PART - I

Grade 10 Maharashtra Math 2020

General Instructions:

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

(iii) The numbers to the right of the questions indicate full marks.

(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.

(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

Q1. (A) For every sub question 4 alternative answers are given. Choose the correct answer and write the alphabet of it:

(i) In the formal of GSTIN there are ______ alpha-numerals.
(A) 15
(B) 10
(C) 16
(D) 9
Solution: (A) 15 alpha-numerals

(ii) From the following equations, which one is the quadratic equation?
(A) $\frac{5}{x} - 3 = x^2$
(B) $x(x+5) = 4$
(C) $n - 1 = 2n$
(D) $\frac{1}{x^2}(x+2) = x$
Solution: (B)$x(x+5) = 4$

(iii) For simultaneous equations in variables x and y, if $D_x = 49, D_y = -63, D = 7$, then what is the value of x ?
(A) 7
(B) -7

(C) $\frac{1}{7}$

(D) $-\frac{1}{7}$

Solution: (A) 7

(iv) If $n(A) = 2, P(A) = \frac{1}{5}$, then $n(S) = ?$

(A) $\frac{2}{5}$

(B) $\frac{5}{2}$

(C) 10

(D) $\frac{1}{3}$

Solution: (C) 10

(B) Solve the following sub questions:

(i) Find second and third term of an A.P. whose first term is -2 and common difference is -2.

Solution:

Given,

First term, $a = -2$

Common difference, $d = -2$

We know that Second term $= a + d$

$= -2 + (-2)$

$= -4$

And Third Term $= a + 2\,d$

$= -2 + 2(-2)$

$= -2 - 4$

$= -6$

∴ The second term is -4 and third term is -6.

(ii) Pawan Medicals supplies medicines. On some medicines the rate of GST is 12%, then what is the rate of CGST and SGST?

Solution:

Rate of CGST = 6%

Rate of SGST = 6%

(iii) Find the values of a and b from the quadratic equation $2x^2 - 5x + 7 = 0$.

Solution:

The given quadratic equation is $2x^2 - 5x + 7 = 0$.

Comparing the given quadratic equation with

$ax^2 + bx + c = 0$

$\therefore$ The values of $a = 2$ and $b = -5$

(iv) If $15x + 17y = 21$ and $17x + 15y = 11$, then find the value of $x + y$.

Solution: The given equations are

$15x + 17y = 21 \ldots (1)$

$17x + 15y = 11 \ldots (2)$

Adding equations (1) and (2)

$15x + 17y = 21$

$17x + 15y = 11$

$32x + 32y = 32$

Dividing both sides by 32, we get

$x + y = 1$

Q2. (A)Complete and write any two activities from the following:

(i) Complete the following table to draw the graph of $2x - 6y = 3$:

X	-5	☐
Y	☐	0
(x, y)	☐	☐

Solution:

X	-5	$\frac{3}{2}$
Y	$-\frac{13}{6}$	0
(x, y)	$-5, -\frac{13}{6}$	$\frac{3}{2}, 0$

(ii) First term and common difference of an A.P. are 6 and 3 respectively. Find S_{27}.

Solution:

First term $= a = 6$, common difference $= d = 3$,

$S_{27} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d] -$ Formula

$S_{27} = \frac{27}{2}[12 + (27-1)3]$

$= \frac{27}{2} \times [90]$

$= 27 \times 45$

$S_{27} = 1215$

(iii) A Card is drawn from a well shuffled pack of 52 playing cards. Find the probability of the event, the card drawn is a red card.

Solution:

Suppose ' S ' is sample space.

$\therefore n(S) = 52$

Event A: Card drawn is a red Card.

$\therefore$ Total Red Cards = 13 hearts +13 diamonds

$\therefore n(A) = 26$

$\therefore p(A) = \frac{n(A)}{n(S)} -$ Formula

$\therefore p(A) = \frac{26}{52}$

$\therefore p(A) = \frac{1}{2}$

(B) Solve any four sub questions from the following:

(i) Find the value of the determinant:

$$\begin{vmatrix} \frac{7}{5} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$$

Solution:

$$\begin{vmatrix} \frac{7}{5} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix} = \frac{7}{10} - \frac{5}{2}$$

$$= \frac{7 - 5(5)}{10} = \frac{-18}{10} = \frac{-9}{5}$$

(ii) Solve the quadratic equation by factorisation method: $x^2 - 15x + 54 = 0$.

Solution:

The given quadratic equation is

$x^2 - 15x + 54 = 0$

$\Rightarrow x^2 - x - 6x + 54 = 0$

$\Rightarrow x(x-9) - 6(x-9) = 0$

$\Rightarrow (x-9)(x-6) = 0$

$\Rightarrow (x-9) = 0$ or $((x-6) = 0$

$\therefore x = 9$ or $x = 6$

$\therefore$ 9 and 6 are the roots of the given quadratic equation.

(iii) Decide whether the following sequence is an A.P. if so, find the 20^{th} term of the progression:

$-12, -5, 2, 9, 16, 23, 30, \ldots$

Solution:

Here $a = t_1 =$ first term $= -12, t_2 = -5,$

Common difference $= d_d = t_2 - t_1$

$d = -5 - (-12)$

$= -5 + 12$

$\therefore d = 7$

We know that $t_n = a + (n-1)d$

Here, $n = 20, a = -12, d = 7$

$t_{20} = -12 + (20-1)7$

$= -12 + 133$

$t_{20} = 121$

$\therefore 20^{\text{th}}$ term of the progression is 121.

(iv) A two-digit number is formed with digits 2,3,5,7, 9 without repetition. What is the probability that the number formed is an odd number?

