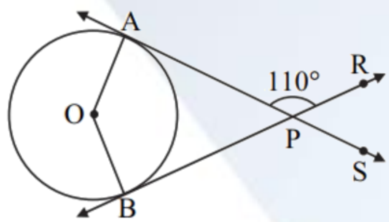
$AB^2 = (\frac{1}{2})AC^2$

$AB = (\frac{1}{\sqrt{2}})AC$

And $AB = BC = (\frac{1}{\sqrt{2}})AC$

Hence proved.

(ii) Find the angle between two radii at the centre of the circle as shown in the figure. Lines PA and PB are tangents to the circle at other ends of the radii and $\angle APR = 110°$.



Solution:

From the given,

$\angle APR + \angle APB = 180°$ (linear pair: BPR is a straight line)

$110° + \angle APB = 180°$

$\angle APB = 180° - 110°$

$\angle APB = 70°$

We know that the radius is perpendicular to the tangent through the point of contact.

$\angle OAP = \angle OBP = 90°$

In quadrilateral OAPB,

$\angle APB + \angle OBP + \angle BOA + \angle OAP = 360°$

$70° + 90° + \angle BOA + 90° = 360°$

$\angle BOA + 250° = 360°$

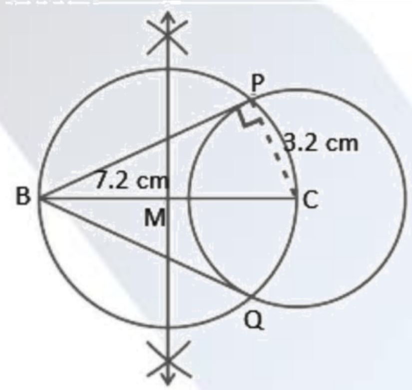
$\angle BOA = 360° - 250°$

$\angle BOA = 110°$

Therefore, the angle between the two radii is 110°.

(iii) Construct tangents to the circle from point B, having a radius of 3.2 cm and centre 'C'. Point B is at a distance of 7.2 cm from the centre.

Solution:

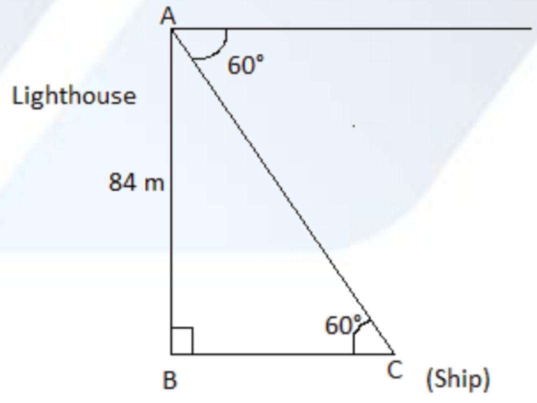


Therefore, BP and BQ are the required tangents to the circle with centre C.

(iv) From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 60°. If the height of the lighthouse is 84 metres, then find how far is that ship from the lighthouse? ($\sqrt{3} = 1.73$)

Solution:

Let AB be the lighthouse and C be the position of the ship.



$AB = 84$ m

In right triangle ABC ,

$$\tan\ 60^{\circ} = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{84}{BC}$$

$$BC = \frac{84}{\sqrt{3}}$$

$$= \left(\frac{84}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$= \frac{(84\sqrt{3})}{3}$$

$$= 28\sqrt{3}$$

$$= 28 \times 1.73$$

$$= 48.44$$

Hence, the distance between the ship and the lighthouse is 48.44 m.

(v) The volume of a cube is 1000 cm^3. Find its total surface area.

Solution:

Given,

Volume of cube $= 1000\ cm^3$

$\Rightarrow (\text{ side })^3 = (10)^3$

$\Rightarrow$ Side $= 10$ cm

Total surface area of cube $= 6 \times (\text{ side })^2$

$= 6 \times (10)^2$

$= 6 \times 100$

$= 600\ cm^2$

Q4. Solve any two sub-questions:

(i) Prove that "The opposite angles of a cyclic quadrilateral are supplementary".

