PART - I

Grade 10 Maharashtra Math 2019

Q1. (A) Solve the following questions (Any four):

i. Find the median of: 66, 98, 54, 92, 87, 63, 72.

Solution:

Arrange the series in ascending order:

$54 < 63 < 66 < 72 < 87 < 92 < 98$

Now, using the median formula,

Median $= \frac{(n+1)}{2}$ term, where n is the number of values in a set of data

$= \left(\frac{7+1}{2}\right)^{th}$ term

$= \left(\frac{8}{2}\right)^{th}$ term

$= 4^{th}$ term

As we can see in the series $54 < 63 < 66 < 72 < 87 < 92 < 98$, the fourth term is 72 .

Therefore, the median of the given data is 72 .

ii. Multiply and write the answer in the simplest form: $5\sqrt{7} \times 2\sqrt{7}$

Solution:

$\sqrt{7}(5 \times 2) = 10\sqrt{7}$

iii. If $3x + 5y = 9$ and $5x + 3y = 7$, then find the value of $x + y$.

Solution:

Add both equations, $3x + 5y = 9$ and $5x + 3y = 7$

$3x + 5y + 5x + 3y = 9 + 7$

$8x + 8y = 16$

$8(x + y) = 16$

$x + y = 2$

iv. Write the ratio of second quantity to first quantity in the reduced form: 5 dozen pens, 120 pens.

Solution:

1 dozen = 12 quantity

So, 5 dozen pen $= 5 \times 12 = 60$

The ratio of second quantity to first quantity is given as;

120 : 60

2 : 1

v. Write the following polynomial in coefficient form: $2x^3 + x^2 - 3x + 4$.

Solution:

Coefficient form of the polynomial is $(2, 1, -3, 4)$

vi. For computation of income tax which is the assessment year of financial year 01-04-2016 to 31-03-2017?

Solution:

For computation of income tax of financial year 01-04-2016 to 31-3-2017, the assessment year is 2017-18.

(B) Solve the following questions (Any two):

i. Find the value of the polynomial $2x^3 + 2x$, when $x = -1$.

Solution: Putting $x = -1$ in the polynomial

$2x^3 + 2x$

$= 2(-1)^3 + 2(-1)$

$= (-2) + (-2)$

$= -4$

Thus, the answer is -4.

ii. If $A = \{11, 21, 31, 41\}$, $B = \{12, 22, 31, 32\}$, then find:

a. $A \cup B$

b. $A \cap B$

Solution:

$A \cup B = \{11,12,21,22,31,32,41\}$

$A \cap B = \{31\}$

iii. Sangeeta's monthly income is Rs. 25,000. She spent 90% of her income and donated 3% for socially useful causes. How much money did she save?

Solution:

90% of Sangeeta's income $= \frac{(90 \times 25,000)}{100} =$ Rs. 22,500

Money denoted for social cause = 3% of 25,000 $= \frac{(3 \times 25,000)}{100} =$ Rs. 750

Money saved by Sangeeta = Income - Money spend by her - Money spend for social cause

Money saved by Sangeeta $= 25,000 - 22,500 - 750$

Money saved by Sangeeta $= 25,000 - 23,250 =$ Rs. 1750

Q2. (A) Choose the correct alternative:

i. In the A.P. $2, -2, -6, -10, \ldots \ldots \ldots \ldots$ common difference (d) is:

(A) -4

(B) 2

(C) -2

(D) 4

Solution:

Common difference $(d) = a_2 - a_1$

$d = (-2) - (2)$

$d = -4$

Hence, option (A) i.e -4 is the answer.

ii. For the quadratic equation $x^2 + 10x - 7 = 0$, the values of a, b, c are:

(A) $a = -1, b = 10, c = 7$

(B) $a = 1, b = -10, c = -7$

(C) $a = 1, b = 10, c = -7$

(D) $a = 1, b = 10, c = 7$

Solution:

Coefficient form of the polynomial is $(1,10,-7)$.

Therefore, option (C) i.e $a = 1, b = 10, c = -7$ is the correct answer.

iii. The tax levied by Central Government for trading within a state is:

(A) IGST

(B) CGST

(C) SGST

(D) UTGST

Solution:

The correct answer is option (B).

iv. If a die is rolled, what is the probability that number appearing on upper face is less than 2 ?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{1}{6}$

Solution:

Probability = Number of favourable outcomes/Total Number of outcomes Total

Number of outcomes $= 1,2,3,4,5,6$

Number of favourable outcomes $= 1$

$P(E) = \left(\frac{1}{6}\right)$

Therefore, the answer is option (D) i.e $\left(\frac{1}{6}\right)$.

(B) Solve the following questions (Any two):

i. First term and common difference of an A.P. are 12 and 4 respectively. If $t_n = 96$, find n.

Solution: $t_1 = 12, d = 4, t_n = 96$

$t_n = t_1 + (n-1)d$

$96 = 12 + (n-1)4$

$96 - 12 = (n-1)4$

$84 = (n-1)4$

$21 = (n-1)$

$21 + 1 = n$

$n = 22$

ii.If $\begin{vmatrix} 4 & 5 \\ m & 3 \end{vmatrix} = 22$, then find the value of m.

Solution:

By solving the above matrix.

$5\,m - 12 = 22$

$5\,m = 22 + 12$

$5\,m = 34$

$m = \left(\frac{34}{5}\right)$

iii. Solve the following quadratic equation:

$x^2 + 8x + 15 = 0$

Solution: $x^2 + 8x + 15 = 0$

$x^2 + 5x + 3x + 15 = 0$

$x(x+5) + 3(x+5) = 0$

$(x+3)(x+5) = 0$

$x = -3, x = -5$

Q3. (A) Complete the following activities (Any two):

i. Smita has invested Rs. 12,000 to purchase shares of FV Rs.10 at a premium of Rs. 2. Find the number of shares she purchased. Complete the given activity to get the answer.

Activity: FV = Rs. 10 , Premium = Rs. 2

$\therefore$ MV=FV+□=□+2=12

$\therefore$ Number of shares $= \dfrac{\text{Totalinvestment}}{\text{MV}}$

$= \dfrac{\square}{12} = \square$ shares

Solution:

$\therefore$ MV = FV + Premium =10+2=12

$\therefore$ Number of shares $= \dfrac{\text{Total investment}}{\text{MV}}$

$= \dfrac{12000}{12} = 1000$ shares

Therefore, the answer is 1000 shares.

ii. The following table shows the daily supply of electricity to different places in a town. To show the information by a pie diagram, measures of central angles of sectors are to be decided. Complete the following activity to find the measures:

Places	Supply of electricity (Thousand units)	Measure of central angle
Roads	4	$\frac{4}{30} \times 360 = 48°$
Factories	12	$\frac{\square}{\square} \times 360 = 144°$
Shops	6	$\frac{6}{30} \times 360 = \square$
Houses	8	$\frac{\square}{\square} \times 360 = \square$
Total	30	

Solution:

Places	Supply of electricity (Thousand units)	Measure of central angle
Roads	4	$\frac{4}{30} \times 360 = 48°$
Factories	12	$\frac{12}{30} \times 360 = 144^0$
Shops	6	$\frac{6}{30} \times 360 = 72^0$
Houses	8	$\frac{8}{30} \times 360 = 96^0$
Total	30	

iii. Two coins are tossed simultaneously. Complete the following activity of writing the sample space (S) and expected outcomes of the events:

a. Event A: to get at least one head.

b. Event B: to get no head.