Solution:

Simple space S: to form two-digit number from 2, 3, 5, 7, 9

$\therefore S = [23,25,27,29,32,35,37,39,52,53,27,59,72,73,75,79,92,93,95,97]$

$\therefore n(S) = 20$

Event A: Number formed is an even number are such that whose unit place is 0,2,4,6,8. One of these numbers should be in even number.

In the given numbers 2,3,5,7,9 only 2 is the even number whose unit place is 2.

$\therefore$ Sample space of even numbers $= [32,52,72,92]$

$\therefore n$ (Even numbers) $= 4$

$\therefore n$ (Odd numbers) $= n(S) - 4$

$= 20 - 4$

$= 16$

$\therefore n(A) = 16$

$p(A) = \frac{n(A)}{n(S)}$

$= \frac{16}{20} = \frac{4}{5}$

(v) If $L = 10, f_1 = 70, f_0 = 58, f_2 = 42, h = 2$, then find the mode by using formula.

Solution:

$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 10 + \left[\frac{70-58}{2(70)-58-42}\right] \times 2$

$= 10 + \left[\frac{12}{140-100}\right] \times 2$

$= 10 + \frac{24}{40}$

$= 10 + \frac{3}{5}$

$= \frac{50+3}{5}$

$= \frac{53}{5}$

$\text{Mode} = \frac{53}{5}$

Q3. (A) Complete and write any one activity from the following:

(i)

Age group (in years)	No. of persons	Measure of Central Angle
$20 - 25$	80	$\frac{\quad}{200} \times 360° =$
$25 - 30$	60	$\frac{60}{200} \times 360° =$
$30 - 35$	35	$\frac{35}{200} \times \quad = 63$
$35 - 40$	25	$\frac{25}{200} \times 360° =$
Total	200	

Solution:

Age group (in years)	No. of persons	Measure of Central Angle
$20-25$	80	$\frac{80}{200}\times 360^\circ = 144^\circ$
$25-30$	60	$\frac{60}{200}\times 360^\circ = 108^\circ$
$30-35$	35	$\frac{35}{200}\times 360^\circ = 63^\circ$
$35-40$	25	$\frac{25}{200}\times 360^\circ = 45^\circ$
Total	200	

(ii) Shri Shantilal has purchased 150 shares of EV ₹ 100, for MV of ₹ 120, company has paid dividend at 7%, then to find the rate of return on his investment, complete the following activity:

Solution:

Shantilal investment $=$ (No. of shares $\times$ MV) $= 150 \times 120 =$ Rs.18, 000

Dividend per share $=$ 7% of 100$= \frac{7}{100} \times 100 = 7$

Total dividend $= 7 \times 150=$ Rs. 1050

Rate of return $= \frac{1050}{18{,}000} \times 100 = 5.83$

$\therefore$ rate of return will be 5.83%.

(B) Attempt any two sub questions from the following:

(i) A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets:

1. a red balloon.   2. a blue balloon.

Solution:

Available balloons are 2 red, 3 blue and 4 green.

Sample space S : one balloon to be choose on random basis,

$\therefore n(S) = 2 + 3 + 4 = 9$

Event A: Probability that a red balloon is chosen.

$\therefore n(A) = 2$

$\therefore p(A) = \frac{n(A)}{n(S)}$

$p(A) = \frac{2}{9}$

Event B: Probability that a blue balloon is chosen.

$\therefore n(B) = 3$

$\therefore p(B) = \frac{n(B)}{n(S)}$

$\frac{3}{9} = \frac{1}{3}$

$\therefore p(B) = \frac{1}{3}$

Probability that a red balloon is chosen is $\frac{2}{9}$ and probability that a blue balloon is chosen is $\frac{1}{3}$.

(ii) The denominator of fraction is 4 more than twice its numerator. Denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6, find the fraction.

Solution:

Suppose numerator is x , then denominator will be $2x + 4$

$\therefore$ Fraction is $\frac{x}{2x+4}$

According to the given information we can write,

$\frac{x-6}{(2x+4)-6} = \frac{1}{12}$

$\therefore \frac{x-6}{2x-2} = \frac{1}{12}$

$\therefore 12(x-6) = 2x - 2$

$\therefore 12x - 72 = 2x - 2$

$\therefore 12x - 2x - 72 + 2 = 0$

$\therefore 10x - 70 = 0$

$\therefore x = \frac{70}{10} = 7$

$\therefore x = 7$

But fractions

$\frac{x}{2x+4} = \frac{7}{2(7)+4}$

$= \frac{7}{14+4} = \frac{7}{18}$

$\therefore$ The faction is $\frac{7}{18}$.

(iii) A milk center sold milk to 50 customers. The table below gives the number of customers and the milk they purchased. Find the mean of the milk sold by direct method:

Milk sold (liter)	No. of customers
1 – 2	17
2 – 3	13
3 – 4	10
4 – 5	7
5 – 6	3

Solution:

Class (Milk sold in litres)	**Class Mark x_i**	**Frequency (Number of customers) f_i**	**Class mark × Frequency $f_i x_i$**
1 – 2	1.5	17	25.5
2 – 3	2.5	13	32.5
3 – 4	3.5	10	35
4 – 5	4.5	7	31.5
5 – 6	5.5	3	16.5
Total	–	$\sum f_i = 50$	$\sum f_i x_i = 141$

Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$= \dfrac{141}{50}$

$= 2.82$ litres

Hence, the mean of the milk sold is 2.82 litres.

(iv) In an A.P. sum of three consecutive terms is 27 and their products is 504. Find the terms. (Assume that three consecutive terms in an A.P. are $a-d, a, a+d$.)

