Grade 10 Mathematics Maharashtra 2018

PAPER-I (ALGEBRA)

Note: -

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

Q1. Attempt any five of the following sub questions:

(i) Find next two terms of an A.P.

$4,9,14, \ldots \ldots$

Solution:

The given sequence is $4,9,14, \ldots$

Here, first term $= \mathrm{a} = 4$

$\Rightarrow t_1 = 4$

$\Rightarrow t_2 = 9$

Now, $\mathrm{t}_2 - \mathrm{t}_1 = 9 - 4 = 5$

$\Rightarrow$ Common difference $= \mathrm{d} = 5$

Hence,

$\mathrm{t}_4 = \mathrm{a} + (4-1)\mathrm{d} = 4 + 3 \times 5 = 4 + 15 = 19$

$\mathrm{t}_5 = \mathrm{a} + (5-1)\mathrm{d} = 4 + 4 \times 5 = 4 + 20 = 24$

Therefore, the next two terms of the given A.P. are 19 and 24 .

(ii) State whether the given equation is quadratic or not. Give reason.

$$\frac{5}{4} m^2 - 7 = 0$$

Solution:

Given equation is $\frac{5}{4} \mathrm{~m}^2 - 7 = 0$

$$\therefore \frac{5 \mathrm{~m}^2 - 28}{4} = 0$$

$\therefore 5 m^2 - 28 = 0$

Here, the maximum index of variable m is 2 .

Comparing with general form of quadratic equation $ax^2 + bx + c$,

we have $a = 5$, $b = 0$ and $c = -28$, which are real numbers and $a \neq 0$.

Hence, it is a quadratic equation in variable m.

(iii) If $D_x = 25, D = 5$ are the values of the determinants for certain simultaneous equations in x and y, find x.

Solution:

. $D_x = 25, D = 5$

$\therefore x = \frac{D_x}{D} = \frac{25}{5} = 5$

(iv) If $S = \{2,4,6,8,10,12\}$ and $A = \{4,8,12\}$. Find A'.

Solution:

$S = \{2,4,6,8,10,12\}$ and $A = \{4,8,12)$

Hence, $A' =$ outcomes in sample space S which are not in $A = \{2,6,10\}$

(v) Write any one solution of equation $x + 2y = 7$.

Solution:

Given equation is $x + 2y = 7$

Substituting $x = 1$ in the given equation, we get

$1 + 2y = 7$

$\therefore 2y = 7 - 1$

$\therefore 2y = 6$

$\therefore y = 3$

Hence, (1,3) is a solution of the given equation.

(vi) If $S_5 = 15$ and $S_6 = 21$ find t_6.

Solution:

$S_5 = 15$ and $S_6 = 21$

$\therefore t_6 = S_6 - S_5 = 21 - 15 = 6$

Q2. Attempt any four of the following sub questions:

(i) Find ' n ' if the nth term of the following A.P. is 64: 1,4,7,10, … …

Solution:

Given A.P. is 1,4,7,10, … …

$t_n = 64$

Here, $a = 1$

$d = 4 - 1 = 3$

Now, $t_n = a + (n - 1)d$

$\therefore 64 = 1 + (n - 1) \times 3$

$\therefore (n - 1) \times 3 = 63$

$\therefore n - 1 = 21$

$\therefore n = 22$

(ii) If one of the roots of the quadratic equation $x^2 - 10x + k = 0$ is 2 , then find the value of k .

Solution:

Given equation is $x^2 - 10x + k = 0$.

$x = 2$ is a root of the given equation.

Thus, it satisfies the given equation.

Hence, substituting $x = 2$ in the given equation, we have

$(2)^2 - 10 \times 2 + k = 0$

$\therefore 4 - 20 + k = 0$

$\therefore -16 + k = 0$

$\therefore k = 16$

(iii) A box contains 20 cards marked with the numbers 1 to 20 . One card is drawn at random, where t is such that the number on the card is multiple of 3 . Write S, n(s), A and n(A).

Solution:

Sample space, $S = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$

$\therefore$ n(S) = 20

$A =$ Event that the number on the card is a multiple of 3

$\therefore A = \{3,6,9,12,15,18\}$

Hence, $n(A) = 6$

(iv) Find the value of $x - y$ if $6x + 5y = 17, 5x + 6y = 13$.

Solution:

$6x + 5y = 17$

$5x + 6y = 13$

Subtracting equation (ii) from equation (i), we have

$x - y = 4$

(v) If the roots of quadratic equation are 5 and -6 , form the quadratic equation.

Solution:

Let $\alpha = 5$ and $\beta = -6$

$\therefore \alpha + \beta = 5 + (-6) = 5 - 6 = -1$

And, $\alpha\beta = 5 \times (-6) = -30$

Hence, the required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2 - (-1)x + (-30) = 0$

i.e. $x^2 + x - 30 = 0$

(vi) For a certain frequency distribution mean is 72 and median is 78 , find the mode.

Solution:

Let the mode of the data be x .

Then Mean - Mode = 3 (Mean - Median)

$\therefore$ 72 − x = 3 (72-78)

$\therefore$ 72 − x = 3 × (−6)

$\therefore 72 - x = -18$

$\therefore x = 72 + 18$

$\therefore x = 90$

Hence, the value of the mode is 90 .

