Grade10 Maths Maharashtra 2022

PART – I(ALGEBRA)

Note: -

(1) All questions are compulsory.

(2) Use of calculator is not allowed.

Q1. (A) For every sub question four alternative answers are given. Choose the correct answer and write the alphabet of it:

(i) Which one is the quadratic equation?

A) $\frac{5}{x}-3=x^2$

B) $x(x+5)=2$

C) $n-1=2n$

D) $\frac{1}{x^2}(x+2)=x$

Solution:

The general form of a quadratic equation is $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$.

Option A: $\frac{5}{x}-3=x^2 \Rightarrow x^3+3x-5=0$

We can see it is not in the form of $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$.

Hence, it is not a quadratic equation.

Option B: $\mathrm{x}(\mathrm{x}+5)=2 \Rightarrow \mathrm{x}^2+5x-2=0$

We can see it is in the form of $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$, with $\mathrm{a}=1$, $\mathrm{b}=5$, and $\mathrm{c}=-2$.

Hence, it is a quadratic equation.

Option C: $\mathrm{n}-1=2\mathrm{n} \Rightarrow \mathrm{n}+1=0$

We can see it is not in the form of $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$.

Hence, it is not a quadratic equation.

Option D: $\frac{1}{x^2}(x+2)=x \Rightarrow x^3-x-2=0$

We can see it is not in the form of $ax^2+bx+c=0$.

Hence, it is not a quadratic equation.

(ii) First four terms of an A.P. are whose first term is -2 and common difference is -2.

A) $-2,0,2,4$

B) $-2,4,-8,16$

C) $-2,-4,-6,-8$

D) $-2,-4,-8,-16$

Solution:

Let the first four terms be $a, a+d, a+2d$ and $a+3d$.

Given, first term $a=-2$ and common difference, $\mathrm{d}=-2$, then AP would be:

$\mathrm{a}, \mathrm{a}+\mathrm{d}, \mathrm{a}+2\mathrm{~d}$ and $\mathrm{a}+3\mathrm{~d}$

$\Rightarrow -2,-2+(-2),2+2\times(-2),2+3(-2)$

$\Rightarrow -2,-4,-6,-8$

(iii) For simultaneous equations in variable x and y, $D_x = 49$, $D_y = -63$, and $\mathrm{D} = 7$, then what is the value of y ?
A) 9
B) 7
C) -7
D) -9
Solution:
Given, $D_y = -63$, and $D = 7$
We know that, $y = \frac{D_y}{D} = \frac{-63}{7} = -9$

(iv) Which number cannot represent probability?
A) 1.5
B) $\frac{2}{3}$
C) 15%
D) 0.7
Solution:
$\frac{2}{3} = 0.67$, and $15\% = 0.15$
We know that $0 \leq$ Probability of an event ≤ 1.
So, among 1.5,0.67,0.15, and 0.7,1.5 cannot represent probability.

(B) Solve the following subquestions:
(i) To draw a graph of $4x + 5y = 19$, find y when $x = 1$.
Solution:
Given,
Equation of the graph $4x + 5y = 19$
Considering the value of x to be 1 ,
$\Rightarrow 4 \times 1 + 5y = 19$
$\Rightarrow 4 + 5y = 19$
$\Rightarrow 5\mathrm{y} = 19 - 4$
$\Rightarrow y = \frac{15}{5} = 3$
$\therefore y = 3$

(ii) Determine whether 2 is a root of quadratic equation $2m^2 - 5m = 0$.
Solution:
Given quadratic equation,
$2m^2 - 5m = 0$
Substituting $\mathrm{m} = 2$, in $2m^2 - 5m = 0$
$\Rightarrow 2(2)^2 - 5(2) = 0$
$\Rightarrow 8 - 10 = 0$
$\Rightarrow -2 \neq 0$
$\therefore$ We can observe 2 is not a root of the equation.

(iii) Write second and third term of an A.P. whose first term is 6 and common difference is -3.

Solution:

Given,

First term, $a=6$

Common difference, $d=-3$

We know that Second term $=a+d=6+-3=6-3=3$

Third term $=a+2d=6+2\times-3=6+(-6)=6-6=0$

So,

Second term $=3$

Third term $=0$

(iv) Two coins are tossed simultaneously. Write the sample space ' S '.

Solution:

Since 2 coins are tossed the sample space

$\therefore \mathrm{S}=\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$

Q2. (A) Complete and write any two activities from the following:

(i) Complete the activity to find the value of the determinant.

$$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$$

$=2\sqrt{3}\times\_\_\_ \quad -9\times\_\_\_$

$=\_\_\_\_-18$

$=0$

Solution:

$$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$$

$=2\sqrt{3}\times3\sqrt{3}-9\times2$

$=18-18$

$=0$

(ii) Complete the activity to find the 19th term of an A.P.: 7, 13, 19, 25.

Given A.P.: 7, 13, 19, 25,

Here first term $a=7$; $t_{19}=?$

$t_n=a+(_)$d .(formula)

$\therefore t_{19}=7+(19-1)$

$\therefore t_{19}=7+$

$\therefore t_{19}=$

Solution:

Given A.P.: 7, 13, 19, 25,

Here first term $a=7$; $t_{19}=?$

$t_n=a+(n-1)d$..(formula)

$\therefore t_{19}=7+(19-1)6$
$\therefore t_{19}=7+108$
$\therefore t_{19}=115$

(iii) If one die is rolled, then to find the probability of an event to get prime number on upper face, complete the following activity.
One die is rolled.
' S ' is the sample space.
S = {__________________}
$\therefore n(\mathrm{~S})=6$
Event A : Prime number on the upper face.
A = {__________________}
$\therefore n(\mathrm{~A})=3$
$P(A)=\frac{-}{n(S)}$. ....(formula)
$\therefore P(A)=$__________________

Solution:
One die is rolled.
' S ' is the sample space.
$S=\{1,2,3,4,5,6\}$
$\therefore n(\mathrm{~S})=6$
Event A : Prime number on the upper face.
$\mathrm{A}=\{2,3,5\}$
$\therefore n(\mathrm{~A})=3$
$P(A)=\frac{3}{n(S)} \ldots \ldots \ldots \ldots$ (formula)
$\therefore P(A)=\frac{1}{2}$

(B) Solve any four sub questions from the following;
(i) To solve the following simultaneous equations by Cramer's rule, find the values of D_x and D_y.
$3x+5y=26, x+5y=22$

Solution:
By Cramer's rule.
$D_x=\begin{vmatrix}2\sqrt{3} & 9\\ 2 & 3\sqrt{3}\end{vmatrix}$
$=26\times 5-22\times 5=130-110=20$
$D_y=\begin{vmatrix}2\sqrt{3} & 9\\ 2 & 3\sqrt{3}\end{vmatrix}$
$=3\times 22-1\times 26=66-26=40$

(ii) A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

Solution:

Total number of pens $= 5 + 8 + 3 = 16$
So, the sample space, $n(\mathrm{S}) = 16$
Let A be the event Rutuja picks a blue pen.
Number of blue pens $= 8$
So, the number of favourable outcomes, $(A) = 8$
Probability of the pen picked randomly to be blue,
$P(A) = \frac{n(S)}{n(A)} = \frac{8}{16} = \frac{1}{2}$

(iii) Find the sum of first ' n ' even natural numbers.
Solution:
First n even natural numbers are $2,4,6, \ldots \ldots \ldots, 2n$.
$t_1 =$ first term $= 2$
$t_n =$ last term $= 2n$
$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2 + 2n)$
$= \frac{n}{2} \times 2 \times (1 + n)$
$= n \times (1 + n)$

(iv) Solve the following quadratic equation by factorisation method:
$x^2 + x - 20 = 0$
Solution:
$x^2 + x - 20 = 0$
$\Rightarrow x^2 + 5x - 4x - 20 = 0$
$\Rightarrow x(x + 5) - 4(x + 5) = 0$
$\Rightarrow (x - 4)(x + 5) = 0$
$\Rightarrow (x - 4) = 0, (x + 5) = 0$
$\therefore x = 4, x = -5$

(v) Find the values of $(x + y)$ and $(x - y)$ of the following simultaneous equations:
$49x - 57y = 172, 57x - 49y = 252$
Solution:
Adding the given equations, we get
$106x - 106y = 424$
$106(x - y) = 424$
$(x + y) = \frac{424}{106}$
$\therefore x - y = 4$
Subtracting the given equations, we get
$-8x - 8y = -80$
$-8x + y = -80$
$(x + y) = \frac{-80}{-8}$
$\therefore x + y = 10$

Q3. (A) Complete the following activity and rewrite it (any one):

(i) One of the roots of equation $kx^2 - 10x + 3 = 0$ is 3 . Complete the following activity to find the value of k :

One of the roots of equation $kx^2 - 10x + 3 = 0$ is 3

Putting $x =$ in the above equation

$\therefore k(\ldots)^2 - 10 \times \ldots + 3 = 0$

$\therefore -30 + 3 = 0$

$\therefore 9k =$

$\therefore k =$

Solution:

One of the roots of equation $kx^2 - 10x + 3 = 0$ is 3

Putting $x = 3$ in the above equation

$\therefore k(3)^2 - 10 \times 3 + 3 = 0$

$\therefore 9\mathrm{k} - 30 + 3 = 0$

$\therefore 9k = 27$

$\therefore k = 3$

(ii) A card is drawn at random from a pack of well shuffled 52 playing cards. Complete the following activity to find the probability tha the card drawn is -

Event A: The card drawn is an ace.

Event B: The card drawn is a spade.

' S ' is the sample space.

$\therefore n(S) = 52$

Event A: The card drawn is an ace.

$\therefore n(\mathrm{~A}) =$ ____________

$\therefore P(A) =$_______________ .... (formula)

$\therefore P(A) = \frac{-}{52}$

$\therefore P(A) = \frac{13}{13}$

Event B : The card drawn is a spade.

$\therefore n(B) =$_______

$P(B) = \frac{\overline{n(B)}}{n(S)} = \frac{13}{52}$

$\therefore n(B) = \frac{-}{4}$

Solution:

' S ' is the sample space.

$\therefore (S) = 52$

Event A: The card drawn is an ace.

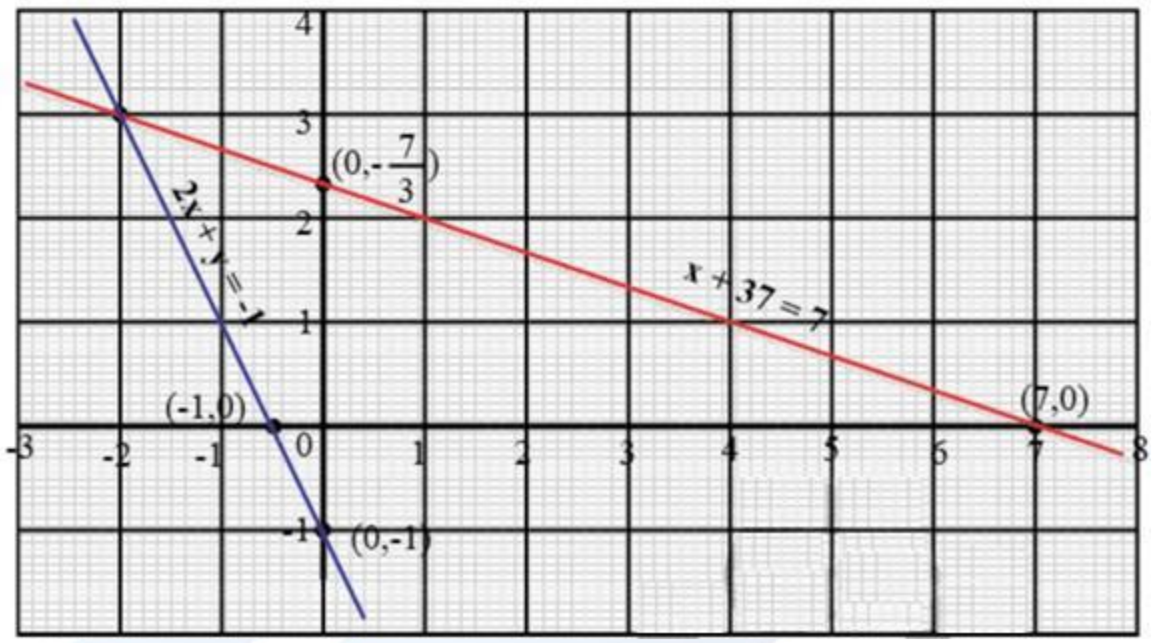
$\therefore (\mathrm{A}) = 4$

$\therefore P(A) = \frac{n(A)}{n(S)}.$

$\therefore P(A) = \frac{4}{52}$

$\therefore P(A) = \frac{1}{13}$
Event B : The card drawn is a spade.
$\therefore n(B) = 13$
$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$
$\therefore n(B) = \frac{1}{4}$

(B) Solve the following sub questions (any two):
(i) Solve the simultaneous equations by using graphical method:
$x + 3y = 7, 2x + y = -1$
Solution:
Plotting the points (7,0), $\left(0, \frac{7}{3}\right)$ and $\left(\frac{-1}{2}, 0\right)$, $(0, -1)$, we get the graph. (next slide)
We observe that both the graphs are intersecting at $-2,3$.
$\therefore x = -2$ and $y = 3$ is the solution.



