Grade 10 Maths Maharashtra 2024

SECTION- I

Time: 2 Hours **Max.Marks: 40**

Note:

1. All questions are compulsory.
2. Use of a calculator is not allowed.
3. The numbers to the right of the questions indicate full marks.
4. In case of MCQs [Q. No. $1(A)$] only the first attempt will be evaluated and will be given credit.
5. Draw proper figures wherever necessary.
6. The marks of construction should be clear. Do not erase them.
7. Diagram is essential for writing the proof of the theorem.

Q1. (A) Choose the correct alternative from given:

i. If 3 is one of the roots of the quadratic equation $\mathrm{kx}^2-7\mathrm{x}+12=0$, then $\mathrm{k}=$

Solution:

The given quadratic equation is $\mathrm{kx}^2-7\mathrm{x}+12=0$.

$\therefore$ Put $\mathrm{x}=3$

$\therefore \mathrm{k}(3)^2-7(3)+12=0$

$\therefore 9\mathrm{k}-21+12=0$

$\therefore 9\mathrm{k}-9=0$

$\therefore 9\mathrm{k}=9$

$\therefore \mathrm{k}=1$

ii. To draw the graph of $x+2y=4$, find x when $y=1$:

Solution:

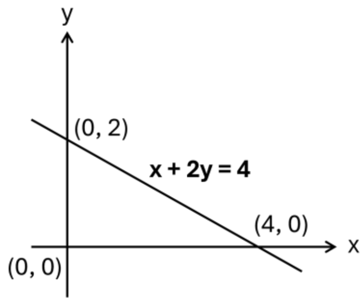
Given equation, $x+2y=4$.

When $\mathrm{y}=0$

$\therefore \mathrm{x}+2(0)=4$

$\therefore \mathrm{x}=4$

$\therefore$ To draw the graph of $x+2y=4$, when $y=0$, the value of x is 4.



iii. For a given A.P., $t_7 = 4, d = -4$, then $a =$

Solution:

Given: $t_7 = 4, d = -4$

Now, $t_n = a + (n - 1)d$

$t_7 = a + (7 - 1)d$

$\Rightarrow 4 = a + 6(-4)$

$\Rightarrow 4 = a - 24$

$\Rightarrow a = 4 + 24$

$\Rightarrow a = 28$

iv. In the format of GSTIN, there are ______ alpha numerals.

Solution:

GSTIN begins with two digits of state code, then ten alpha-numeric characters of the owner's PAN number, then the state registration number and letter Z, and finally the checksum. So, there are a total of 15 alpha-numerals.

Q1. (B) Solve the following sub questions:

i. If $17x + 15y = 11$ and $15x + 17y = 21$, then find the value of $x - y$.

Solution:

The given equations are:

$17x + 15y = 11 \quad ...(1)$

$15x + 17y = 21 \quad ...(2)$

Now, let's subtract equation (2) from (1), we get:

$2x - 2y = -10$

Dividing the above equation by 2, we get:

$x - y = -5$

ii. Find first term of the sequence $t_n = 3n - 2$.

Solution:

Given: $t_n = 3n - 2$

Put $n = 1$

$\therefore t_1 = 3(1) - 2$

$\therefore t_1 = 3 - 2$

$\therefore t_1 = 1$

Therefore, the first term of the sequence is 1.

iii. If the face value of a share is ₹ 100 and the market value is ₹ 150. If rate of brokerage is 2%, find brokerage paid on one share.

Solution:

Given: Face value = ₹100

Market value = ₹150

Rate of brokerage = 2%

Brokerage paid on one share = 2% of Market value

$= \frac{2}{100} \times 150$

$= \frac{300}{100}$

= ₹3

Therefore, the brokerage paid on one share = ₹3.

iv. For the following experiment write sample space ' S ' and number of sample points n(S).

Two-digit numbers are formed using digits 2,3 and 5 without repeating a digit.

Solution:

Let 'S' be the sample space.

Sample Space $(S) = \{23,25,32,35,52,53\}$

$\therefore n(S) = 6$

Hence, the required number of sample spaces is 6.

Q2. (A) Complete the following activities and rewrite it (any two):

i. If (0,2) is the solution of $2x + 3y = k$ then find the value of k, by completing the activity:

Solution:

Activity:

(0,2) is solution of the equation $2x + 3y = k$.

Put $\mathrm{x} = 0$ and $\mathrm{y} = 2$ in the given equation.

$\therefore 2 \times 0 + 3 \times 2 = \mathrm{k}$

$\therefore 0 + 6 = \mathrm{k}$

$\therefore \mathrm{k} = -6$

ii. If 2 and 5 are the roots of the quadratic equation, then complete the following activity to form quadratic equation:

Activity:

Let $\alpha = 2$ and $\beta = 5$ are the roots of the quadratic equation.

Then quadratic equation is:

$x^2 - (\alpha + \beta)\mathrm{x} + \alpha\beta = 0$

$\therefore x^2 - (2 + \square)x + \square \times 5 = 0$

$\therefore x^2 - \square\ x + \square = 0$

Solution:

Let $\alpha = 2$ and $\beta = 5$ are the roots of the quadratic equation.

Then quadratic equation is:

$\mathrm{x}^2 - (\alpha + \beta)\mathrm{x} + \alpha\beta = 0$

$\therefore \mathrm{x}^2 - (2 + 5)\mathrm{x} + 2 \times 5 = 0$

$\therefore \mathrm{x}^2 - 7\mathrm{x} + 10 = 0$

iii. Two coins are tossed simultaneously. Complete the following activity to write the sample space and the given events A and B in the set form:

Event A: To get at least one head.

Event B: To get no head.

Activity:

Two coins are tossed simultaneously.

$\therefore$ Sample space is

$\mathrm{S} = \{\square, HT, TH, \square\}$

Event A: To get at least one head.

$\therefore$ A $= \{\square, HT, TH\}$

Event B: To get no head.

$\therefore B = \{\square\}$

Solution:

Two coins are tossed simultaneously.

$\therefore$ Sample space is

S $= \{HH, \text{HT}, \text{TH}, TT\}$

Event A: To get at least one head.