Q3. Attempt any three of the following sub questions:

(i) For an A.P., find S_9 if $a = 4$ and $d = 2$.

Solution:

For an A.P., $a = 4$ and $d = 2$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore S_9 = \frac{9}{2}[2 \times 4 + 8 \times 2]$$
$$= \frac{9}{2}[8 + 16]$$
$$= \frac{9}{2} \times 24$$
$$= 108$$

(ii) Solve the following quadratic equation by formula method:

$3x^2 + 2x = 1$

Solution:

Given quadratic equation is $3x^2 + 2x = 1$

$\Rightarrow 3x^2 + 2x - 1 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$, we have

$a = 3, b = 2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4 \times 3 \times (-1)}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{4 + 12}}{6}$$

$= \dfrac{-2 \pm \sqrt{16}}{6}$

$= \dfrac{-2 \pm 4}{6}$

$\therefore$ x $= \frac{-2+4}{6}$ or x $= \frac{-2-4}{6}$

$\therefore$ x $= \frac{2}{6}$ or x $= -\frac{6}{6}$

$\therefore$ x $= \frac{1}{3}$ or x $= -1$

$\therefore \frac{1}{3}, -1$ are the roots of the given equation.

(iii) Solve the following simultaneous equations by using Cramer's rule:

$4x + 3y = 4$
$3x + 5y = 8$

Solution:

Given simultaneous equations are

$4x + 3y = 4$

$6x + 5y = 8$

Now,

$D = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} = 20 - 18 = 2 \neq 0$

$D_x = \begin{vmatrix} 4 & 3 \\ 8 & 5 \end{vmatrix} = 20 - 24 = -4$

$D_y = \begin{vmatrix} 4 & 4 \\ 6 & 8 \end{vmatrix} = 32 - 24 = 8$

$x = \dfrac{D_x}{D} = \dfrac{-4}{2} = -2$

$y = \dfrac{D_y}{D} = \dfrac{8}{2} = 4$

So, $x = -2$ and $y = 4$

(iv) A die is thrown, find the probability of the event of getting a square number.

Solution:

When a die is thrown, the sample space S is given by

$S = \{1,2,3,4,5,6\}$
$\Rightarrow n(S) = 6$

Let A = event of getting a square number

$\Rightarrow A = \{1,4\}$

$\Rightarrow \mathrm{n(A)} = 2$

$\Rightarrow \mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(S)}} = \frac{2}{6} = \frac{1}{3}$

(v) The marks obtained by a student in an examination out of 100 are given below. The total marks obt

Subject	Marks
Marathi	80
English	80
Science	100
Mathematics	100
Total	360

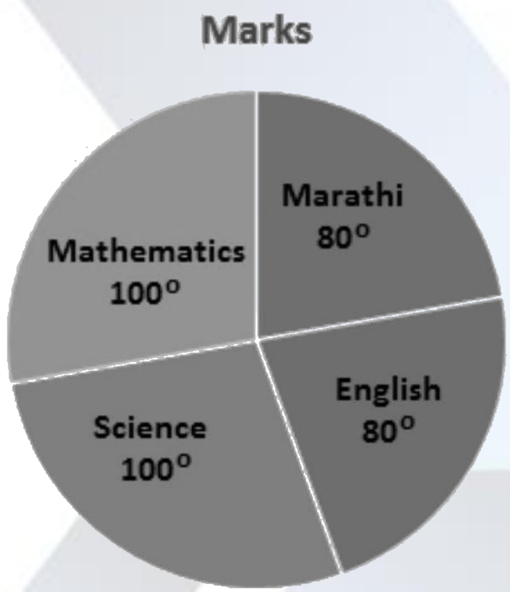
Represent the above data using pie diagram.

Solution:

Subject	Marks	Central Angle
Marathi	80	$\frac{80^\circ}{360^\circ} \times 360^\circ = 80^\circ$
English	80	$\frac{80^\circ}{360^\circ} \times 360^\circ = 80^\circ$
Science	100	$\frac{100^\circ}{360^\circ} \times 360^\circ = 100^\circ$

Mathematics	100	$\frac{100^\circ}{360^\circ} \times 360^\circ = 100^\circ$
Total	360	360°

The pie diagram is as follows:



Q4. Attempt any two of the following sub questions:

(i) If $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$, find the quadratic equation whose roots are α and β.

Solution:

α and β are the roots of the quadratic equation.

We have $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$

Now,

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\therefore 35 = (5)^3 - 3\alpha\beta(5)$

$\therefore 35 = 125 - 15\alpha\beta$

$\therefore 7 = 25 - 3\alpha\beta$ .....(Dividing by 3)

$\therefore 3\alpha\beta = 25 - 7 = 18$

$\therefore \alpha\beta = 6$

Hence, the required quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$

i.e. $x^2-5x+6=0$

(ii) Two dice are thrown. Find the probability of getting:

(a) The sum of the numbers on their upper faces is at least 9 .

(b) The sum of the numbers on their upper faces is 15.

(c) The number on the upper face of the second die is greater than the number on the upper face of the first die.