(ii) There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in second row, 4 seats in the third row and so on. Find how many total numbers of seats in the auditorium?
Solution:
The number of seats arranged row-wise is as follows:
20,22,24, ...
This sequence is an A. P. with $a = 20, d = 22 - 20 = 2$, and $n = 27$

We know, $S_n = \frac{n}{2}(2a + d \times (n-1))$

$\Rightarrow S_{27} = \frac{27}{2}(2 \times 20 + 2 \times (27-1))$

$\Rightarrow S_{27} = \frac{27}{2}(40 + 52)$

$\Rightarrow S_{27} = \frac{27}{2}(92) = 1242$

Total seats in the auditorium are 1242.

(iii) Sum of the present ages of Manish and Savitha is 31 years. Manish's age 3 years ago was 4 times the age of Savitha at that time. Find their present ages.

Solution:

Suppose the present age of Manish is x years and Savitha be y years.
According to the first condition, the sum of their present ages is 31 .
So, $x + y = 31$ ... (i)
Three years ago;
Age of Manish $= x - 3$ years
Age of Savitha $= y - 3$ years
$\therefore$ According to the second condition, 3 years ago Manish's age was 4 times the age of Savitha's. So,
$x - 3 = 4y - 3.$
$x - 3 = 4y - 12$
$\therefore x - 4y = -9$
$x - 4y = -9$
Subtracting equation (ii) from (i), We get
$5y = 40$
$\Rightarrow y = 8$
Substituting $y = 8$ equation (i), We get
$x + y = 31$
$x + 8 = 31$
$\Rightarrow x = 23$
Therefore, present age of Manish is 23 years and Savitha is 8 years.

(iv) Solve the following quadratic equation using formula:
$x^2 + 10x + 2 = 0$

Solution:

Comparing the given equation $x^2 + 10x + 2 = 0$ with $ax^2 + bx + c = 0$

$\therefore$ We get, $a = 1, b = 10, c = 2$

We know the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$

$\Rightarrow x = \frac{-10 \pm \sqrt{100 - 4 \times 2}}{2}$

$\Rightarrow x = \dfrac{-10 \pm \sqrt{92}}{2}$

$\Rightarrow x = \dfrac{-10 \pm \sqrt{23 \times 4}}{2}$

$\Rightarrow x = \dfrac{-5 \times 2 \pm 2\sqrt{23}}{2}$

$\Rightarrow x = -5 \pm \sqrt{23}$

$\therefore$ Roots of the quadratic equation are $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$

Q4. Solve the following subquestions (any two):

(i) If 460 is a natural number, then quotient is 2 more than nines times the divisor and remainder is 5. Find the quotient and divisor

Solution:

Let the divisor be x.

Them, according to question quotient $= 9x + 2$

We know, Dividend = Divisor × Quotient + Remainder

$\Rightarrow 460 = x \times (9x + 2) + 5$

$\Rightarrow 460 = 9x^2 + 2x + 5$

$\Rightarrow 455 = 9x^2 + 2x$

$\Rightarrow 9x^2 + 2x - 4500 = 0$

$\Rightarrow 9x^2 + 65x - 63x - 455 = 0$

$\Rightarrow 9(x - 7) + 65(x - 7) = 0$

$\Rightarrow (9x + 65)(x - 7) = 0$

$\Rightarrow (9x + 65) = 0$ or $(x - 7) = 0$

$\Rightarrow x = \frac{-65}{9}$ or $x = 7$

However, 460 is divided by a natural number so $x = 7$.

$\therefore$ Divisor $= 7$

And quotient $= 9(7) + 2 = 65$.

(ii) If the 9th term of an $A.P.$ is zero, then prove that the $29th$ term is double the 19th term.

Solution:

We know, nth term of a sequence is $tn = a + (n - 1)\text{d}$

$\therefore t_9 =$ ninth term $= a + 9 - 1d = a + 8d = 0$ (Given)

And $t_{29} = 29$ th term $= a + 29 - 1d = a + 28\text{ d}$

$= a + 8d + 20\text{ d} = 0 + 20d = 20\text{ d}(\because a + 8d = 0)$

$\Rightarrow t_{29} = 20d$

And $t_{19} = 19$ th term $= a + 19 - 1d = a + 18\text{ d}$

$= a + 8d + 10\text{ d} = 0 + 10d = 10\text{ d}(a + 8d = 0)$

$\Rightarrow t_{19} = 10d$

So, we have, $t_{29} = 20$ d and $t_{19} = 10$ d
Observe that
$t_{29} = 2 \times t_{19}$ as $20d = 2 \times 10d$
Hence proved that if the 9th term of an $A.P.$ is zero, then prove that the 29th term is double the 19th term.

(iii) The perimeter of an isosceles triangle is 24 cm . The length of its congruent sides is 13 cm less than twice the length of its base. Find the lengths of all sides of the triangle.

Solution:

Let the length of the base of isosceles triangle $= x$ cm

Length of congruent sides $= 2x - 13$ cm (Given)

Perimeter of isosceles triangle $=$ 24 cm (Given)

Perimeter $=$ Length of base + length of congruent sides

$\Rightarrow 24 = x + 2x - 13 + 2x - 13$

$\Rightarrow 24 = 5x - 26$

$\Rightarrow 50 = 5x$

$\Rightarrow x = 10$

So, the length of Base $=$ 10 cm

Congruent side $= 2x - 13 = 20 - 13 = 7$

$\therefore$ The length of base is 10 cm and the length of congruent side are 7 cm and 7 cm .

Q5. Solve the following sub questions (any one):

(i) A bag contains 8 red and some blue balls. One ball is drawn at random from the bag. If ratio of probability of getting red ball and blue ball is 2: 5, then find the number of blue balls.

Solution:

Suppose the number of blue balls $= x$
$\Rightarrow$ (Blue ball) $= x$
Number of red balls = 8
$\Rightarrow$ (Red ball) $=$ 8
Total number of balls $= 8 + x$
$\Rightarrow$ (Total) $= 8 + x$
$\therefore$ P(Blue ball drawn) $= \frac{n(\text{ Blue ball })}{n(\text{ Total })} = \frac{x}{8+x}$
According to the given condition,
$$\frac{P(\text{ Blue ball drawn })}{P(\text{ Red ball drawn })} = \frac{5}{2}$$

$$\frac{P(\text{ Blue ball drawn })}{P(\text{ Red ball drawn })}=\frac{\left(\frac{x}{8+x}\right)}{\left(\frac{8}{8+x}\right)}=\frac{x}{8}$$

$\Rightarrow \frac{x}{8}=\frac{5}{2}$

$\Rightarrow x=20$

Hence, the number of blue balls is 20 .

(ii) Measures of angles of a triangle are in A.P. The measure of smallest angle is five times of common difference. Find the measures of all angles of a triangle. (Assume the measures of angles as $a, a+d, a+2d$.)

Solution:

Let the angles of triangles be $a, a+d, a+2d$.