Solution:

Assume that the three consecutive terms are $a-d, a,$ and a +d

According to first condition,

$(a-d)+a+(a+d)=27$

$\therefore\ 3a=27$

$\therefore\ a=9$

According to second condition,

$(a-d)(a)(a+d)=504$

Putting the value of $a=9$ in above equation, we get

$\therefore (9-d)(9)(9+d)=504$

$\therefore (9^2-\mathrm{d}^2)\times 9=504$

$\therefore (81-\mathrm{d}^2)=56$

$\therefore 81-\mathrm{d}^2=56$

$\therefore \mathrm{d}^2=81-56$

$\therefore \mathrm{d}^2=25$

$\therefore d=5$

$\therefore$ First term $=a-d=(9-5)=4$

Second term $=a=9$

Third term $=a+d=9+5=14$

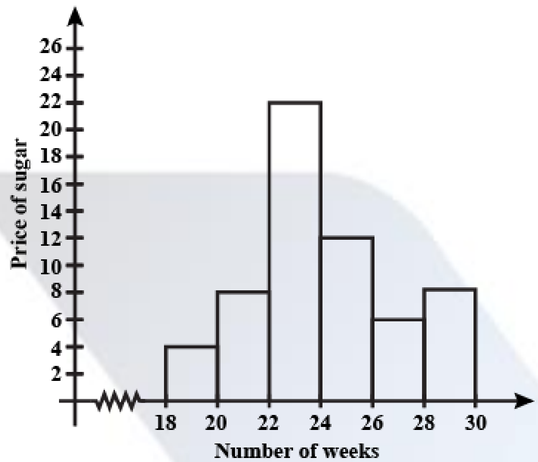
$\therefore$ the three terms are 4,9,14.

Q4. Complete and write any two activity from the following:

(i) Represent the following data by histogram:

Price of Sugar (per kg in ₹)	Number of weeks
18 – 20	4
20 – 22	8
22 – 24	22
24 – 26	12
26 – 28	6
28 – 30	8

Solution:



(ii) One person borrows ₹4,000 and agrees to repay with a total interest of ₹500 in 10 installments. Each installment being less than the preceding installment by ₹ 10. What should be the first and the last installments?

Solution:

Number of installments, $n = 10$

Let the first installment be ₹A

As per the given data each further installment is less than the preceding one by ₹10.

∴ These installments are in A.P.

∴ First term $= a$

And common difference, $d = -10$

Here the negative sign indicates that the next term of A.P. is less than that the preceding term.

∴ Repayment of loan is a below:

∴ $S_n =$ Loan + Total interest

∴ $S_n = 4000 + 500$

∴ $S_n = 4500$

Here $n = 10$

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ $4500 = \frac{10}{2}[2a + (10-1)(-10)]$

∴ $4500 = 5[2a - 90]$

∴ $4500 = 10a - 450$

∴ $10a = 4500 + 450$

∴ $a = \frac{4950}{10} = 495$

∴ first installment $= a =$ ₹495
The last installment is the 10^{th} installment.
$\therefore n = 10$
We know that, $a_n = a + (n-1)d$
Here $n = 10, a = 495, d = -10$
$\therefore a_{10} = 495 + (10-1)(-10) = 495 - 90$
$a_{10} = 405$
Last installment $= a_{10} =$ ₹405.
∴ First installment is ₹495 and the last installment is ₹405.

(iii) The sum of the areas of two squares is 400 sq.m. if the difference between their perimeters is 16 m, find the sides of two square.

Solution:

Let the side of first square be x meter and the side of second square be y meter
As per the first given condition,
$\therefore x^2 + y^2 = 400 \ldots (i)$
As per the second given condition,
$4x - 4y = 16$
$\therefore x - y = 4$
$\therefore x = y + 4 \ldots (ii0$
Put the value of $x = y + 4$ in equation (i), we get
$\therefore (y+4)^2 + y^2 = 400$
$\therefore y^2 + 8y + 16 + y^2 = 400$
$\therefore 2y^2 + 8y + 16 - 400 = 0$
$\therefore 2y^2 + 8y - 384 = 0$
Dividing both sides by 2 we get
$y^2 + 4y - 192 = 0$
$\therefore y^2 + 16y - 12y - 12 = 0$
$\therefore y(y+16) - 12(y+16) = 0$
$\therefore (y+16)(y-12) = 0$
$\therefore (y+16) = 0$ or $(y-12) = 0$
$\therefore\ y = -16$ or $y = 12$
But side of square is never negative.
$\therefore y \neq -16$
$\therefore y = 12$
Putting the value of $y = 12$ in equation (ii), we get
$x = 12 + 4 = 16$
∴ Side of first square, $x = 16\ m$ and side of second square, $y = 12\ m$.

Q5. Attempt any one sub question from the following:

(i) Convert the following equations into simultaneous equations and solve:

$\sqrt{\frac{x}{y}} = 4, \qquad \frac{1}{x} + \frac{1}{y} = \frac{1}{xy}$

Solution:

$\sqrt{\frac{x}{y}} = 4$

Squaring on both sides, we get

$\frac{x}{y} = 16$

$\therefore x = 16y$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{xy}$

Multiplying both sides by xy, we get

$y + x = 1$

i.e., $x + y = 1$

Substituting $x = 16y$ in equation (ii), we get

$16y + y = 1$

$\therefore 17y = 1$

$\therefore y = \frac{1}{17}$

Substituting $y = \frac{1}{17}$ in equation (i), we get

$x = 16y = \frac{16}{17}$

$\therefore (x, y) = \left(\frac{16}{17}, \frac{1}{17}\right)$ is the solution of the given equations.

(ii) A dealer sells a toy for ₹ 24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.