Activity: If two coins are tossed simultaneously

∴ S = { ☐ , HT, TH, ☐}

a. Event A : at least getting one head.

∴ A = {HH, ☐ , TH }.

b. Event B : to get no head.

B = {☐}

Solution:

Activity: If two coins are tossed simultaneously

∴ $S = \{HH, HT, TH, TT\}$

a. Event A : at least getting one head.

∴ $A = \{HH, HT, TH\}$.

b. Event B : to get no head.

B = {TT}.

(B) Solve the following questions (Any two):

i. Find the 19th term of the A.P. 7,13,19,25, …….

Solution:

$t_n = a + (n - 1)d$

In the given A.P, $a = 7, n = 19$

$d = t_2 - t_1$

$d = 13 - 7 = 6$

$t_{19} = 7 + (19 - 1)6$

$t_{19} = 7 + 108$

$t_{19} = 115$

ii. Obtain a quadratic equation whose roots are −3 and -7.

Solution:

Let $\alpha = -3, \beta = -7$

$\alpha + \beta = (-3) + (-7) = -10$

$\alpha \times \beta = (-3) \times (-7) = 21$

Quadratic Equation is given by;

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (-10)x + 21 = 0$

$x^2 + 10x + 21 = 0$

iii. Two numbers differ by 3 . The sum of the greater number and twice the smaller number is 15. Find the smaller number.

Solution:

Let the greater number be "a" and the smaller number be "b".

It is given that;

$a - b = 3$

$a = 3 + b$

$a + 2b = 15$

Putting the value of "a" in equation (ii)

$3 + b + 2b = 15$

$3 + 3b = 15$

$3(1 + b) = 15$

$1 + b = 5$

$b = 4$

Now, finding the value of a

$a = 3 + b$

$a = 3 + 4 = 7$

Therefore, the smaller number is $b = 4$.

Q4. Solve the following questions (Any three):

i. Amit saves certain amount every month in a specific way. In the first month he saves Rs. 200, in the second month Rs. 250, in the third month Rs. 300 and so on. How much will be his total savings in 17 months?

Solution:

It forms an A.P

200, 250, 300,

$a = 200$

$d = 250 - 200 = 50$

$n = 17$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{17} = \frac{17}{2}[2(200) + (17 - 1)50]$

$= \frac{17}{2}[1200]$

$= 17 \times 600$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Therefore, Amit's total saving in 17 months will be Rs. 10200.

ii. A two digit number is to be formed using the digits 0,1,2,3. Repetition of the digits is allowed. Find the probability that a number so formed is a prime number.

Solution:

Total two digit number that can be formed using the digit 0, 1, 2, 3 are 10, 20, 30,12,21,13,31,23,32,11,22,33

Total prime number formed are 13,31,23,11

Required Probability $= \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$

$$P = \frac{4}{12}$$

$$P = \frac{1}{3}$$

iii. Smt. Malhotra purchased solar panels for the taxable value of Rs. 85,000. She sold them for Rs. 90,000. The rate of GST is 5%. Find the ITC of Smt. Malhotra. What is the amount of GST payable by her?

Solution:

Taxable purchase value of solar panel = Rs. 85000

Rate of GST $= 5\%$

Input Tax Credit (ITC) $= 5\%$ of $85000 =$ Rs. 4250

Selling price of the solar panel = Rs. 90000

Output Tax $= 5\%$ of $90000 =$ Rs. 4500

GST payable by Malhotra = Output Tax - Input Tax Credit (ITC)

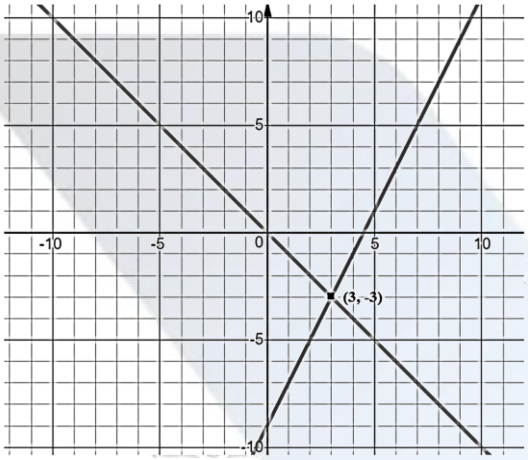
GST payable by Malhotra $= 4500 - 4250 =$ Rs. 250

So, the tax paid by Malhotra is Rs. 250 .

iv. Solve the following simultaneous equations graphically:

$x + y = 0; 2x - y = 9$

Solution:



Q5. Solve the following questions (Any one):

i. The following frequency distribution table shows marks obtained by 180 students in Mathematics examination:

Marks	Number of Students
$0 - 10$	25
$10 - 20$	x
$20 - 30$	30
$30 - 40$	2x
$40 - 50$	65

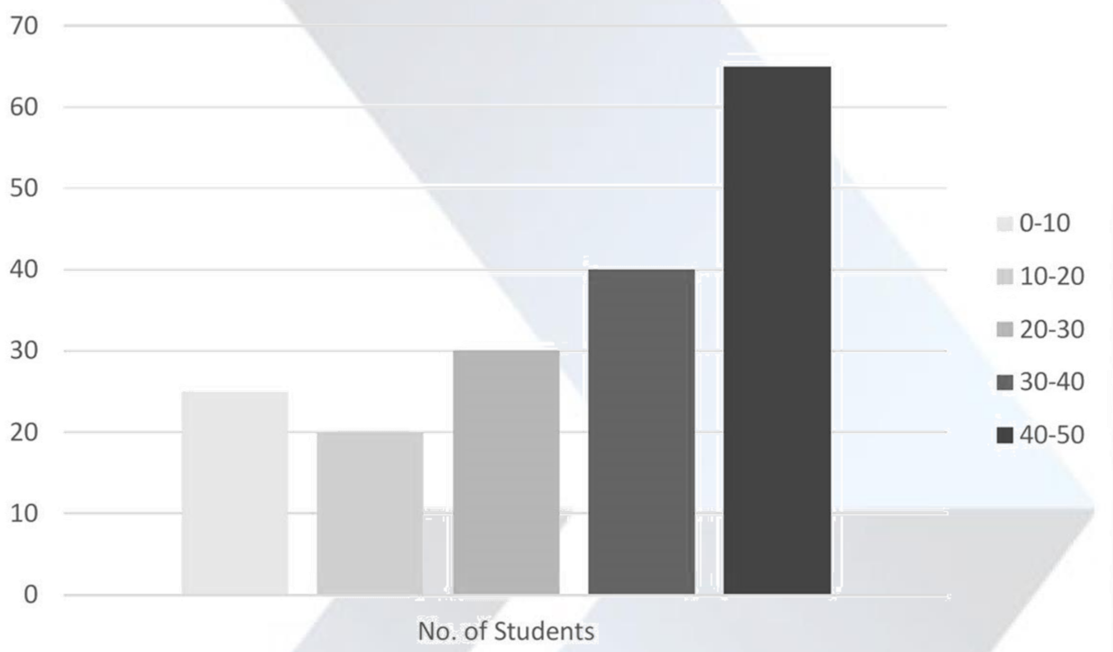
Find the value of **x.**

Also draw a histogram representing the above information.