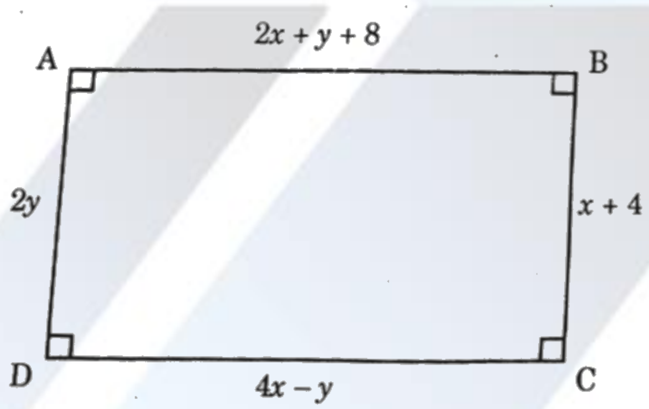
$\therefore$ A $= \{HH, \text{HT}, \text{TH}\}$

Event B: To get no head.

$\therefore$ B $= \{TT\}$

Q2. (B) Solve the following subquestions (any four):

i. $\square\ ABCD$ is a rectangle. Write two simultaneous equations using information given below in the diagram, in the form of $ax + by = c$:



Solution:

In $\square$ ABCD,

AB $=$ CD ... [Opposite sides of rectangle]

$2x + y + 8 = 4x - y$

$2x - 4x + y + y = -8$

$-2x + 2y = -8$

$2x - 2y = 8$

$x - y = 4$

BC $=$ AD ... [Opposite sides of rectangle]

$x + 4 = 2y$

$x-2y=-4$

$x-y=4$ and $x-2y=-4$ are the required simultaneous equations.

ii. Solve the following quadratic equation by factorisation method:

$x^2+x-20=0$

Solution:

$$\begin{aligned}
&x^2+x-20=0\\
&\therefore x^2+5x-4x-20=0\\
&\therefore x+5x-4x-20=0\\
&\therefore x(x+5)-4(x+5)=0\\
&\therefore (x+5)(x-4)=0\\
&\therefore (x+5)=0 \text{ or } (x-4)=0\\
&\therefore x+5=0 \text{ or } x-4=0\\
&\therefore x=-5 \text{ or } x=4
\end{aligned}$$

$\therefore -5$ and 4 are roots of the given quadratic equation.

iii. Find the 19^{th} term of the following A.P.:

7,13,19,25, … …

Solution:

The sequence given is 7,13,19,25, ….

Here,

First term $=a=t_1=7, t_2=13, t_3=19, t_4=25$,

Common difference $=d=t_2-t_1=13-7=6$

To find the 19th term, we have to use the formula, i.e.,

$t_n=a+(n-1)d$

$\therefore t_{19}=7+(19-1)\times 6$ … (On substituting value)

$\therefore t_{19}=7+18\times 6$

$=7+108$

$\therefore t_{19}=115$

$\therefore 19^{\text{th}}$ term $=t_{19}=115$

Hence, the 19^{th} term of the progression is 115.

iv. A card is drawn from well shuffled pack of 52 playing cards. Find the probability that the card drawn is a face card.

Solution:

There are 52 cards.

$n(S) = 52$

Let 'A' be the event of getting face cards.

Event A: Getting face cards. There are 12 face cards in the pack of playing cards.

$n(A) = 12$

$$P(A) = \frac{n(A)}{n(s)}$$

$$P(A) = \frac{12}{52}$$

$$P(A) = \frac{3}{13}$$

v. The following table shows classification of number of workers and number of hours they work in software company. Prepare less than upper limit type cumulative frequency distribution table:

Number of hours daily	Number of workers
$8 - 10$	150
$10 - 12$	500
$12 - 14$	300
$14 - 16$	50

Solution:

Number of hours daily	Number of workers	Cumulative Frequency
$8 - 10$	150	150
$10 - 12$	500	$150 + 500 = 650$
$12 - 14$	300	$650 + 300 = 950$
$14 - 16$	50	$950 + 50 = 1000$

Q3. (A) Solve the following subquestions (any one):

i. The following frequency distribution table shows the classification of the number of vehicles and the volume of petrol filled in them. To find the mode of the volume of petrol filled, complete the following activity:

Class (Petrol filled in Liters)	Frequency (Number of Vehicles)
0.5 − 3.5	33
3.5 − 6.5	40
6.5 − 9.5	27
9.5 − 12.5	18
12.5 − 15.5	12

Activity:

From the given table,

Modal class $= \square$

$\therefore$ Mode $= \square + \left[\dfrac{f_1 - f_0}{2f_1 - f_0 - \square}\right] \times h$

$\therefore$ Mode $= 3.5 + \left[\dfrac{40 - 33}{2(40) - 33 - 27}\right] \times \square$

$\therefore$ Mode $= 3.5 + \left[\frac{7}{80-60}\right] \times 3$

$\therefore$ Mode $= \square$

$\therefore$ The mode of the volume of petrol filled is $\square$.

Solution:

From the given table,

Modal class $= \underline{3.5 - 6.5}$ ... ($\because$ This class has max. frequency)

$\therefore$ Mode $= \underline{L} + \left[\frac{f_1 - f_0}{2f_1 - f_0 - \underline{f_2}}\right] \times h$

L = Lower class boundary of the modal class

$f_1 = 40$ (Frequency of modal class)

$f_2 = 27$ (Frequency of the class succeding the modal class)

$f_0 = 33$ (frequency of the class preceding the modal class)

$\therefore$ Mode $= 3.5 + \left[\frac{40-33}{2(40)-33-27}\right] \times \underline{3}$

$\therefore$ Mode $= 3.5 + \left[\frac{7}{80-60}\right] \times 3$

$\therefore$ Mode $= \underline{4.55}$

$\therefore$ The mode of the volume of petrol filled is $\underline{4.55}$.

ii. The total value (with GST) of remote controlled toy car is ₹ 2360 . Rate of GST is 18% on toys. Complete the following activity to find the taxable value for the toy car.

Activity:

Total value for toy car with GST = ₹2360

Rate of GST = 18%

Let taxable value for toy car be ₹ x

$$\therefore \text{GST} = \frac{18}{100} \times x$$

$\therefore$ Total value for toy car = (taxable value for toy car) + $\square$ .... Formula

$$\therefore 2360 = \square + \frac{\square}{100} \times x$$

$$\therefore 2360 = \frac{\square}{100} \times x$$

$$\therefore 2360 \times 100 = 118x$$

$$\therefore x = \frac{2360 \times 100}{\square}$$

$\therefore$ Taxable value for toy car is ₹ $\square$.