Solution:

Selling price of the toy = ₹24

Let the cost price of the toy be ₹x

Gain % = x% (Given)

$$\text{Gain } \% = \left(\frac{\text{Selling price-Cost price}}{\text{Cost Price}}\right) \times 100$$

$$\therefore x = \left(\frac{24 - x}{x}\right) \times 100$$

$\therefore x^2 = 2400 - 100x$

$\therefore x^2 + 100x - 2400 = 0$

$\therefore x^2 + 120x - 20x - 2400 = 0$

$\therefore x(x + 120) - 20(x + 120) = 0$

$\therefore (x + 120)(x - 20) = 0$

$\therefore (x + 120) = 0$ or $(x - 20) = 0$

$\therefore x = -120$ or $x = 20$

$x \neq -120$, because cost cannot be negative

$\therefore x = 20$

∴ Cost price = ₹ 20

∴ The cost price of the toy is ₹ 20

PART - II

Grade 10 Maharashtra Math 2020

General Instructions:

(i) All questions are compulsory.

(ii) Use of calculator is not allowed.

(iii) The numbers to the right of the questions indicate full marks.

(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.

(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

Q1. (A) Four alternative answer are given for every sub - question. Select the correct alternative and write the alphabet of that answer:

(i) Out of the following which is the Pythagorean triple?

(A) (1,5,10)

(B) (3,4,5)

(C) (2,2,2)
(D) (5,5,2)
Solution: (B) (3, 4, 5)

(ii)Two circles of radii 5.5 cm and 3.3 cm respectively touch each other externally. What is the distance between their centres?
(A) 4.4 cm
(B) 2.2 cm
(C) 8.8 cm
(D) 8.9 cm
Solution: (C) 8.8 cm

(iii) Distance of point $(-3,4)$ from the origin is:
(A) 7
(B) 1
(C) -5
(D) 5
Solution: (D) 5

(iv) Find the volume of a cube of side 3 cm:
(A) $27 cm^3$
(B) $9\ cm^3$
(C) $81\ cm^3$
(D) $3 cm^3$
Solution: (A) $27 cm^3$

(B) Solve the following questions:

(i) The ratio of corresponding sides of similar triangles is 3: 5, then find the ratio of their areas.

Solution:

Ratio of areas of similar triangle = (Ratio of corresponding sides of similar triangle)^2

$$= \frac{3^2}{5^2}$$

Ratio of their areas $= \frac{9}{25}$

(ii) Find the diagonal of a square whose side is 10 cm.

Solution:

Let □ ABCD is a square

$l(\mathrm{AB}) = l(\mathrm{BC}) = l(\mathrm{CD}) = l(\mathrm{AD}) = 10\ cm (Given)$

In $\triangle ABC$,

$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$ (Pythagoras theorem)

$\therefore\ \mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{AB}^2\ (\because AB = BC)$

$\therefore\ \mathrm{AC}^2 = 2\mathrm{AB}^2$

$\therefore\ AC = \sqrt{2}AB$

$= \sqrt{2}(10)cm(AB = 10\ cm)$

$\therefore\ AC = 10 \times 1.414 = 14.14\ cm$

$\therefore$ Diagonal of the square $AC = 14.14\ cm$

(iii) □ $ABCD$ is cyclic. If $\angle B = 110^\circ$, then find measure of $\angle D$.

Solution:

ABCD is cyclic

$\therefore\quad \mathrm{m}\angle \mathrm{B} + \mathrm{m}\angle \mathrm{D} = 180^\circ$

$\therefore\quad 110^\circ + \mathrm{m}\angle \mathrm{D} = 180^\circ$

$\therefore\quad$ (Given, $\mathrm{m}\angle\ \mathrm{B} = 110^\circ$)

$\therefore\quad \mathrm{m}\angle \mathrm{D} = 180^\circ - 110^\circ$

$\quad \mathrm{m}\angle \mathrm{D} = 70^\circ$

(iv) Find the slope of the line passing through the points $A(2,3)$ and $B(4,7)$.

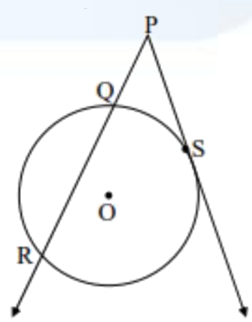
Solution:

Slope of line passing through the points (x_1, y_1) and (x_2, y_2) is given by, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Therefore, the slope of line passing through the points $A(2,3)$ and $B(4,7)$ is

$= \frac{7-3}{4-2} = \frac{4}{2} = 2$

Q2. (A) Complete and write the following activities (Any two):



(i) In the figure given, ' O ' is the centre of the circle, seg PS is a tangent segment and S is the point of contact. Line PR is a secant. If $PQ = 3.6$, $QR = 6.4$, find PS.

Solution:

$PS^2 = PQ \times PR$ … ….. (tangent secant segments theorem)

$= PQ \times (PQ + QR)$
$= 3.6 \times (3.6 + 6.4)$
$= 3.6 \times 10$
$= 36$

$\therefore PS = 6$ …….(by taking square roots)

(ii) If $se\,c\,\theta = \frac{25}{7}$, find the value of $ta\,n\,\theta$.