Solution:

The marks obtained by all students is given $= 180$

$$25 + x + 30 + 2x + 65 = 180$$
$$3x + 120 = 180$$
$$3x = 60$$
$$x = 20$$



ii. Two taps together can fill a tank completely in $3\frac{1}{13}$ minutes. The smaller tap takes 3 minutes more than the bigger tap to fill the tank. How much time does each tap take to fill the tank completely?

Solution:

Two tap can fill the tank completely in $\frac{40}{13}$ minutes.

Let us consider that 1 tap can fill the tank in $=$ x min

So, the smaller tap fill the tank in $(x + 3)$ min

Work done by both tap in 1 min ;

$$\left(\frac{1}{x}\right) + \frac{1}{(x+3)} = \left(\frac{13}{40}\right)$$

$$\frac{(2x+3)}{x(x+3)} = \left(\frac{13}{40}\right)$$

$$\frac{(2x+3)}{(x^2+3x)} = \left(\frac{13}{40}\right)$$

$80x + 120 = 13x^2 + 39x$

$13x^2 - 41x - 120 = 0$

$13x^2 - 65x + 24x - 120 = 0$

$13x(x-5) + 24(x-5) = 0$

$(13x + 24)(x - 5) = 0$

$x = \left(-\frac{24}{13}\right), x = 5$

Ignoring the negative value.

Therefore, one tap will take 5 minutes and other tap will be 8 min to fill the tank.

Q6. Solve the following questions (Any one):

i. The co-ordinates of the point of intersection of lines $ax + by = 9$ and $bx + ay = 5$ is $(3, -1)$. Find the values of a and b.

Solution:

Given equations are:

$ax + by = 9$ ...(i)

$bx + ay = 5$ ...(ii)

$(3, -1)$ is given as the point of intersection. So, it will satisfy equation (i) and (ii).

$3a - b = 9$ (iii)

$3b - a = 5$

Now solving equation (iii) and (iv) we get $b = 3$ and $a = 4$.

ii. The following frequency distribution table shows the distances travelled by some rickshaws in a day. Observe the table and answer the following questions:

Class (Daily distance travelled in km)	Continous Classes	Frequency (Number of rickshaws)	Cumulative Frequency less than type
$60-64$	$59.5-64.5$	10	10
$65-69$	$64.5-69.5$	34	$10+34=44$
$70-74$	$69.5-74.5$	58	$44+58=102$
$75-79$	$74.5-79.5$	82	$102+82=184$
$80-84$	$79.5-84.5$	10	$184+10=194$
$85-89$	$84.5-89.5$	6	$194+6=200$

a. Which is the modal class? Why?

Solution:

The class where the frequency is maximum is known as the modal class.

In the given table, the highest frequency is 82 . So, the modal class is $74.5-79.5$.

b. Which is the median class and why?

Solution:

Median frequency $=\frac{(10+34+58+82+10+6)}{2}$

Median frequency $=(\frac{200}{2})=100$

Median class is defined as the class where the median frequency falls in the cumulative frequency.

As we can see in the table, 100 falls under 69.5-74.5

Therefore, the median class is $69.5-74.5$

c. Write the cumulative frequency (C.F.) of the class preceding the median class.

Solution:

Median class preceding the median is 64.5-69.5 and the cumulative frequency of it is 44 .

d. What is the class interval (h) to calculate median?

Solution:

Class interval = range of the class element set

$$h = 64.5 - 59.5$$
$$h = 5$$

PART - II

Grade 10 Maharashtra Math 2019

Q1. (A) Solve the following questions (Any four):

(i) If $\triangle$ ABC $\sim \triangle$ PQR and $\angle A = 60^\circ$, then $\angle P = ?$

Solution:

In $\triangle$ ABC $\sim \triangle$ PQR

$\angle A = \angle P$...(c.p.c.t.)

$\therefore \angle P = 60^\circ$

(ii) In right - angled $\triangle ABC$, if $\angle B = 90^\circ, AB = 6, BC = 8$, then find AC.

Solution:

In $\triangle ABC$

$\Rightarrow \angle B = 90^\circ$,

By Pythagoras theorem, we get

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow AC = \sqrt{6^2 + 8^2}$

$\Rightarrow AC = \sqrt{100} = 10$

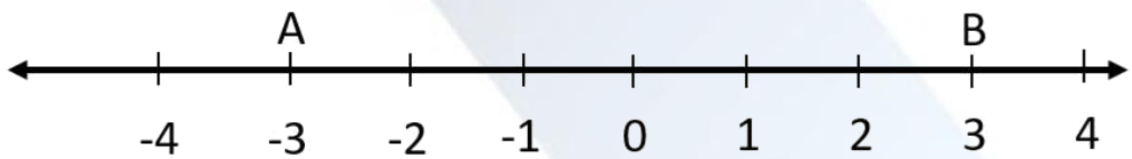
(iii) Write the length of largest chord of a circle with radius 3.2 cm .

Solution:

The largest chord of circle with radius 3.2 cm , will be its diameter.

Therefore, the length of diameter = twice the radius $= 2 \times 3.2 = 6.4$ cm

(iv) From the given number line, find $d(A, B)$:



Solution:

$d(A, B) = (3) - (-3) = 6$

(v) Find the value of $\sin\ 30^\circ + \cos\ 60^\circ$.

Solution:

$$\begin{aligned} & \sin\ 30^\circ + \cos\ 60^\circ \\ & = \frac{1}{2} + \frac{1}{2} \\ & = 1 \end{aligned}$$

(vi) Find the area of a circle of radius 7 cm .

Solution:

Area of circle $= \pi r^2$

$$\begin{aligned} & = \frac{22}{7} \times 7^2 \\ & = 154 \text{ cm}^2 \end{aligned}$$

Q1. (B) Solve the following questions (Any two):

(i) Draw seg AB of length 5.7 cm and bisect it.