We know that, sum of angles of a triangle $=180^{\circ}$

$\Rightarrow a+a+d+a+2d=180^{\circ}$

$\Rightarrow 3a+3d=180^{\circ}$

$\Rightarrow a+d=60^{\circ}$

According to the given conditions,

Smallest angle, $a=5d$

Putting $a=5d$ in $a+d=60^{\circ}$

$\Rightarrow 6d=60^{\circ}$

$\Rightarrow d=10^{\circ}$

Putting $d=10^{\circ}$ in $a+d=60^{\circ}$

$\Rightarrow a+10=60^{\circ}$

$\Rightarrow a=50^{\circ}$

As, angles of triangles are $a, a+d, a+2d$,

Hence, $a=50^{\circ}$

And $a+d=60^{\circ}$

And $a+2d=70^{\circ}$.

$\therefore$ Angles of the given triangle are $50^{\circ}, 60^{\circ}$, and 70°.

PART – II(GEOMETRY)

Q1. (A) For every sub question four alternative answers are given. Choose the correct answer and write the alphabet of it:

(i) If $\triangle ABC \sim \triangle DEF$ and $\angle A=48^{\circ}$, then $\angle D=$ ____________________.

A) 48°

B) 83°

C) 49°

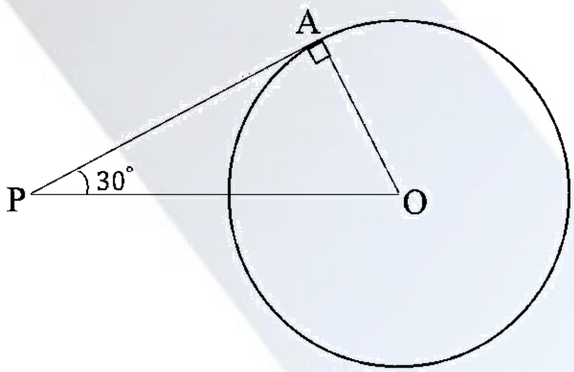
D) 132°

Solution:

If $\triangle \mathrm{ABC} \sim \Delta \mathrm{DEF}$ and $\angle \mathrm{A}=48^{\circ}$, then $\angle \mathrm{D}=48^{\circ}$.

(The corresponding angles in a triangle have the same measure.)

(ii) AP is a tangent at A drawn to the circle with centre O from an external point P. $OP = 12$ cm and $\angle OPA = 30°$, then the radius of a circle is .
A) 12 cm
B) $6\sqrt{3}$ cm
C) 6 cm
D) $12\sqrt{3}$ cm
Solution:



Give $\mathrm{OP} = 12$ cm, $\angle \mathrm{OPA} = 30°$
As tangent will be perpendicular to radius of the circle So, $\angle OPA = 90°$
In $\triangle APO, \angle OAP = 90°$

$$\therefore \sin\ 30° = \frac{OA}{OP}$$

$$\Rightarrow \frac{1}{2} = \frac{OA}{12} \Rightarrow OA = 6 \text{ cm}$$

(iii) Seg AB is parallel to X - axis and co - ordinates of the point A are $(1,3)$, then the coordinates of the power B can be .
A) $(-3,1)$
(B) $(5,1)$
(C) $(3,0)$
(D) $(-5,3)$
Solution:
Co - ordinates of point A are $(1,3)$, then the co - ordinates of the point B can be $(-5,3)$ as y co - ordinate should be same if seg AB is parallel to X - axis.

(iv) The value of $2\tan 45° - 2\sin 30°$ is _______________ .
(A) 2
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$
Solution:

We know that $\tan 45^\circ = 1$ and $\sin 30^\circ = \frac{1}{2}$

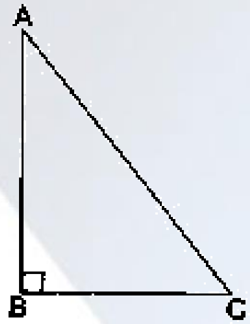
Thus, we get $2\tan\ 45^\circ = 2\sin\ 30^\circ = 2 \times 1 - 2 \times \frac{1}{2}$

$= 2 - 1 = 1$

(B)

(i) In $\triangle ABC, \angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ$. If $AC = 9\sqrt{2}$, then find the value of AB.

Solution:



Given, in $\triangle ABC$

$\angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ, \mathrm{AC} = 9\sqrt{2}$

Now, $\mathrm{AB} = \frac{1}{\sqrt{2}} \times AC$ [Property of $45^\circ - 45^\circ - 90^\circ$ triangle]

$$\therefore AB = \frac{1}{\sqrt{2}} \times 9\sqrt{2}$$

$\therefore AB = 9$

(ii) Chord AB and chord CD of a circle with centre O are congruent. If $m(\mathrm{arc}AB) = 120^\circ$, then find the m (arc CD.)

Solution:

Given, chord AB = Chord CD

$m(\mathrm{arc}AB) = 120^\circ$

We know that,

Arc $AB \cong \mathrm{arc}CD$

[Corresponding arcs of congruent chord of a circle are congruent]

$\Rightarrow m(\mathrm{arc}AB) = m(\mathrm{arc}CD)$

$\Rightarrow 120^\circ = \mathrm{m(arcCD)}$

$\therefore \mathrm{m(arcCD)} = 120^\circ$

(iii) Find the Y-coordinate of the centroid of a triangle whose vertices are (4, -3), (7,5), and (-2, 1).

Solution:

Vertices of the triangle,

$(4, -3), (7,5)$, and $(-2,1)$ [Given]

$x_1 = 4, x_2 = 7, x_3 = -2$

$y_1 = -3, y_2 = 5, y_3 = 1$

By using the centroid formula,
Co - ordinate of centroid $= \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$
Now, Y - coordinate of centroid $= \frac{y_1+y_2+y_3}{3}$
$= \frac{-3+5+1}{3} = \frac{3}{3} = 1$
$\therefore$ Y $-$ coordinate of centroid $= 1$.

(iv) If $\sin\ \theta = \cos\ \theta$, then what will be the measure of angle θ ?
Solution:
Given, $\sin\ \theta = \cos\ \theta$

We know that,

$\sin\ \theta = \cos\ (90^\circ - \theta)$

$\therefore \cos\ \theta = \cos\ (90^\circ - \theta)$

$\Rightarrow \theta = 90^\circ - \theta$

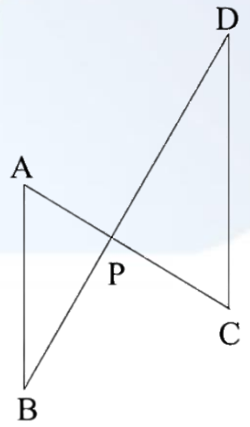
$\Rightarrow \theta + \theta = 90^\circ$

$\Rightarrow 2\theta = 90^\circ$

$\Rightarrow \theta = \frac{90^\circ}{2}$

$\therefore \theta = 45^\circ$

Q2. (A)
(i) In the given figure, seg AC and seg BD intersect each other in point P. If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove $\triangle$ ABP $\sim\triangle$ CDP.
Solution:

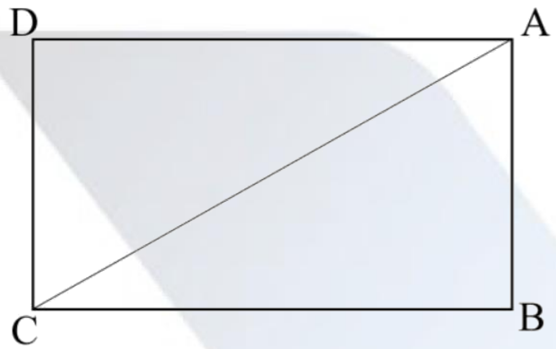


In $\triangle$ APB and $\triangle$ CDP
$\frac{AP}{CP} = \frac{BP}{DP}$ Given

$\therefore \angle APB \cong \angle DPC$ … …. Vertically opposite angles
$\therefore \angle ABP \sim \triangle CDP$ test of similarity

(ii) In the given figure, $\square$ ABCD is a rectangle. If AB $= 5$, AC $= 13$, then complete the following activity to find BC.

Solution:



$\triangle$ APB is
right - angled triangle.
$\therefore$ By Pythagoras theorem
$AB^2 + BC^2 = AC^2$
$\therefore 25 + BC^2 = 169$
$\therefore BC^2 = 144$
$\therefore BC = 12$

(iii) Complete the following activity to prove: $\cot\ \theta + \tan\ \theta = \operatorname{cosec}\theta \times \sec\ \theta$

Solution:

L.H.S $= \operatorname{Cot}\theta + \tan\ \theta$

$= \frac{\cos\ \theta}{\sin\ \theta} + \frac{\sin\ \theta}{\cos\ \theta} = \frac{\cos^2\ \theta + \sin^2\ \theta}{\sin\ \theta \times \cos\ \theta}$

$= \frac{1}{\sin\ \theta \times \cos\ \theta}$ … … … . . ($\because \cos^2\theta + \sin^2\theta = 1$)

$= \frac{1}{\sin\ \theta} \times \frac{1}{\cos\ \theta}$

$= \operatorname{cosec}\theta \times \sec\ \theta$

$\because$ L. H. S = R. H. S

(B)

(i) If $\triangle$ ABC $\sim \Delta$PQR, AB: PQ $= 4:5$ and A($\triangle$ PQR) $= 125\ \text{cm}^2$, then find A($\triangle$ ABC).

Solution:

Given: $\triangle$ ABC $\sim \triangle$ PQR

We know that,

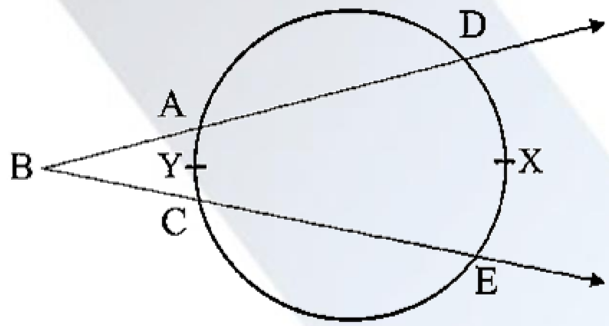
$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2}$ … … … … [Theorem of area of similar triangles]

$\Rightarrow \frac{A(\triangle\ ABC)}{125} = \frac{(4)^2}{(5)^2}$

$$\Rightarrow \frac{A(\triangle ABC)}{125} = \frac{16}{25}$$
$$\Rightarrow A(\triangle ABC) = \frac{16}{25} \times 125$$
$$\therefore A(\triangle ABC) = 80 \text{ cm}^2$$

(ii) In the given figure, $m(\text{arc}DXE) = 105°$, $\text{m}(\text{arc}AYC) = 47°$ then find the measure of $\angle DBE$.

Solution:



From Figure.

Chord AD and CE intersect externally at point B.

$\therefore m(\text{arc}DEX) = \frac{1}{2}[m(\text{arc}DXE) - m(\text{arc}AYC)]$

.... [By inscribed angle theorem]

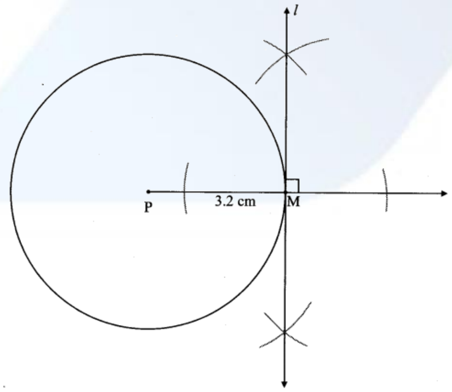
$\therefore m(\text{arc}DEX) = \frac{1}{2}[105° - 47°]$

$\therefore m(\text{arc}DEX) = \frac{1}{2}[58°]$

$\therefore m(\text{arc}DEX) = 29°$

(iii) Draw a circle of radius 3.2 cm and centre O. Take any point P on it. Draw tangent to the circle through Point P using the centre of the circle.

Solution:



Given: Radius of the circle = 3.2 cm

Construction:
(a) With O as the centre draw a circle of radius 3.2 cm .
(b) Take a point P on the circle and draw ray OP .
(c) Draw line 1 Perpendicular to ray OX through point
(d) Line 1 is the required tangent to the circle at point P .

(iv) If $\sin\ \theta=\frac{11}{61}$, then find the value of $\cos\ \theta$ using trigonometric identity.

Solution:

We know $\sin^2\ \theta+\cos^2\ \theta=1$

$\Rightarrow \cos^2\ \theta=1-\sin^2\ \theta$

$$\Rightarrow \cos^2\ \theta=1-\left(\frac{11}{61}\right)^2$$

$$\Rightarrow \cos^2\ \theta=1-\left(\frac{121}{3721}\right)=\frac{3721-121}{3721}=\frac{3600}{3721}$$

$$\Rightarrow \cos^2\ \theta=\sqrt{\left(\frac{60}{61}\right)^2}=\frac{60}{61}$$

Thus the value of $\cos\ \theta$ is $\frac{60}{61}$.

(v) In $\triangle ABC, AB=9$ cm, $BC=40$ cm, and $AC=41$ cm. State whether $\triangle ABC$ is a right angled triangle or not? Write reason.

Solution:

Side of $\triangle ABC$ are $AB=9$ cm, $BC=40$ cm, $AC=41$ cm

The triangle's longest side measures 41 cm .