Solution:

$1 + \tan^2 \theta = \sec^2 \theta$

$$\therefore 1 + \tan^2 \theta = \left(\frac{25}{7}\right)^2$$

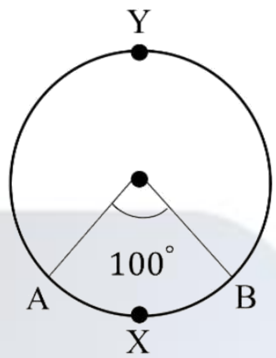
$$\therefore \tan^2 \theta = \frac{625}{49} - 1$$

$$= \frac{625 - 49}{49} = \frac{576}{49}$$

$$\tan \theta = \frac{24}{7} \quad \text{…. by taking square roots}$$

(iii) In the figure given, O is the centre of the circle. Using given information complete the following table:

Type of arc	Name of the arc	Measure of the arc
Minor arc	□	□
Major arc	□	□



Solution:

Type of arc	Name of the arc	Measure of the arc
Minor arc	Arc AXB	100°
Major arc	Arc AYB	260°

(B) Solve the following sub - questions (Any four):

(i) In $\triangle PQR, NM \| RQ$. If $PM = 15, MQ = 10, NR = 8$, then find PN.

Solution:

Given NM || RQ

$\therefore \frac{PN}{NR} = \frac{PM}{MQ}$ (Basic proportionality theorem)

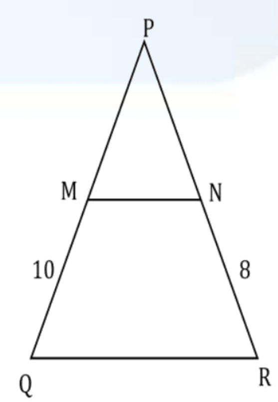
But $PM = 15, MQ = 10, NR = 8$ (Given)

$\therefore$ Equation (i) becomes,

$$\frac{PN}{8} = \frac{15}{10}$$

$$\therefore PN = \frac{15 \times 8}{10} = \frac{15 \times 4}{5} = 3 \times 4$$

$\therefore PN = 12$ Unit



(ii) In $\triangle MNP, \angle MNP = 90^{\circ} segNQ \perp segMP$. If $MQ = 9, QP = 4$, then find NQ.

Solution:

In $\triangle MNP, \angle MNP = 90^{\circ}$, seg NQ ⊥ seg MP

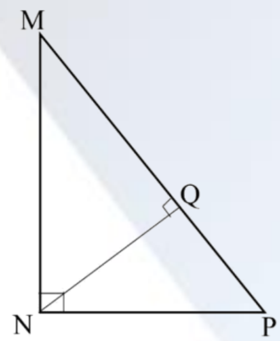
∴ According to right angled triangle geometric mean sub theorem

$N^2 = MQ \times QP$

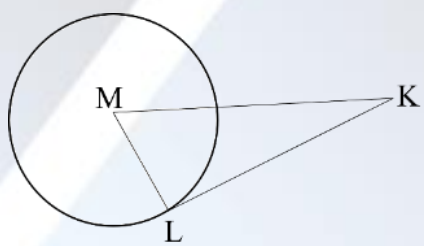
$= 9 \times 4 = 36$

$\therefore NQ = \sqrt{36}$

$= 6$ unit



(iii) In the figure given above, M is the centre of the circle and seg KL is a tangent segment. L is a point of contact. If $MK = 12, KL = 6\sqrt{3}$, then find the radius of the circle.



Solution:

In given figure, radius ML ⊥ tangent Segment KL ... (Tangent theorem)

$\therefore m\angle MLK = 90^{\circ}$

In right - angled $\triangle$ MLK

$MK^2 = ML^2 + LK^2$ (According to Pythagoras theorem)

$\therefore (12)^2 = ML^2 + (6\sqrt{3})^2$

$\therefore 144 = ML^2 + 108$

$\therefore ML^2 = 144 - 108 = 36$

$\therefore ML = 6$

∴ Radius ML $= 6$ unit

(iv) Find the co-ordinate of midpoint of the segment joining the points (22,20) and (0,16).

Solution:

Given points are (22,20) and (0,16)

Let, $x_1 = 22, x_2 = 0, y_1 = 20, y_2 = 16$

We know, Midpoint

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

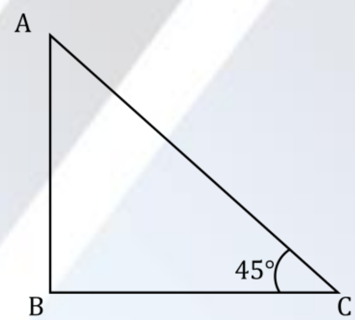
$$= \left(\frac{22 + 0}{2}, \frac{20 + 16}{2}\right)$$

$$= \left(\frac{22}{2}, \frac{36}{2}\right)$$

$$= (11,18)$$

(v) A person is standing at a distance of 80 metres from a Church and looking at its top. The angle of elevation is of 45° . Find the height of the Church.

Solution:



Let, AB be the height of the church.

$\angle ACB = 45°, BC = 80\ m$

In right angled $\triangle ABC$, we have,

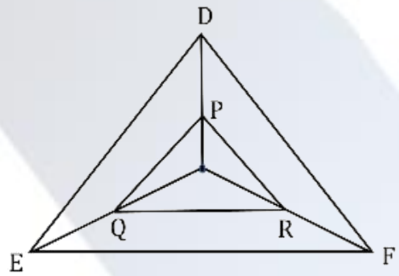
$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{AB}{80}$$

$$\Rightarrow AB = 80\ m$$

Q3. (A) Complete and write the following activities (Any one):

(i) In the given figure, X is any point in the interior of the triangle. Point X is joined to the vertices of triangle. seg PQ || seg DE, seg QR || seg EF. Complete the activity and prove that seg PR || seg DF.