Solution:

When two dice are thrown, the sample space S is given by

$S=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),$
$(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),$
$(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$\therefore \mathrm{n(S)}=36$

(a)Let A be the event that the sum of the numbers on the upper faces is at least 9 .

$A=\{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$\therefore \mathrm{n(A)}=10$

$\therefore \mathrm{P(A)}=\frac{\mathrm{n(A)}}{\mathrm{n(S)}}=\frac{10}{36}=\frac{5}{18}$

(b)Let B be the event that the sum of the numbers on the upper faces is 15 .

This is an impossible event as maximum sum when two dice are rolled is $6+6=12$

$\therefore \mathrm{P(B)}=0$

(c)Let C be the event that the number on the upper face of the 2^{nd} die is greater than the number on the upper face of the first die..

$C=\{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,4),(3,5),$
$(3,6),(4,5),(4,6),(5,6)\}$

$\therefore \mathrm{n(C)}=15$

$\therefore \mathrm{P(C)}=\frac{\mathrm{n(C)}}{\mathrm{n(S)}}=\frac{15}{36}=\frac{5}{12}$

(iii) Frequency distribution of daily commission received by 100 salesman is given below:

Daily commission (in Rs.)	No. of Salesmen
$100 - 120$	20
$120 - 140$	45
$140 - 160$	22
$160 - 180$	09
$180 - 200$	04

Find mean daily commission received by salesmen, by assumed mean method.

Solution:

We take 150 as the assumed mean

$A = 150$ and Deviation $d_i = x_i - A = x_i - 150$

Daily Commission (in Rs.)	Class marks x_i	Deviations d_i	No. of Salesmen f_i	$f_i d_i$
$100 - 120$	110	-40	20	-800
$120 - 140$	130	-20	45	-900
$140 - 160$	150	0	22	0
$160 - 180$	170	20	09	180
$180 - 200$	190	40	04	160
Total			$\sum f_i = 100$	$\sum f_i d_i = -1360$

$$\mathbf{\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{-1360}{100} = -13.6}$$

$$\mathbf{\bar{x} = A + \bar{d} = 150 + (-13.6) = 136.4}$$

Q5. Attempt any two of the following sub questions:

(i) A boat takes 10 hours to travel 30 km upstream and 44 km downstream, but it takes 13 hours to travel 40 km upstream and 55 km downstream. Find the speed of the boat in still water and the speed of the stream.

Solution:

Let the speed of the boat in still water be x km/h.

Let the speed of the stream be ykm/h.

The speed of the boat downstream = speed of the boat + speed of the stream.

$\therefore$ The speed of the boat downstream $= (x + y)$km/h

The speed of boat upstream = speed of boat - speed of stream.

$\therefore$ The speed of the boat upstream $= (x - y)$km/h

From the first condition, time taken by boat:

To travel 30 km upstream $= \frac{30}{x-y}$

To travel 44 km downstream $= \frac{44}{x+y}$

$$\therefore \frac{30}{x-y} + \frac{44}{x+y} = 10$$

From the second condition, time taken by the boat:

To travel 40 km upstream $= \frac{40}{x-y}$

To travel 55 km downstream $= \frac{55}{x+y}$

$$\therefore \frac{40}{x-y} + \frac{55}{x+y} = 13$$

Substituting, $\frac{1}{x-y}$ for m and $\frac{1}{x+y}$ for n in (i) and (ii)

$30m + 44n = 10$

$40 \text{ m} + 55\text{n} = 13$

Multiplying (iiii) by 40 and (iv) by 30 , we get

$120 \text{ m} + 1760\text{n} = 400$ ....(v)

$120 \text{ m} + 1650\text{n} = 390$ .....(vi)

Subtracting (vi) from (v), wehave

$110\text{n} = 10$

$$\Rightarrow \text{n} = \frac{10}{110} = \frac{1}{11}$$

Substituting $\text{n} = \frac{1}{11}$ in (iii),

$$30\text{ m} + 44 \times \frac{1}{11} = 10$$

$\Rightarrow 30\text{ m} + 4 = 10$

$\Rightarrow 30\text{ m} = 6$

$$\Rightarrow \text{m} = \frac{6}{30}$$

$$\Rightarrow \text{m} = \frac{1}{5}$$

Resubstituting m for $\frac{1}{x-y}$ and n for $\frac{1}{x+y}$

$$\frac{1}{x-y} = \frac{1}{5}$$

$\therefore \text{x} - \text{y} = 5$

$$\frac{1}{x+y} = \frac{1}{11}$$

$\therefore \text{x} + \text{y} = 11$

Adding (vii) and (viii)

$2\text{x} = 16$

$\Rightarrow \text{x} = 8$

$\Rightarrow 8 + y = 11$

$\Rightarrow \text{y} = 3$

Thus, the speed of the boat in still water is 8 km/h and the speed of the stream is 3 km/h.

(ii) If the 9^{th} term of an A.P. is zero, then prove that 29^{th} term is double of 19^{th} term.