Solution:

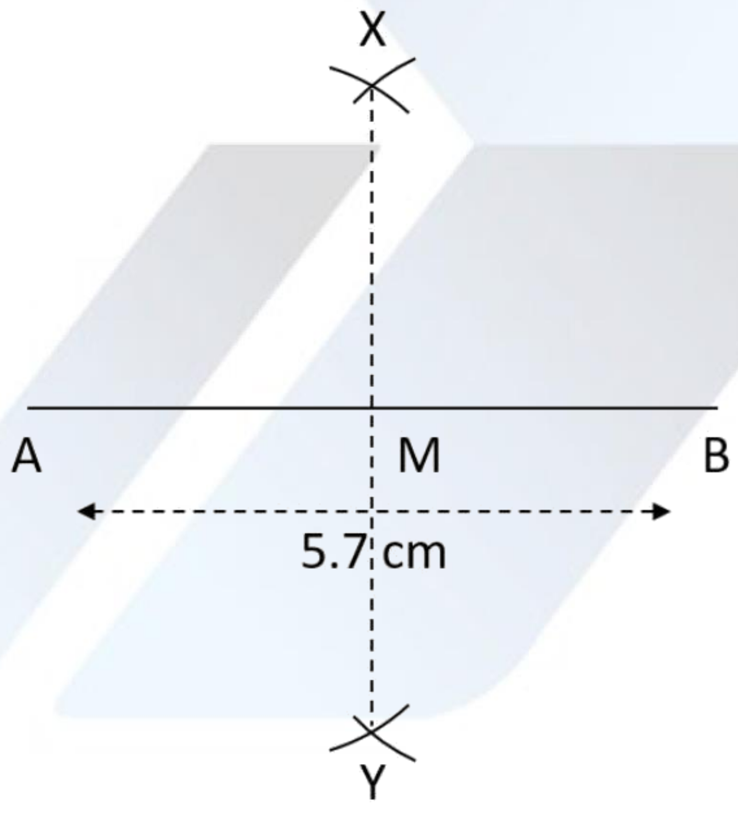
a. Draw a line segment AB , measuring 5.7 cm

b. Take compass with the measure more than half of AB, put the steel end on A and make arcs above and below.

c. Take the same compass measurement and by keeping the steel head on B, cut the previously made arcs, name the point of intersection X and Y.

d. Join XY and mark the point M, where the line XY cuts the line segment AB

e. M is the midpoint of the line AB and XY its bisector.



(ii) In right-angled triangle PQR, if $\angle P = 60°, \angle R = 30°$ and $PR = 12$, then find the values of PQ and QR.

Solution:

In $\triangle$ PQR

$\Rightarrow \angle P = 60^\circ$

$\Rightarrow \angle R = 30^\circ$

$\Rightarrow \angle Q = 180^\circ - (\angle P + \angle R) = 90^\circ$ ... (Sum of all the angles of Δ is 180°)

$\therefore \triangle$ PQR is $30^\circ - 60^\circ - 90^\circ$ triangle.

PR is hypotenuse $= 12$ cm

$\Rightarrow QR = \frac{\sqrt{3}}{2} PR$ ... (side opp. to 60°)

$\Rightarrow QR = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$ cm

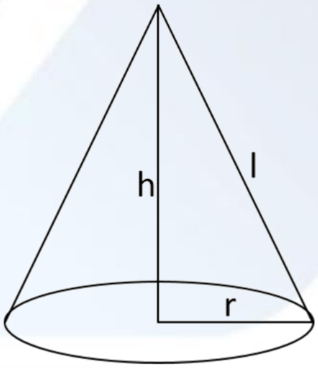
$\Rightarrow PQ = \frac{1}{2} PR$... (side opp. to 30°)

$\Rightarrow PQ = \frac{1}{2} \times 12 = 6$ cm

(iii) In a right circular cone, if perpendicular height is 12 cm and radius is 5 cm , then find its slant height.

Solution:

In cone we have,



Given that

$h = 12$ cm

$r = 5$ cm

So, by Pythagoras theorem, we can write

$l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

Therefore the slant height of a right circular cone is 13 cm .

Q2. (A) Choose the correct alternative:

(i) ΔABC and $\triangle$ DEF are equilateral triangles. If $A(\triangle ABC): A(\triangle DEF) = 1: 2$ and $AB = 4$, then what is the length of DE ?

(a) $2\sqrt{2}$

(b) 4

(c) 8

(d) $4\sqrt{2}$

Solution:

$\triangle$ ABC and $\triangle$ DEF are equilateral triangles.

So, they are similar

By the theorem of areas of similar triangles, we have

$$\Rightarrow \frac{A(\triangle ABC)}{A(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{1}{2} = \frac{4^2}{DE^2}$$

$$\Rightarrow DE^2 = 4 \times 2$$

$$\Rightarrow \boldsymbol{DE = 2\sqrt{2}}$$

Correct option (a)

(ii) Out of the following which is a Pythagorean triplet?

(a) (5,12,14)

(b) (3,4,2)

(c) (8,15,17)

(d) (5,5,2)

Solution:

A Pythagorean triplet should satisfy the condition that is

Square of large number = sum of squares of other two numbers

From the options given only option (c) satisfies the condition

$17^2 = 289$

$8^2 + 15^2 = 289$

$\Rightarrow 8^2 + 15^2 = 17^2$

Correct option (c)

(iii) $\angle ACB$ is inscribed in arc ACB of a circle with centre 0 . if $\angle ACB = 65°$, find $m(\text{arc}ACB)$:

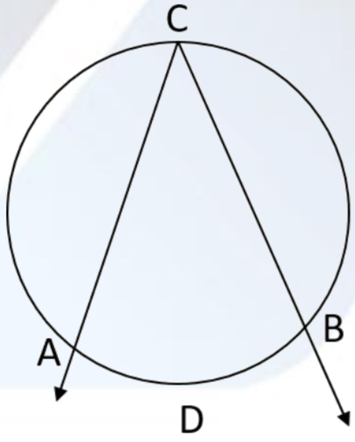
(a) 130°

(b) 295°

(c) 230°

(d) 65°

Solution:



By inscribed angle theorem,

$2\angle \text{ACB} = \text{m}(\text{arcADB})$

$\Rightarrow \text{m}(\text{arcADB}) = 2 \times 65° = 130°$

$\Rightarrow \text{m}(\text{arcADB}) + \text{m}(\text{arcACB}) = 360°$ ... (Total sum of arc in a circle)

$\Rightarrow$ m(arcACB) $= 360^\circ - 130^\circ = 230^\circ$

Correct option (c)

(iv) $1 + \tan^2\ \theta = ?$

(a) $\text{Sin}^2\theta$

(b) $\sec^2\ \theta$

(c) $\text{Cosec}^2\theta$

(d) $\cot^2\ \theta$

Solution:

By trigonometric identities we know that,

$1 + \tan^2\ \theta = \sec^2\ \theta$

Correct option (b)

Q2. (B) Solve the following questions (Any two):

(i) Construct tangent to a circle with centre A and radius 3.4 cm at any point P on it.

Solution:

a. Take 3.4 cm in compass and draw a circle with center A.