Proof: In $\triangle XDE$

PQ | DE ..... (Given)

$\therefore \frac{xP}{PD} = \frac{\square}{QE}$ ...... (Basic proportionality theorem) .... (i)

$\therefore \frac{XP}{PD} = \frac{\square}{\square}$ .... [From (i) and (ii)

.... [From (i) and (ii)]

∴ Seg PR || seg DF .. (By converse of basic proportionality throrem)

Solution:

In $\triangle XEF$

QR || EF .... (Given)

$\therefore \frac{XQ}{QE} = \frac{XR}{\underline{RF}}$ .... (Basic proportionality theorem)

$\therefore \frac{XP}{PD} = \frac{XR}{\underline{RF}}$ .... [[From (i) and (ii)]

∴ Seg PR || seg DF ... (By converse of basic proportionality theorem)

(ii) If $A(6,1), B(8,2), C(9,4)$ and $D(7,3)$ are the vertices of □ $ABCD$, show that □ $ABCD$ is a parallelogram.

Solution:

Given A (6,1), B (8,2), $C(9,4)$ and D (7,3)

$AB = \sqrt{(8-6)^2 + (2-1)^2}$ $\left[\because \text{ Distance } = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{2^2 + 1^2} = \sqrt{5}$

$BC = \sqrt{(9-8)^2 + (4-2)^2}$

$= \sqrt{1^2 + 2^2} = \sqrt{5}$

$CD = \sqrt{(7-9)^2 + (3-4)^2}$

$= \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$DA = \sqrt{(7-6)^2 + (3-1)^2}$

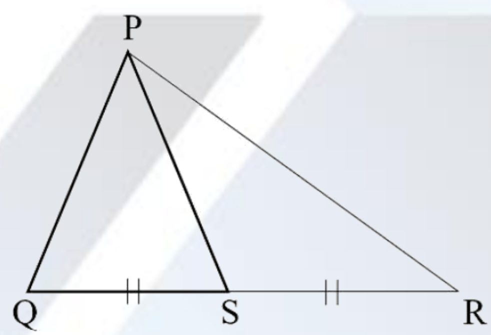
$= \sqrt{1^2 + (2)^2} = \sqrt{5}$

$\therefore AB = BC = CA = DA$

Hence, ABCD is a parallelogram.

(B) Solve the following sub - questions (Any two):

(i) If $\triangle PQR$, point S is the mid - point of side QR. If $PQ = 11, PR = 17, PS = 13$, find QR



Solution:

In $\triangle PQR$, point S is the mid - point of side QR.

$\therefore$ Segment PS is median of $\triangle PQR$

According to Apollonius's theorem

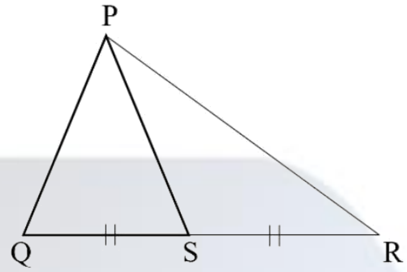
$PQ^2 + PR^2 = 2PS^2 + 2QS^2$

As per given values,

$\therefore (11)^2 + (17)^2 = 2(13)^2 + 2QS^2$

$\therefore 121 + 289 = 2(169) + 2\,QS^2$

$\therefore 410 = 338 + 2\,QS^2$



$\therefore 2Q^2 = 410 - 338 = 72$

$\therefore QS^2 = \frac{72}{2} = 36$

$\therefore QS = 6$ unit .... (i)

We know, point S is the mid - point of side QR

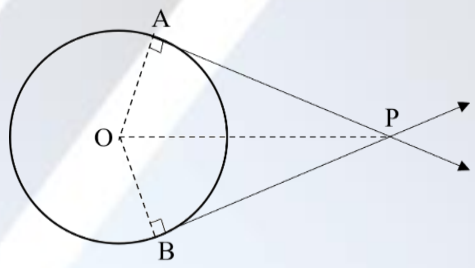
$\therefore 2QS = QR$ ($\because$QS=SR)

$\therefore QR = 2 \times (6)$ [From equation (i)]

$\therefore QR = 12$ unit

$\therefore$ Length of side QR is 12 unit.

(ii) Prove that, tangent segments drawn from an external point to the circle are congruent.



Solution:

Point O is the centre of the circle and point P is external to the circle. Segment PA and segment PB are tangent segments to the circle. Point A and point B are touch points of the tangent segments.

Prove: PA $\cong PB$

Construction: Draw OA, OB and OP.

Proof: $\because$ Each tangent of a circle is perpendicular to the radius drawn through the point of contact .... (Theorem)

$\therefore$ Radius $OA \perp AP$ and, Radius $OB \perp BP$ .....(i)

$\therefore m\angle PAO = 90^\circ$ and $m\angle PBO = 90^\circ$

$\therefore \triangle PAO$ and $\triangle PBO$ are right - angled triangles.

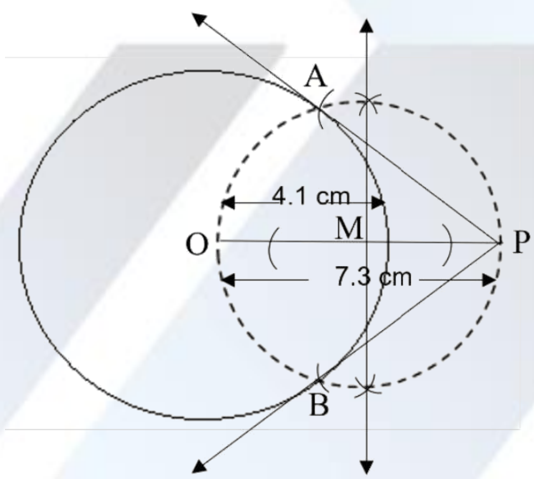
Now in $\triangle$ PAO and $\triangle PBO$,

$OA = OB$ (∵ Radius of same circle)
$\angle PAO = \angle PBO$ [Using (i)]
Hypotenuse OP = Hypotenuse OP (∵ common side)
$\therefore \triangle PAO \cong \triangle PBO$ (RHS conguruency criterion)
∴ line $PA \cong$ line PB (∵ corresponding sides of Congruent triangles)
Line PA and line PB are tangent.
Hence proved.