Solution:

$\text{t}_9 = 0$

$\Rightarrow a+8 d=0 . .(i)$

Now, $t_{29}=a+28 d$

$=-8 d+28 d \ \ldots .[$ From (i)]

$=20 d$

$t_{19}=a+18 d$

$=-8 d+18 d \ \ldots .[$ From (i)]

$=10 d$

$\Rightarrow 2 \times t_{19}=2 \times 10 d=20 d$

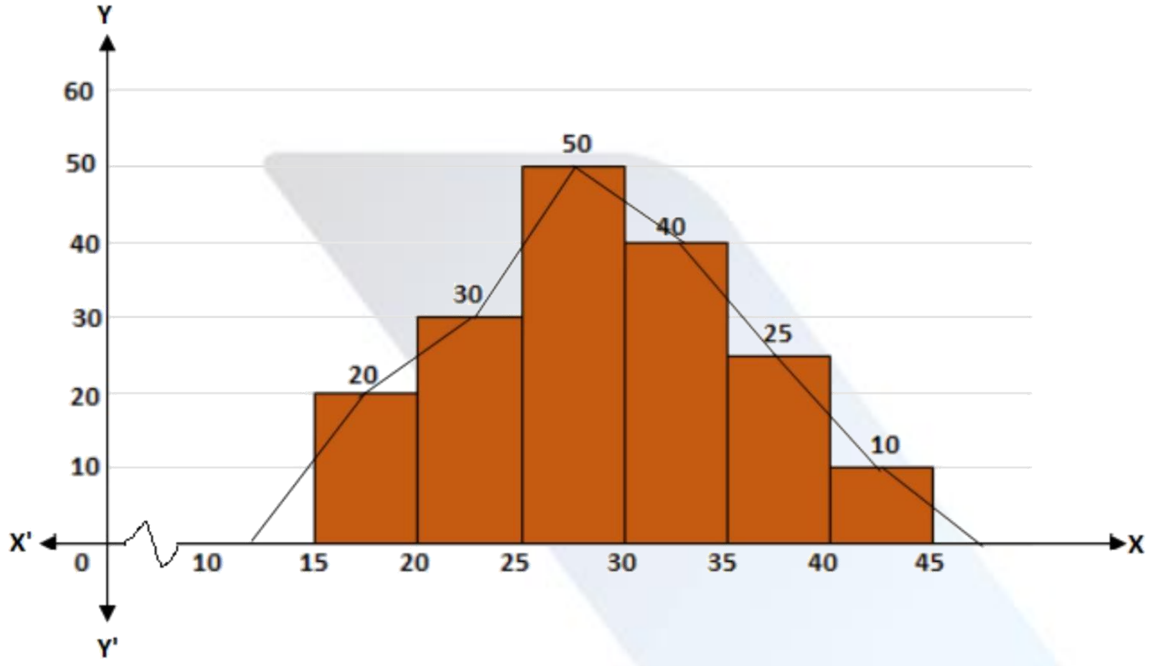
$\therefore t_{29}=2 \times t_{19}$

(iii) Draw histogram and frequency polygon on the same graph paper for the following frequency distribution:

Class	Frequency
15 – 20	20
20 – 25	30
25 – 30	50
30 – 35	40
35 – 40	25
40 – 45	25

Solution:

The histogram with frequency polygon is as follows:



PAPER-II (GEOMETRY)

Q1. Attempt any five sub-questions from the following:

(i) $\Delta DEF \sim \Delta MNK$. If $DE = 5$ and $MN = 6$, then find the value of $\left(\frac{A(\triangle DEF)}{A(\triangle MNK)}\right)$.

Solution:

Given,

$\triangle DEF \sim \triangle MNK$

$DE = 5, MN = 6$

We know that the ratio of areas of similar triangles is equal to squares of ratio of their corresponding sides.

$$\frac{A(\triangle DEF)}{A(\triangle MNK)} = \frac{DE^2}{MN^2}$$

$$= \frac{(5)^2}{(6)^2}$$

$$= \frac{25}{36}$$

(ii) If two circles with radii 8 cm and 3 cm respectively touch externally, then find the distance between their centres.

Solution:

Given,

Two circles with radii 8 cm and 3 cm respectively touch externally.

Distance between their centre = Sum of the radii

$= 8 + 3$

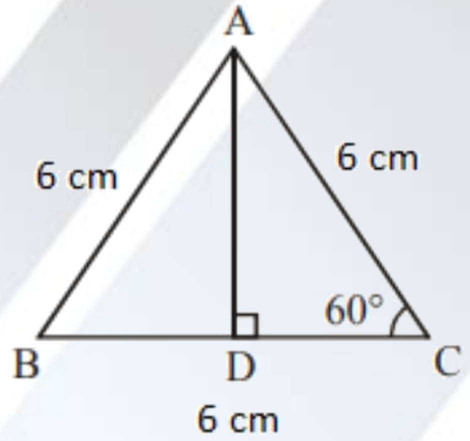
$= 11$ cm

(iii) Find the length of the altitude of an equilateral triangle with side 6 cm.

Solution:

Let ABC be an equilateral triangle with side 6 cm.

AD be the altitude of triangle ABC.