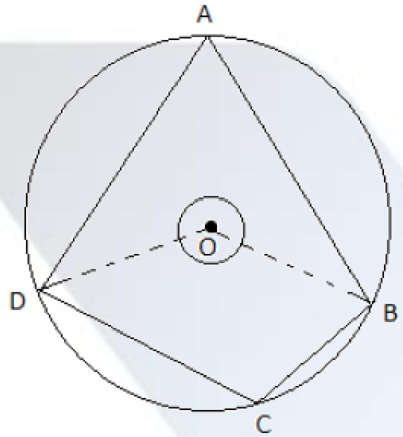
Solution:

Given,

ABCD is a cyclic quadrilateral of a circle with centre O.

Construction: Join OB and OD.

To prove: $\angle BAD + \angle BCD = 180°$



Proof:

We know that the angle subtended by the arc at the centre is twice the angle subtended by it at the remaining part of the circle.

$\angle BOD = 2\angle BAD$... (i)

Also,

reflex $\angle BOD = 2\angle BCD$....(ii)

Adding (i) and (ii),

$2\angle BAD + 2\angle BCD = \angle BOD + \text{reflex } \angle BOD$

$2(\angle BAD + \angle BCD) = 360°$

$\angle BAD + \angle BCD = \dfrac{360°}{2}$

$\angle BAD + \angle BCD = 180°$

Hence proved.

(ii) Eliminate θ, if

$x = 3\operatorname{cosec}\theta + 4\cot\ \theta$

$y = 4\operatorname{cosec}\theta - 3\cot\ \theta$

Solution:

Given,

$x = 3\text{cosec}\theta + 4\cot\ \theta$ ......(i)

$y = 4\text{cosec}\theta - 3\cot\ \theta$ .....(ii)

(i) $\times$ 4 $-$ (ii) $\times$ 3,

$4x - 3y = 12\text{cosec}\theta + 16\cot\ \theta - (12\text{cosec}\theta - 9\cot\ \theta)$

$4x - 3y = 25\cot\ \theta$

$$\cot\ \theta = \frac{(4x - 3y)}{25}$$

Squaring on both sides,

$\cot^2\ \theta = [\frac{(4x-3y)}{25}]^2$ .....(iii)

Now,

(i) $\times$ 3 + (ii) $\times$ 4,

$3x + 4y = 9\text{cosec}\theta + 12\cot\ \theta + 16\text{cosec}\theta - 12\cot\ \theta$

$3x + 4y = 25\text{cosec}\theta$

$$\text{cosec}\theta = \frac{(3x + 4y)}{25}$$

Squaring on both sides,

$\text{cosec}^2\theta = [\frac{(3x+4y)}{25}]^2$ .... (iv)

We know that,

$\text{cosec}^2\theta - \cot^2\ \theta = 1$

$$[\frac{(3x + 4y)}{25}]^2 - [\frac{(4x - 3y)}{25}]^2 = 1$$

$$(\frac{1}{625})[(3x + 4y)^2 - (4x - 3y)^2] = 1$$

$$(3x + 4y)^2 - (4x - 3y)^2 = 625$$

(iii) A toy is a combination of a cylinder, hemisphere and a cone, each with a radius of 10 cm as shown in the figure. The height of the conical part is 10 cm, and the total height is 60 cm. Find the total surface area of the toy. ($\pi = 3.14, \sqrt{2} = 1.41$)

Solution:

Given,

Radius of cylinder = Radius of hemisphere = Radius of cone = r = 10 cm

Height of the conical part = h=10 cm

Total height of the toy = 60 cm

Height of the cylinder = H = 60 − 10 − 10 = 40 cm

Slant height of cone = l = $\sqrt{(r^2 + h^2)}$

$= \sqrt{(10^2 + 10^2)}$

$= \sqrt{(100 + 100)}$

$= \sqrt{200}$

$= 10\sqrt{2}$ cm

Total surface area of the toy = CSA of cone + CSA of cylinder + CSA of hemisphere

