Grade-10-Maharastra-Math-2023

PART - I

Question 1: (A) Choose the correct answer and write the alphabet of it in front of the sub-question number:

i. To draw the graph of $4x + 5y = 19$, find y when $x = 1$:

(A) 4

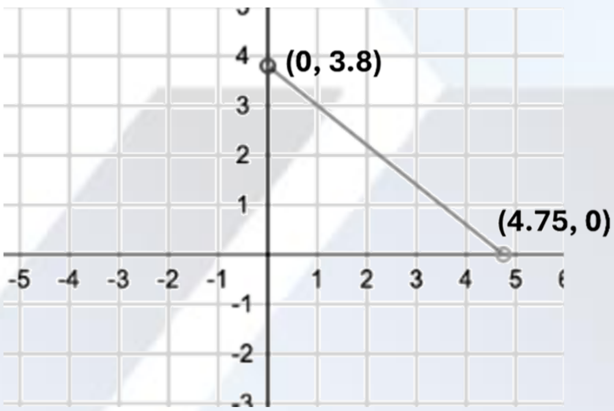
(B) 3

(C) 2

(D) -3

Answer: (B) 3

Solution:



$4x + 5y = 19$

When $x = 1$, then y will be:

$$4(1) + 5y = 19$$
$$\Rightarrow 4 + 5y = 19$$
$$\Rightarrow 5y = 19 - 4 = 15$$
$$\Rightarrow 5y = 15$$

$$\Rightarrow y = \frac{15}{5} = 3$$

Hence, the correct answer is 3.

ii. Out of the following equations, which one is not a quadratic equation?

(A) $x^2 + 4x = 11 + x^2$

(B) $x^2 = 4x$

(C) $5x^2 = 90$

(D) $2x - x^2 = x^2 + 5$

Answer: (A) $x^2 + 4x = 11 + x^2$

Solution:

Option A: $x^2 + 4x = 11 + x^2 \Rightarrow 4x = 11$

Thus, $x^2 + 4x = 11 + x^2$ is not a quadratic equation.

Option B: $x^2 = 4x$ can be written as $x^2 - 4x + 0 = 0$. So, $x^2 - 4x$ is a quadratic equation.

Option C: $5x^2 = 90$ can be written as $5x^2 - 90 + 0 = 0$. So, $5x^2 - 90 + 0 = 0$ is a quadratic equation.

Option D: $2x - x^2 = x^2 + 5$ can be written as $2x^2 - 2x + 5 = 0$. So, it also forms a quadratic equation.

Hence, the correct answer is $x^2 + 4x = 11 + x^2$.

iii. For the given A.P. $a = 3.5, d = 0$, then $t_n =$

(A) 0

(B) 3.5

(C) 103.5

(D) 104.5

Answer: (B) 3.5

Solution:

Given: $a = 3.5, d = 0$

$t_n = a + (n - 1)d$

$= 3.5 + (n - 1)0$

$= 3.5 + 0 = 3.5$

iv. If $n(A) = 2, P(A) = \frac{1}{5}$, thèn $n(S) = ?$

(A) 10

(B) $\frac{5}{2}$

(C) $\frac{2}{5}$

(D) $\frac{1}{3}$

Answer: (A) 10

Solution:

We know that $n(A) = 2$ and $P(A) = \frac{1}{5}$. We can use the formula:

$$\mathrm{P(A)} = \frac{n(A)}{n(S)}$$

where $P(A)$ is the probability of A, $n(A)$ is the number of elements in A, and $n(S)$ is the number of elements in the sample space S.

Substituting the given values, we get:

$$\frac{1}{5} = \frac{2}{n(S)}$$

Multiplying both sides by n(S), we get:

$$\mathrm{n(S)} \times \frac{1}{5} = 2$$

$$\mathrm{n(S)} = 2 \times 5 = 10$$

Therefore, the answer is 10.

(B) Solve the following sub questions:

(i) Find the value of the following determinant:

$$\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$$

Answer: 22

Solution:

$$|A| = \begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$$

$$|A| = (4 \times 7) - (3 \times 2)$$

$$|A| = 28 - 6$$

$$|A| = 22$$

(ii) Find the common difference of the following A.P:

2, 4, 6, 8, ….

Answer: 2

Solution:

Common Difference $= a_2 - a_1 = 4 - 2 = 2$

Therefore, the common difference of the given AP is 2.

(iii) On certain article if rate of CGST is 9%, then what is the rate of SGST?

Solution:

Given:

Rate of CGST $= 9\%$

Rate of SGST $=$ Rate of CGST $= 9\%$

(iv) If one coin is tossed, write the sample space ' S '.

Solution:

Sample space $(S) = \{H, T\}$

Question 2: (A) Complete any two given activities and rewrite it.

(i) Complete the following activity, find the value of x:

$5x + 3y = 9$ ...(I)

$2x - 3y = 12$ ...(II)

Add equations (I) and (II)

$$\begin{array}{r} 5x + 3y = 9 \\ +\ \ 2x - 3y = 12 \\ \hline 7x = \boxed{} \end{array}$$

$$x = \frac{\boxed{}}{\boxed{}}$$

$$x = \boxed{}$$

Solution:

Given equations are:

$5x + 3y = 9$

$2x - 3y = 12$

Add equations (I) and (II),

$$\begin{array}{r} 5x + 3y = 9 \\ +\ 2x - 3y = 12 \\ \hline 7x = \boxed{21} \end{array}$$

$$x = \frac{\boxed{21}}{\boxed{7}}$$

$$x = \boxed{3}$$

(ii) Complete the following activity to determine the nature of the roots of the quadratic equation $x^2 + 2x - 9 = 0$:

Answer: Roots of the given equation are real and unequal.

Solution:

Compare $x^2 + 2x - 9 = 0$ with $ax^2 + bx + c = 0$

$a = 1, b = 2, c = -9$

$\therefore b^2 - 4ac = (2)^2 - 4 \times 1 \times (-9)$

$\Delta = 4 + 36 = 40$

$\therefore b^2 - 4ac > 0$

$\therefore$ The roots of the equation are real and unequal.