In the right triangle ADC, $\sin\ 60^\circ = \frac{AD}{AC}$

$(\frac{\sqrt{3}}{2}) \times AC = AD$

$\Rightarrow AD = (\frac{\sqrt{3}}{2}) \times 6$

$= 3\sqrt{3}$ cm

Alternative method:

Altitude of an equilateral triangle $= (\frac{\sqrt{3}}{2}) \times$ side

$= \frac{\sqrt{3}}{2} \times 6$

$= 3\sqrt{3}$ cm

(iv) If $\theta = 45^\circ$, then find $\tan\theta$.

Solution:

Given,

$\theta = 45^\circ$

$\tan\ \theta = \tan\ 45^\circ = 1$

(v) Slope of a line is 3 and y intercept is -4. Write the equation of a line.

Solution:

Given,

Slope of a line $= m = 3$

y intercept $= c = -4$

Equation of line having slope m and y -intercept c is

$y = mx + c$

Hence, the required equation of line is $y = 3x - 4$.

(vi) Using Euler's formula, find V, if $E = 30$, $F = 12$.

Solution:

Given,

$E = 30, F = 12$

Using Euler's formula,

$F + V = E + 2$

$12 + V = 30 + 2$

$V = 32 - 12$

$V = 20$

Q2. Attempt any four sub-questions from the following:

(i) The ratio of the areas of two triangles with common base is 4: 3. The height of the larger triangle is 6 cm, then find the corresponding height of the smaller triangle.

Solution:

Let H be the height of the larger triangle and h be the height of the smaller triangle.

We know that the ratio of the areas of two triangles with common base is equal to the ratio of their corresponding heights.

$\Rightarrow \left(\frac{4}{3}\right) = \left(\frac{H}{h}\right)$

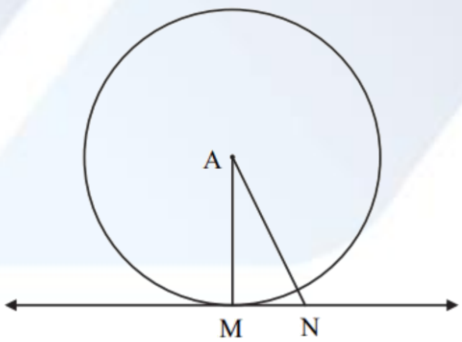
$\Rightarrow \left(\frac{4}{3}\right) = \left(\frac{6}{h}\right)$ (given height of the larger triangle is 6 cm)

$\Rightarrow h = \frac{(6 \times 3)}{4}$

$\Rightarrow h = \frac{9}{2} = 4.5$ cm

Hence, the corresponding height of the smaller triangle is 4.5 cm .

(ii) In the following figure, point ' A ' is the centre of the circle. Line MN is tangent at point M. If AN = 12 cm and MN = 6 cm, determine the radius of the circle.



Solution:

Given,

AN = 12 cm

MN = 6 cm

We know that the radius is perpendicular to the tangent through the point of contact.

Thus, $\angle AMN = 90^\circ$

In right triangle AMN,

$AN^2 = AM^2 + MN^2$

$(12)^2 = AM^2 + (6)^2$

$\Rightarrow AM^2 = 144 - 36$

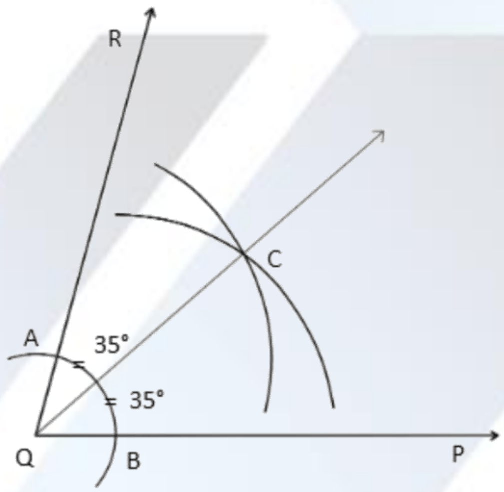
$= 108$

$\Rightarrow AM = \sqrt{108} = 6\sqrt{3}$

Hence, the radius of the circle is $6\sqrt{3}$ cm.

(iii) Draw $\angle PQR$ of measure 70° and bisect it.

Solution:



Therefore, $\angle PQR = 70^\circ$ and QC is the bisector of it.

(iv) If $\cos\ \theta = \frac{3}{5}$, where ' θ ' is an acute angle. Find the value of $\sin\ \theta$.

Solution:

Given,

$$\cos\ \theta = \frac{3}{5}$$

$$\sin\ \theta = \sqrt{(1 - \cos^2\ \theta)}$$

$= \sqrt{\left[1-(\frac{3}{5})^2\right]}$

$= \sqrt{[1-(\frac{9}{25})]}$

$= \sqrt{\frac{(25-9)}{25}}$

$= \sqrt{(\frac{16}{25})}$

$= \frac{4}{5}$ (given that θ is an acute angle.)

Therefore, $\sin\ \theta = \frac{4}{5}$

(v) The volume of a cube is 1000 cm^3. Find its side.