(iii) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

Solution:

Steps of construction:
Step 1: Draw a circle of radius 4.1 cm with centre O.
Step 2: Take a point P in the exterior of the circle such that $OP = 7.3\ cm$
Step 3: Draw segment OP, draw perpendicular bisector of segment OP to get its midpoint M.



Step 4: Draw a circle with radius OM and centre M.
Step 5: Name the point of intersection of the two circles as A and B.
Step 6: Join PA and PB.
Thus, PA and PB are required tangents.

(iv) A metal cuboid of measures 16 cm 11 cm 10 cm was melted to make coins. How many coins were made, if the thickness and diameter of each coin was 2 mm and 2 cm respectively? $(\pi = 3.14)$

Solution:

Radius of each coin, $r = \frac{2}{2} = 1\ cm$

Thickness of each coin, $h = 2\ mm = \frac{2}{10} = 0.2\ cm (1\ cm = 10\ mm)$

Let the number of coins made be n.

It is given that a metal parallelopiped is melted to make the coins.

$\therefore n \times$ Volume of metal in each coin = Volume of the metal cuboid

$$\Rightarrow n = \frac{\text{Volume of the metal cuboid}}{\text{Volume of metal in each coin}}$$

$$\Rightarrow n = \frac{16 \times 11 \times 10}{\pi r^2 h}$$

$$\Rightarrow n = \frac{16 \times 11 \times 10}{\frac{22}{7} \times 1 \times 1 \times 0.2} = 2800$$

Thus, the number of coins made are 2800.

Q4. Solve the following sub - questions (Any two):

(i) In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that seg $PQ \parallel$ seg BC. If PQ divides $\triangle ABC$ into two equal parts having equal areas, find $\frac{BP}{AB}$.

Solution:

In above figure $\triangle ABC$, $PQ \parallel BC$

$A - P - B$ and $A - Q - C$

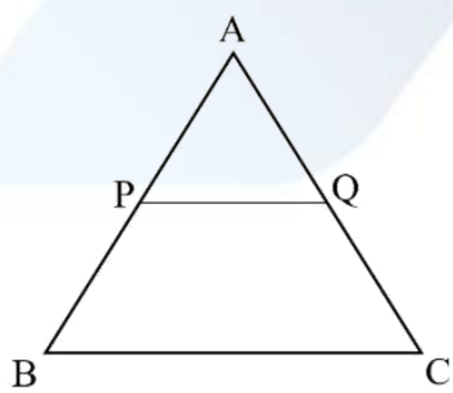
and $ar(\triangle APQ) = ar(\square PBCQ)$

In $\triangle APQ$ and $\triangle ABC$

$\angle A = \angle A$ ..... (common angle)

$\angle APQ = \angle ABC$ .... $(corresponding angle)$

$\therefore \Delta APQ \sim \triangle ABC$ ... ($A - A$ similarity test)



$\therefore \frac{ar(\triangle ABC)}{ar(\triangle APQ)} = \frac{\text{AB}^2}{\text{AP}^2}$ (Theorem of areas of similar triangles) ..... (i)

Now,

ar (△ APQ) = $ar(\square \text{ PBCQ})(Given)$

$\therefore \frac{ar[\square\ PBCQ]}{ar[\triangle\ APQ]} = \frac{1}{1}$

Adding 1 on both sides,

$\therefore \frac{ar[\square\ PBCQ] + ar(\triangle\ APQ)}{ar(\triangle\ APQ)} = \frac{1+1}{1} = \frac{2}{1}$

$\therefore \frac{ar(\triangle\ ABC)}{ar(\triangle\ APQ)} = \frac{2}{1} \ldots (ii)[\because ar(\triangle\ APQ) + ar(\square\ PBCQ) = ar(\triangle\ ABC)]$

∴ From (i) and (ii)

$\therefore \frac{ar(\triangle\ ABC)}{ar(\triangle\ APQ)} = \frac{2}{1} = \frac{\text{AB}^2}{\text{AP}^2}$

$\frac{\text{AB}}{\text{AP}} = \frac{\sqrt{2}}{1}$ (by taking square roots on both sides)

Let $AB = \sqrt{2}x$ .... (iii)

and $AP = 1x$

Now, $BP = AB - AP$

$\therefore BP = \sqrt{2}x - 1x = (\sqrt{2} - 1)x$ .... (iv)

From (iii) and (iv)

$\therefore \frac{\text{BP}}{\text{AB}} = \frac{(\sqrt{2} - 1)}{\sqrt{2}}$

(ii) Draw a circle of radius 2.7 cm and draw a chord PQ of length 4.5 cm. Draw tangents at point P and Q without using centre.

Solution:

Step of construction:

Step 1: Draw a circle of with centre O and radius 2.7 cm

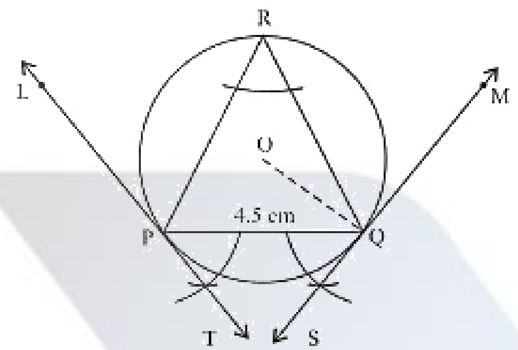
Step 2: Draw a chord PQ of length 4.5 cm

Step 3: Taking a point R on the major arc QP, join PR and QR.