(iii) Complete the following table using the given information:

Sr. No.	FV	Share is at	MV
1.	₹100	Par	☐
2.	☐	Premium ₹ 500	₹575
3.	₹10	☐	₹5
4.	₹200	Discount ₹ 50	☐

Answer: ₹ 100, ₹ 75, Discount ₹ 5, ₹ 150

Solution:

(1) When share is at par, MV (Market value) = FV (Face value)

$\therefore MV = FV = ₹100$

(2) $FV = MV -$ Premium $= ₹575 - ₹500 = ₹75$

(3) $FV = ₹10$ and $MV = ₹5$

Since $MV < FV$, so the share is at discount.

Discount $= FV - MV = ₹10 - ₹5 = ₹5$

(4) Discount $= FV - MV$

$50 = 200 - MV$

$MV = ₹150$

The complete table is given below:

Sr. No	FV	Share is at	MV
1.	₹ 100	Par	**₹ 100**
2.	**₹ 75**	premium ₹ 500	₹ 575
3.	₹ 10	**Discount ₹ 5**	₹ 5
4.	₹ 200	Discount ₹ 50	**₹ 150**

(B) Solve the following sub-questions (any four):

1. Solve the following simultaneous equations:

$x + y = 4; 2x - y = 2$

Answer: $x = 2$ and $y = 2$

Solution:

Given: $x + y = 4;\ 2x - y = 2$

Adding both the equations, we get:

$3x = 6$

$x = \frac{6}{3} = 2$

Now, substitute the value of x in $x + y = 4$,

$2 + y = 4$

$y = 4 - 2 = 2$

Therefore, $x = 2$ and $y = 2$

2. Write the following equation in the form $ax^2 + bx + c = 0$, then write the values of a, b and c:

$2y = 10 - y^2$

Answer: $a = 1, b = 2, c = -10$

Solution:

$2y = 10 - y^2$

$y^2 + 2y - 10 = 0$

Now, compare the above equation with $ax^2 + bx + c = 0$,

Therefore, $a = 1, b = 2, c = -10$

3. Write an A.P. whose first term is a $=$ 10 and common difference d $=$ 5.

Answer: 10, 15, 20, ...

Solution:

Given that, $a = 10$ and $d = 5$

Then AP is,

$a, a + d, a + 2d, \ldots$

$= 10, (10 + 5), (10 + 10), \ldots$

$= 10,15,20, \ldots$

Therefore, A.P is 10,15,20, ...

4. Courier service agent charged total ₹ 590 to courier a parcel from Nashik to Nagpur. In the tax invoice the taxable value is ₹ 500 on which CGST is ₹ 45 and SGST is ₹45. Find the rate of GST charged for this service.

Answer:18%

Solution:

Total GST = CGST + SGST = 45 + 45 = ₹ 90.

Rate of GST = $\frac{90}{500} \times 100 = 18\%$

$\therefore$ Rate of GST charged by agent is 18%.

5. Observe the following table and find Mean:

Solution:

Assumed mean $A = 300$

Class	*Class mark* x_i	$d_i = x_i - A$ $d_i = x_i - 300$	*Frequency* f_i	*Frequency* $\times$ *Deviation* $f_i\, d_i$
$200 - 240$	220	-80	5	-400
$240 - 280$	260	-40	10	-400
$280 - 320$	$300 \to A$	0	15	0
$320 - 360$	340	40	12	480
$360 - 400$	380	80	8	640
Total			$\Sigma f_i = 50$	$\Sigma f_i\, d_i = 320$

Mean $\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 300 + \frac{320}{50} = 300 + 6.4 = 306.4$

Question 3 (A) Complete any one activity and rewrite it:

1. Form a 'Road Safety Committee' of two, from 2 boys (B_1, B_2) and 2 girls (G_1, G_2).

   Complete the following activity to write the sample space:

   (a) Committee of 2 boys = $\cdots$ ... ...

   (b) Committee of 2 girls = $\cdots$ ... ...

   (c) Committee of one boy and one girl = $\{B_1 G_1{'}, B_1 G_2{'}, \square, \square\}$

   (d) $\therefore$ Sample space $(S) = \{(B_1 B_2), (B_1 G_1), \square, \square\ (B_2 G_2), (G_1 G_2)\}$

   Solution:

   (a) Committee of 2 boys = B_1, B_2

   (b) Committee of 2 girls = $G_1{'} G_2$

(c) Committee of one boy and one girl = $(B_1, G_1), (B_1, G_2), (B_2, G_1), (B_2, G_2)$

(d) Sample space $= \{(B_1, B_2), (G_1, G_2), (B_1, G_1), (B_1, G_2), (B_2, G_1), (B_2, G_2)\}$

or $n(S) = 6$

(ii) Fill in the boxes with the help of given information:

Tax invoice of services provided (Sample)								
Food Junction, Khed - Shivapur, Pune Mob no. 7588580000, email- ahar.khed@yahoo.com GSTIN: 27AAAAA5555B1ZA				Invoice no. 58 Invoice Date 25 Feb., 2020				
SAC	Food Items	Qty	Rate (in ₹)	Taxable amount	CGST		SGST	
9963	Coffee	1	20	20.00	2.5%	₹0.50	2.5%	
9963	Masala Tea	1	10	10.00		₹0.25	2.5%	
9963	Masala Dosa	2	60		2.5%	□	2.5%	₹ 3.00
			Total	150.00				₹3.75
Grand total							157.50	

Solution:

a) Coffee:

$2.5\% \text{ of } 20 = \frac{2.5}{100} \times 20 = 0.50$

So, SGST = ₹0.50

b) Masala Tea

CGST:

$x\%$ of $10 = 0.25$

$$\frac{x}{100} \times 10 = 0.25$$

$x = 2.5$

So, CGST $= 2.5\%$

SGST:

2.5% of $10 = y$

$$\frac{2.5}{100} \times 10 = y$$

$y = 0.25$

So, SGST = ₹0.25

c) Masala Dosa

Taxable amount $=$ Rate $\times$ Qty $= 60 \times 2 =$ Rs. 120

CGST: 2.5% of $120 = \frac{2.5}{100} \times 120$ = ₹ 3

Tax invoice of services provided (Sample)								
Food Junction, Khed - Shivapur, Pune Mob no. 7588580000, email- ahar.khed@yahoo.com GSTIN: 27AAAAA5555B1ZA				Invoice no. 58 Invoice Date 25 Feb., 2020				
SAC	Food Items	Qty	Rate (in ₹)	Taxable amount	CGST		SGST	
9963	Coffee	1	20	20.00	2.5%	₹0.50	2.5%	₹0.50
9963	Masala Tea	1	10	10.00	2.5%	₹ 0.25	2.5%	₹0.25
9963	Masala Dosa	2	60	120.00	2.5%	₹3.00	2.5%	₹3.00
			Total	150.00		₹3.75		₹3.75

(B) **Solve the following sub-questions (any two):**

(i) Solve the following simultaneous equations using Cramer's rule:

$4m + 6n = 54; 3m + 2n = 28$

Answer: (6,5)

Solution:

$$4m + 6n = 54; 3m + 2n = 28$$

$$D = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix} = 4 \times 2 - 6 \times 3 = 8 - 18 = -10$$

$$D_m = \begin{vmatrix} 54 & 6 \\ 28 & 2 \end{vmatrix} = 54 \times 2 - 6 \times 28 = 108 - 168 = -60$$

$$D_n = \begin{vmatrix} 4 & 54 \\ 3 & 28 \end{vmatrix} = 4 \times 28 - 54 \times 3 = 112 - 162 = -50$$

$$m = \frac{D_m}{D} = \frac{-60}{-10} = 6$$

$$n = \frac{D_n}{D} = \frac{-50}{-10} = 5$$

$$(m, n) = (6,5)$$

(ii) Solve the following quadratic equation by formula method:

$x^2 + 10x + 2 = 0$

Answer: $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where, $a = 1, b = 10$, and $c = 2$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - 8}}{2}$$

$$x = \frac{-10 \pm \sqrt{92}}{2}$$

$$x = \frac{-10 \pm 2\sqrt{23}}{2}$$

$$x = \frac{2(-5 \pm \sqrt{23})}{2}$$

$$x = -5 \pm \sqrt{23}$$

Therefore, the roots of the given quadratic equation are

$-5 + \sqrt{23}$ and $-5 - \sqrt{23}$

(iii) A two-digit number is formed with digits 2,3,5,7,9 without repetition. What is the probability of the following events?

Event A: The number formed in an odd number.

Event B: The number formed is a multiple of 5.

Answer: $\frac{4}{5}$ and $\frac{1}{5}$

Solution:

Digits are

{22,23,25,27,29,32,33,35,37,39,52,53,55,57,59,72,73,75,77,79,92,93,95,97,99}

∴ Total digits are 25.

Odd numbers are 20.

∴ probability that an odd number is formed is $\frac{20}{25} = \frac{4}{5}$.

Numbers which are multiple of 5 are {25,35,55,75,95}.

∴ probability of multiple of 5 is $\frac{5}{25} = \frac{1}{5}$.

(iv) The frequency distribution table shows the number of mango trees, in a grove and their yield of mangoes. Find the median data:

No. of Mangoes	No. of Trees
$50-100$	33
$100-150$	30
$150-200$	90
$200-250$	80
$250-300$	17

Answer: 184 mangoes

Solution:

Class interval	Frequency	Cumulative frequency
$50-100$	33	33
$100-150$	30	63
$150-200$	90	153
(Median Class)	80	233
$200-250$	17	250
$250-300$	$N=250$	

From the above table, we get:

L (Lower class limit of the median class) $=150$

N (Sum of frequencies) $=250$

h (Class interval of the median class) $=50$

f (Frequency of the median class) $=90$

cf (Cumulative frequency of the class preceding the median class) $=63$

Now, Median $=L+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h$

$$=150+\left(\frac{\frac{250}{2}-63}{90}\right)\times 50=150+34.44$$

$=184.44$ mangoes

$=184$ mangoes

Hence, the median of data is 184 mangoes.

Question 4. Solve the following sub-questions (any two):

(i) If the first term of an A.P. is p, second term is q and last term is r, then show that sum of all terms is $(q+r-2p)\times\frac{(p+r)}{2(q-p)}$.

Solution:

Given First term, $a=p$ Common difference $d=q-p$

According to the question, $r=p+(n-1)(q-p)$

$$\frac{r-p}{q-p}=n-1$$

$$\frac{r-p}{q-p}+1=n$$

$$\frac{r-p+q-p}{q-p}=n$$

$$\frac{r+q-2p}{q-p}=n$$

We know:

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_n=\frac{r+q-2p}{2(q-p)}\left[2p+\left(\frac{r-p}{q-p}\right)\times(q-p)\right]$$

$$S_n=\frac{r+q-2p}{2(q-p)}[2p+r-p]$$

$$S_n=\frac{r+q-2p}{2(q-p)}[r+p]$$

$$S_n=[q+r-2p]\times\frac{(p+r)}{2(q-p)}$$

Hence proved.

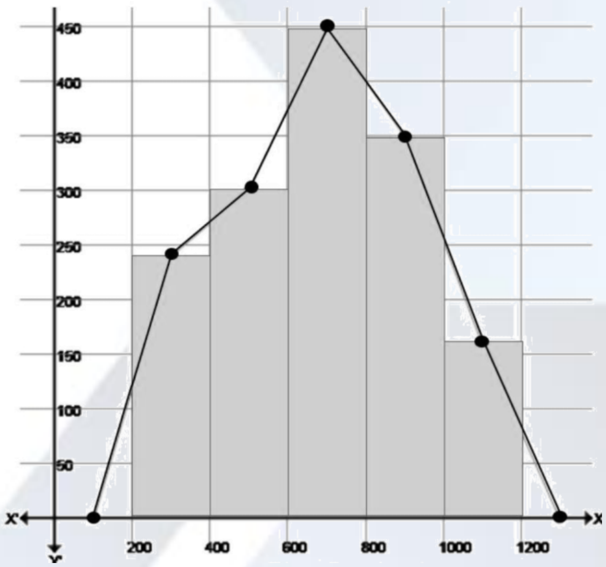
(ii) Show the following data by a frequency polygon:

Electricity bill (₹)	Families
200 – 400	240
400 – 600	300
600 – 800	450
800 – 1000	350
1000 – 1200	160

Solution:

To draw frequency Polygon, we first prepare the following table

Class	Mid	Frequency
$0-200$	100	0
$200-400$	300	240
$400-600$	500	300
$600-800$	700	450
$800-1000$	900	350
$1000-1200$	1100	160
$1200-1400$	1300	0



(iii) The sum of the squares of five consecutive natural numbers is 1455. Find the numbers.