Solution:

Given,

Volume of cube $= 1000\ cm^3$

$\Rightarrow (\text{ side })^3 = (10)^3$

$\Rightarrow$ Side $= 10$ cm

(vi) The radius and slant height of a cone are 4 cm and 25 cm respectively. Find the curved surface area of that cone. $(\pi = 3.14)$

Solution:

Given,

Radius of cone $= r = 4$ cm

Slant height $= l = 25$ cm

Curved surface area of cone $= \pi rl$

$= 3.14 \times 4 \times 25$

$= 314\ cm^2$

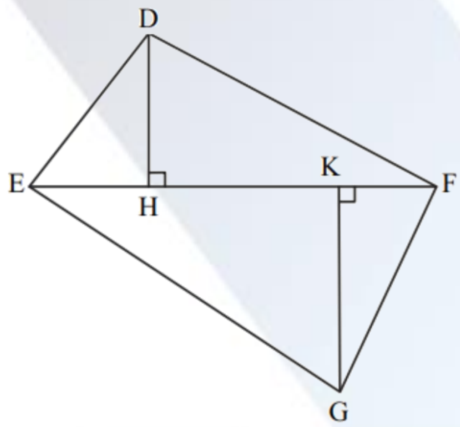
Q3. Attempt any three sub-questions from the following:

(i) In the following figure, seg DH ⊥ seg EF and seg GK ⊥ seg EF. If DH = 6 cm, GK = 10 cm and A(△ DEF) = 150 cm^2, then find:

i. EF

ii. A(Δ GEF)

iii. A(△ DFGE).



Solution:

Given,

DH = 6 cm

GK = 10 cm

A(△ DEF) = 150 cm^2

i. A(△ DEF) = $(\frac{1}{2}) \times EF \times DH$

$150 = (\frac{1}{2}) \times EF \times 6$

$\Rightarrow EF \times 3 = 150$

$\Rightarrow EF = \frac{150}{3}$

$\Rightarrow EF = 50$ cm

ii. △ DEF and △ GEF have the common base EF.

Therefore, their areas are proportional to their corresponding heights.

$\frac{A(\triangle DEF)}{A(\Delta GEF)} = \frac{DH}{GK}$

$\frac{150}{A(\Delta GEF)} = \frac{6}{10}$

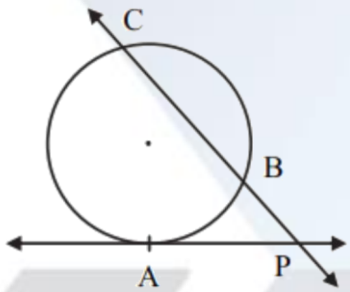
$\Rightarrow A(\triangle\ GEF) = \frac{150 \times 10}{6} = 250$

$A(\triangle\ GEF) = 250\ cm^2$

iii. $A(DFGE) = A(\Delta DEF) + A(\Delta GEF)$

$= 150 + 250 = 400\ cm^2$

(ii) In the following figure, ray PA is the tangent to the circle at point A and PBC is a secant. If $AP = 14, BP = 10$, then find BC.



Solution:

Given,

PA is the tangent to the circle at point A and PBC is a secant.

$AP = 14, BP = 10$

We know that,

$PB \times PC = PA^2$

$10 \times PC = (14)^2$

$PC = \left(\frac{196}{10}\right) = 19.6$

Now,

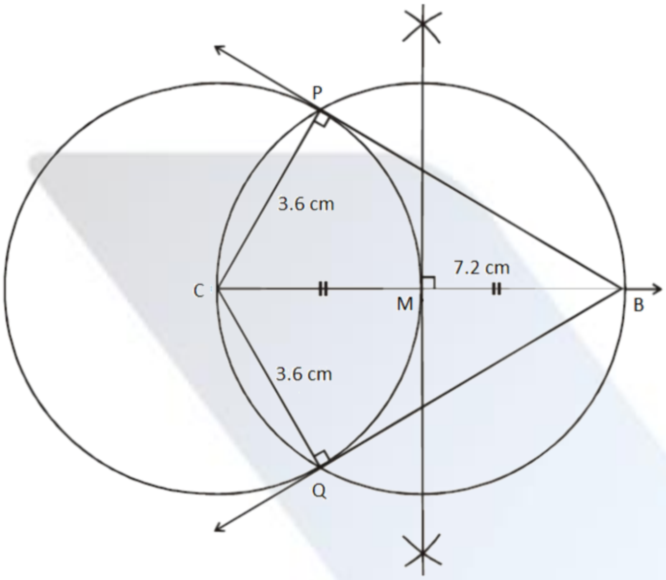
$PB + BC = PC$

$10 + BC = 19.6$

$BC = 19.6 - 10 = 9.6\ cm$

(iii) Draw the circle with centre C and radius 3.6 cm. Take point B which is at distance 7.2 cm from the centre. Draw tangents to the circle from point B.

Solution:



Therefore, BP and BQ are the required tangents to the circle.