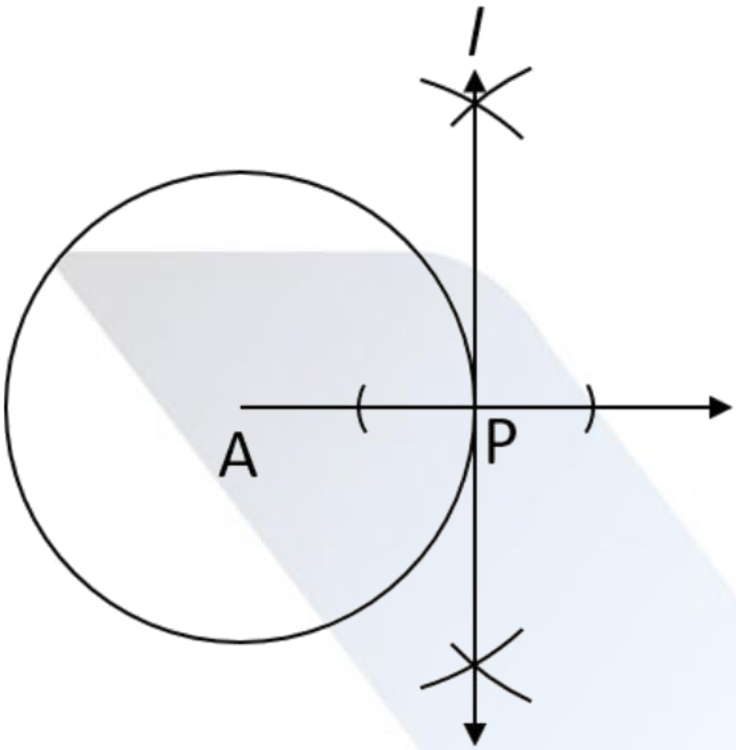
b. Take any point P on it.

c. Draw a ray AP and extend it.

d. Now take an appropriate measurement in compass and make arcs on both sides of P.

e. From this arcs make arcs above and below and get intersection of the arcs.

f. Join the intersection of arcs, and extend them, this is the required tangent.



(ii) Find slope of a line passing through the points $A(3,1)$ and $B(5,3)$.

Solution:

The slope of line passing through points $(\mathbf{x_1}, \mathbf{y_1})$ and $(\mathbf{x_2}, \mathbf{y_2})$ is given by slope $= \frac{y_2 - y_1}{x_2 - x_1}$

So, slope of $\boldsymbol{AB}$ is given by

slope $= \frac{\mathbf{3-1}}{\mathbf{5-3}} = \frac{\mathbf{2}}{\mathbf{2}} = \mathbf{1}$

(iii) Find the surface area of a sphere of radius 3.5 cm .

Solution:

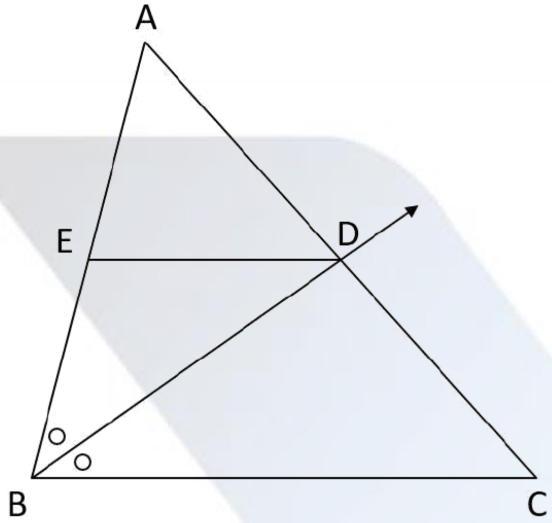
Surface area of sphere is given by $4\pi r^2$.

$\mathrm{r} = 3.5 \mathrm{~cm}$

$\mathrm{S.A.} = 4 \times \frac{22}{7} \times 3.5^2 = 154 \mathrm{~cm}^2$

Q3. (A) Complete the following activities (Any two) :

(i)



In △ ABC, ray BD bisects ∠ABC.

If $A-D-C, A-E-B$ and seg ED || side BC, then prove that:

$$\frac{AB}{BC}=\frac{AE}{EB}$$

Proof:

In △ ABC, ray BD is bisector of ∠ABC.

$\therefore \frac{AB}{BC}=\frac{\ldots\ldots.}{\ldots\ldots..}$ (I) (by angle bisector theorem)

In △ ABC, seg DE|| side BC.

$\therefore \frac{AE}{EB}=\frac{AD}{DC}$ (II) □

$\therefore \frac{AB}{\square}=\frac{\square}{EB}$ … ….

(From I and II)

Solution:

Proof:

In △ ABC, ray BD is bisector of ∠ABC.

$\therefore \frac{AB}{BC}=\frac{AD}{DC}$ (I) (by angle bisector theorem)

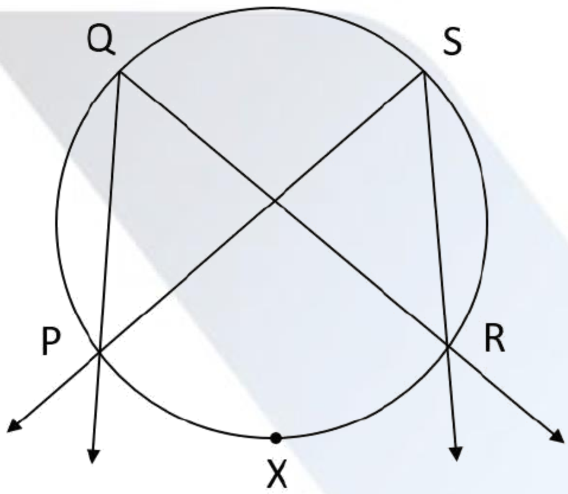
In △ ABC, seg DE||| side BC .

$\therefore \frac{AE}{EB}=\frac{AD}{DC}$ (II) by Basic proportionality theorem

$\therefore \frac{AB}{BC} = \frac{AE}{EB} \ldots \ldots$

(From I and II)

(ii)



Prove that, angles inscribed in the same arc are congruent.

Given: ∠PQR and ∠PSR are inscribed in the same arc.

Arc PXR is intercepted by the angles

To prove:

∠PQR ≅ ∠PSR

Proof

$$m\angle PQR = \frac{1}{2}\ m(arcPXR) \ldots \ldots . . (I)\ \square$$

$$m\angle \square = \frac{1}{2}\ m(arcPXR) \ldots \ldots\ (II)\ \square$$

$\therefore m\angle \square = m\angle PSR$ (from I and II)

$\therefore \angle PQR \cong \angle PSR$ (Angles equal in measure are congruent)

Solution:

Proof

$$m\angle PQR = \frac{1}{2}\ m(arcPXR) \ldots \ldots . . (I) \text{ inscribed angle theorem}$$

$$m\angle PSR = \frac{1}{2}\ m(arcPXR) \ldots \ldots\ (II) \text{ inscribed angle theorem}$$

$\therefore m\angle PQR = m\angle PSR$ (from I and II)

$\therefore \angle PQR \cong \angle PSR$ (Angles equal in measure are congruent)

(iii) How many solid cylinders of radius 6 cm and height 12 cm can be made by melting a solid sphere of radius 18 cm ?