Solution:

Total value for toy car with GST = ₹2360

Rate of GST = 18%

Let taxable value for toy car be ₹ x

$$\therefore \text{GST} = \frac{18}{100} \times x$$

$\therefore$ Total value for toy car = (taxable value for toy car) + GST.... Formula

$$\therefore 2360 = \underline{x} + \frac{18}{100} \times x$$

$$\therefore 2360 = \frac{118}{100} \times x$$

$$\therefore 2360 \times 100 = 118x$$

$$\therefore x = \frac{2360 \times 100}{118}$$

$\therefore$ Taxable value for toy car is ₹ $\underline{2000}$.

Q3. (B) Solve the following subquestions (any two):

i. Solve the following quadratic equation by formula method:

$3m^2 - m - 10 = 0$

Solution:

Given: $3\mathrm{m}^2 - \mathrm{m} - 10 = 0$

Comparing with standard form $ax^2 + bx + c = 0$.

$a = 3, b = -1, c = -10$

$D = b^2 - 4ac$

$= (-1)^2 - 4(3)(-10)$

$= 1 + 120$

$= 121 > 0$

By formula method,

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-(-1) \pm \sqrt{121}}{2(3)}$

$m = \frac{1 \pm \sqrt{121}}{6}$

$m = \frac{1 \pm 11}{6}$

$m = \frac{1+11}{6}$ or $m = \frac{1-11}{6}$

$m = \frac{12}{6}$ or $m = \frac{-10}{6}$

$\mathrm{m} = 2$ or $m = \frac{-5}{3}$

The roots of the quadratic equation are $\mathrm{m} = 2$ or $m = \frac{-5}{3}$.

ii. Solve the following simultaneous equations using Cramer's rule.

$3x - 4y = 10; 4x + 3y = 5$

Solution:

Given equations,

$3x - 4y = 10 \quad \ldots(1)$

$4x + 3y = 5$

$a_1 = 3, a_2 = 4, \; b_1 = -4, \; b_2 = 3, c_1 = 10, c_2 = 5$

$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$D = \begin{vmatrix} 3 & -4 \\ 4 & 3 \end{vmatrix}$

$= 3 \times 3 - (-4) \times 4$

$= 9 + 16$

$= 25$

$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

$D_x = \begin{vmatrix} 10 & -4 \\ 5 & 3 \end{vmatrix}$

$= 10 \times 3 - (-4) \times 5$

$= 30 + 20 = 50$

$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$D_y = \begin{vmatrix} 3 & 10 \\ 4 & 5 \end{vmatrix}$

$= 3 \times 5 - 4 \times 10$

$= 15 - 40$

$= -25$

By using cramer's rule,

$x = \frac{D_x}{D} = \frac{50}{25} = 2$

$y = \frac{D_y}{D}$

$= \frac{-25}{25}$

$= -1$

$(x, y) = (2, -1)$ is the solution of the given simultaneous equation.

iii. 50 shares of face value ₹ 10 were purchased for market value of ₹ 25 . Company declared 30% dividend on the shares, then find:

a. Sum invested

b. Dividend received

c. Rate of return.

Solution:

Given: FV = ₹10; MV = ₹25

Number of shares = 50

Rate of dividend = 30%

a. Sum invested $= \text{No. of shares} \times \text{MV}$

Sum invested $= 50 \times 25 = ₹1250$

b. Dividend received,

$$\text{Dividend} = \frac{\text{Rate of dividend}}{100} \times \text{FV}$$

$$= \frac{30}{100} \times 10$$

$= ₹3$

Total dividend $= \text{No. of shares} \times \text{Dividend}$

$= 50 \times 3 = ₹150$

Dividend received $= ₹150$

c. Rate of return,

$$\text{Rate of return} = \frac{\text{Dividend}}{\text{Sum invested}} \times 100$$

$$= \frac{150}{1250} \times 100$$

$$= \frac{1500}{125}$$

$= 12\%$

Rate of dividend $= 12\%$

iv. One coin and a die are thrown simultaneously. Find the probability of the following event:

Event A: To get a head and a prime number.

Event B: To get a tail and an odd number.

Solution:

One coin and a die are thrown simultaneously.

Sample space,

$S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$

$\therefore n(S) = 12$

Event A: To get a head and a prime number.

$\therefore \mathrm{A}=\{(\mathrm{H}, 2),(\mathrm{H}, 3),(\mathrm{H}, 5)\}$
$\therefore \mathrm{n}(\mathrm{A})=3$
$\therefore \mathrm{P}(\mathrm{A})=\frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{S})}$
$\therefore \mathrm{P}(\mathrm{A})=\frac{3}{12}$
$\therefore \mathrm{P}(\mathrm{A})=\frac{1}{4}$

The Probability of event A is $\frac{1}{4}$.

One coin and a die are thrown simultaneously.

Sample space,

$\mathrm{S}=\{(\mathrm{H}, 1),(\mathrm{H}, 2),(\mathrm{H}, 3),(\mathrm{H}, 4),(\mathrm{H}, 5),(\mathrm{H}, 6),(\mathrm{T}, 1),(\mathrm{T}, 2),(\mathrm{T}, 3),(\mathrm{T}, 4),(\mathrm{T}, 5),(\mathrm{T}, 6)\}$
$\therefore \mathrm{n}(\mathrm{S})=12$

Event B: To get a tail and an odd number.

$\therefore B=\{(T, 1),(T, 3),(T, 5)\}$
$\therefore \mathrm{n}(\mathrm{B})=3$
$\therefore \mathrm{P}(\mathrm{B})=\frac{\mathrm{n}(\mathrm{B})}{\mathrm{n}(\mathrm{S})}$
$\therefore \mathrm{P}(\mathrm{B})=\frac{3}{12}$
$\therefore \mathrm{P}(\mathrm{B})=\frac{1}{4}$

$\therefore$ The probability of event B is $\frac{1}{4}$.

Q4. Solve the following subquestions (any two):

i. A tank can be filled up by two taps in 6 hours. The smaller tap alone takes 5 hours more than the bigger tap alone. Find the time required by each tap to fill the tank separately.

Solution:

Let the bigger tap alone take x hours to fill the tank.

Then the smaller tap alone takes $(x+5)$ hours to fill the tank.

The bigger tap fills $\frac{1}{x}$ part of the tank in 1 hour, and the smaller tap fills $\frac{1}{x+5}$ part of the tank in 1 hour.