Solution:

Let the five consecutive integers be $n, n+1, n+2, n+3, n+4$ then, $n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2=1455$

$5n^2+20n+30-1455=0$

$5n^2+20n-1425=0$

$n^2+4n-285=0$

$$n=\frac{-4\pm\sqrt{16+1140}}{2}=\frac{-4\pm 34}{2}=15,-17$$

Hence, the numbers are 15,16,17,18,19.

Question 5 Solve the following sub-questions (anyone):

1. Draw the graph of the equation $x + 2y = 4$. Find the area of the triangle formed by the line intersecting to X-axis and Y-axis.

   Answer: 4 square units

   Solution:

   To graph the equation $x + 2y = 4$, we can solve for y to get it in slope-intercept form:

   $x + 2y = 4$

   $2y = -x + 4$

   $y = \left(-\frac{1}{2}\right)x + 2$

   Now we can plot this line on a coordinate plane by finding the y-intercept of 2 and then using the slope of $-\frac{1}{2}$ to find additional points. We can also find the x -intercept by setting $\mathrm{y} = 0$ and solving for x:

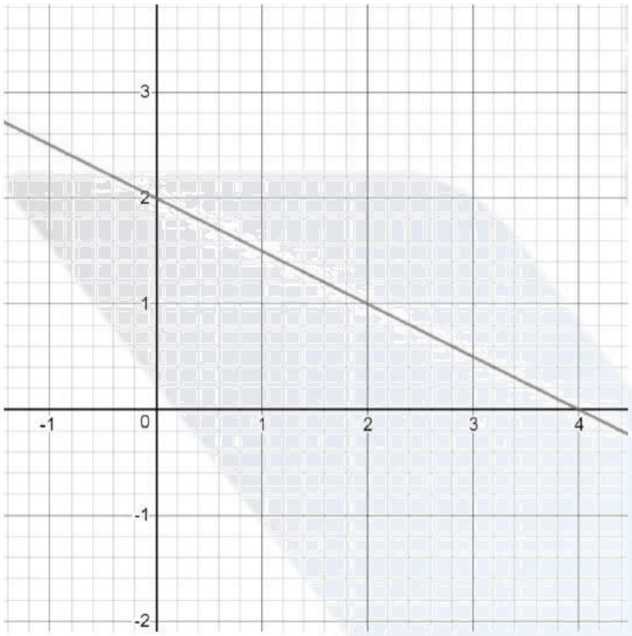
   $0 = \left(-\frac{1}{2}\right)x + 2$

   $x = 4$

   So, the x-intercept is (4,0). We can plot this point and draw a line through it with the slope we found:

   To find the area of the triangle formed by this line and the x and y axes, we need to find the x - and y-intercepts of the line. We already found the x-intercept to be (4,0), and to find the y-intercept we can set $x = 0$ and solve for y:

   So, the y-intercept is (0,2). We can now draw the triangle formed by the x - and y-intercepts and the point (0,0):



The base of the triangle is the x-intercept, which has a length of 4. The height of the triangle is the y-coordinate of the y-intercept, which is 2. Therefore, the area of the triangle is:

$A = \left(\frac{1}{2}\right) bh$

$A = \left(\frac{1}{2}\right)(4)(2)$

$A = 4$

So, the area of the triangle formed by the line $x + 2y = 4$ and the x and y axes is 4 square units.

2. A survey was conducted for 180 people in a city. 70 ate Pizza, 60 ate burgers and 50 ate chips. Draw a pie diagram for the given information.

Solution:

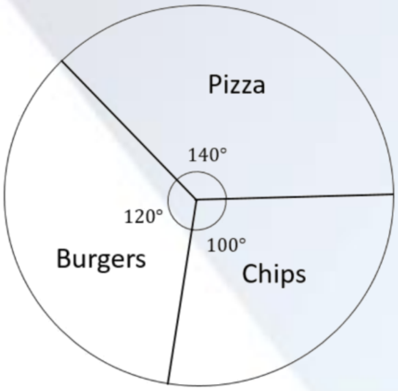
Total number of people $= 180$

Converting the number of people prefer various food items into components part of $360°$

Central angle of a component $= \frac{\text{Value of the component}}{\text{Total value}} \times 360°$

Item	No. of people	Central angle
Pizza	70	$\frac{70}{180} \times 360^\circ = 140^\circ$
Burgers	60	$\frac{60}{180} \times 360^\circ = 120^\circ$
Chips	50	360°
Total	180	

Food items preferred by people:



Part II

Question 1: (A) Four alternative answers are given for every sub question, Select the correct alternative and write the alphabet of that answer:

1. If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle:

   (A) Obtuse angled triangle

   (B) Acute angled triangle

   (C) Right angled triangle

   (D) Equilateral triangle

   Answer: (C) Right angled triangle

   Solution:

   In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is right angled triangle.

   Hence, the correct option is the Right-angled triangle.

2. Chords AB and CD of a circle intersect inside the circle at point E. If $AE = 4$, $EB = 10, CE = 8$, then find ED :

   (A) 7

   (B) 5

   (C) 8

   (D) 9

   Answer: B

   Solution:

   $AE \times EB = CE \times ED$

   Plugging in the given values, we get:

   $4 \times 10 = 8 \times ED$

   Simplifying, we get:

   $40 = 8 \times ED$

   $ED = 5$

3. Co-ordinates of origin are

(A) (0,0)

(B) (0,1)

(C) (1,0)

(D) (1,1)

Answer: A

Solution:

The coordinates of the origin are (0,0).