Activity: Radius of the sphere, r = 18 cm

For cylinder, radius R = 6 cm, height H = 12 cm

$\therefore$ Number of cylinders can be made $= \frac{\text{Volume of the sphere}}{\square}$

$$= \frac{\frac{4}{3}\pi r^3}{\square}$$

$$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{\square}$$

$$= \square$$

Solution:

Activity: Radius of the sphere, r = 18 cm

For cylinder, radius R = 6 cm, height H = 12 cm

$\therefore$ Number of cylinders can be made $= \frac{\text{Volume of the sphere}}{\text{Volume of a soild cylinder}}$

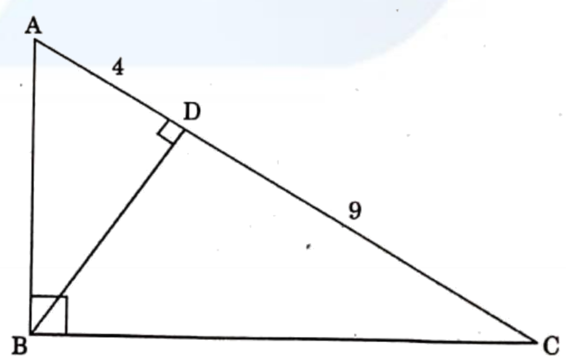
$$= \frac{\frac{4}{3}\pi r^3}{\pi r^2 h}$$

$$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{6 \times 6 \times 12}$$

$= 18$ cylinders

Q3. (B) Solve the following questions (Any two):

(i)



In right-angled $\triangle$ ABC; BD $\perp$ AC.

If AD $= 4$, DC $= 9$, then find BD .

Solution:

We are given a right angled triangle ABC, right angle at B and BD perpendicular to hypotenuse AC.

So, by similarity in right angled triangles we can say that $\triangle ABC \sim \triangle ADB$

$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB} \ldots$ (c.p.c.t.)

$\Rightarrow \frac{\mathrm{AB}}{4} = \frac{\mathrm{BC}}{\mathrm{DB}}$

$\Rightarrow AB = \frac{BC}{DB} \times 4 \ldots (1)$

Also

$\triangle$ ABC $\sim \triangle$ BDC

$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC} \ldots$ (c.p.s.t.)

$\Rightarrow \frac{AB}{BD} = \frac{BC}{9}$

$\Rightarrow \frac{\frac{BC}{DB}\times 4}{BD} = \frac{BC}{9} \ldots$ (from 1)

$\Rightarrow \frac{BC \times 4}{\mathrm{BD}^2} = \frac{\mathrm{BC}}{9}$

$\Rightarrow 4 \times 9 = \mathrm{BD}^2$

$\Rightarrow BD = 2 \times 3 = 6$

(ii) Verify whether the following points are collinear or not :

$A(1, -3), B(2, -5), C(-4,7)$

Solution:

If the sum of any two distances out of $d(A, B), d(B, C)$ and $d(A, C)$ is equal to the third, then the three points A, B and C are collinear.

Therefore, we will find $d(A, B), d(B, C)$ and $d(A, C)$.

Given $A(1, -3), B(2, -5), C(-4,7)$

$\therefore \ d(AB) = \sqrt{(1-2)^2 + (-3+5)^2} = \sqrt{5}$

$\therefore \ d(AC) = \sqrt{(1+4)^2 + (-3-7)^2} = 5\sqrt{5}$

$\therefore \ d(BC) = \sqrt{(2+4)^2 + (-5-7)^2} = 6\sqrt{5}$

So,

$d(BC) = d(AB) + d(AC)$

Therefore A, B and C are collinear

(iii) if $\sec \ \theta = \frac{25}{7}$, then find the value of $\tan \ \theta$

Solution:

Given that

$$\sec \ \theta = \frac{25}{7}$$

Using trigonometric identity

$1 + \tan^2 \ \theta = \sec^2 \ \theta$

we get,

$\tan^2 \ \theta = \sec^2 \ \theta - 1$

$$= \left(\frac{25}{7}\right)^2 - 1$$

$$= \frac{576}{49}$$

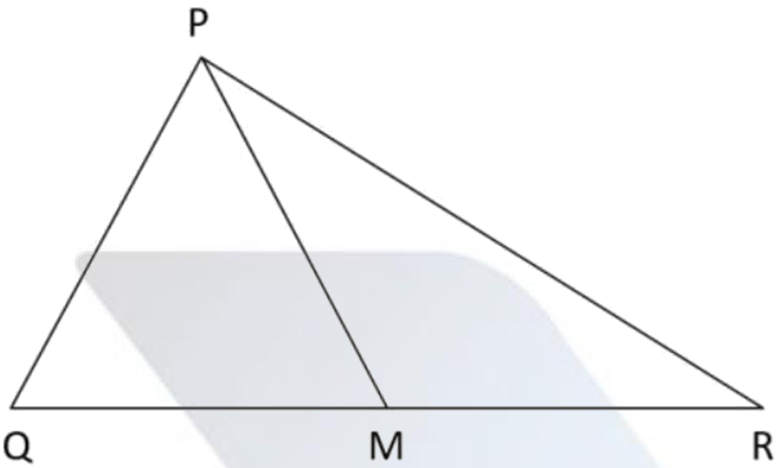
$$\Rightarrow \tan^2 \ \theta = \frac{576}{49}$$

$$\therefore \tan \ \theta = \frac{24}{7}$$

Q4. Solve the following questions (Any three):

(i) In $\triangle$ PQR, seg PM is a median, PM $= 9$ and $PQ^2 + PR^2 = 290$. Find the length of QR .

Solution:



In △ PQR, we have PM as median

QM=MR……(1)

so, by Apollonius theorem we have,

$\mathrm{PQ}^2 + \mathrm{PR}^2 = 2\mathrm{PM}^2 + 2\mathrm{QM}^2$

$\therefore 290 = 2(9)^2 + 2\mathrm{QM}^2$

$\therefore 2Q^2 = 128$

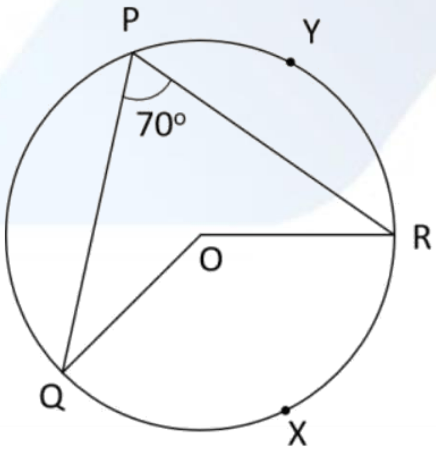
$\therefore \mathrm{QM}^2 = 64$

$\therefore \mathrm{QM} = 8$ units

QR = QM+MR ...From (1)

QR=16 units

(ii)



In the given figure, 0 is centre of circle. $\angle QPR = 70°$ and $m(arc PYR) = 160°$, then find the value of each of the following:

(a) m (arc QXR)

(b) $\angle QOR$

(c) $\angle PQR$

Solution:

Given that

$\angle QPR = 70°$ and $m(arcPYR) = 160°$

(a) $m(arc QXR) = 2\angle QPR$ ... inscribed angle theorem

$m(arc QXR) = 2(70°)$

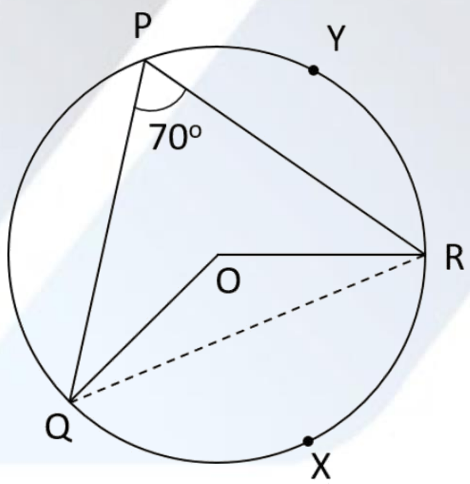
$m(arc QXR) = 140°$

(b) $\angle QOR = m(arcQXR)$ ... measure of arc is equal to the measure of its central angle $\angle QOR = 140°$

(c) $\angle PQR = \frac{1}{2}$ m(arcPYR) ... inscribed angle theorem

$$\angle PQR = \frac{1}{2}(160°)$$

$$\angle PQR = 80°$$



(iii) Draw a circle with radius 4.2 cm . Construct tangents to the circle from a point at a distance of 7 cm from the centre.