$\therefore$ both the taps together fill $\left(\frac{1}{x}+\frac{1}{x+5}\right)$ part of the tank in 1 hour. Both the taps together fill the tank in 6 hours. (Given)

$\therefore$ Both the taps together fill $\frac{1}{6}$ part of the tank in 1 hour.

$$\therefore \frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$\therefore \frac{x+5+x}{x(x+5)} = \frac{1}{6}$$

$$\therefore \frac{2x+5}{x^2+5x} = \frac{1}{6}$$

$\therefore 6(2x+5) = x^2 + 5x$

$\therefore 12x + 30 = x^2 + 5x$

$\therefore x^2 + 5x - 12x - 30 = 0$

$\therefore x^2 - 7x - 30 = 0$

$\therefore x^2 - 10x + 3x - 30 = 0$

$\therefore x(x-10) + 3(x-10) = 0$

$\therefore (x-10)(x+3) = 0$

$\therefore x - 10 = 0$ or $x + 3 = 0$

$\therefore x = 10$ or $x = -3$

But the time cannot be negative.

$\therefore x = -3$ is unacceptable.

$\therefore x = 10$ and $x + 5 = 10 + 5 = 15$.

The bigger tap alone fills the tank in 10 hours and the smaller tap alone in 15 hours.

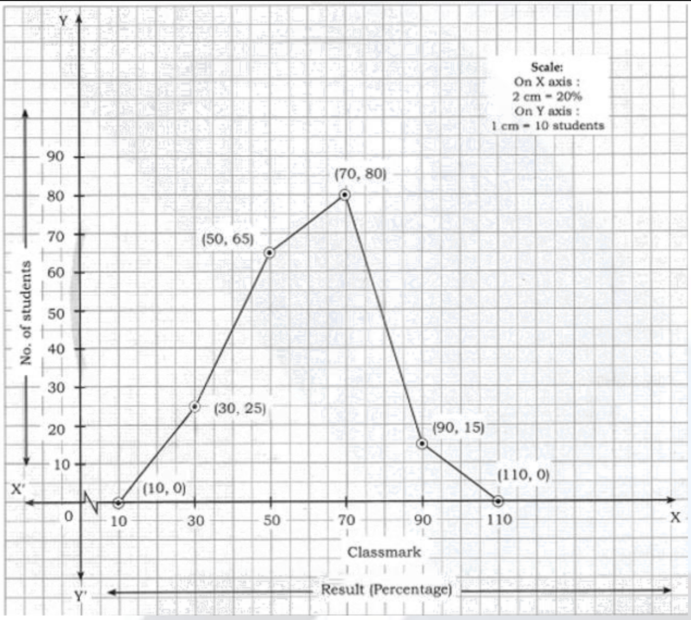
ii. The following table shows the classification of percentage of marks of students and the number of students. Draw frequency polygon from the table without drawing histogram:

Result (Percentage)	Number of Students
20 – 40	25
40 – 60	65
60 – 80	80
80 – 100	15

Solution:

Result (Percentage)	Class Mark	Number of Students	Coordinates of points

20 – 40	30	25	(30,25)
40 – 60	50	65	(50,65)
60 – 80	70	80	(70,80)
80 – 100	90	15	(90,15)



iii. In a 'Mahila Bachat Gat', Kavita invested from the first day of month ₹ 20 on first day, ₹ 40 on second day and ₹ 60 on third day. If she saves like this, then what would be her total savings in the month of February 2020?

Solution:

2020 is a leap year.

In February month there are 29 days.

n = 29

On the first day of the month Kavita invested ₹ 20.

On the second day, she invested ₹ 40.

On the third day, she invested ₹ 60.

Kavita's investment is in A.P. 20, 40, 60, ...

$a = 20, n = 29, d = 20$

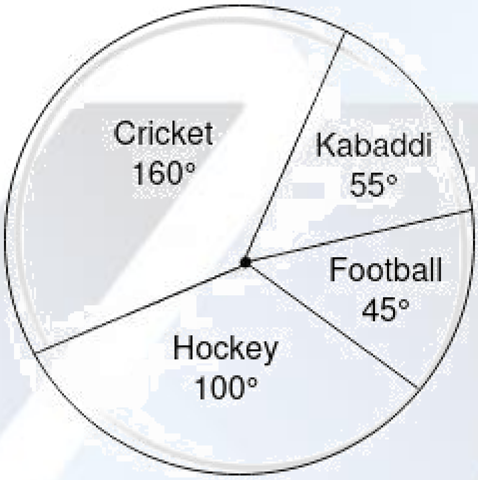
Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore S_{29} = \frac{29}{2}[2(20) + (29-1)(20)]$$
$$= \frac{29}{2}[40 + 28 \times 20]$$
$$= \frac{29}{2}[40 + 560]$$
$$= \frac{29}{2}[600]$$
$$= 29 \times 300$$
$$= 8700$$

Therefore, total savings of Kavita in the month of February 2020 is ₹ 8,700.

Q5. Solve the following subquestions (any one):

i. In the given figure, the pie diagram represents the amount spent on different sports by a school administration in a year. If the money spent on football is ₹ 9,000, answer the following questions:



a. What is the total amount spent on sports?

b. What is the amount spent on cricket?

Solution:

a. Let total amount spent on sports be x.

Central angle for football $= 45^\circ$

Amount spent on football $= 9{,}000$

$$\text{Central angle} = \frac{\text{Amount spent on football } \times 360}{\text{Total amount spent on sports}}$$

$$45 = \frac{9{,}000 \times 360}{45}$$

$x = 200 \times 360$

$x = 72{,}000$

Total amount spent on sports is 72,000.

b. Let amount spent on cricket by y.

Central angle for cricket $= 160^\circ$

Total amount spent on sports = ₹72,000

$$\text{Central angle} = \frac{\text{Amount spent on cricket}}{\text{Total amount spent on sports}} \times 360$$

$$160 = \frac{y}{72000} \times 360$$

$$y = \frac{160 \times 72000}{360}$$

$y =$ ₹32,000

Amount spent on cricket is ₹ 32,000.

ii. Draw the graph of the equation $x + y = 4$ and answer the following questions:

a. Which type of triangle is formed by the line with X and Y -axes based on its sides.

b. Find the area of that triangle.