(iv) Show that: $\sqrt{\frac{(1-\sin\ x)}{(1+\sin\ x)}} = \sec\ x - \tan\ x$

Solution:

$$\begin{aligned}
\text{LHS} &= \sqrt{\frac{1-\sin\ x}{1+\sin\ x}} \\
&= \sqrt{\frac{(1-\sin\ x)^2}{(1+\sin\ x)(1-\sin\ x)}} \\
&= \sqrt{\frac{(1-\sin\ x)^2}{1-\sin^2\ x}} \\
&= \frac{\sqrt{(1-\sin\ x)^2}}{\sqrt{\cos^2\ x}} \\
&= \frac{1-\sin\ x}{\cos\ x} \\
&= \frac{1}{\cos\ x} - \frac{\sin\ x}{\cos\ x} \\
&= \sec\ x - \tan\ x \\
&= \text{RHS}
\end{aligned}$$

(v) Write the equation of the line passing through points $C(4, -5)$ and $D(-1, -2)$ in the form of ax + by +c = 0.

Solution:

Let the given points be:

$C(4, -5) = (x_1, y_1)$

$D(-1, -2) = (x_2, y_2)$

Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is:

$$\frac{(x - x_1)}{(x_2 - x_1)} = \frac{(y - y_1)}{(y_2 - y_1)}$$

$$\frac{(x - 4)}{(-1 - 4)} = \frac{(y + 5)}{(-2 + 5)}$$

$$\frac{(x - 4)}{(-5)} = \frac{(y + 5)}{3}$$

$3(x - 4) = -5(y + 5)$

$3x - 12 = -5y - 25$

$3x + 5y - 12 + 25 = 0$

$3x + 5y + 13 = 0$

This is of the form $ax + by + c = 0$

Hence, the required equation of the line CD is $3x + 5y + 13 = 0$.

Q4. Attempt any two sub-questions from the following:

(i) Prove that "the lengths of the two tangent segments to a circle drawn from an external point are equal".

Solution:

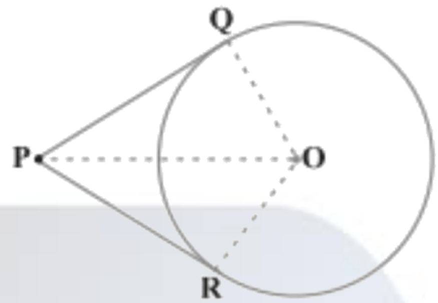
Given,

PQ and PR are the tangents to the circle with centre O from an external point P.

To prove: $PQ = PR$

Construction:

Join OQ, OR and OP.



Proof:

We know that the radius is perpendicular to the tangent through the point of contact.

$\angle OQP = \angle ORP = 90^\circ$

In right $\triangle$ OQP and ORP,

$OQ = OR$ (radii of the same circle)

OP = OP (common)

By RHS congruence criterion,

$\triangle OQP \cong \triangle ORP$

By CPCT,

$PQ = PR$

Hence proved.

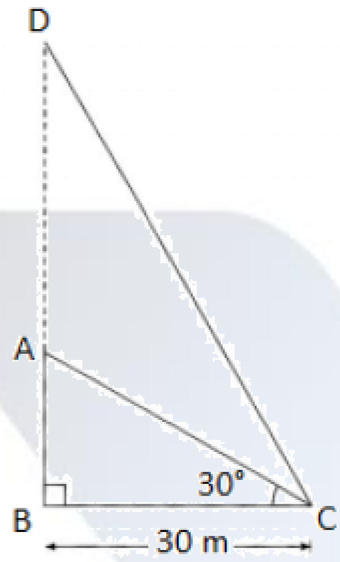
(ii) A tree is broken by the wind. The top of that tree struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the height of the whole tree.

$(\sqrt{3} = 1.73)$

Solution:

Let D be the top of the tree and AB be the unbroken part of the tree.

DA = AC = Broken part of the tree



BC = 30 m

In right triangle ABC, tan $30^\circ = \left(\frac{AB}{BC}\right)$

$\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{AB}{30}\right)$

$\Rightarrow AB = \left(\frac{30}{\sqrt{3}}\right)$

Again in triangle ABC,

$\cos 30^\circ = \left(\frac{BC}{AC}\right)$

$\frac{\sqrt{3}}{2} = \left(\frac{30}{AC}\right)$

$\Rightarrow AC = \frac{(30 \times 2)}{\sqrt{3}}$

$\Rightarrow AC = \frac{60}{\sqrt{3}}\,\text{m}$

Also, $AC = AD = \frac{60}{\sqrt{3}}\,\text{m}$

Total height of the tree = BD

= DA + AB

$= \left(\frac{60}{\sqrt{3}}\right) + \left(\frac{30}{\sqrt{3}}\right)$

$= (\frac{90}{\sqrt{3}})$

$= (\frac{90}{\sqrt{3}}) \times (\frac{\sqrt{3}}{\sqrt{3}})$

$= \frac{(90\sqrt{3})}{3}$

$= 30\sqrt{3}$

$= 30 \times 1.73$

$= 51.9$ m

Hence, the height of the whole tree is 51.9 m.

(iii) $A(5,4), B(-3,-2)$ and $C(1,-8)$ are the vertices of a triangle ABC. Find the equation of median AD.

Solution:

Given,

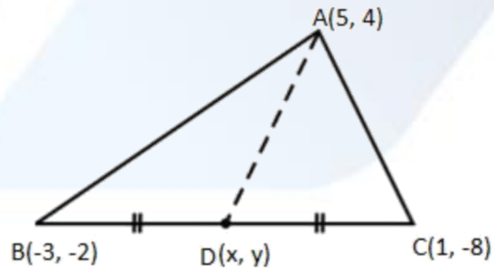
Vertices of a triangle ABC are $A(5,4), B(-3,-2)$ and $C(1,-8)$.