Solution:

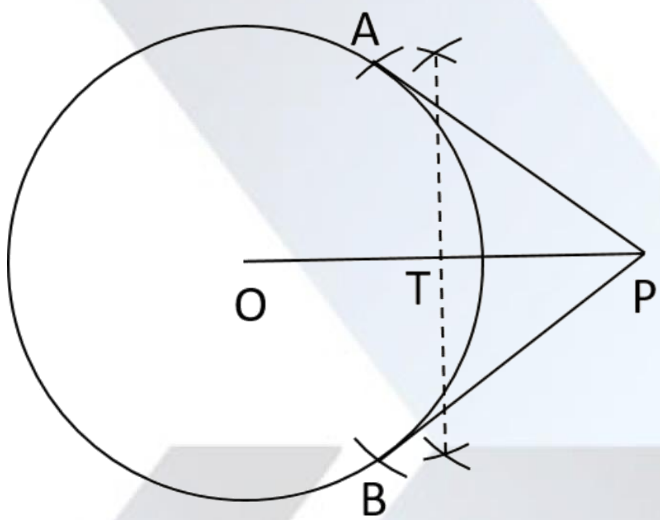
(a) Take 4.2 cm in compass as a radius and draw a circle with center 0 .

(b) Take a point P such that $d(OP) = 7$ cm

(c) Join OP and find its midpoint using perpendicular bisector, name it T .

(d) Take measure OT in compass, keep steel head at T and make 2 arcs on circle, above and below.

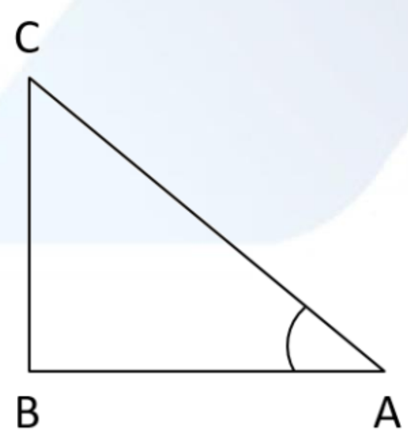
(e) Mark the intersection A and B, then join AP and BP which are the required tangents



(iv) When an observer at a distance of 12 cm m from a tree looks at the top of the tree, the angle of elevation is 60°. What is the height of the tree?

$(\sqrt{3} = 1.73)$

Solution:



Let BC represent the height of tree,

Observer is at A, such that $\angle \mathrm{CAB}=60^{\circ}$

$BA=12 \mathrm{~m}$

So in $\triangle$ ABC

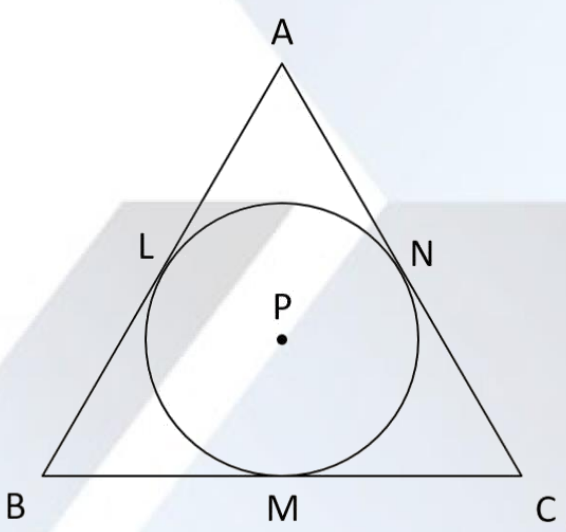
$$\tan 60^{\circ}=\frac{CB}{AB}$$

$$\Rightarrow \sqrt{3}=\frac{CB}{12}$$

$$\Rightarrow \mathrm{CB}=12 \sqrt{3} \mathrm{~m}=20.76 \mathrm{~m}$$

Q5. Solve the following questions (Any one) :

(i)



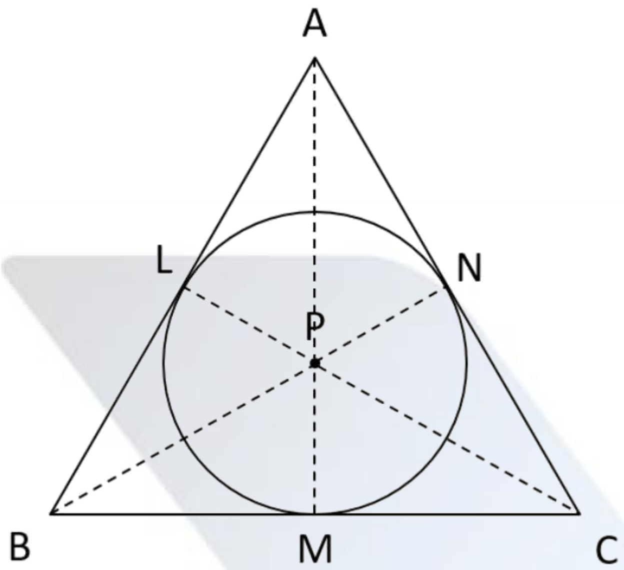
A circle with centre P is inscribed in the ΔABC. Side AB , side BC and side AC touch the circle at points L, M and N respectively. Radius of the circle is r.

Prove that:

$$A(\triangle ABC) \frac{1}{2}(AB+BC+AC) \times r$$

Solution:

Construction: join PA, PL, PN, PC, PM and PB



Now, AB, AC and BC can be considered tangents to the in circle, since it touches the circle at a single point.

PL, PM and PN being the radius of this circle

Also, tangent is perpendicular to radius

Hence,

PL ⊥ AB

PN ⊥ AC

PM ⊥ BC

Now in △ APB,

$$\text{Area}\ (\triangle\ \text{APB}) = \frac{1}{2} \times \text{PL} \times \text{AB}$$

Similarly in △ APC and △ BPC, we can say

Area (△ APC) $= \frac{1}{2} \times r \times AC$..

Area $(\triangle\ BPC) = \frac{1}{2} \times r \times BC$..