$= \pi rl + 2\pi rh + 2\pi r^2$

$= \pi[10 \times 10\sqrt{2} + 2 \times 10 \times 40 + 2 \times 10 \times 10]$

$= 3.14[100\sqrt{2} + 800 + 200]$

$= 3.14\ [100 \times (1.41) + 1000]$

$= 3.14\ [141 + 1000]$

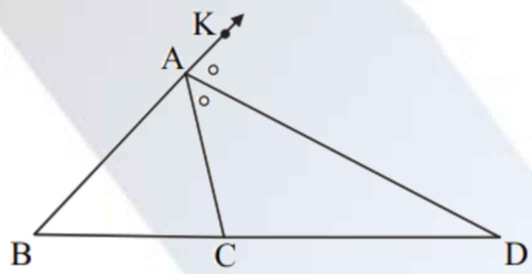
$= 3.14 \times 1141$

$= 3582.74\ cm^2$

Hence, the total surface area of the toy is $3582.74\ cm^2$.

Q5. Solve any two sub-questions:

(i) In the given figure, AD is the bisector of the exterior ∠A of △ ABC. Seg AD intersects the side BC produced in D. Prove that: $\left(\frac{BD}{CD}\right) = \left(\frac{AB}{AC}\right)$



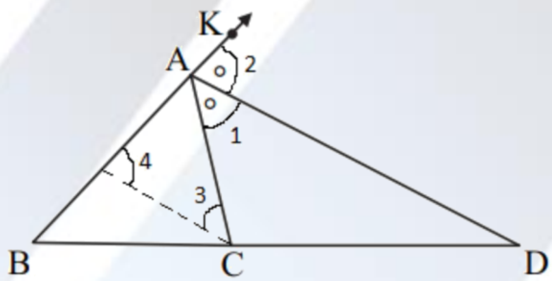
Solution:

Given,

AD is the bisector of the exterior ∠A of △ ABC.

Also, Seg AD intersects the side BC produced in D.

Draw EC || AD.



EC || AD

Thus, AC is the transversal.

$\angle 1 = \angle 3$ (alternate interior angles)

$\angle 2 = \angle 4$ (corresponding angles on the same side of the transversal)

$\angle 1 = \angle 2$ (AD os the angle bisector of ∠A)

Also, $\angle 4 = \angle 3$

Therefore, $AC = AE$ ....(i) (sides opposite to equal angles are equal)

In triangle ABD,

EC || AD

By BPT,

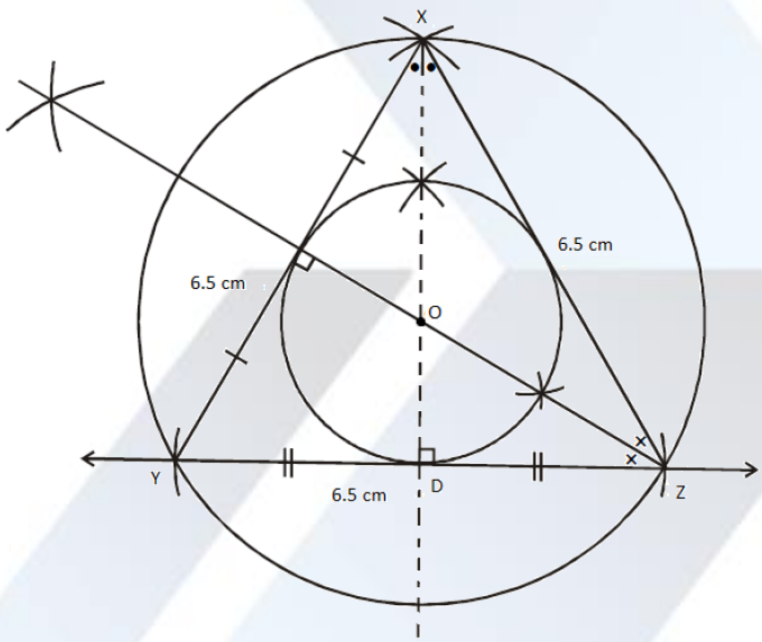
$$\frac{BD}{CD} = \frac{AB}{AE}$$

$\frac{BD}{CD} = \frac{AB}{AC}$ [From (i)]