$\therefore 41^2=1681$

Now, the sum of the square of the remaining sides is

$9^2+40^2=81+1600$

$=1681$

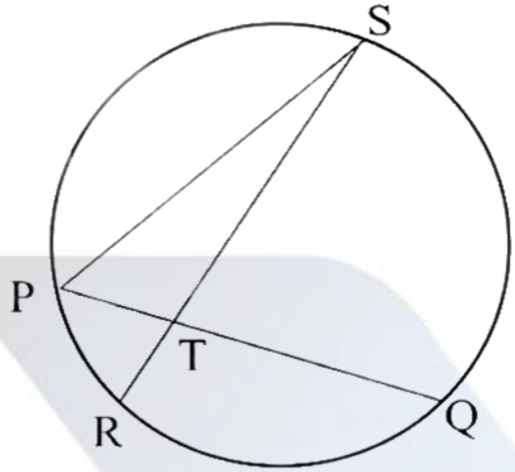
From equations (i) and (ii), as the square of the longest side equals the sum of the squares of the remaining two sides, by suing converse of Pythagoras theorem the given sides from a right - angle triangle.

Q3. (A)

(i) In the given figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ=58^\circ$ and $\angle PSR=24^\circ$, then complete the following activity to verify:

$$\angle STQ=\frac{1}{2}[\mathrm{m}(\mathrm{arc}PR)+m(\mathrm{arc}SQ)]$$

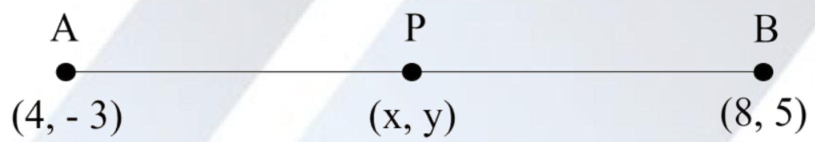
Solution:



In △ PTS,
∠SPQ = ∠STQ − ∠PSR ∵ Exterior angle theorem
∠SPQ = 34°
∴ $m(\text{arc}QS) = 2 \times 34^\circ = 68^\circ$ …..∵ Inscribed angle theorem
Similarly m(arcPR) $= 2\angle PSR = 48$
$\therefore \frac{1}{2}[m(\text{arc}QS) + m(\text{arc}PR)] = \frac{1}{2} \times 116^\circ = 58^\circ$
But ∠STQ = 58° … …. (II)given
$\therefore \frac{1}{2}[\mathrm{m}(\text{arc}PR) + \mathrm{m}(\text{arc}QS)] = \angle \mathrm{STQ}$ … … … [From (I)and (II)]

(ii) Complete the following activity to find the co - ordinates of point P which divides seg AB in the ratio 3 : 1 where $(4,-3)$ and $B(8,5)$.

Solution:



∴ By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

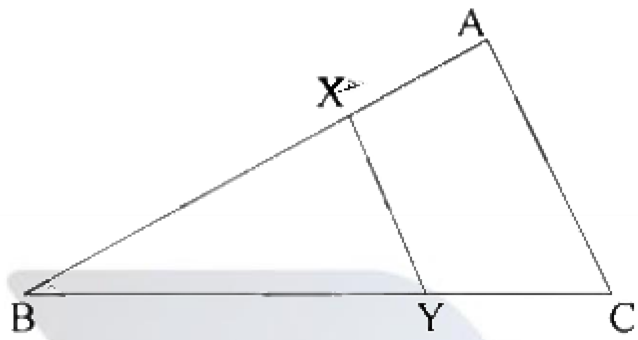
$$\therefore x = \frac{3 \times 8 + 1 \times 4}{3+1}, y = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

$$\therefore x = \frac{24+4}{4}, y = \frac{15-3}{4},$$

$$\therefore x = 7 \therefore y = 3$$

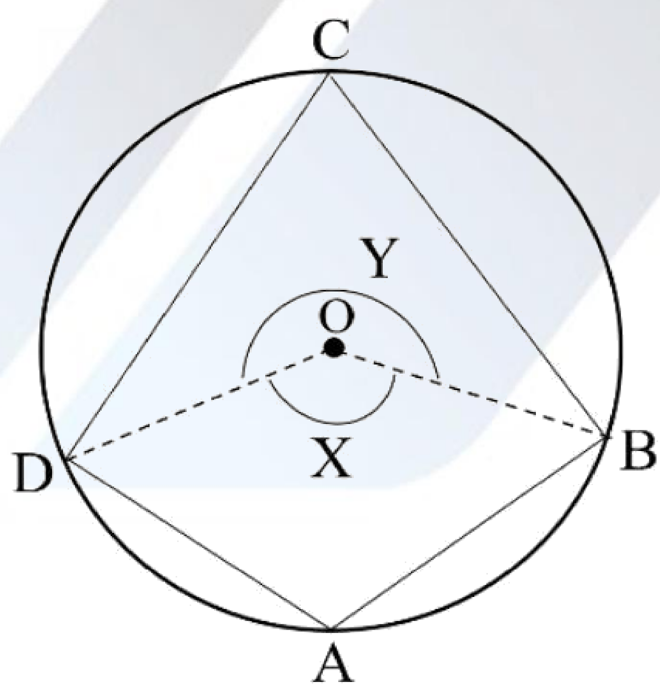
(B)
(i) In △ ABC, seg XY || side AC. If $2AX = 3BX$ and $XY = 9$, then find the value of AC.

Solution:



Given seg XY || seg AC, $2AX = 3BX$ and $XY = 9$
Consider, $2\mathrm{AX} = 3\mathrm{BX}$
$\therefore \frac{AX}{BX} = \frac{3}{2}$
$\Rightarrow \frac{AX+BX}{BX} = \frac{3+2}{2}$ ...... (By Compenendo)
$\Rightarrow \frac{AB}{BX} = \frac{5}{2} .. (\mathrm{I})$
$\triangle \mathrm{BCA} \sim \triangle \mathrm{BYX}$ SAS test of similarity
$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ corresponding sides of similar triangles
$\therefore \frac{5}{2} = \frac{\mathrm{AC}}{9}$ from (I)
$\therefore \mathrm{AC} = 22.5$

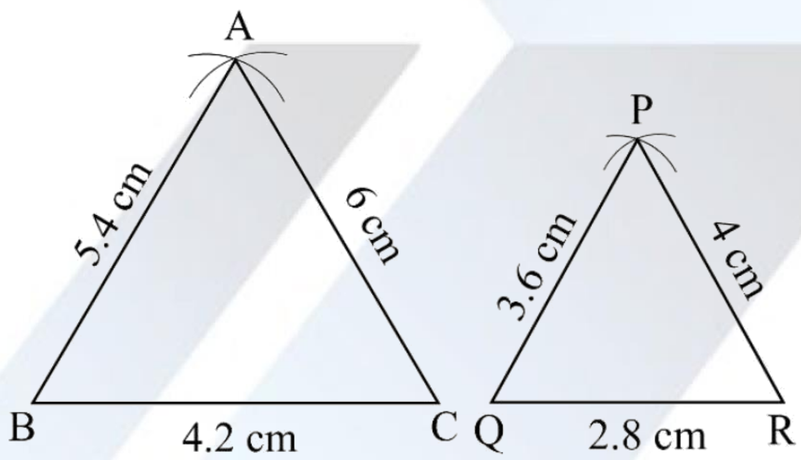
(ii) Prove that, "Opposite angles of cyclic quadrilateral are supplementary".
Solution:



Let O be the centre of the circle. Join O to B and D.
Let the angle subtended by the minor arc and the major arc at the centre be x and y respectively.
Proof: $x = 2\angle C$
[Angle at centre theorem]
and y $= 2\angle A$
Adding (i) and (ii), we get
$x + y = 2\angle C + 2\angle A$
But, $x + y = 360^\circ$
From (iii) and (iv), we get
$2\angle C + 2\angle A = 360^\circ$
$\Rightarrow \angle C + \angle A = 180^\circ$
But we know that angle sum property of quadrilateral
$\angle A + \angle B + \angle C + \angle D = 360^\circ$
$\Rightarrow \angle B + \angle D + 180^\circ = 360^\circ$
$\Rightarrow \angle B + \angle D = 180^\circ$
Hence proved that opposite angles of cyclic quadrilateral are supplementary.