Adding (1), (2) and (3)

Area (△ APB) + Area(△ APC) + Area(△ BPC) =

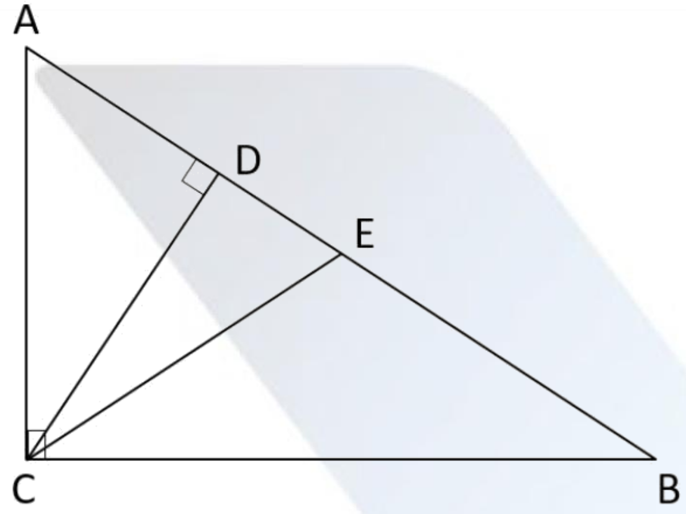
$$\frac{1}{2} \times r \times AB + \frac{1}{2} \times r \times AC + \frac{1}{2} \times r \times BC$$

Now,

Area (△ APB) + Area(△ APC) + Area(△ BPC) = Area(△ ABC)

Area $(\triangle ABC) = \frac{1}{2} \times r \times (AB + AC + BC)$

(ii)



In $\triangle$ ABC, $\angle$ACB $= 90°$. Seg CD $\perp$ side AB and seg CE is angle bisector of $\angle$ACB.

Prove that:

$$\frac{AD}{BD} = \frac{AE^2}{BE^2}$$

Solution:

In $\triangle$ ABC, CD $\perp$ AD

So by the property of similar triangles in a right angled triangle

we can say that

$\triangle$ ACB $\sim \triangle$ ADC

$\frac{AC}{AD} = \frac{CB}{DC} = \frac{AB}{AC}$ ... (c.p.s.t.)

$\Rightarrow \frac{AC}{AD} = \frac{AB}{AC}$

$\Rightarrow AC^2 = AB \times AD$

Also,

$\triangle$ ACB $\sim \triangle$ CDB

$\frac{AC}{CD} = \frac{CB}{DB} = \frac{AB}{CB}$ ... (c.p.s.t.)

$\Rightarrow \frac{CB}{DB} = \frac{AB}{CB}$

$\Rightarrow CB^2 = AB \times DB$

Also, CE is angle bisector hence,

$$\frac{AC}{CB} = \frac{AE}{EB}$$

Squaring on both the sides, we get

$$\therefore \frac{AC^2}{CB^2} = \frac{AE^2}{EB^2}$$

$$\therefore \frac{AB \times AD}{AB \times DB} = \frac{AE^2}{EB^2}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AE^2}{EB^2}$$

Q6. Solve the following questions (Any one) :

(i) Show that the points $(2,0)$, $(-2,0)$ and $(0,2)$ are the vertices of triangle. Also state with reason the type of the triangle.

Solution:

Let A$(2,0)$, B$(-2,0)$ and C$(0,2)$

So by distance formula, we get

$$d(AB) = \sqrt{(2+2)^2 + (0-0)^2} = 4 \text{ cm}$$
$$d(AC) = \sqrt{(2-0)^2 + (0-2)^2} = 2\sqrt{2} \text{ cm}$$
$$d(BC) = \sqrt{(-2-0)^2 + (0-2)^2} = 2\sqrt{2} \text{ cm}$$
$$d(AB) + d(AC) > d(BC)$$
$$d(AB) + d(BC) > d(AC)$$
$$d(BC) + d(AC) > d(AB)$$

As, sum of any two distance is greater than the third, points A, B and C denote a triangle

Also,

$d(AC) = d(BC)$

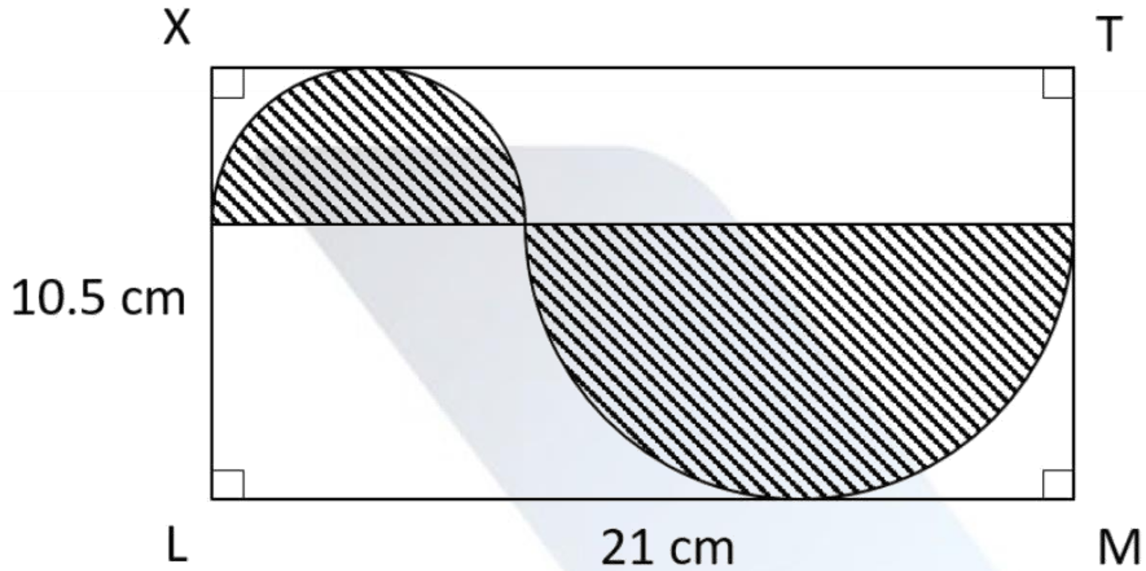
$AB^2 = 16$

$AC^2 + BC^2 = 8 + 8 = 19$

$AB^2 = AC^2 + BC^2$

So, we can say $\triangle$ ABC is isosceles right angled triangle.

(ii)



In the above figure, □ XLMT is a rectangle is a rectangle. LM = 21 cm, XL = 10.5 cm. diameter of the smaller semicircle is half the diameter of the larger semicircle. Find the area of non-shaded region.

Solution:

Let the diameter of smaller circle be d .

then the diameter of larger circle will be 2d.

hence

d + 2 d = 21

⇒ d = 7 cm

Hence area of smaller semi-circle is given by

$$\begin{aligned} \text{Area } &= \frac{\pi}{8} d^2 \\ &= \frac{22}{7} \times \frac{1}{8} \times 7^2 \\ &= 19.25 \text{ cm}^2 \end{aligned}$$

And area of larger semi-circle is given by

$$\text{Area } = \frac{\pi}{8}(2\text{ d})^2$$
$$= \frac{22}{7} \times \frac{1}{8} \times 14^2$$
$$= 77\text{ cm}^2$$

Now, area of non-shaded region = Area of rectangle - Area of shaded region

$\therefore$ Area of shaded region $= \mathrm{l} \times \mathrm{b} - (\text{Area of two semi circles})$

$= 220.5 - (19.25 + 77) \ldots (1 \times b = 21 \times 10.5 = 220.5)$

$= 124.25\text{ cm}^2$