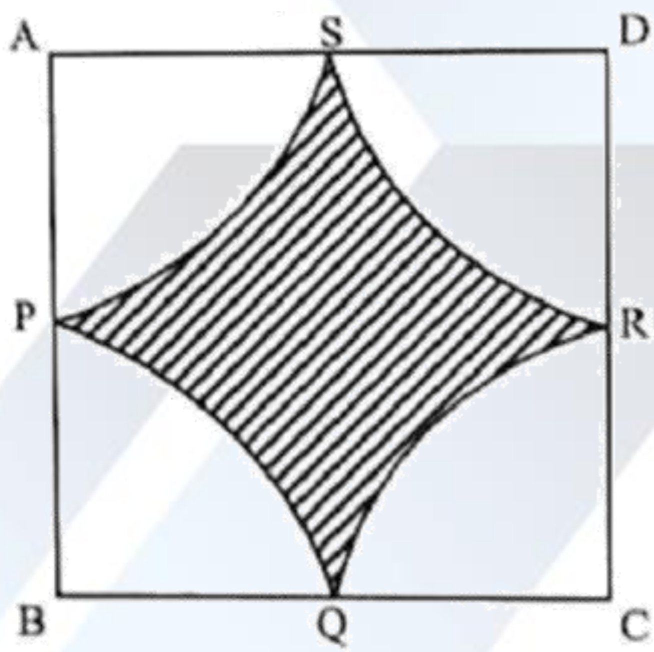
Step 4: Make $\angle QPT = \angle PRQ$ and $\angle PQS = \angle PRQ$.

Step 5: Produce TP to L and SQ to M.

Hence, TPL and SQM are the required tangents.



(iii) In the figure given □ $ABCD$ is a square of side 50 m. Points P, Q, R, S are midpoints of side AB, side BC, side CD, side CD, side AD respectively. Find area of shaded region.



Solution:

Area of square $ABCD = (\text{ side })^2$

$= (50)^2$

$= 2500\ m$

Radius of sector $A - SP = \frac{1}{2} \times 50 = 25\ m$

$\theta = 90^{\circ}$ … … [Angle of a square]

Area of sector $A-SP=\frac{\theta}{360}\times\pi r^2$

$$=\frac{90}{360}\times\frac{22}{7}\times(25)^2$$

$$=\left(\frac{1}{4}\times\frac{13750}{7}\right)m^2$$

A(shaded region) = Area of square $ABCD-4$ (Area of sector $A-SP$)

$$=2500-4\left(\frac{1}{4}\times\frac{13750}{7}\right)$$

$$=2500-\frac{13750}{7}$$

$$=\frac{17500-13750}{7}$$

$$=\frac{3750}{7}$$

$$\approx 535.71\ m^2$$

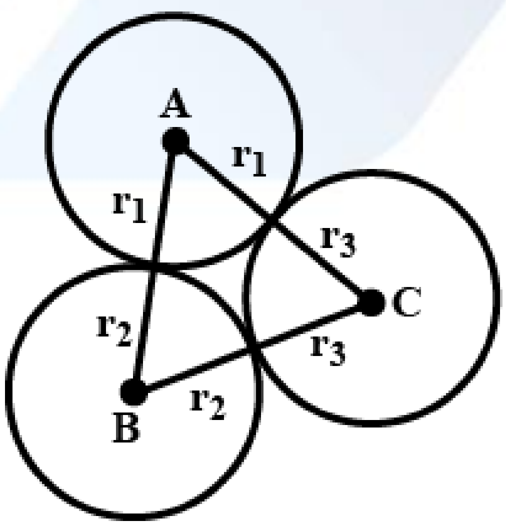
$\therefore$ Area of the shaded region is $535.71\ m^2$.

Q5. Solve the following sub - questions (Any one):

(i) Circles with centres A, B and C touch each other externally. If $AB=3\ cm$, $BC=3cm, CA=4\ cm$, then find the radii of each circle.

Solution:

Suppose radius of circle with centre A is x cm



$\therefore$ Radius of circle with centre $B = (3 - x)cm$ $(\because AB = 3cm)$
and radius of circle with centre $C = (4 - \text{x})cm (\because CA = 4cm)$
$\therefore (3 - x) + (4 - x) = BC = 3$
$\therefore 3 - x + 4 - x = 3$
$\therefore 7 - 2x = 3$
$\therefore 2x = 7 - 3$
$\therefore 2x = 4$
$\therefore x = 2$
$\therefore$ Radius of circle with centre $A = 2\ cm$
$\therefore$ Radius of circle with centre $B = (3 - x)$
$= (3 - 2)$
$= 1$ cm
$\therefore$ Radius of circle with centre $C = (4 - x)$
$= (4 - 2)$
$= 2$ cm

(ii) If $\sin\theta + \sin^2\theta = 1$
Show that: $\cos^2\theta + \cos^4\theta = 1$
Solution:
$\sin\theta + \sin^2\theta = 1$ .... (Given)
But $\sin^2\theta + \cos^2\theta = 1$
$\therefore$ Putting the value 1 in given relation we get.
$\sin\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta$

$\therefore \quad \sin\theta = \cos^2\theta$
Now as per given relation
$\sin\theta + \sin^2\theta = 1$
$\therefore \cos^2\theta + (\cos^2\theta)^2 = 1$
$\therefore \cos^2\theta + \cos^4\theta = 1$
Hence Proved.