Hence proved.

(ii) Construct the circumcircle and incircle of an equilateral $\triangle$ XYZ with side 6.5 cm and centre O. Find the ratio of the radii of incircle and circumcircle.

Solution:



Radius of incircle $=$ OD $= 2$ cm

Radius of circumcircle $=$ OX $= 4$ cm

Radius of incircle/ Radius of circumcircle $= \frac{2}{4} = \frac{1}{2}$

Hence, the required ratio is 1: 2.

(iii) $A(5,4), B(-3,-2)$ and $C(1,-8)$ are the vertices of a triangle ABC. Find the equation of median AD and the line parallel to AB passing through point C.

Solution:

Given,

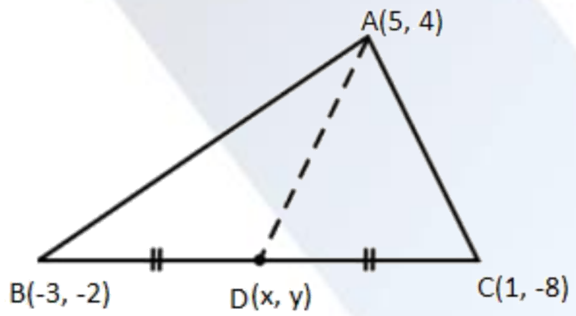
Vertices of a triangle ABC are $A(5,4), B(-3,-2)$ and $C(1,-8)$.

$A(5,4) = (x_1, y_1)$

$B(-3,-2) = (x_2, y_2)$

$C(1,-8) = (x_3, y_3)$

Let $D(x, y)$ be the midpoint of BC .



D is the midpoint of BC .

$D(x, y) = \left[\frac{(x_2 + x_3)}{2}, \frac{(y_2 + y_3)}{2}\right]$

$= [\frac{(-3+1)}{2}, \frac{(-2-8)}{2}]$

$= (\frac{-2}{2}, \frac{-10}{2})$

$= (-1, -5)$

$D(-1,-5) = (x_4, y_4)$

Equation of median AD is

$\frac{(x - x_1)}{(x_4 - x_1)} = \frac{(y - y_1)}{(y_4 - y_1)}$

$\frac{(x-5)}{(-1-5)} = \frac{(y-4)}{(-5-4)}$

$\frac{(x-5)}{(-6)} = \frac{(y-4)}{(-9)}$

$-9(x-5) = -6(y-4)$

$-9x + 45 = -6y + 24$

$9x - 45 - 6y + 24 = 0$

$9x - 6y - 21 = 0$

$3(3x - 2y - 7) = 0$

$3x - 2y - 7 = 0$

Hence, the required equation of median AD is $3x - 2y - 7 = 0$.

We know that the line parallel to AB = Slope of AB

Slope of AB $= \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$$= \frac{(-2-4)}{(-3-5)}$$

$$= (-\frac{6}{-8})$$

$$= \frac{3}{4}$$

Thus, $m = \frac{3}{4}$

Equation of the line parallel to AB and passing through the point $C(1, -8)$ is $y - y_3 = m(x - x_3)$

$$y - (-8) = (\frac{3}{4})(x - 1)$$

$4(y + 8) = 3(x - 1)$

$4y + 32 = 3x - 3$

$3x - 3 - 4y - 32 = 0$

$3x - 4y - 35 = 0$