4. If radius of the base of cone is 7 cm and height is 24 cm, then find its slant height:

(A) 23 cm

(B) 26 cm

(C) 31 cm

(D) 25 cm

Answer: D

Solution:

The slant height of a cone can be calculated using the Pythagorean theorem, which states that the square of the slant height is equal to the sum of the square of the radius and the square of the height.

So, slant height $= \sqrt{\text{radius}^2 + \text{height}^2}$

Substituting the given values, we get:

slant height $= \sqrt{7^2 + 24^2}$

slant height $= \sqrt{49 + 576}$

slant height $= \sqrt{625}$

slant height $= 25$ cm

Therefore, the answer is 25 cm .

B) Solve the following sub-questions:

1. If $\triangle ABC \sim \triangle PQR$ and $\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{16}{25}$, then find $AB:PQ$.

   Answer: 4:5

   Solution:

   $$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2}$$
   $$\frac{16}{25} = \frac{AB^2}{PQ^2}$$
   $$\frac{AB}{PQ} = \sqrt{\frac{16}{25}}$$
   $$\frac{AB}{PQ} = \frac{4}{5}$$
   $$\therefore AB:PQ = 4:5$$

2. In $\triangle RST, \angle S = 90^\circ, \angle T = 30^\circ, RT = 12$ cm, then find RS.

   Answer: 6 cm

   Solution:

   $$\sin\ 30^\circ = \frac{RS}{RT}$$
   $$\frac{1}{2} = \frac{RS}{12}$$
   $$\therefore RS = 6 \text{ cm}$$

3. If the radius of a circle is 5 cm, then find the length of the longest chord of a circle.

   Answer: 10 cm

   Solution:

   The longest chord of a circle is the diameter of the circle.

   $\therefore$ Diameter $= 2 \times$ Radius

   $= 2 \times 5$

   $= 10$ cm

   $\therefore$ Diameter $= 10$ cm

4. Find the distance between the points O(0,0) and P(3,4).

Answer: 5 units

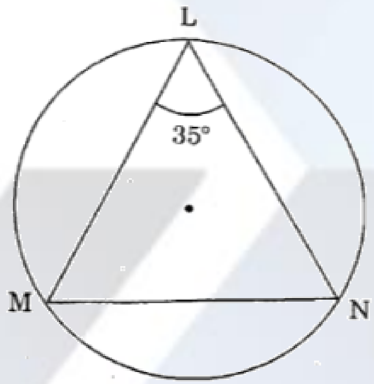
Solution:

Distance between a point and origin $= \sqrt{x^2 + y^2}$

$= \sqrt{3^2 + 4^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$ units

$\therefore$ The distance between 2 points is 5 units.

Question 2: (A) Complete the following activities (any two):



1. In the above figure, $\angle L = 35°$, find:

(i) $m(\text{arc}MN)$

(ii) $m(\text{arc}MLN)$

Solution:

$\angle L = \frac{1}{2} m(\text{arc}MN)$ ... (By inscribed angle theorem)

$\therefore 35 = \frac{1}{2} m(\text{arc}MN)$

$\therefore 2 \times 35 = m(\text{arc}MN)$

$\therefore m(\text{arc}MN) = 70°$

$m(\text{arc}MLN) = 360° - m(\text{arc}MN)$ [Definition of measure of arc]

$= 360° - 70°$

$\therefore m(\text{arc}MLN) = 290°$

2. Show that, $\cot\ \theta + \tan\ \theta = \operatorname{cosec}\theta \times \sec\ \theta$

Solution:

L.H.S. $= \cot\ \theta + \tan\ \theta = \frac{\cos\ \theta}{\sin\ \theta} + \frac{\sin\ \theta}{\cos\ \theta}$

$= \frac{\sin^2\ \theta + \theta}{\sin\ \theta \times \cos\ \theta}$

$= \frac{1}{\sin\ \theta \times \cos\ \theta} \ldots \ldots \ldots \ldots (\sin^2\ \theta + \theta = 1)$

$= \frac{1}{\sin\ \theta} \times \frac{1}{\cos\ \theta}$

$= \operatorname{cosec}\theta \times \sec\ \theta$

$=$ R.H.S.

$\therefore \cot\ \theta + \tan\ \theta = \operatorname{cosec}\theta \times \sec\ \theta$

3. Find the surface area of a sphere of radius 7 cm.

Answer: 616 cm^2

Solution:

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 7^2$

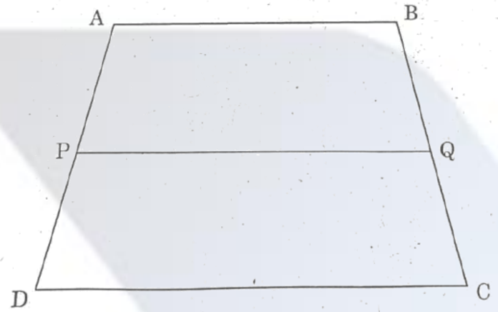
$= 4 \times \frac{22}{7} \times 49$

$= 88 \times 7$

$\therefore$ Surface area of sphere $= 616\ cm^2$.