Solution:

a. The isosceles triangle is formed by a line with X and Y axes.

b. $1 = 4$ and $b = 4$

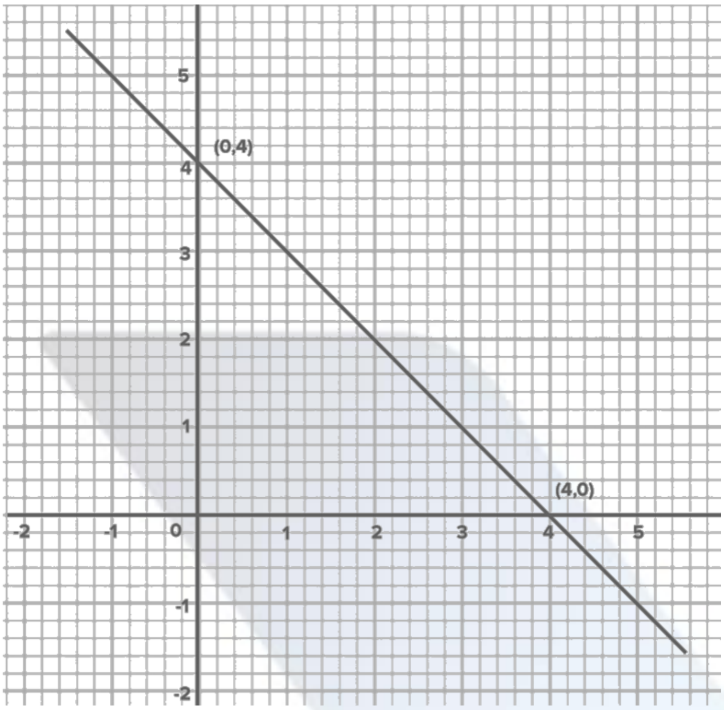
Area of triangle $= \frac{1}{2} \times l \times b$

$$= \frac{1}{2} \times 4 \times 4$$

$= 8$ sq. units

Graph:

x	0	2	4
y	4	2	0
(x, y)	(0,4)	(2,2)	(4,0)



SECTION- II

Q1. (A) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

i. Out of the dates given below which date constitutes a Pythagorean triplet?

(A) 15/8/17

(B) 16/8/16

(C) 3/5/17

(D) 4/9/15

Solution:

A Pythagorean triplet consists of three numbers (a, b, c) that satisfy the equation

$a^2 + b^2 = c^2$

For the given dates, we check if the day, month, and year form a Pythagorean triplet:

$15^2 + 8^2 = 17^2$

$225 + 64 = 289$ True

So, 15/8/17 is a Pythagorean triplet.

ii. sin 0 × cosec 0 = ?

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $\sqrt{2}$

Solution:

Since, by reciprocal relation $\sin\ \theta = \frac{1}{\text{cosec}\ \theta}$, we obtain:

$\sin 0 \times \text{cosec}\ 0 = \frac{1}{\text{cosec}\ 0} \times \text{cosec}\ 0 = 1$

iii. Slope of X-axis is …………

(A) 1

(B) -1

(C) 0

(D Cannot be determined

Solution:

The slope of the X-axis is 0 because the X-axis is horizontal, and the slope of any horizontal line is always 0.

So, the correct answer is 0.

iv. A circle having radius 3 cm, then the length of its largest chord is ……

(A) 1.5 cm

(B) 3 cm

(C) 6 cm

(D) 9 cm

Solution:

The largest chord of a circle is its diameter. The diameter is twice the radius.

Given radius = 3 cm, the diameter (largest chord) = 2 × 3 = 6 cm.

So, the correct answer is 6 cm.

Q1. (B) Solve the following sub-questions:

i. If AABC ~ APQR and AB : PQ = 2:3, then find the value of $\frac{A(\triangle ABC)}{A(\triangle PQR)}$.

Solution:

Given $\Delta ABC \sim \Delta PQR$

AB : PQ = 2 : 3

Since, $\frac{\mathbf{A}(\triangle ABC)}{\mathbf{A}(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2}$ [Area of similar triangles]

$$\frac{\mathbf{A}(\triangle ABC)}{\mathbf{A}(\triangle PQR)} = \frac{(2)^2}{(3)^2}$$

$$\frac{\mathbf{A}(\triangle ABC)}{\mathbf{A}(\triangle PQR)} = \frac{4}{9}$$

ii. Two circles of radii 5 cm and 3 cm touch each other externally. Find the distance between their centres.

Solution:

Let the radii of two circles be $r_1 = 5$ cm and $r_2 = 3$ cm.

Then, distance between their centres = $r_1 + r_2 = 5 + 3 = 8$ cm

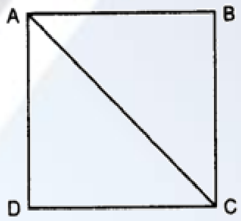
Hence, distance between their centres is 8 cm.

iii. Find the side of a square whose diagonal is $10\sqrt{2}$ cm.

Solution:

Let ABCD be the square and AC be the diagonal of length $10\sqrt{2}$ cm.

Let side of square be x.



In ADC,

$\angle ADC = 90°$ (By Pythagoras theorem)

$AC^2 = AD^2 + DC^2$

$\Rightarrow (10\sqrt{2})^2 = x^2 + x^2$

$\Rightarrow 100 \times 2 = 2x^2$

$\Rightarrow 200 = 2x^2$

$\Rightarrow x^2 = 100$

Taking square root on both sides, we get: x = 10 cm

Hence, side of the square = 10 cm.

iv. Angle made by the line with the positive direction of X-axis is 45°. Find the slope of that line.

Solution:

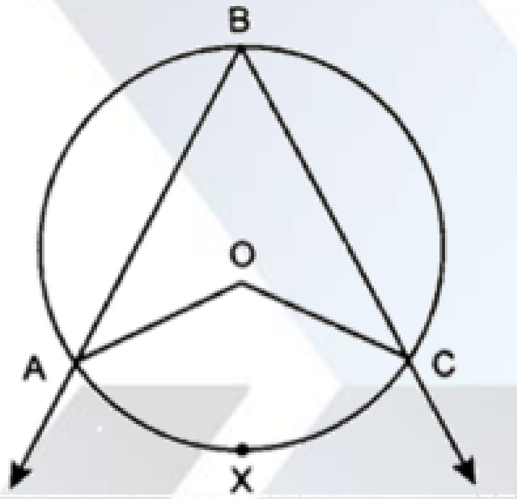
Since Slope = tan θ

Here, θ = 45°

Therefore, slope = tan 45° = 1

Hence, slope = 1

Q2. (A) i. Complete any two activities and rewrite it:



In the above figure, ∠ABC is inscribed in arc ABC.