(iii) $\triangle$ ABC $\sim\triangle$ PQR. In $\triangle$ ABC, AB $=$ 5.4 cm, BC $=$ 4.2 cm, AC $=$ 6.0 cm, and AB: PQ $=$ 3: 2, then construct $\triangle ABC$ and $\triangle PQR$.

Solution:



$\triangle ABC \sim\triangle PQR$ … … … [Given]
We know that corresponding sides of triangle which are similar are in proportion.
$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2}$
$\Rightarrow \frac{AB}{PQ} = \frac{3}{2}$
Also, $\frac{BC}{QR} = \frac{3}{2}$
Also, $\frac{AC}{PR} = \frac{3}{2}$
$\Rightarrow \frac{5.4}{PQ} = \frac{3}{2}$

$\Rightarrow \frac{4.2}{PQ} = \frac{3}{2}$
$\Rightarrow \frac{6}{PR} = \frac{3}{2}$
$\Rightarrow PQ = \frac{5.4 \times 2}{3} = 3.6 \text{ cm} \Rightarrow QR = \frac{4.2 \times 2}{3}$
$\Rightarrow PR = \frac{6 \times 2}{3} = 4 \text{ cm}$
$\Longrightarrow QR = 2.8 \text{ cm}$
Now, draw angle $\triangle$ ABC with sides AB = 5.4 cm, BC = 4.2 cm and AC = 6 cm.
Also draw triangle $\triangle$ PQR with sides PQ = 3.6 cm, QR = 2.8 cm and PR = 4 cm.

(iv) Show that: $\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2} = \sin A \times \cos A$

Solution:

Sol: $\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$

$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2}$ [$\because 1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \operatorname{cosec}^2 A$]

$= \frac{\sin A}{\cos A} \times \cos^4 + \frac{\cos A}{\sin A} \times \sin^4 A$

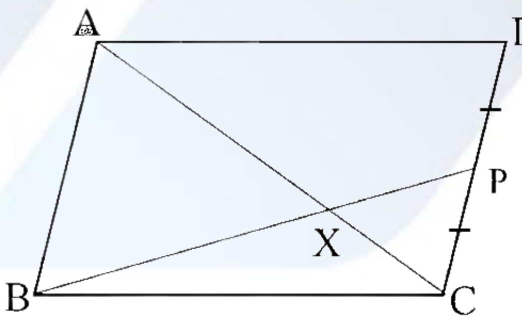
$= \sin A\cos^3 A + \cos A\sin^3 A$

$= \sin A\cos A$

Hence proved.

Q4. (i) $\square ABCD$ is parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that: $3AX = 2AC$

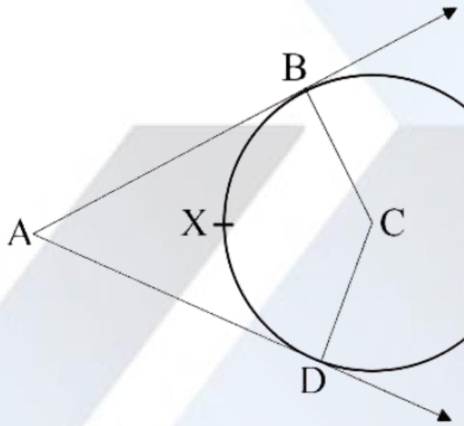
Solution:



From the figure, in $\triangle ABX$ and $\triangle$ CPX
As, $AB \| CD$
$\angle BAX = \angle PCX$ [Alternate angle]
$\angle BXA = \angle PXC$ [Vertically opposite angles]
$\therefore \triangle$ ABX $\sim \Delta$CPX [By AA similarity theorem]

We know that,
Similar triangles have comparable side ratios that are similar to or equal.
$\therefore \frac{AX}{CX} = \frac{AB}{CP}$
But $CD = AB$ and P is mid - point of CD.
$\therefore AB = 2CP$
$\Rightarrow \frac{AX}{AC - AX} = \frac{2CP}{CP} = 2$
$\Rightarrow AX = 2(AC - AX)$
$\Rightarrow AX = 2AC - 2AX$
$\Rightarrow AX + 2AX = 2AC$
$\Rightarrow 3AX = 2AC$
Hence proved.

(ii) In the given figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that:
$\angle A = \frac{1}{2}[\mathrm{m}(\mathrm{arc}BYD) - m(\mathrm{arc}BXD)]$
Solution:



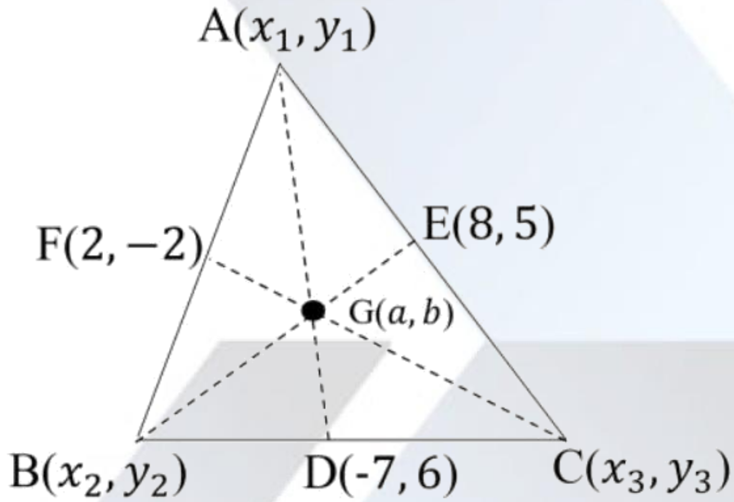
Proof: From figure
Seg $AB \perp$ segBC and seg$AD \perp$ segCD … …
[By tangent theorem]
$\therefore \angle ABC = \angle ADC = 90^\circ$
In □ ABCD,
$\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Angle of the square]
$\therefore \angle A + 90^\circ + \angle C + 90^\circ = 360^\circ$
$\therefore \angle A + \angle C = 360^\circ - 180^\circ$
$\therefore \angle A + \angle C = 180^\circ$
$\therefore \angle A + \mathrm{m}(\mathrm{arcBXD}) = 180^\circ$ [Central angle] (i)
Now, $m(\mathrm{arc}BXD) + m(\mathrm{arc}BYD) = 360^\circ$
….. [Two arcs contribute a complete circle]
Now, multiply equation (i) by 2 on both sides
$2[\angle A + m(\mathrm{arc}BXD)] = 2 \times 180^\circ$
$\Rightarrow 2\angle A + 2 \times m(\mathrm{arcBXD}) = 360^\circ$