B. Solve the following sub-questions (Any four):

1. In trapezium $ABCD$, side $AB \|$ side $PQ \|$ side DC. $AP = 15, PD = 12, QC = 14$, find BQ.



Answer: 17.5

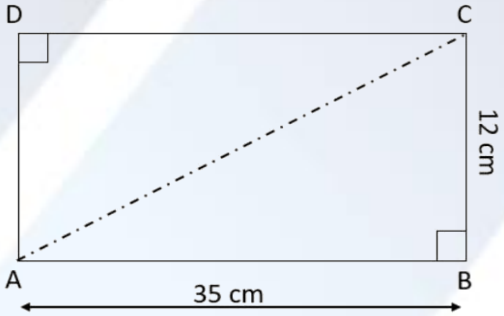
Solution: According to property of three parallel lines and their transversals if $AB \| PQ \| DC$ then:

$$\frac{AP}{PD} = \frac{BQ}{QC} \Rightarrow \frac{15}{12} = \frac{BQ}{14} \Rightarrow BQ = 17.5$$

2. Find the length of the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

Answer: 37 cm

Solution:



In $\triangle ABC$, according to Pythagoras theorem,

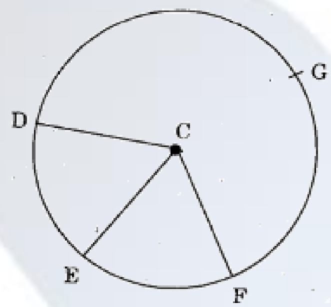
$$AB^2 + BC^2 = AC^2$$
$$\Rightarrow 35^2 + 12^2 = AC^2$$
$$\Rightarrow 1225 + 144 = AC^2$$
$$\Rightarrow AC^2 = 1369$$
$$\Rightarrow AC = 37 \text{ cm}$$

Hence, the length of the diagonal is 37 cm.

3. In the given figure points G, D, E, F are points of a circle with centre $C, \angle ECF = 70^\circ, m(\text{arc DGF}) = 200^\circ$.

   Find: (i) $m(\text{arc}DE)$

   (ii) m (arc DEF).



   Answer: 90° and 160°

   Solution:

   (i) $m(\text{arc}DE) = 360^\circ - [m(\text{arc}DGF) + m(\text{arcECF})]$

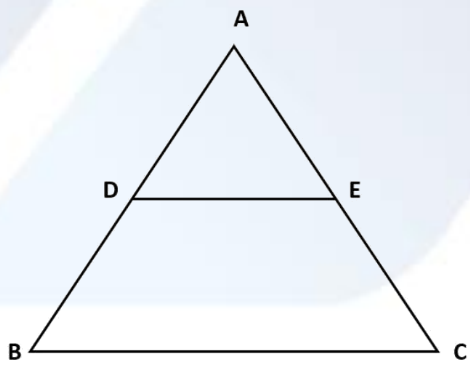
   $m(\text{arc}DE) = 360^\circ - [200^\circ + 70^\circ]$
   $m(\text{arc}DE) = 90^\circ$

   (ii) $m(\text{arc}DEF) = m(\text{arc}DE) + m(\text{arc}EF)$

   $m(\text{arc}DEF) = 90^\circ + 70^\circ$
   $m(\text{arc}DEF) = 160^\circ$

4. Show that points $A(-1,-1), B(0,1), C(1,3)$ are collinear.

   Solution:



   $A(-1,-1), B(0,1), C(1,3)$

Slope of $AB = \frac{1-(-1)}{0-(-1)} = \frac{2}{1} = 2$

Slope of $BC = \frac{3-1}{1-0} = \frac{2}{1} = 2$

Slope of $AB =$ Slope of $BC = 2$

Thus, the given points are collinear.

5. A person is standing at a distance of 50 m from a temple looking at its top. The angle of elevation is of 45°, Find the height of the temple.

   Answer: $50\sqrt{2}$ m

   Solution:

   Let ABC be the triangle right angled at B and angle C be 45°.

   Given that, $BC = 50$ m

   To find, AB (height).

   $$\cos\ 45° = \frac{BC}{AB}$$

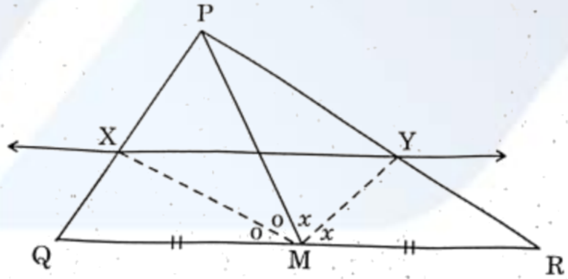
   $$\frac{1}{\sqrt{2}} = \frac{50}{AB}$$

   $\therefore AB = 50\sqrt{2}$ m

   $\therefore$ Height of the temple $= 50\sqrt{2}$ m.

Question 3A. Complete the following activities (any one):

1.



In $\triangle PQR$, seg PM is a median. Angle bisectors of $\angle PMQ$ and $\angle PMR$ intersect side PQ and side PR in points X and Y respectively. Prove that $XY \| QR$.

Complete the proof by filling in the boxes.

Solution:

In $\triangle PMQ$,

Ray MX is the bisector of $\angle PMQ$

$\therefore \frac{MP}{MQ} = \frac{XQ}{PX}$ (I) [Theorem of angle bisector]

Similarly, in $\triangle$ PMR, Ray MY is bisector of $\angle PMR$

$\therefore \frac{MP}{MR} = \frac{YR}{PY}$ (II) [Theorem of angle bisector]

But $\frac{MP}{MQ} = \frac{MP}{MR}$ (III) [As M is the midpoint of QR]

Hence $MQ = MR$

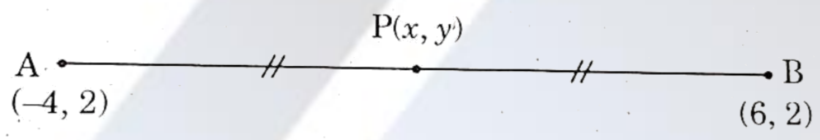
$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$ [From (I), (II) and (III)]

$\therefore XY \| QR$ … … … [Converse of basic proportionality theorem]

2. Find the coordinates of point P where P is the midpoint of a line segment AB with $A(-4,2)$ and $B(6,2)$.

Answer: (1, 2)

Solution:



Suppose $(-4,2) = (x_1, y_1)$ and $(6,2) = (x_2, y_2)$ and co-ordinates of P are (x, y)

$\therefore$ According to midpoint theorem,

$$x = \frac{x_1 + x_2}{2} = \frac{-4 + 6}{2} = \frac{2}{2} = 1$$
$$y = \frac{y_1 + y_2}{2} = \frac{2 + 2}{2} = \frac{4}{2} = 2$$

$\therefore$ Co-ordinates of midpoint P are $(1,2)$.