If ∠ABC = 60°, find m∠AOC.

Solution:

∠ABC = $\frac{1}{2}$ m (arc AXC)

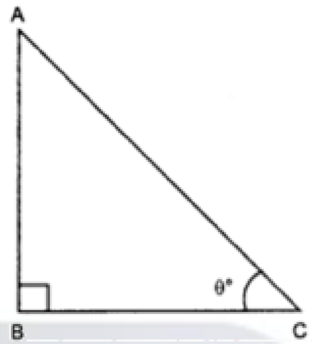
60° = $\frac{1}{2}$ m (arc AXC)

<u>120°</u> = m (arc AXC)

But m∠AOC = m(arc <u>AXC</u>)

Therefore, m∠AOC = <u>120°</u>

ii. Find the value of $sin^2\theta + cos^2\theta$.



Solution:

In ΔABC, ∠ABC = 90°, ∠C = 0°

$AB^2 + BC^2 = \underline{AC^2}$ ……(Pythagoras theorem)

Divide both sides by AC^2

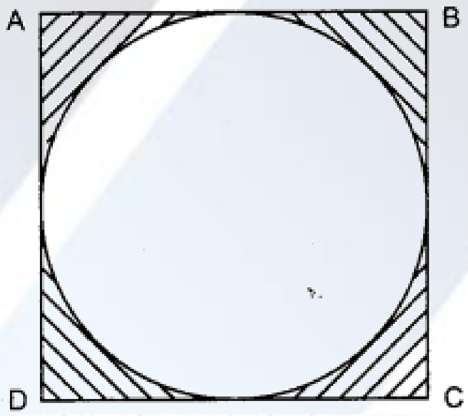
$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

Therefore, $\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = 1$

But $\frac{AB}{AC} = sin\theta$ and $\frac{BC}{AC} = cos\theta$

∴ $\sin^2 \theta + \cos^2 \theta = 1$

iii.



In the figure given above, ABCD is a square and a circle is inscribed in it. All sides of a square touch the circle. If AB = 14 cm, find the area of shaded region.

Solution:

Area of square = $(\text{Side})^2 = 14^2 = 196 \text{ cm}^2$

Area of circle = πr^2

$= \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

(Area of shaded portion) = (Area of square) − (Area of circle)

= 196 – 154

= 42 cm^2

Q2. (B) Solve any four of the following sub-questions:

i. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.

Solution:

Radius of circle, r = 3.5 cm

Length of its arc, l = 2.2 cm

Then, Area of sector $= \dfrac{l \times r}{2}$

$= \dfrac{2.2 \times 3.5}{2}$

$= 1.1 \times 3.5$

$= 3.85 \text{ cm}^2$

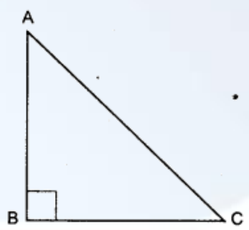
Hence, the area of sector = 385 cm^2

ii. Find the length of the hypotenuse of a right-angled triangle if remaining sides are 9 cm and 12 cm.

Solution:

Let Δ ABC be the right-angled triangle, right angled at B.

Then, BC = 9 cm, AB = 12 cm



By Pythagoras theorem,

$AC^2 = BC^2 + AB^2$

$AC^2 = (9)^2 + (12)^2$

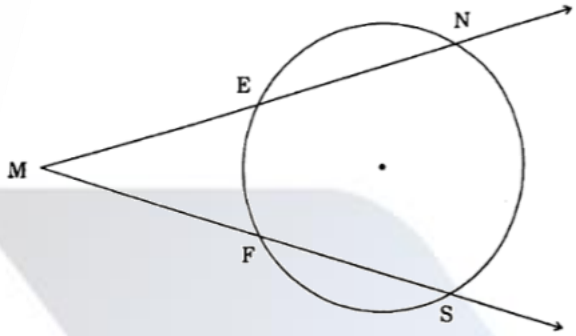
$AC^2 = 81 + 144$

$AC^2 = 225$

Taking square root on both sides, we get:

AC = 15 cm

So, the length of the hypotenuse = 15 cm



iii. In the above figure, m(arc NS) = 125°, m(arc EF) = 37°. Find the measure of ∠NMS.

Solution:

Given, m(arc NS) = 125°

m(arc EF) = 37°

m∠NMS = $\frac{1}{2}$ [m(arc NS) – m(arc EF)]

= $\frac{1}{2}$ [125°- 37°]

= $\frac{1}{2}$ × 88°

= 44°

∴ m∠NMS = 44°

iv. Find the slope of the line passing through the points A(2,3), B(4, 7).

Solution:

For the given points A(2,3), B(4, 7).

A(2, 3) = A(x_1, y_1)

B(4, 7) = B(x_2, y_2)

Slope of line AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{7-3}{4-2} = \frac{4}{2} = 2$

Hence, the slope of line AB will be 2.

v. Find the surface area of a sphere of radius 7 cm.

Solution:

Given: Radius of sphere, r = 7 cm

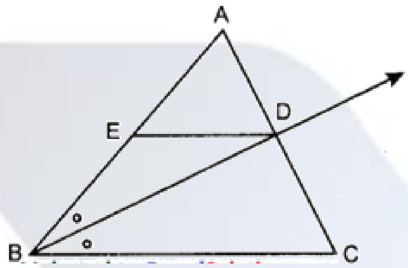
Then, surface area of sphere = $4\pi r^2$

= 4 × $\frac{22}{7}$ × 7 × 7

$= 88 \times 7 = 616 \text{ cm}^2$

Therefore, the surface area of sphere is 616 cm^2.

Q3. (A) Complete any one activity of the following and rewrite it:



i. In ΔABC, ray BD bisects ∠ABC, A – D – C, seg DE || side BC, A – E – B, then for showing $\frac{AB}{BC} = \frac{AE}{EB}$, complete the following activity:

Proof: In △ABC, ray BD bisects ∠B

$\therefore \frac{\quad}{BC} = \frac{AD}{DC}$ ...(I) ($\square$)

In △ABC, DE || BC

$\therefore \frac{\quad}{EB} = \frac{AD}{DC}$ ...(II) ($\square$)

$\frac{AB}{\quad} = \frac{\quad}{EB}$ [from (I)and (II)]

Solution:

Proof: In △ABC, ray BD bisects ∠B.