$A(5,4) = (x_1, y_1)$

$B(-3,-2) = (x_2, y_2)$

$C(1,-8) = (x_3, y_3)$

Let $D(x,y)$ be the midpoint of BC .



D is the midpoint of BC .

$D(x,y) = \left[\frac{(x_2 + x_3)}{2}, \frac{(y_2 + y_3)}{2}\right]$

$= \left[\frac{(-3+1)}{2}, \frac{(-2-8)}{2}\right]$

$= (\frac{-2}{2}, \frac{-10}{2})$

$= (-1, -5)$

$D(-1, -5) = (x_4, y_4)$

Equation of median AD is

$\frac{(x - x_1)}{(x_4 - x_1)} = \frac{(y - y_1)}{(y_4 - y_1)}$

$\frac{(x - 5)}{(-1 - 5)} = \frac{(y - 4)}{(-5 - 4)}$

$\frac{(x - 5)}{(-6)} = \frac{(y - 4)}{(-9)}$

$-9(x - 5) = -6(y - 4)$

$-9x + 45 = -6y + 24$

$9x - 45 - 6y + 24 = 0$

$9x - 6y - 21 = 0$

$3(3x - 2y - 7) = 0$

$3x - 2y - 7 = 0$

Hence, the required equation of median AD is $3x - 2y - 7 = 0$.

Q5. Attempt any two sub-questions from the following:

(i) Prove that, in a right-angled triangle, the square of hypotenuse is equal to the sum of the square of remaining two sides.

Solution:

Given:

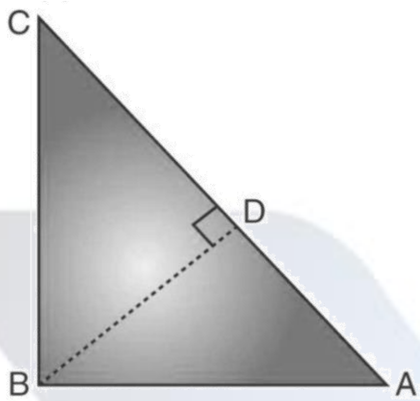
In a right triangle ABC, $\angle B = 90^\circ$

To prove:

$AC^2 = AB^2 + BC^2$

Construction:

Draw a perpendicular BD onto the side AC.



We know that,

$\triangle ADB \sim \triangle ABC$

Therefore, $\left(\frac{AD}{AB}\right) = \left(\frac{AB}{AC}\right)$ (by similarity)

$AB^2 = AD \times AC.....(i)$

Also, $\triangle BDC \sim \triangle ABC$

Therefore, $\left(\frac{CD}{BC}\right) = \left(\frac{BC}{AC}\right)$ (by similarity)

$BC^2 = CD \times AC...$ (ii)

Adding (i) and (ii),

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC(AD + CD)$

Since, $AD + CD = AC$

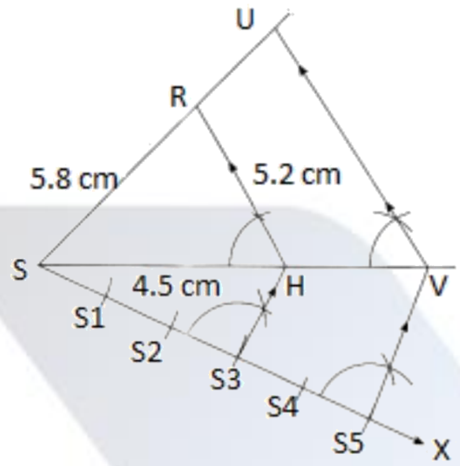
Therefore, $AC^2 = AB^2 + BC^2$

Hence proved.

(ii) $\Delta SHR \sim \Delta SVU$, in ΔSHR, $SH = 4.5$ cm, $HR = 5.2$ cm, $SR = 5.8$ cm and $\left(\frac{SH}{SV}\right) = \frac{3}{5}$.

Construct ΔSVU.

Solution:



(iii) If 'V' is the volume of a cuboid of dimensions a × b × c and ' S ' is its surface area, then prove that: $\frac{1}{V} = \frac{2}{S}\left[\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right) + \left(\frac{1}{c}\right)\right]$

Solution:

Given,

Dimensions of the cuboid $= a \times b \times c$

Volume of cuboid $= V = abc$

$\Rightarrow \frac{1}{V} = (\frac{1}{abc})$....(i)

Surface area of cuboid = S = 2(ab + bc + ca)....(ii)

RHS $= \frac{2}{S}\left[\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right) + \left(\frac{1}{c}\right)\right]$

$= \frac{2}{S}\left[\frac{(bc + ca + ab)}{abc}\right]$

$= \frac{2(ab + bc + ca)}{S(abc)}$

$= \left(\frac{S}{S(abc)}\right)$ [From (ii)]

$= \frac{1}{abc}$

$= \left(\frac{1}{V}\right)$ [From (i)]

= LHS

Hence proved.