(B) Solve the following sub-questions (any two):

1. In $\triangle ABC$, seg AP is a median. If $BC = 18, AB^2 + AC^2 = 260$, find AP.

   Answer: 7 cm

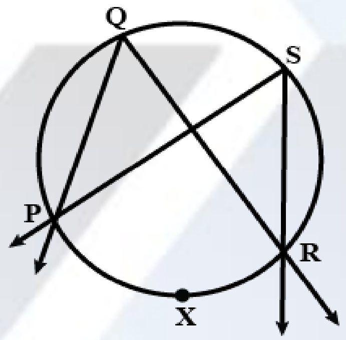
   Solution:

   $AB^2 + AC^2 = 2AP^2 + 2BP^2$(by Apollonius theorem)
   $\Rightarrow 260 = 2AP^2 + 2(9^2)$
   $\Rightarrow 260 = 2AP^2 + 2(81)$
   $\Rightarrow 260 = 2AP^2 + 162$
   $\Rightarrow 2AP^2 = 260 - 162$
   $\Rightarrow 2AP^2 = 98$
   $\Rightarrow AP^2 = 49$
   $\Rightarrow AP = 7$

2. Prove that "Angles inscribed in the same arc are congruent."

   Solution:

   Proof:



   $m\angle PQR = \frac{1}{2} m($ are $P \times R) - (i)$ (inscribed angle theorem)

   $m\angle PSR = \frac{1}{2} m(\text{arc}PXR) - (i)$ (Inscribed angle theorem)

   $\therefore m\angle PQR = m\angle PSR -$ [form $(i)\&(ii)$]

   $\therefore \angle PQR \cong \angle PSR$. (Angles equal in measure are congruent).

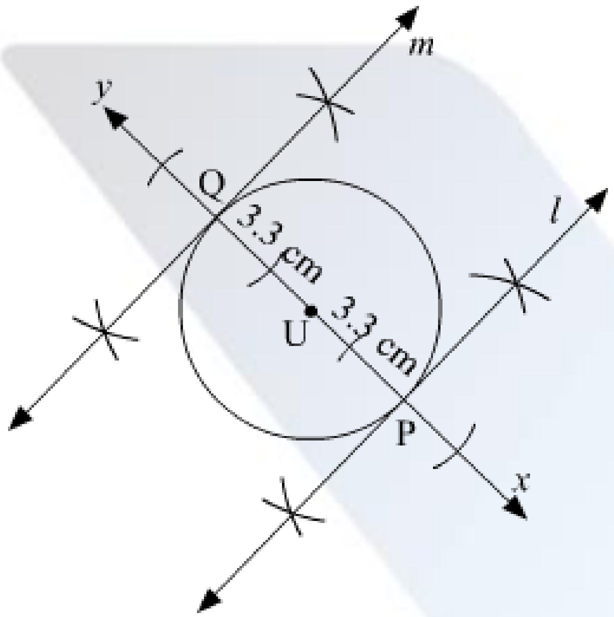
3. Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q.

   Solution:

Steps of Construction:

Step 1: Draw a circle with radius 3.3 cm. Mark any point P on it.

Step 2: Draw chord $PQ = 6.6$ cm(PQ is the diameter of the circle).



Step 3: Draw rays CX and CY.

Step 4: Draw line I perpendicular to ray CX through point P.

Step 4: Draw line m perpendicular to ray CY through point Q.

Here, line I and line m are the required tangents to the circle at points P and Q, respectively. It can be observed that the tangents I and m are parallel to each other.

4. The radii of circular ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its curved surface area. $(\pi = 3.14)$

Answer: 628 sq. cm

Solution:

Here, $r_1 = 14$ cm, $r_2 = 6$ cm and $h = 6$ cm.

Slant height of the frustum, $I = \left[\sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{6^2 + (14-6)^2} =\right.$

$\left.\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}\right] = 10$ cm

Curved surface area of frustum $= \pi(r_1 + r_2)l$

$= 3.14 \times (14 + 6) \times 10$

$= 3.14 \times 20 \times 10$

$= 628 \text{ cm}^2$

$\therefore$ The curved surface area of frustum is 628 sq. cm.

Question 4: Solve the following sub-questions (any two):

1. In $\triangle ABC$, seg $DE \parallel$ side BC. If $2A(\triangle ADE) = A(\square DBCE)$, find $AB : AD$ and show that $BC = \sqrt{3}DE$.

Solution:

Given:

In $\triangle ABC$, segment $DE \|$ side BC

$2A(\triangle ADE) = A(DBCE)$

To Find: $BC = \sqrt{3} \times DE$

Since DE is parallel to BC. In $\triangle ADE$ and $\triangle ABC$, $\angle A$ is common.

The corresponding angles of a triangle are equal.

$\therefore \angle ADE = \angle ABC$

So, $\triangle ADE$ is similar to $\triangle ABC$ by AA similarity criteria.

$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad (1)$

Then, $\frac{A(\text{ triangle } ADE)}{A(\text{ triangle } ABC)} = \frac{AD^2}{AB^2} = \frac{DE^2}{BC^2} = \frac{AE^2}{AC^2} \quad (\text{II})$

Also, $2A(\triangle ADE) = A(DBCE) \quad$ (III) [Given]

Since, $A(\triangle ABC) = A(\triangle ADE) + A(DBCE)$

$A(\triangle ABC) = A(\triangle ADE) + 2A(\triangle ADE)$

$\therefore A(\triangle ABC) = 3A(\triangle ADE)$

$$\frac{A(\text{ triangle } ADE)}{A(\text{ triangle } ABC)} = \frac{1}{3}$$

Using (II) and (III), we get:

$$\frac{AD^2}{AB^2} = \frac{DE^2}{BC^2} = \frac{1}{3}$$

$\therefore \frac{AD^2}{BD^2} = \frac{1}{3}$ and $\frac{DE^2}{BC^2} = \frac{1}{3}$

$\frac{AD}{BD} = \frac{1}{\sqrt{3}} \cdot (IV)$ and $\frac{DE}{BC} = \frac{1}{\sqrt{3}}.$ (V)

Now, by applying for an inverted in (IV)