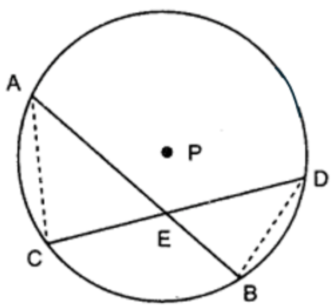
$\frac{AB}{BC} = \frac{AD}{DC}$ ...(I) [Angle Bisector theorem]

In ΔABC, DE || BC

$\therefore \frac{AE}{EB} = \frac{AD}{DC}$ ...(II) [Basic proportionality theorem]

$\therefore \frac{AB}{BC} = \frac{AE}{EB}$ ....[from (I) and (II)]

ii.



Given: Chords AB and CD of a circle with centre P intersect at point E.

To prove: AE × EB = CE × ED

Construction:

Draw seg AC and seg BD.

Fill in the blanks and complete the proof.

Proof: In ΔCAE and ΔBDE,

$\angle AEC \cong \angle DEB$ .... $\square$

$\square \cong \angle BDE$ (angles inscribed in the same arc)

$\therefore \triangle CAE \sim \triangle BDE$ .... $\square$

$\therefore \frac{\quad}{DE} = \frac{CE}{\quad}$ ... ($\square$)

$\therefore AE \times EB = CE \times ED.$

Solution:

Proof: In ΔCAE and ΔBDE,

∠AEC ≅ ∠DEB ...(Vertically opposite angle)

∠CAE ≅ ∠BDE .....(angles inscribed in the same arc)

∴ ΔCAE ~ ΔBDE ... [By AA test of similarity]

$\therefore \frac{AE}{DE} = \frac{CE}{EB}$ ... (Corresponding sides of similar triangle)

$\therefore AE \times EB = CE \times ED.$

Q3. (B) Solve any two of the following sub-questions:

i. Determine whether the points are collinear.

A(1, −3), B(2, −5), C(−4, 7)

Solution:

Let A(1, – 3) = A(x_1, y_1)

$B(2, -5) = B(x_2, y_2)$

$C(-4, 7) = C(x_3, y_3)$

$d(A, B) = \sqrt{(2-1)^2 + (-5+3)^2}$

$= \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$d(B, C) = \sqrt{(-4-2)^2 + (7+5)^2}$

And $d(A, C) = \sqrt{(-4-1)^2 + (7+3)^2}$
$= \sqrt{(-5)^2 + (10)^2}$
$= \sqrt{25 + 100}$
$= \sqrt{125}$
$= 5\sqrt{5}$

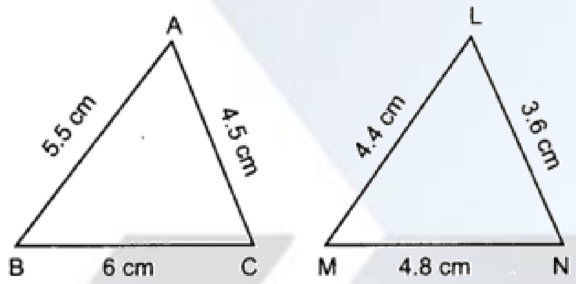
Therefore, d(A, B) + d(A, C) = d(B, C)

Hence, points A, B and C are collinear.

ii. ΔABC ~ ΔLMN. In ΔABC, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm. Construct ΔABC and ΔLMN such that $\frac{BC}{MN} = \frac{5}{4}$.

Solution:

Given, ΔABC ~ ΔLMN



$\frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN}$

$\frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN}$

$\frac{5.5}{LM} = \frac{5}{4}$

$\text{LM} = \frac{5.5 \times 4}{5}$

LM = 1.1 × 4

LM = 4.4 cm

$\frac{BC}{MN} = \frac{5}{4}$

$\frac{5}{MN} = \frac{5}{4}$

$\text{MN} = \frac{6 \times 4}{5}$

$\text{MN} = \frac{24}{5} = 4.8 \text{ cm}$

$\frac{AC}{LN} = \frac{5}{4}$

$$\frac{4.5}{LN} = \frac{5}{4}$$

$$LN = \frac{4.5 \times 4}{5}$$

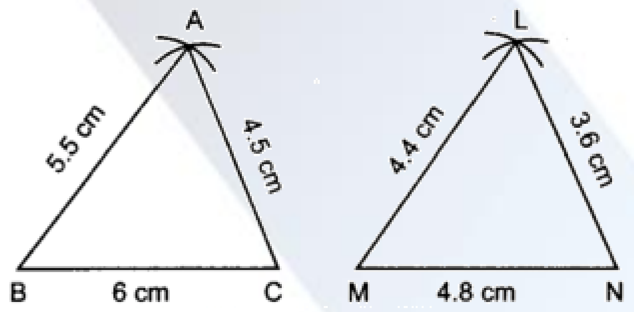
LN = 0.9 × 4

LN = 3.6 cm

In ΔABC, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm

Then, in ΔLMN, LM = 4.4 cm, MN = 4.8 cm, LN = 3.6 cm.

Construction:

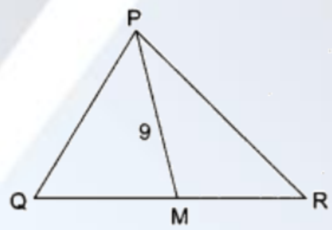


iii. Seg PM is a median of ΔPQR, PM = 9 and $PQ^2 + PR^2 = 290$, then find QR.

Solution:

In ΔPQR, M is the mid-point of line QR.

∴ QM = MR ...(i)



By Apollonius theorem,

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$

$\Rightarrow 290 = 2(9)^2 + 2QM^2$

$\Rightarrow 290 = 2(81) + 2QM^2$

$\Rightarrow 290 = 162 + 2QM^2$

$\Rightarrow 290 - 162 = 2QM^2$

$\Rightarrow 128 = 2QM^2$

$\Rightarrow QM^2 = \frac{128}{2} = 64$

⇒ QM = 8 units

QM = MR = 8 units [from (i)]

QR = QM + MR [∵ QNR is a straight line]

QR = 8 + 8

QR = 16 units

iv. Prove that 'If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the side in the same proportion'.