$\Rightarrow 2\angle A = 360^\circ - 2 \times m(\text{arc}BXD)$
$\Rightarrow 2\angle \text{ A} = \text{m}(\text{arc}BXD) + m(\text{arc}BYD) - 2m(\text{arc}BXD)$
$\Rightarrow 2\angle A = m(\text{arc}BYD) - m(\text{arc}BXD)$.....[From (ii)]
$\Rightarrow \angle A = \frac{1}{2}[m(\text{arc}BYD) - m(\text{arc}BXD)]$
Hence proved.

(iii) Find the co-ordinates of centroid of a triangle if points $D(-7,6), E(8,5)$, and $F(2,-2)$ are the mid - points of the sides of the that triangle.

Solution:

Suppose $\text{A}(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of triangle.
$\text{D}(-7,6), \text{E}(8,5)$ and $F(2,-2)$ are the midpoints of sides BC, AC and AB respectively.
Let G be the centroidof $\triangle$ ABC. D is the midpoint of seg BC .



By midpoint formula,
Co - ordinates of D $= \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$
$\Rightarrow (-7,6) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$
$\Rightarrow \frac{x_2+x_3}{2} = -7, \frac{y_2+y_3}{2} = 6$
$\Rightarrow x_2 + x_3 = -14$ .... (i), $y_2 + y_3 = 12$ ...
E is the midpoint of seg AC ;
Co - ordinates of E $= \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$
$\Rightarrow (8,5) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$
$\Rightarrow \frac{x_1+x_3}{2} = 16, \frac{y_1+y_3}{2} = 10$
$\Rightarrow x_1 + x_3 = 16$ ... ... (iii), $y_1 + y_3 = 10$ ... (iv)
Similarly, as F is the midpoint of seg AB;
Co - ordinates of E $= \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$
$\Rightarrow x_1 + x_2 = 4$ ... ... (v), $y_1 + y_3 = -4$

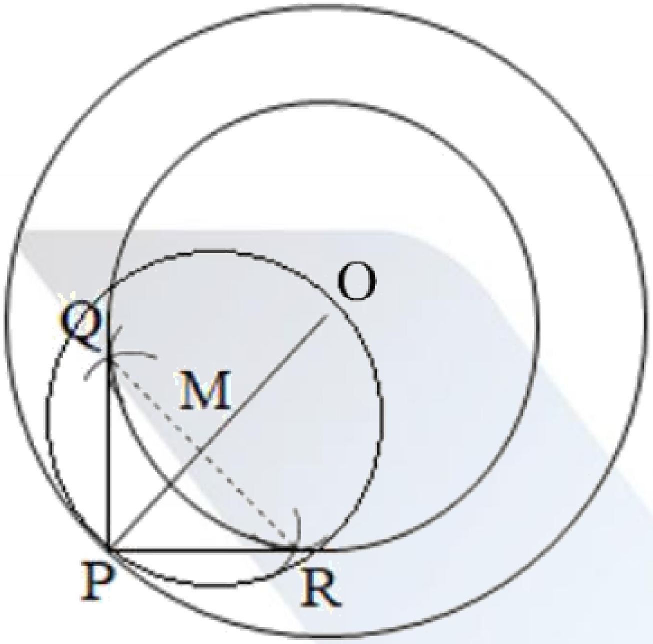
Adding (i), (iii) and (v),
$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -14 + 16 + 4$
$\Rightarrow 2x_1 + 2x_2 + 2x_3 = 6 \Rightarrow x_1 + x_2 + x_3 = 3 \ldots$
Adding (ii), (iv) and (vi),
$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 12 + 10 - 4$
$\Rightarrow 2y_1 + 2y_2 + 2y_3 = 18 \Rightarrow y_1 + y_2 + y_3 = 9$ ... (viii)
G is the centroid of $\triangle ABC$. By centroid formula,
Co- ordinates of G $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
$= \left(\frac{3}{3}, \frac{9}{3}\right)$ ... .From (vii)and (viii)
$= (1,3)$
$\therefore$ The Co-ordinates of the centroid of the triangle are (1,3)

Q5. (i) If a and b are natural numbers and $a > b$ If $(a^2 + b^2)$, $(a^2 - b^2)$ and $2ab$ are the sides of the triangle, then prove that the triangle is right angled. Find out two pythagorean triplets by taking suitable values of a and b.

Solution:
$a^2 + b^2, a^2 - b^2, 2ab$ are sides of triangle.
By Pythagoras' theorem,
$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$
$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$
AS L.H.S. = R.H.S.
$\therefore$ Triangle is a right - angle triangle as it follows Pythagorean triplets As a $>$ b
..... [Given]
Let a $=$ 4, b $=$ 3
$a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$
$a^2 - b^2 = 16 - 9 = 7$
$2ab = 2 \times 4 \times 3 = 4$
$\therefore$ (25,7,24) is a Pythagorean triplet.
Let a $=$ 2, b $=$ 1
$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$
$\therefore$ (5,3,4) is another Pythagorean triplet.

(ii). Construct two concentric circles with centre O with radii 3 cm and 5 cm . construct tangent to a smaller circle from any point A on the larger circle. Measure and write the length of tangent segment. Calculate the length of tangent segment using Pythagoras theorem.

Solution:



Following are the steps to draw tangents on the given circle:
Step 1: Draw a circle of 3 cm radius with centre O on the given plane.
Step 2: Draw a circle of 5 cm radius, taking O as its centre. Locate a point P on this circle and join OP.
Step 3: Bisect OP. Let M be the midpoint of PO.
Step 4: Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at points Q and R .
Step 5: Join PQ and PR. PQ and PR are the required tangents.
It can be observed that PQ and PR are of length 4 cm each.
Since PQ is a tangent,
$\therefore \angle PQO = 90^{\circ}$ and $PO = 5$ cm and $QO = 3$ cm
Applying Pythagoras theorem in $\triangle PQO$, we obtain
$\mathrm{PQ}^2 + \mathrm{QO}^2 = \mathrm{PQ}^2$
$\Rightarrow \mathrm{PQ}^2 + (3)^2 = (5)^5$
$\Rightarrow PQ^2 + 9 = 25$
$\Rightarrow PQ^2 = 25 - 9 = 16$
$\Rightarrow PQ = 4$ cm