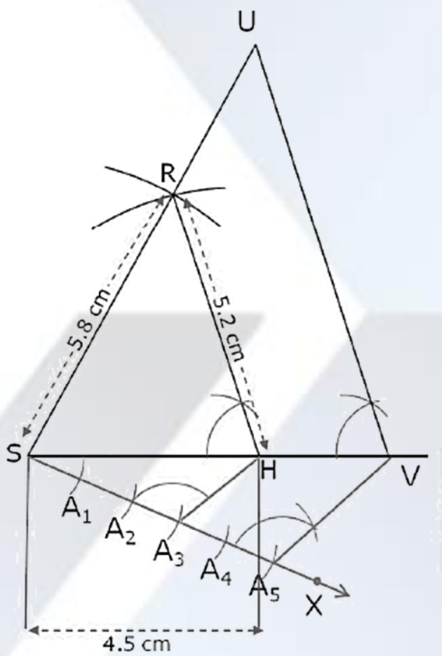
We get: $\frac{AB}{BD} = \sqrt{3}$ and

Using equation (V), we get: $BC = \sqrt{3} \times DE$

Hence proved.

2. $\triangle SHR \sim \triangle SVU$. In $\triangle SHR, SH = 4.5$ cm, $HR = 5.2$ cm, $SR = 5.8$ cm and $\frac{SH}{SV} = \frac{3}{5}$, construct $\triangle SVU$.

Solution:



Steps of Construction:

1. Construct the $\triangle$ SHR with the given measurements. For this draw SH of 4.5 cm.
2. Taking S as the center and radius equal to 5.8 cm draw an arc above SH.
3. Taking H as the center and radius equal to 5.2 cm draw an arc to intersect the previous arc. Name the point of intersection as R.
4. Join SR and HR. $\triangle$ SHR with the given measurements is constructed. Extend SH and SR further on the right side.

5. Draw any ray SX making an acute angle (i.e.; 45°) with SH on the side opposite to the vertex R.
6. 6 Locate 5 points. (the ratio of old triangle to new triangle is 3/5 and 5 > 3). Locate A_1, A_2, A_3, A_4 and A_5 on AX so that $SA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
7. Join A_3H and draw a line through A_5 parallel to A_3H_1, intersecting the extended part of SH at V.
8. Draw a line VU through V parallel to HR. $\triangle SVU$ is the required triangle.

3. An ice cream pot has a right circular cylindrical shape. The radius of the base is 12 cm and height is 7 cm. This pot is completely filled with ice-cream. The entire ice cream is given to the students in the form of right circular ice-cream cones, having a diameter of 4 cm and height is 3.5 cm. If each student is given one cone, how many students can be served?

Answer: 647 students

Solution:

Volume of ice cream in the cylindrical pot $= \pi \times (\text{radius})^2 \times (\text{height})$

$= \pi(12\text{ cm}^2)(7\text{ cm}) = 3{,}024\pi\text{ cm}^3$

Volume of one ice-cream cone $= \frac{1}{3}\pi\ (\text{radius})\ ^2\ (\text{height})$

$$= \frac{1}{3}\pi(2\text{ cm})^2(3.5\text{ cm})$$

$= 4.67\pi\text{ cm}^3$ (Rounded to two decimal places)

Number of cones that can be filled with the ice-cream in the pot = (Volume of icecream in the cylindrical pot) ÷ (Volume of one ice-cream cone)

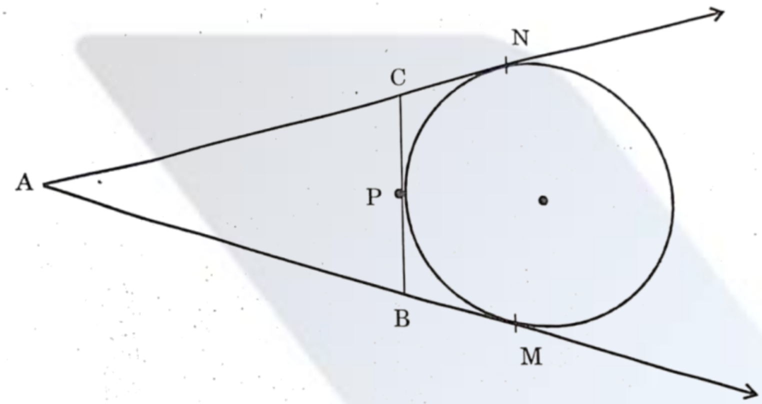
$= (3{,}024\pi\text{ cm}^3) \div (4.67\pi\text{ cm}^3)$

$= 647.46$ (Rounded to two decimal places)

Therefore, 647 students can be served with one cone each. Note that this is an approximation and assumes that no ice cream is lost or wasted in the process of filling the cones.

Question 5. Solve the following sub-questions (anyone):

1.



A circle touches side BC at point P of the $\triangle ABC$, from out-side of the triangle. Further extended lines AC and AB are tangents to the circle at N and M respectively. Prove that: $AM = \frac{1}{2}(\text{ Perimeter of } \triangle ABC)$

Solution:

Lengths of triangle drawn from an external point to a circle are equal.

$\Rightarrow AM = AN, BM = BP, CP = CN + (AN - CN)$

Perimeter of $\triangle ABC = AB + BC + CA$

$= AB + (BP + PC)$

$= (AB + BM) + PC + (AM - PC)$

$= AM + AM = 2AM$

$$AM = \frac{1}{2}(\text{ perimeter of } \triangle ABC)$$

Hence proved.

2. Eliminate θ if $x = r\cos\ \theta$ and $y = r\sin\ \theta$.

Solution:

$x = r\cos\ \theta, y = r\sin\ \theta$

Squaring on both terms:

$x^2 = r^2\cos^2\theta$ ...(1)

$y^2 = r^2\sin^2\theta$ ...(2)

Adding (1) and (2), we get:

$x^2 + y^2 = r^2\sin^2\theta + r^2\cos^2\theta$
$x^2 + y^2 = r^2(\sin^2\theta + \cos^2\theta)$

But we know $(\sin^2\theta + \cos^2\theta) = 1$

$\therefore x^2 + y^2 = r^2$