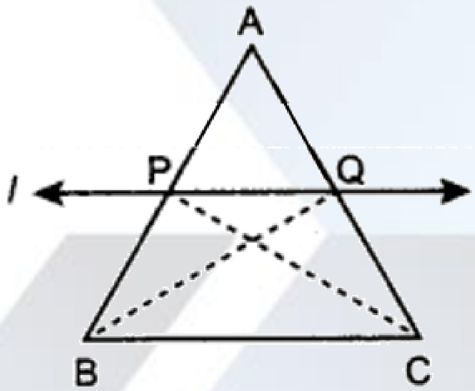
Solution:

Given: In ΔABC, line l || line BC and line l intersects AB and AC at point P and Q respectively.

To Prove:

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Construction: Draw seg PC and seg BQ.



Proof: ΔAPQ and ΔPQB have equal heights.

$\frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{AP}{PB}$ ...(areas proportionate to bases) ...(i)

Similarly, $\frac{A(\Delta APQ)}{A(\Delta PQC)} = \frac{AQ}{QC}$ ...(areas proportionate to bases) ...(ii)

Seg PQ is common base of ΔPQB and ΔPQC, seg PQ || seg BC.

Hence, ΔPQB and ΔPQC have equal areas.

A(ΔPQB) = A(ΔPQC) ......(iii)

$\frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{A(\Delta APQ)}{A(\Delta PQC)}$ [From (i), (ii) and (iii)]

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Hence Proved.

Q4. Solve any two of the following sub-questions:

(1) $\frac{1}{sin^2\theta} - \frac{1}{cos^2\theta} - \frac{1}{tan^2\theta} - \frac{1}{cot^2\theta} - \frac{1}{sec^2\theta} - \frac{1}{cosec^2\theta} = -3$, then find the value of θ.

Solution:

$$\frac{1}{sin^2\theta}-\frac{1}{cos^2\theta}-\frac{1}{tan^2\theta}-\frac{1}{cot^2\theta}-\frac{1}{sec^2\theta}-\frac{1}{cosec^2\theta}=-3$$

$(cosec^2\theta - sec^2\theta) - (cot^2\theta + tan^2\theta) - (cos^2\theta + sin^2\theta) = -3$

We know that: $cos^2\theta + sin^2\theta = 1$

$(cosec^2\theta - sec^2\theta) - (cot^2\theta + tan^2\theta) - 1 = -3$

$(cosec^2\theta - sec^2\theta) - (cot^2\theta + tan^2\theta) = -2$

$(cosec^2\theta - cot^2\theta) - (sec^2\theta + tan^2\theta) = -2$

$1 - (sec^2\theta + tan^2\theta) = -2$

$1 - (1 + tan^2\theta + tan^2\theta) = -2$

$(2tan^2\theta) = 2$

$tan^2\theta = 1$

$tan\theta = 1$

$\theta = 45^o$

(2) A cylinder of radius 12 cm contains water up to the height 20 cm. A spherical iron ball is dropped into the cylinder and thus water level raised by 6.75 cm. What is the radius of iron ball?

Solution:

Given: Radius of cylinder = 12 cm = r_1

Water level in cylinder = 20 cm

On dropping sphere ball,

rise in height = 6.75 cm = h

Radius of sphere = r_2

Now, Volume of water raised in cylinder = Volume of the sphere

$\pi r_1^2 h = \frac{4}{3}\pi r_2^3$

$(12 \times 12 \times 6.75) = \frac{4}{3} r_2^3$

$12\times12\times6.75\times\frac{3}{4} = r_2^3$

$r_2^3 = 729$

Taking the cube roots on both sides,

$r_2 = 9$

Therefore, the radius of the iron ball is 9 cm.

(3) Draw a circle with centre O having radius 3 cm. Draw tangent segments PA and PB through the point P outside the circle such that ∠APB = 70°.

Solution:

Construction:

(i) Construct a circle of radius 3 cm with centre O.

(ii) Join OA, OB and increase it.

(iii) Now draw the perpendicular bisectors of these two lines.

(iv) Join A to P and B to P. They meet at point P which is of 70°.

Q5. Solve any one of the following sub-questions:

(1) ABCD is trapezium, AB || CD diagonals of trapezium intersects in point P.

Write the answers of the following questions:

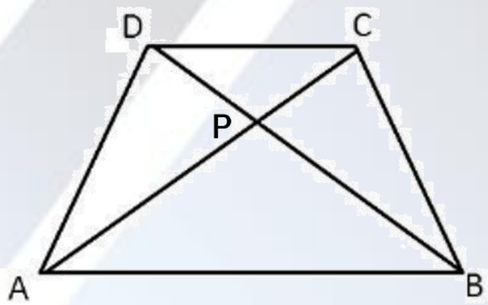
(a) Draw the figure using given information.

(b) Write any one pair of alternate angles and opposite angles.

(c) Write the names of similar triangles with test of similarity.

Solution:

(a)



(b) Alternate angles, ∠BAP = ∠PCD (Since, AB || DC and BD are their transversal.)

Opposite angles, ∠APB = ∠CPD (Vertically opposite angles)

(c) ΔAPB ~ ΔCPD (By AA test of similarity)

(2) AB is a chord of a circle with centre O. AOC is diameter of circle, AT is a tangent at A.

Write answers of the following questions:

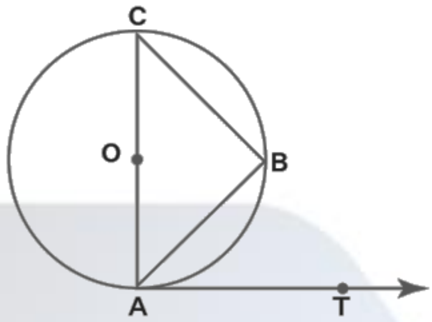
(a) Draw the figure using given information.

(b) Find the measures of ∠CAT and ∠ABC with reasons.

(c) Whether ∠CAT and ∠ABC are congruent? Justify your answer.

Solution:

(a)



(b) ∠CAT = ∠OAT = ∠90° (By tangent theorem)

and ∠ABC = 90° (Angle in a semi-circle is a right angle.)

(c) ∠CAT = ∠ABC = 90°

∠CAT ≅ ∠ABC

[∵ The angle between a tangent of a circle and a chord drawn from the point of contact is congruent to the angle inscribed in the arc opposite to the arc intercepted by that angle.]